Hadron Tomography

Matthias Burkardt

burkardt@nmsu.edu

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

Outline

- GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2)$
    $\hookrightarrow \perp$ deformation of unpol. PDFs in $\perp$ pol. target
    - Sivers effect
  - $2\tilde{H}_T + E_T \longrightarrow \perp$ deformation of $\perp$ pol. PDFs in unpol. target
    - correlation between quark angular momentum and quark transversity
    - Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$
\begin{aligned}
\int dx H_q(x,\xi,t) &= F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t) \\
\int dx E_q(x,\xi,t) &= F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),
\end{aligned}
$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)

Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$
$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- GPDs are **form factor** for only those quarks in the nucleon carrying a certain **fixed momentum fraction** x

$\hookrightarrow$ t dependence of GPDs for fixed x, provides information on the **position space distribution** of quarks carrying a certain momentum fraction x

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:
  [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ relate GPDs to distribution of partons in $\perp$ plane

$$
\begin{aligned}
q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\mathbf{\Delta}_\perp^2), \\
\Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2),
\end{aligned}
$$

- **no rel. corrections** to this result! (Galilean subgroup of $\perp$ boosts)
- $q(x, \mathbf{b}_\perp)$ has **probabilistic interpretation**, e.g.

$$
\begin{aligned}
&q(x, \mathbf{b}_\perp) \geq |\Delta q(x, \mathbf{b}_\perp)| \geq 0 \quad \text{for} \quad x > 0 \\
&q(x, \mathbf{b}_\perp) \leq |\Delta q(x, \mathbf{b}_\perp)| \leq 0 \quad \text{for} \quad x < 0
\end{aligned}
$$

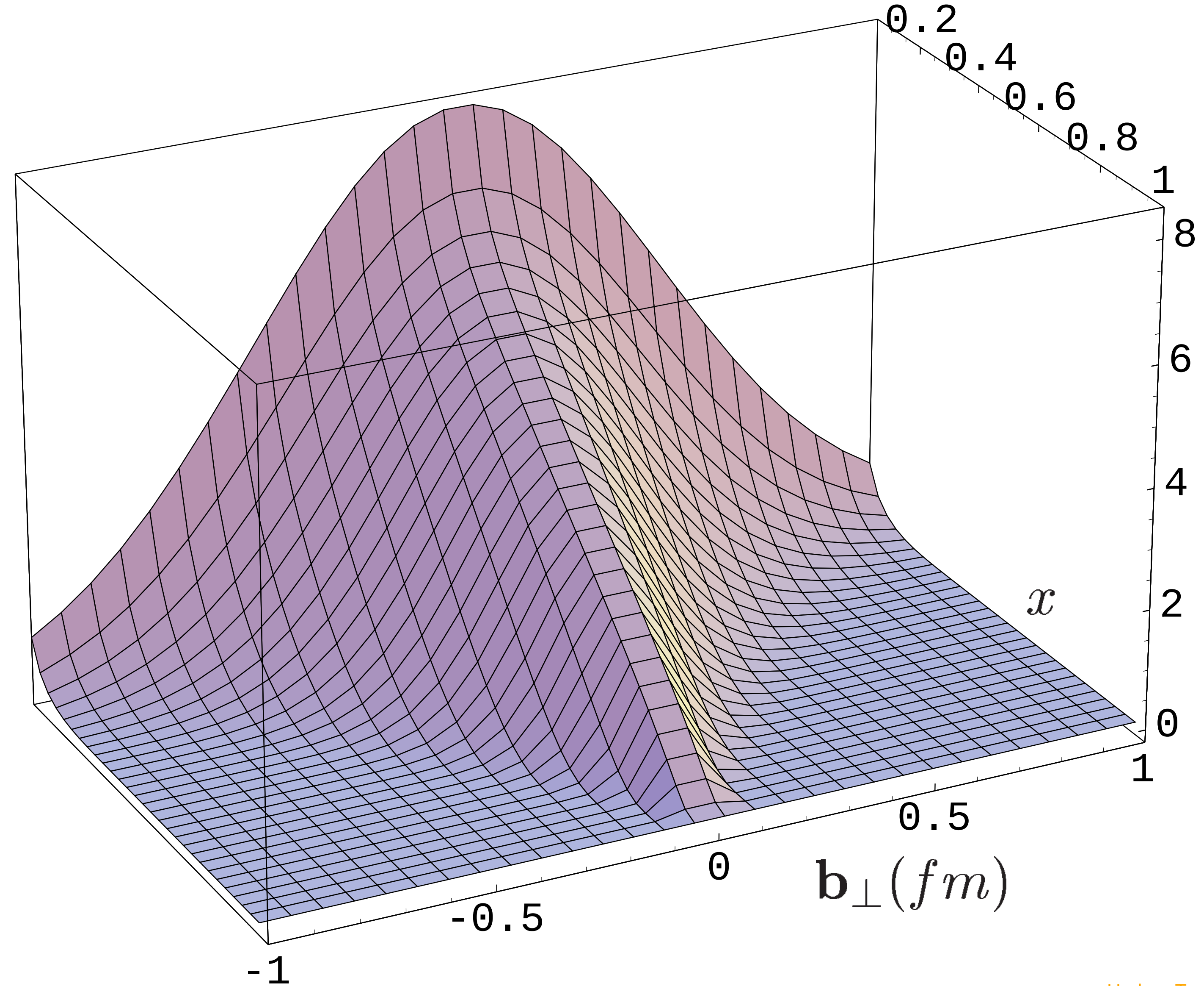
- Note that x already measures longitudinal momentum of quarks

$\hookrightarrow$ no simultaneous measurement of long. position of quarks (Heisenberg) $\longrightarrow$ no FT w.r.t. ξ

GPDs $\longleftrightarrow$ $q(x, \mathbf{b}_\perp)$

- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
  $\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \to 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
  $\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp^2$-indep. as $x \to 1$. Confirmed by recent lattice studies (QCDSF, LHPC)
- Anticipated shape of $q(x, \mathbf{b}_\perp)$:
  large x: quarks from localized valence 'core',
  small x: contributions from larger ' meson cloud'
  $\hookrightarrow$ expect a gradual increase of the t-dependence ($\perp$ size) of $H(x, 0, t)$ as x decreases

$q(x, \mathbf{b}_\perp)$ in a simple model



Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow |\bar{q}(0)\, \gamma^+ q(x^-)|\, P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

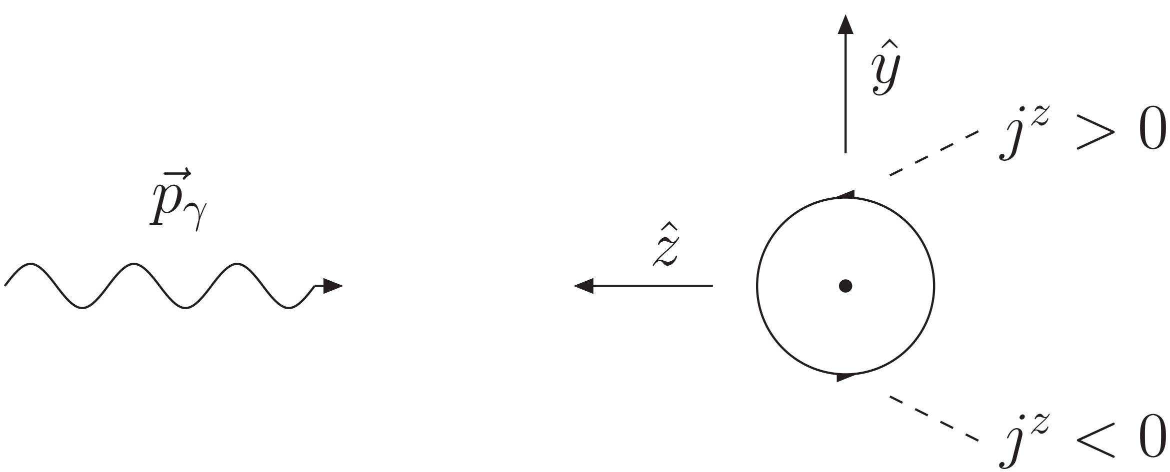
$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 !
  [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



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$\hookrightarrow$ j^+ is deformed not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) "see" the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side.

- $\perp$ deformation described by $E_q(x, 0, t)$

$\hookrightarrow$ not surprising to find that $E_q(x, 0, t)$ enters the Ji relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$

Transversely Deformed Distributions and $E(x, 0, -\boldsymbol{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

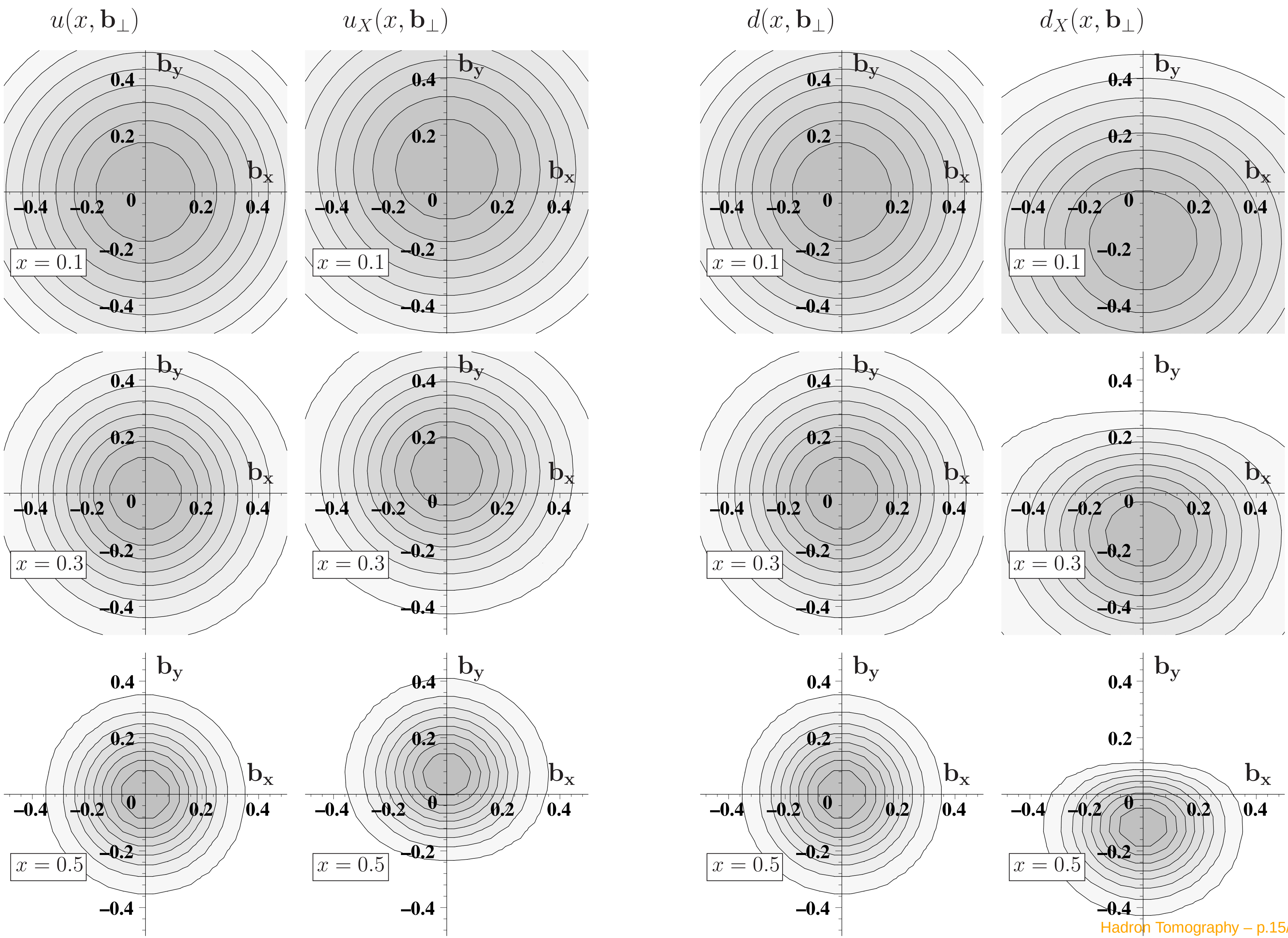
with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

$$\begin{aligned} E_u(x, 0, -\boldsymbol{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\boldsymbol{\Delta}_\perp^2) \\ E_d(x, 0, -\boldsymbol{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\boldsymbol{\Delta}_\perp^2) \end{aligned}$$

with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!



u(x, b⊥)
u_X(x, b⊥)
d(x, b⊥)
d_X(x, b⊥)
b_y
b_x
0.4
0.2
0
−0.2
−0.4
x = 0.1
x = 0.3
x = 0.5

GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_1^q(x,\mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S}{M}$$

- without FSI, $\langle \mathbf{k}_\perp\rangle = 0$, i.e. $f_{1T}^{\perp q}(x,\mathbf{k}_\perp^2) = 0$
- with FSI, $\langle \mathbf{k}_\perp\rangle \neq 0$ (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of Wilson line gauge links in gauge invariant def. of $q(x,\mathbf{k}_\perp)$

$\hookrightarrow$ Qiu, Sterman; Collins; Ji; Boer et al.;..

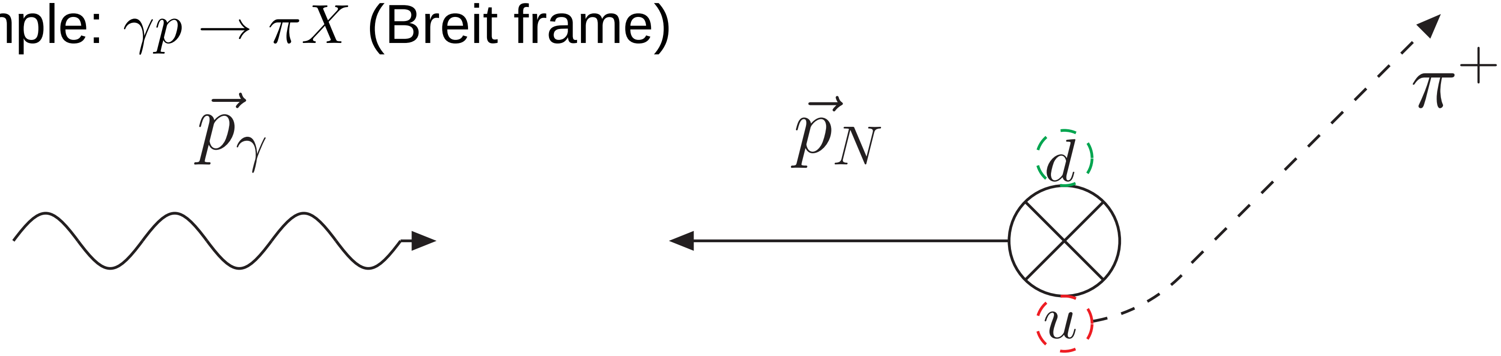
$$\langle \mathbf{k}_\perp\rangle \sim \left\langle P,S\left|\bar{q}(0)\gamma^+\int_0^\infty d\eta^- G^{+\perp}(\eta)q(0)\right|P,S\right\rangle$$

  - $\int_0^\infty d\eta^- G^{+\perp}(\eta)$ is the $\perp$ impulse that the active quark acquires as it moves through color field of "spectators"

- What should we expect for Sivers effect in QCD ?

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$ (Breit frame)



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space deformation (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q$

- $f_{1T}^{\perp q} \sim -\kappa_q$ consistent with HERMES & COMPASS results
- need more results for $f_{1T}^{\perp q}$ $\longrightarrow$ DY@J-PARC

Chirally Odd GPDs

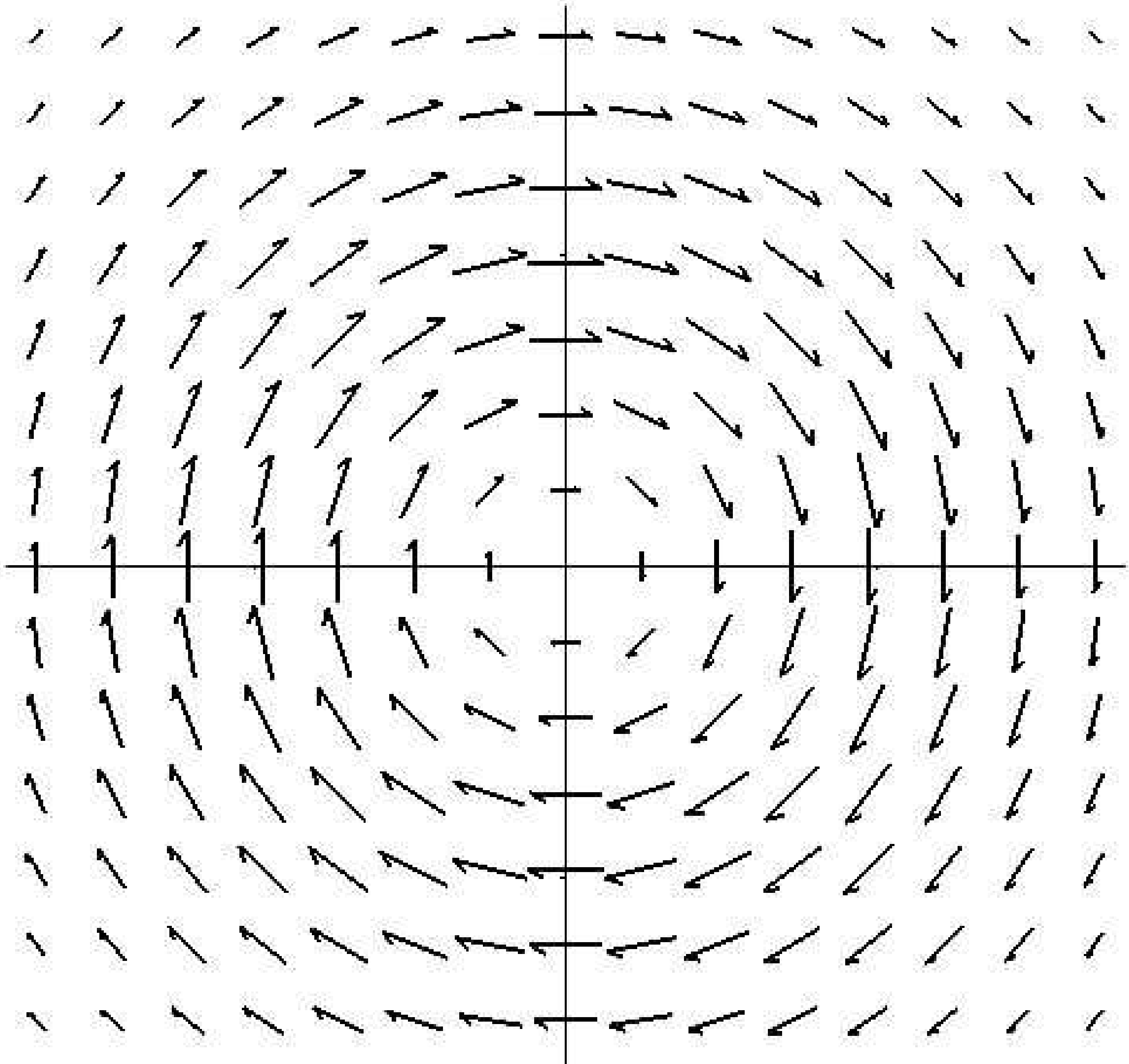
$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \sigma^{+j}\gamma_5 q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H_T \bar{u}\sigma^{+j}\gamma_5 u + \tilde{H}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha P_\beta}{M^2} u$$
$$+ E_T \bar{u} \frac{\varepsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} u + \tilde{E}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta}P_\alpha\gamma_\beta}{M} u$$

- Fourier trafo of $2\tilde{H}_T^q + E_T^q$ for $\xi = 0$ describes distribution of transversity $q^i(x, \mathbf{b}_\perp)$ for <u>un</u>polarized target in $\perp$ plane: (M.Diehl+P.Hägler, hep-ph/0504175)

$$q^i(x, \mathbf{b}_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \left[2\tilde{H}_T^q(x, 0, -\mathbf{\Delta}_\perp^2) + E_T^q(x, 0, -\mathbf{\Delta}_\perp^2) \right]$$

- origin: correlation between quark spin (i.e. transversity) and angular momentum

Transversity Distribution in Unpolarized Target



Transversity Decomposition of J_q^x

- M.B., hep-ph/0505189: $\perp$ deformation of $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized target described by $E_q(x, 0, t)$

$\hookrightarrow$ Ji relation: $\langle J_q^i \rangle = S^i \int dx\, x \left[H_q(x, 0, 0) + E_q(x, 0, 0) \right]$

Transversity Decomposition of J_q^x

- similarily $\perp$ deformation distribution for quarks with given transversity described by $2\tilde{H}_T + E_T$

$\hookrightarrow$ new relation: transversity decomposition of $J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$

$$\begin{aligned} \langle J_{q,\pm\hat{x}}^x \rangle &= \frac{S^x}{2} \int dx\, x \left[H(x,0,0) + E(x,0,0) \right] \\ &\pm \frac{1}{4} \int dx\, x \left[H_T(x,0,0) + 2\tilde{H}_T(x,0,0) + E_T(x,0,0) \right]. \end{aligned}$$

- $J_{q,\pm\hat{x}}^x$ is angular momentum carried by quarks with spin (transversity) in $\pm\hat{x}$-direction; $P_{\pm\hat{x}} = \frac{1}{2}(1 \pm \gamma^x \gamma_5)$
- Decomposition is possible since $J_q^x \sim \int d^3r \left(T_q^{0z} y - T_q^{0y} z \right)$ is diagonal in transversity!
- unpol. target: $J_{q,\pm\hat{x}}^x \pm \frac{1}{4} \int dx\, x \left[H_T + 2\tilde{H}_T + E_T \right]$
- $\int dx\, x \left[2\tilde{H}_T + E_T) \right]$ from lattice (hep-ph/0511047), or ...

Boer-Mulders function

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x,\mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x,\mathbf{k}_\perp^2) - h_1^{\perp q}(x,\mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot S_q}{M}\right]$$

- can be probed in Drell-Yan ($\rightarrow$ J-PARC)
- physical mechanism for BM-function: attractive FSI expected to convert position space asymmetry for $\perp$ polarized quark distributions into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x,\mathbf{k}_\perp)$ and the chirally odd GPD $2\tilde{H}_T + E_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x,\mathbf{k}_\perp)$ and the GPD E.

Transversity Distribution in Unpolarized Target

- attractive FSI expected to convert position space asymmetry into momentum space asymmetry
- $\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction
- $\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $2\tilde{H}_T + E_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.
- $\hookrightarrow$ qualitative predictions for $h_1^\perp(x, \mathbf{k}_\perp)$
  - sign of $h_1^\perp$ opposite to sign of $2\tilde{H}_T + E_T$
  - "$\frac{h_1^\perp}{2\tilde{H}_T + E_T} \approx \frac{f_{1T}^\perp}{E}$"
- use measurement of $h_1^\perp$ to learn about spin-orbit correlation in nucleon wave function
- use LGT calcs. of $2\tilde{H}_T + E_T$ to make qualitative prediction for $h_1^\perp$

Summary

- GPDs provide decomposition of form factors w.r.t. the momentum of the active quark

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlator, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (purely $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$

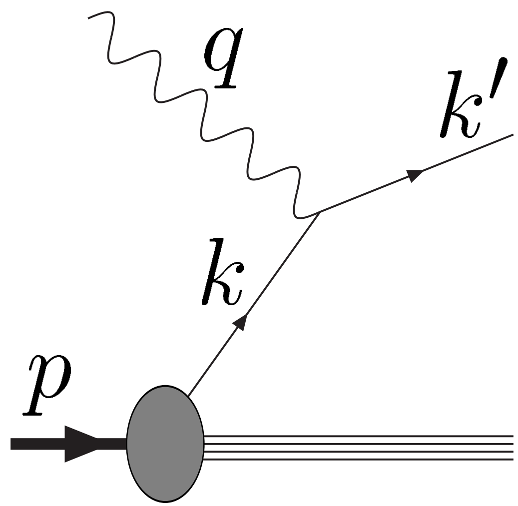
Summary

- $\frac{\Delta_\perp}{2M} E(x, 0, -\mathbf{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely deformed when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction in semi-inclusive DIS converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry

$\hookrightarrow$ simple physical explanation for observed Sivers effect in $\gamma^* p \longrightarrow \pi X$

- $2\tilde{H}_T + E_T$ measures correlation between $\perp$ spin and $\perp$ angular momentum (M.B., hep-ph/0505189)
- physical explanation for Boer-Mulders effect; relation between $h_1^\perp$ and the GPDs $2\tilde{H}_T + E_T$
- GPDs vs. $q(x, \mathbf{b}_\perp)$: M.B., PRD **62**, 71503 (2000), Int. J. Mod. Phys. **A18**, 173 (2003); see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **69**, 057501 (2004); NPA **735**, 185 (2004); PRD **66**, 114005 (2002).

Physical Meaning of $x_{Bj} = \frac{Q^2}{2p\cdot q}$



- Go to frame where $\mathbf{q}_\perp = 0$, i.e.

$$Q^2 = -q^2 = -2q^+q^- \qquad\qquad 2p\cdot q = 2q^-p^+ + 2q^+p^-$$

- Bjorken limit: $q^- \to \infty \quad , \quad q^+$ fixed

$\hookrightarrow$

$$x_{Bj} = -\frac{q^+q^-}{q^-p^+ + q^+p^-} \to -\frac{q^+}{p^+}$$

Physical Meaning of $x_{Bj} = \frac{Q^2}{2p\cdot q}$

- $x_{Bj} = -\frac{q^+}{p^+}$
- LC energy-momentum dispersion relation

$$k^- = \frac{m^2 + \mathbf{k}_\perp^2}{2k^+}$$

$\hookrightarrow$ struck quark with $k^{-\prime} = k^- + q^- \to \infty$ can only be on mass shell if $k^{+\prime} = k^+ + q^+ \approx 0$

$\hookrightarrow$

$$k^+ = -q^+ \qquad \Rightarrow \qquad x \equiv \frac{k^+}{p^+} = x_{Bj}$$

$\hookrightarrow$ x_{Bj} has physical meaning of light-cone momentum fraction carried by struck quark before it is hit by photon

back

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \langle p^+,\mathbf{p}'_\perp|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)|p^+,\mathbf{p}_\perp\rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$\times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+,\mathbf{R}_\perp=\mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+,\mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+,\mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp) \left| p^+,\mathbf{p}_\perp \right\rangle e^{ixp^+x^-} \times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp H\left(x,0,-\left(\mathbf{p}'_\perp-\mathbf{p}_\perp\right)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$\hookrightarrow \quad q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$