



Analysis of cascade production spectra in local optimal Fermi averaging t -matrix

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Cascade-Nucleus potential

Strong coupling dependence of cascade-Nucleus potential (c.f. Rijken's talk):

Nijmegen model D: $V \sim -23$ MeV

Nijmegen model F: $V \sim 28$ MeV

ESC04d: $V \sim -18$ MeV

→ **knowledge of the cascade-nucleus potential would give a valuable information for the YN interaction.**

In Phenomenological side

Old emulsion data



Woods-Saxon type:

Potential depth

$-24 < V < -20$ (MeV)

C. B. Dover and A. Gal, Annals of Phys, 146(1983)309.

Twin lambda emulsion data



$V \sim -15$ (MeV)

S. Aoki *et al.*, Phys. Lett. B **355** (1995) 45.

Production data



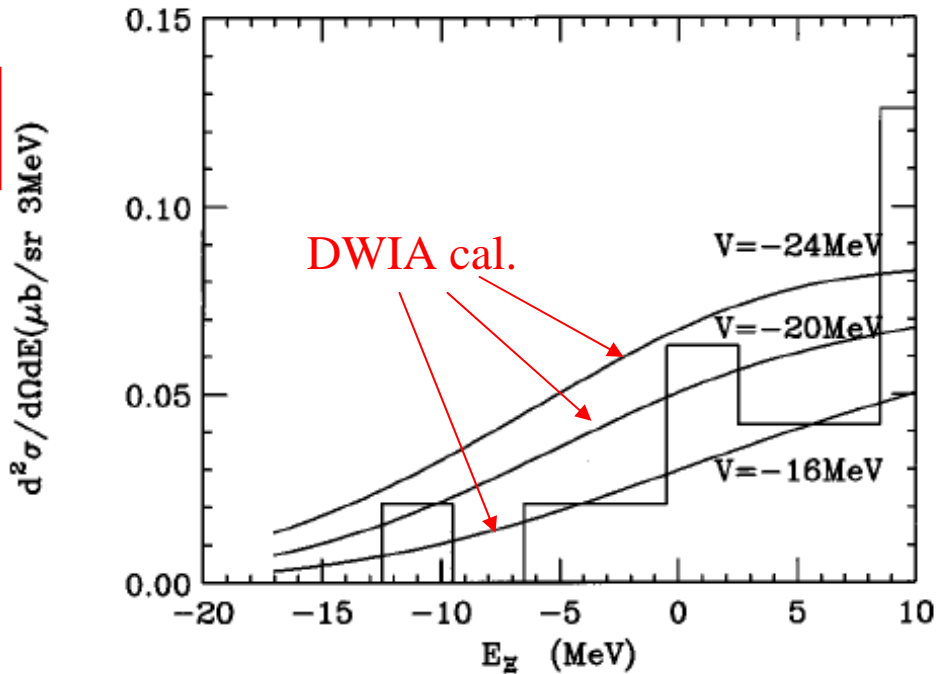
$V \sim -16$ (MeV)

T. Fukuda *et al.*, Phys. Rev. C **58** (1998) 1306.

Ξ -production by (K^-, K^+) reaction in E224 at the KEK-

PS

$^{12}\text{C} (K^-, K^+)$



T. Fukuda et al., Phys. Rev. C58 (1998) 1306.

DWIA calculation suggests that cascade-nucleus potential is about -16 MeV.

Shallow potential depth is suggested from their investigations.

Ξ -production by (K^-, K^+) reaction in E885 at the

AGS

$^{12}\text{C}(K^-, K^+)$

$P_K = 1.80 \text{ GeV}/c$

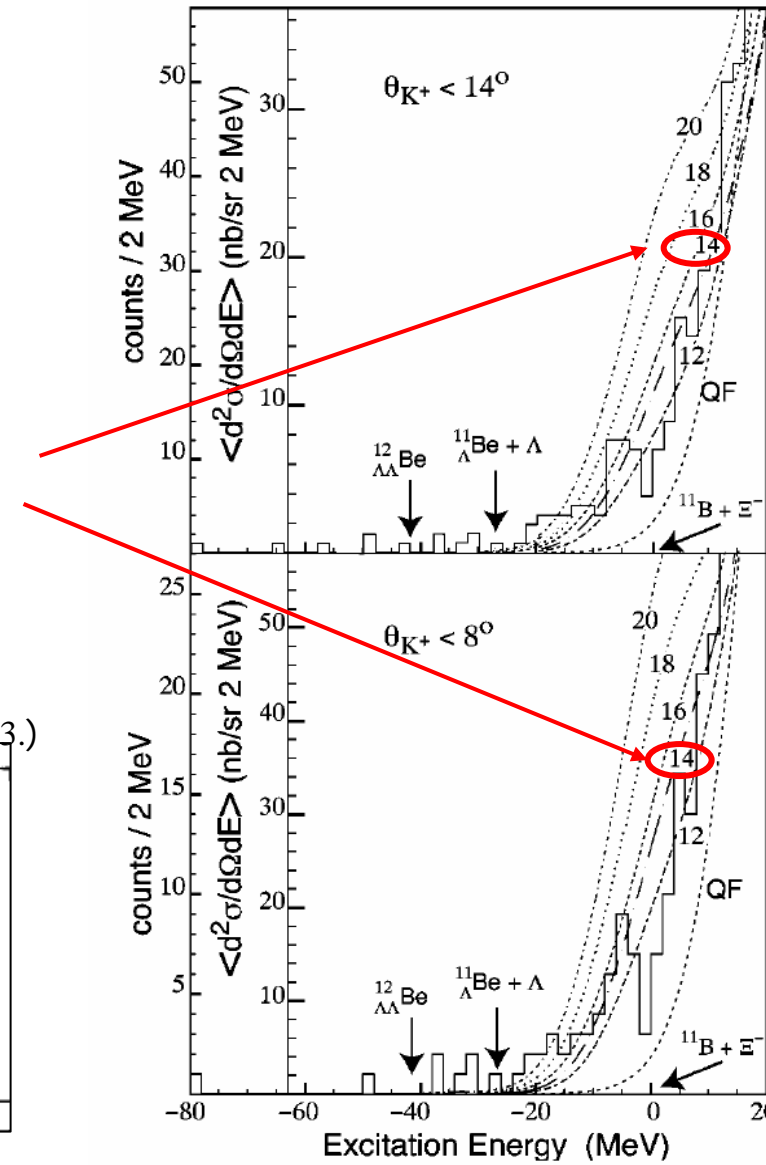
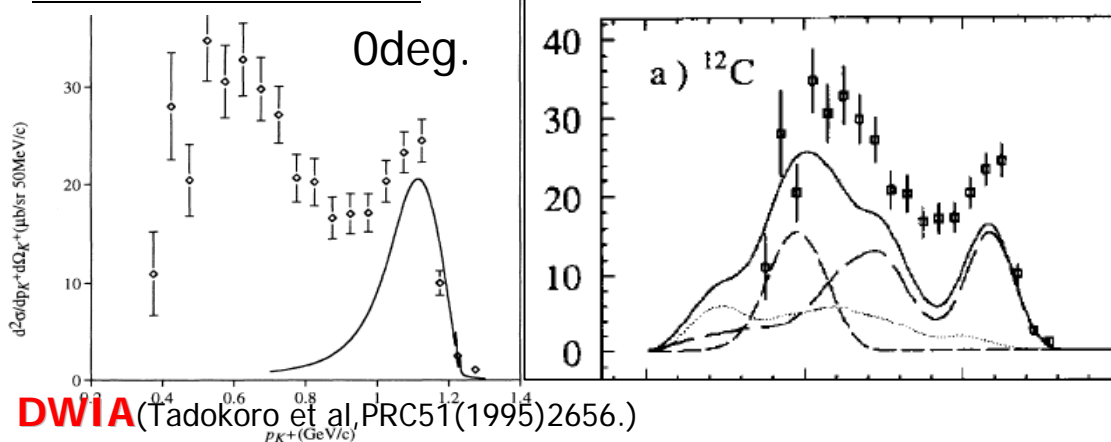
C

P. Khaustov et al., Phys. Rev. C61 (2000) 054603-1.

Reasonable agreement between the experimental data and DWIA calculation is achieved by assuming a cascade-nucleus potential well depth of about **14 MeV** with the

Theoretical curve:

INC (Y. Nara et al., NPA614(1997)433.)



DWIA (Tadokoro et al., PRC51(1995)2656.)

Purpose of our study

1. To describe the cascade production spectrum **in both of Bound and Quasi-Free regions on the same footing** using a Fermi averaging t-matrix with **on-shell condition** (Green's function method).

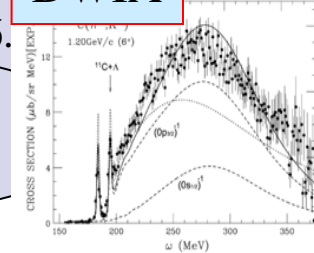
T. Harada, Y. Hirabayashi, Nucl. Phys. A 744 (2004) 323.

M. Kohno, Y. Fujiwara, M. Kawai et al., Phys.Rev.C74:064613,2006.

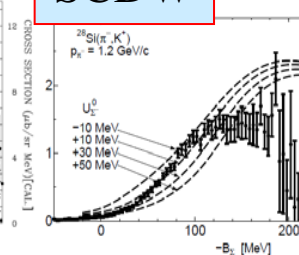
S. Hashimoto, M. Kohho, K. Ogata, M. Kawai, nucl-th/0610126.

H. Maekawa, K. Tsubakihara, A. Ohnishi, nucl-th/0701066.

DWIA

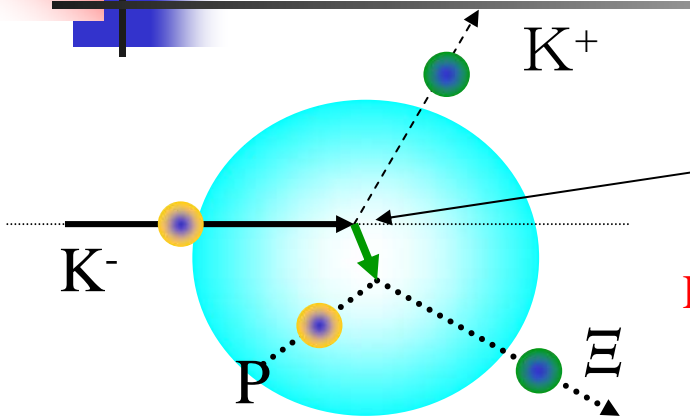


SCDW



2. To predict the cascade production spectra on several targets.

Fermi Averaging of t -matrix for elementary process



We include the potential effects at the cascade production points into the optimal Fermi average.

Energy conservation equation

$$E_{K^-}(r) + E_p(r) = E_{K^+}(r) + E_{\bar{\nu}}(r)$$

Local Energy of Hadrons

$$E_H(r) = \sqrt{\mathbf{p}_H^2 + m_H^{*2}(r)} \approx m_H + \frac{\mathbf{p}_H^2}{2m_H} + V_H(r)$$

Effective mass of Hadrons

$$m_H^{*2}(r) = m_H^2 + 2m_H V_H(r)$$

“potential”

Local Optimal Fermi Averaging t -matrix (LOFAt)

$$\bar{t}(r, \omega, \mathbf{q}) = \frac{\int d\mathbf{p}_N t(s, t) \rho(p_N) \delta^{(4)}(p_1 + p_2(r) - p_3 - p_4(r))}{\int d\mathbf{p}_N \rho(p_N) \delta^{(4)}(p_1 + p_2(r) - p_3 - p_4(r))}$$

Double differential cross section:

$$\frac{d^2\sigma}{dE_K d\Omega_K} = \frac{p_K E_K}{(2\pi)^2 v_{inc}} R(E)$$

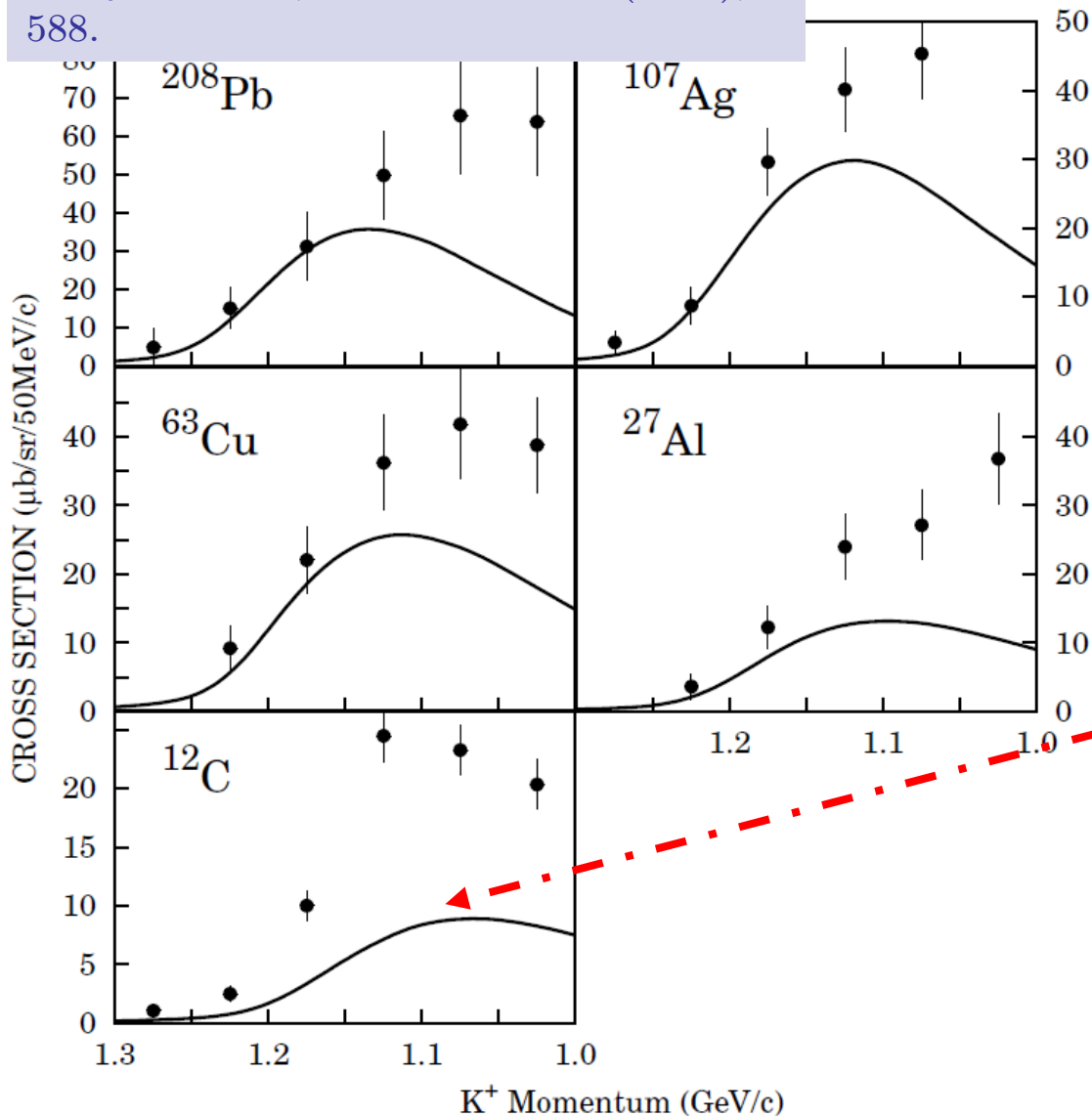
Response function:
$$R(E) = -\frac{1}{\pi} \text{Im} \int d\mathbf{r} d\mathbf{r}' f_{\alpha}^*(\mathbf{r}) \bar{t}_{\alpha}^*(\mathbf{r}) G_{\alpha\alpha'}(E; \mathbf{r}', \mathbf{r}) \bar{t}_{\alpha'}(\mathbf{r}) f_{\alpha'}(\mathbf{r})$$

Non-Factorization form

Cascade production Quasi-Free spectra on several targets

Exp. Data

T. Iijima *et al.*, Nucl. NPA **546** (1992), 588.



$$P_K = 1.65 (\text{GeV}/c)$$

Theory:

$$V = -14 (\text{MeV})$$

$$W = -1 (\text{MeV})$$

In many works, spectra on ^{12}C target are underestimated.

Y. Nara *et al.*, NPA614(1997)433.
 M. Kohno, Y. Fujiwara, M. Kawai *et al.*, Phys.Rev.C74(2006)064613.
 S. Hashimoto, M. Kohho, K. Ogata, M. Kawai, nucl-th/0610126.
 C. Gobbi, C. B. Dover, A. Gal, Phys. Rev. C 50(1994)1594.

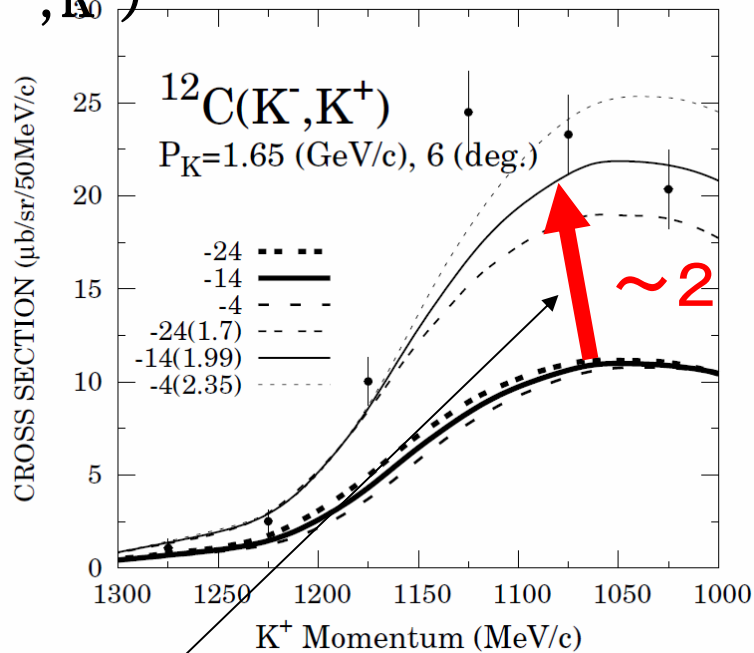
Possible origins: Recoil, t-matrix, ...

These does not change the spectrum shape.

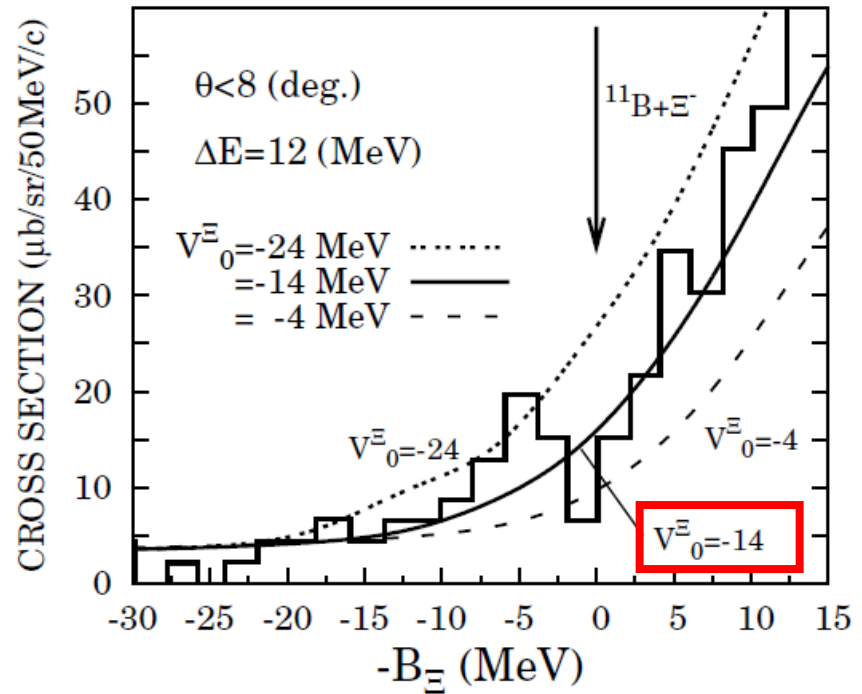
Cascade production QF and BS spectra on ^{12}C

^{12}C (K

$^-, \text{K}^+)$

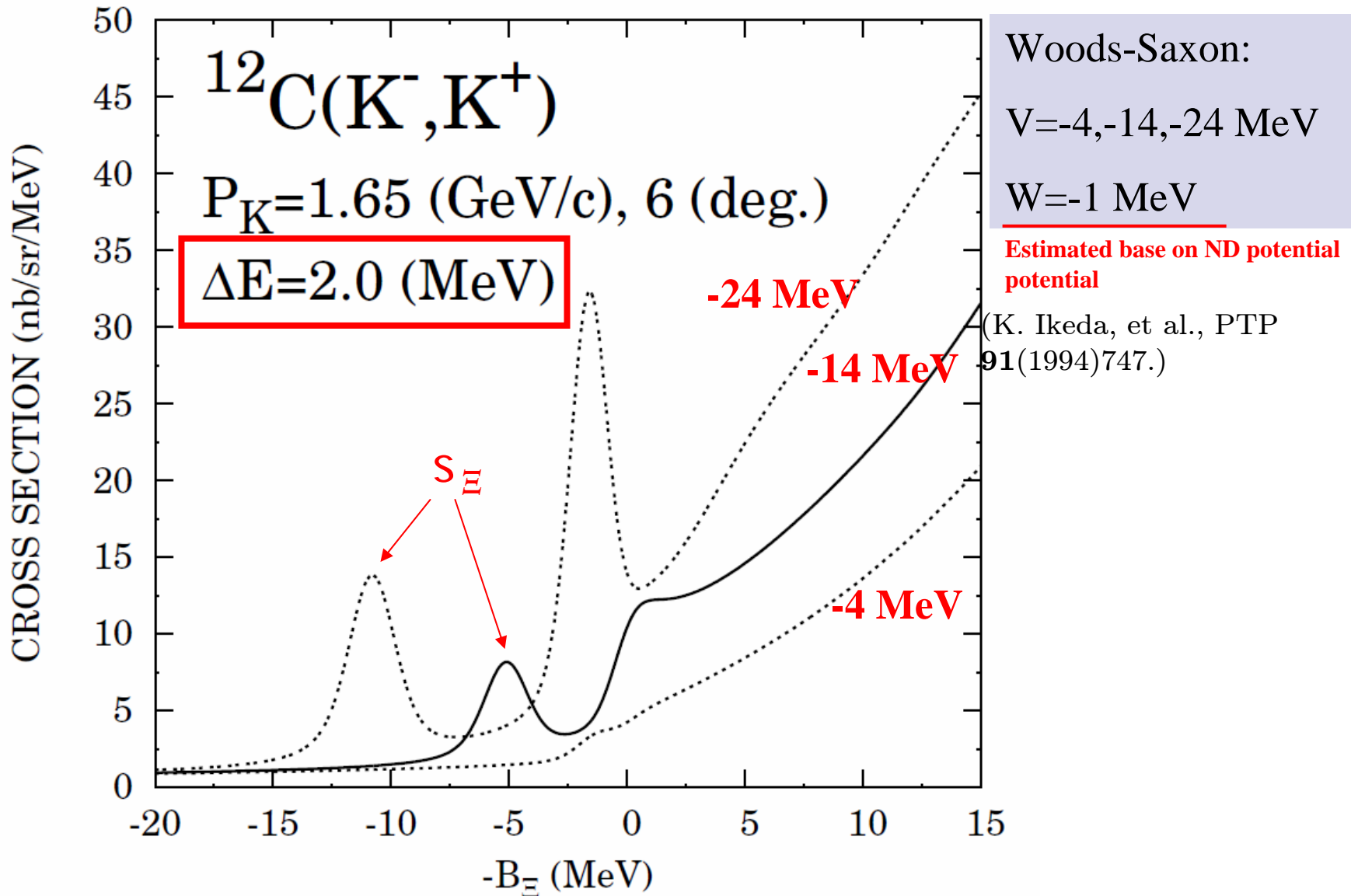


We decide the multiplicative factors in comparison with the QF data.

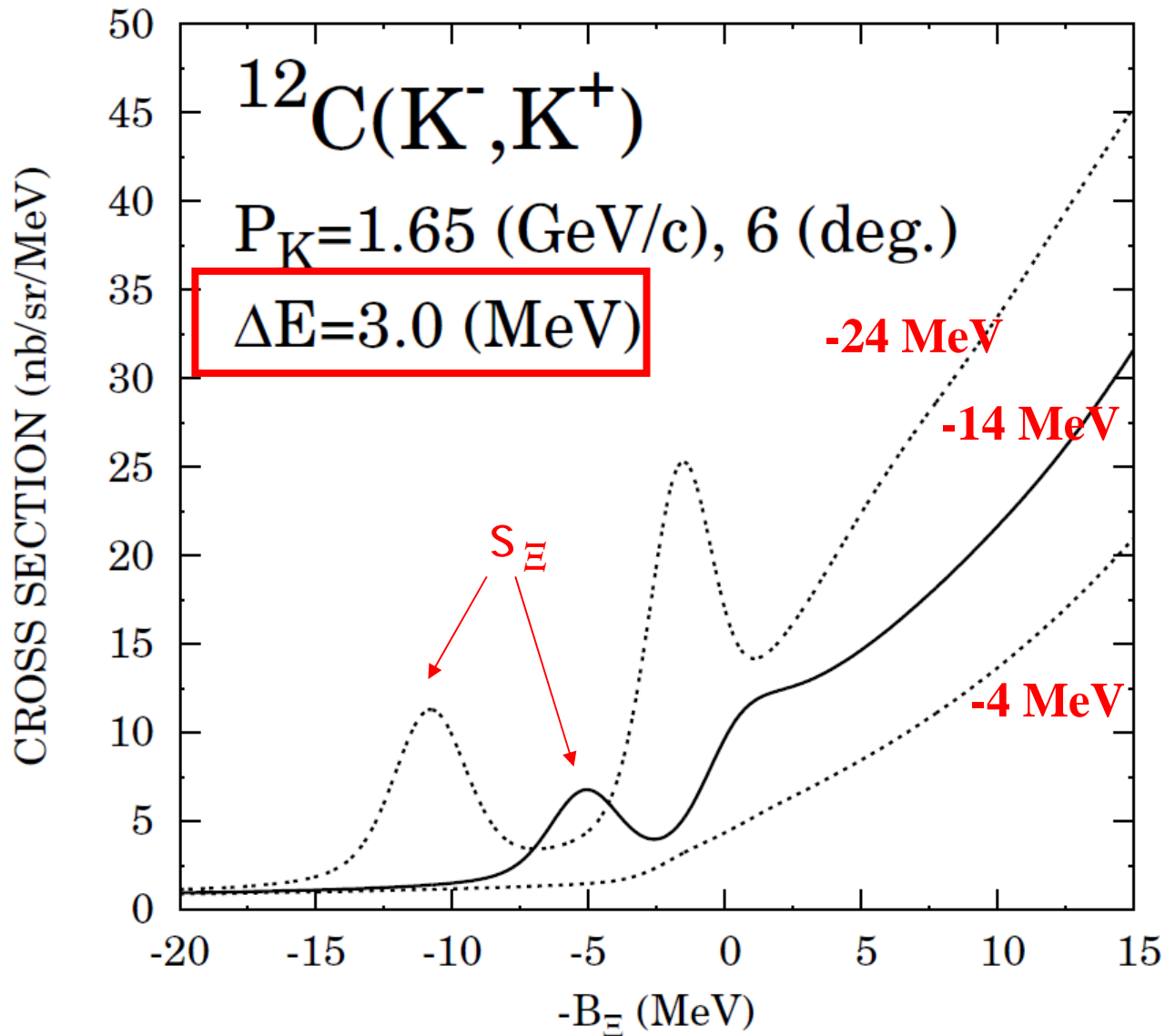


Calculated curve reproduces the low resolution data with cascade-nucleus potential of about -14MeV .

Cascade production BS spectra on ^{12}C target



Cascade production BS spectra on ^{12}C target



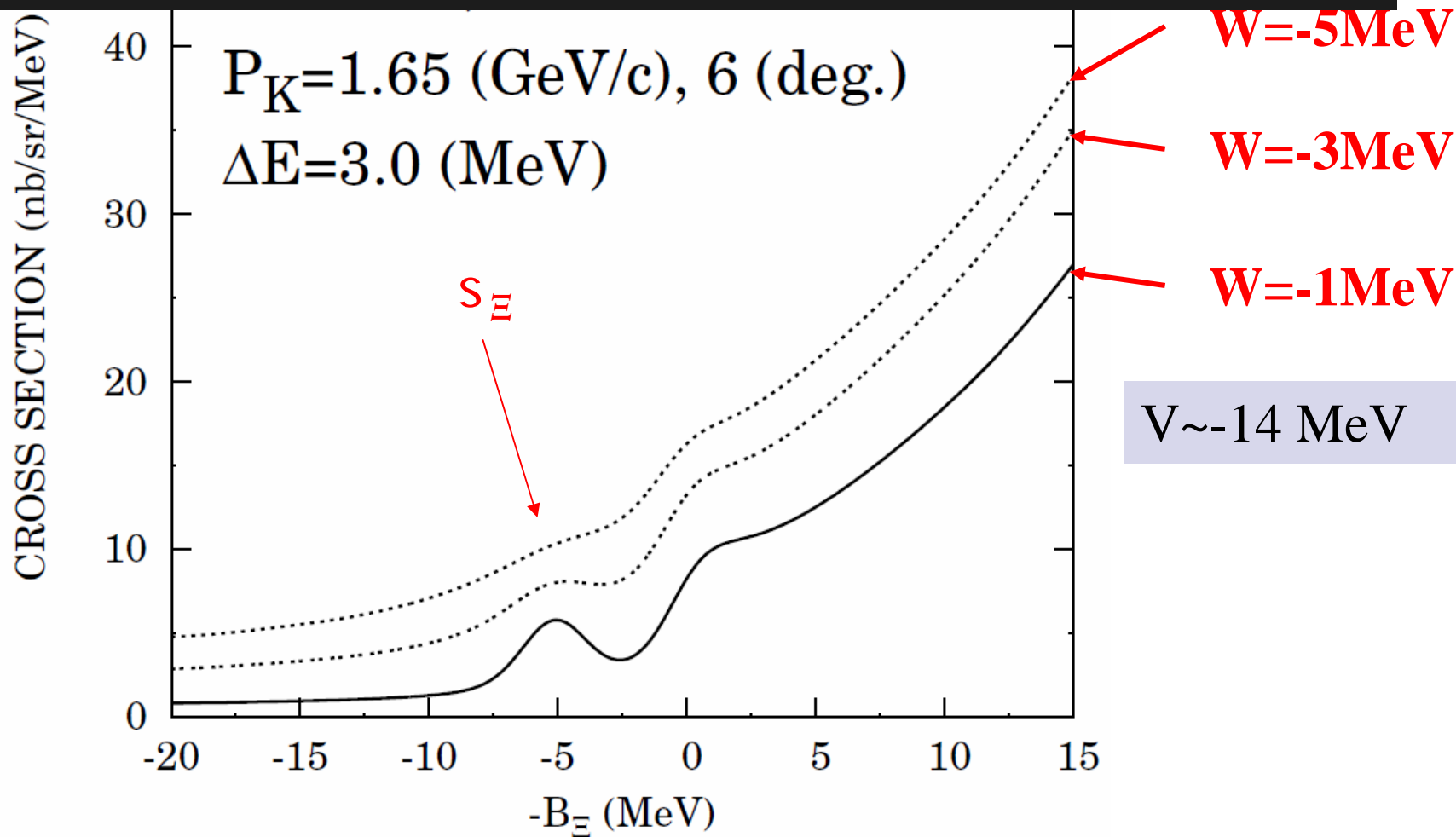
Woods-Saxon:

$V=-4,-14,-24$ MeV

$W=-1$ MeV

Cascade production BS spectra on ^{12}C target

When the conversion width is not very large ($|W| < 3\text{MeV}$), we can identify the bound state peak.



Cascade production BS spectra on several targets

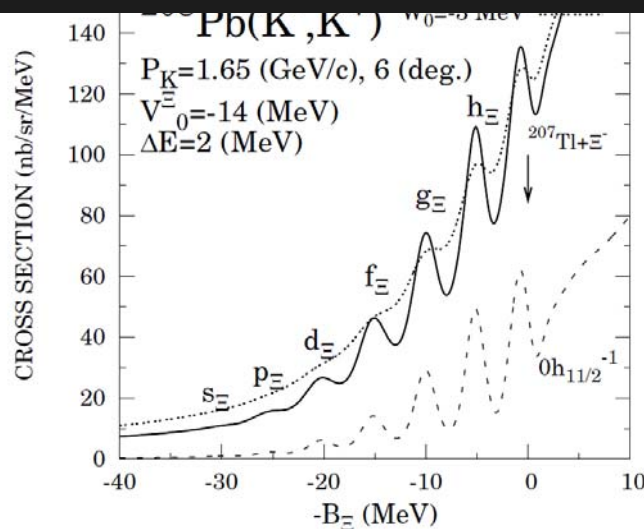
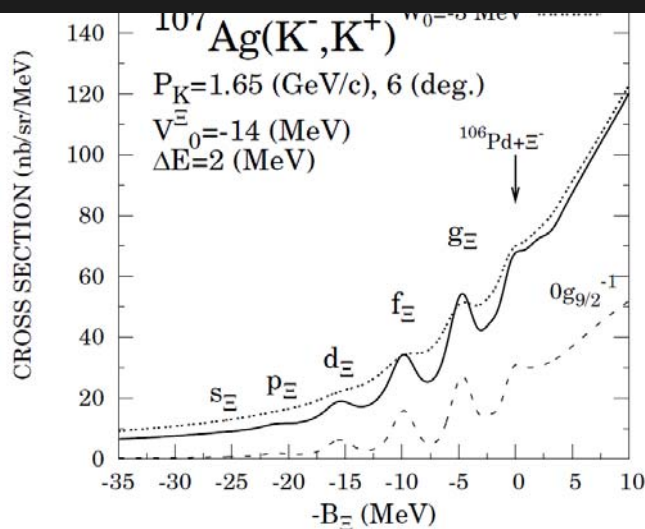
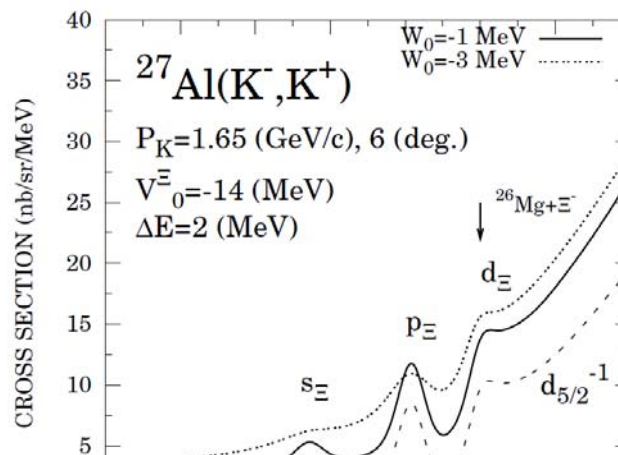
High resolution case

FWHM=2 (MeV)

Potential: Woods-Saxon type

$$U=(V+iW)f(r)$$

For small width ($|W| < 3\text{MeV}$), we can identify the stretched state peak just below the threshold.





Summary and Conclusion

- We have investigated the cascade-nucleus potential through the cascade hypernuclear production spectra by (K^-, K^+) reaction
in both of the bound and continuum regions
in the Green's function method of the distorted wave impulse approximation with the local optimal Fermi averaging t -matrix.
- The calculated Quasi-Free spectra are in good agreement with the experimental data for heavy targets. After fixing the absolute value in QF region, low resolution bound spectrum data on ^{12}C are well described with cascade-nucleus potential **$U_{\Xi} \sim -14\text{MeV}$**
- The cascade bound state peak structure can be found in the (K^-, K^+) spectra on light target such as ^{12}C , as far as
the imaginary part is **not very large ($|W| < 3\text{ MeV}$)**
and
the experimental resolution is improved (**$\Delta E = 2 \sim 3\text{ MeV}$**),
as expected in the J-PARC experiment.

Model: Green function method by Morimatsu and Yazaki

Ref) O.Morimatsu and K.Yazaki, Nucl. Phys. A483(1988)493.

S.Tadokoro, Y.Akaishi, H.Kobayashi. Phys.Rev.C51(1995)2656.

M.T.Lopez-Arias, Nucl. Phys. A582(1995)440.

Double differential cross section

$$\frac{d^2\sigma}{dE_K d\Omega_K} = \beta \left\langle \left(\frac{d\sigma}{d\Omega} \right)_{\pi+n \rightarrow K+Y}^{(ele)} \right\rangle_{Average} S(E)$$

Kinematical factor

$$\beta = \left\{ 1 + \frac{E_K}{E_Y} \left(\frac{p_K - p_\pi \cos \theta}{p_K} \right) \right\} \frac{E_K^M p_K^M}{E_K p_K}$$

Strength function

Fermi averaged Elementary cross section

Strength function

$$S(E) = -\frac{1}{\pi} \text{Im} \int d\mathbf{r} d\mathbf{r}' f_\alpha^*(\mathbf{r}) G_{\alpha\alpha'}(E; \mathbf{r}', \mathbf{r}) f_{\alpha'}(\mathbf{r})$$

$$f_\alpha(\mathbf{r}) = \chi_K^{(-)*}(\mathbf{r}) \chi_\pi^{(+)}(\mathbf{r}) \langle \alpha | \psi_N(\mathbf{r}) | i \rangle$$

Include the hyperon potential in Green function

Meson distorted waves

$$\chi_K^{(+)*}(\mathbf{r}) \chi_\pi^{(-)}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}} \Gamma(\mathbf{r})$$

$$G = \frac{1}{E - T_Y - U_Y - H_{Core} + i\epsilon}$$

Distortion factor

$$\Gamma(\mathbf{r}) = \exp \left[-\bar{\sigma}_{\pi N} \int_{-\infty}^z \rho(z') dz' - \bar{\sigma}_{KN} \int_z^{\infty} \rho(z') dz' \right]$$

"Green function"

Meson distorted waves in eikonal approximation

(π , K) reaction at 1.20 GeV/c

$$\chi_K^{(+)*}(\mathbf{r})\chi_\pi^{(-)}(\mathbf{r}) = e^{i\mathbf{q}\mathbf{r}}\Gamma(\mathbf{r})$$

Distortion factor $\Gamma(r) = \exp\left[-\bar{\sigma}_{\pi N} \int_{-\infty}^z \rho(z')dz' - \bar{\sigma}_{KN} \int_z^{\infty} \rho(z')dz'\right]$

$\sim 35.2\text{mb}$ $\sim 16.3\text{mb}$

Eikonal phase for Kaon

→ $\Gamma(r) = \exp\left[-\bar{\sigma}_{\pi N} \int_{-\infty}^z \rho(z')dz' - \bar{\sigma}_{KN} \int_z^{\infty} \rho(z')dz' + i\chi_{K^+}(b)\right]$

$$\chi_{K^+}(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} dz' U(z')$$

Kaon optical potential(real)

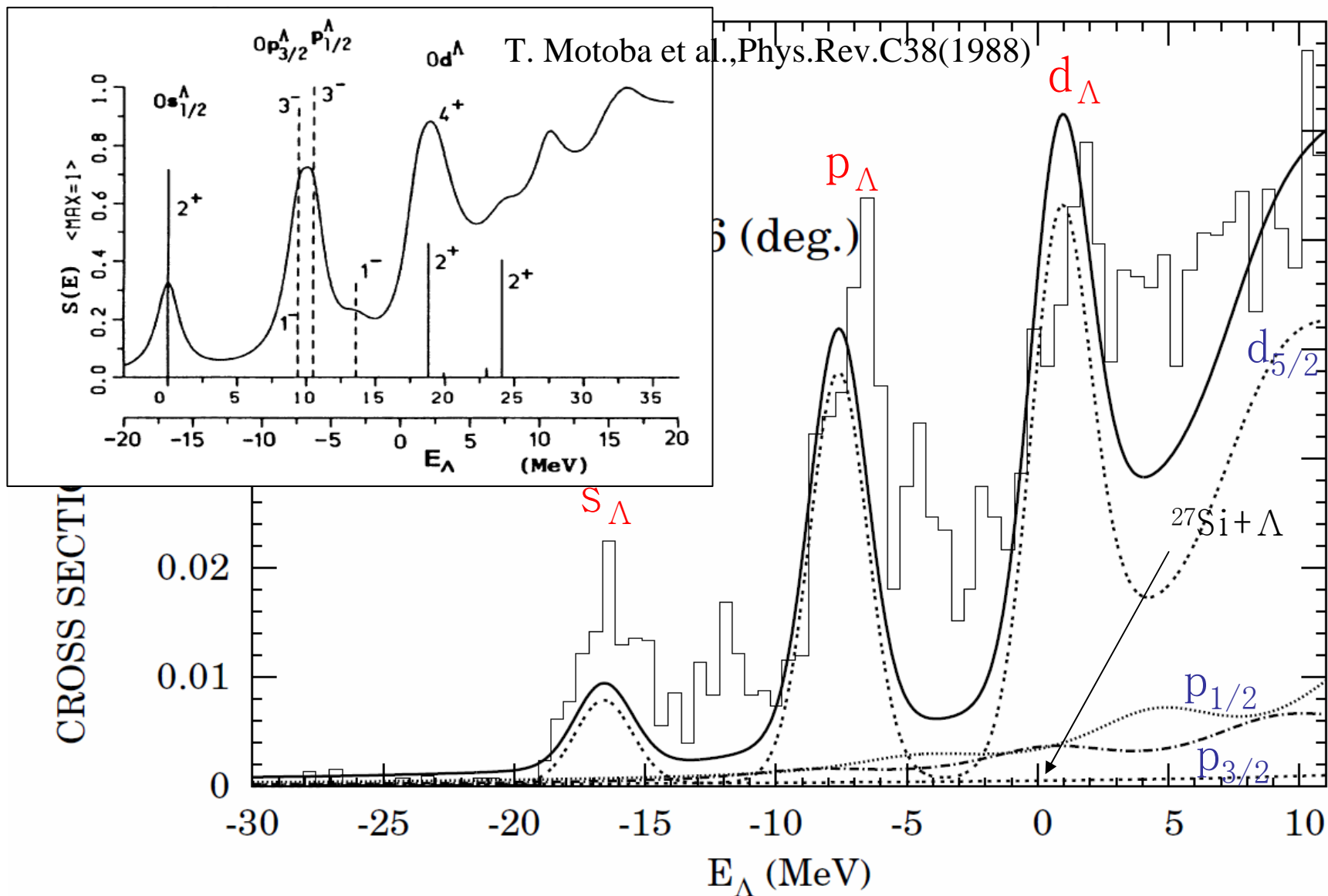
Partial wave decomposition for DW product

$$\left[\begin{aligned} \chi_K^{(+)*}(\mathbf{r})\chi_\pi^{(-)}(\mathbf{r}) &= \sum_{LM} 4\pi i^L \tilde{j}_L(qr) Y_{LM}^*(\hat{k}) Y_{LM}(\hat{r}) \\ \tilde{j}_L(qr) &= i^{-L} (2L+1)^{-1} \sum_{l'} (2l'+1)^l \sqrt{(2l'+1)/4\pi} (l0l'0 | L0)^2 j_l(qr) \Gamma_{l'}(r) \end{aligned} \right.$$

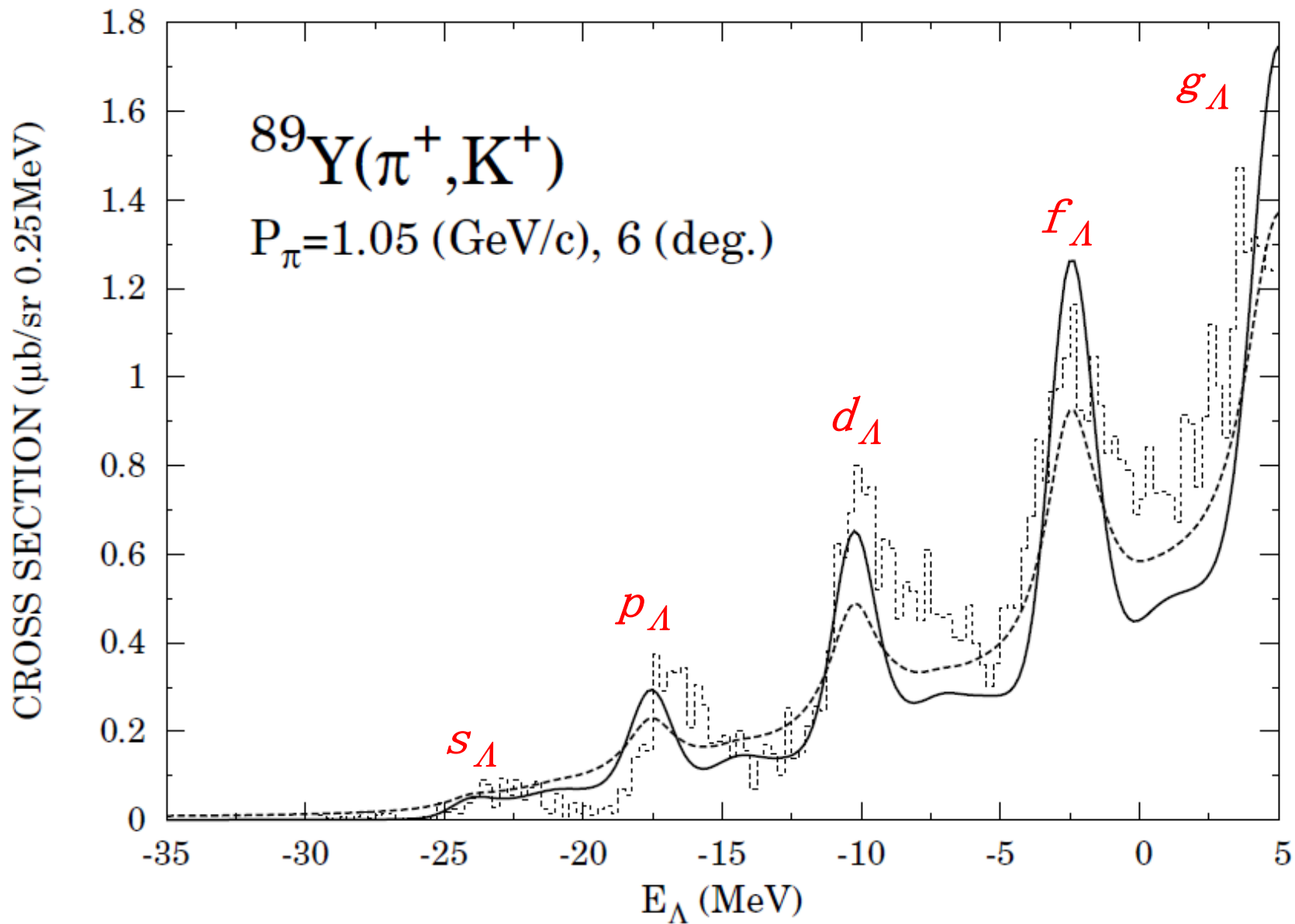
Distorted Bessel function

Distortion factor

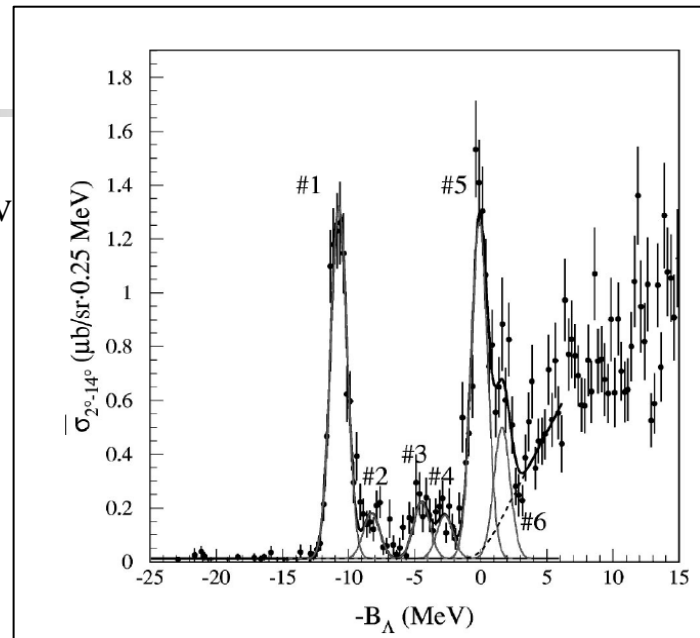
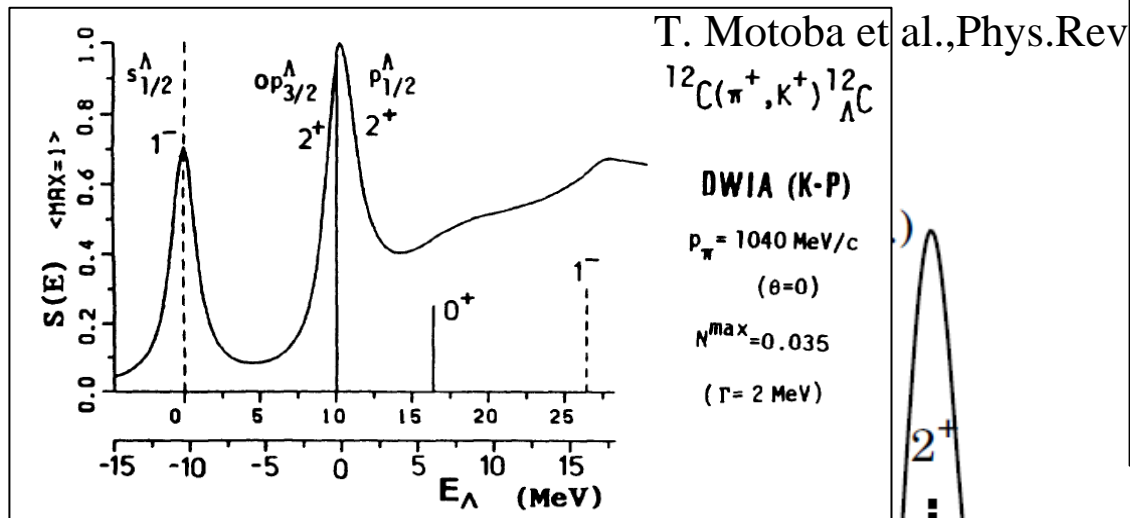
Λ production spectra on ^{28}Si



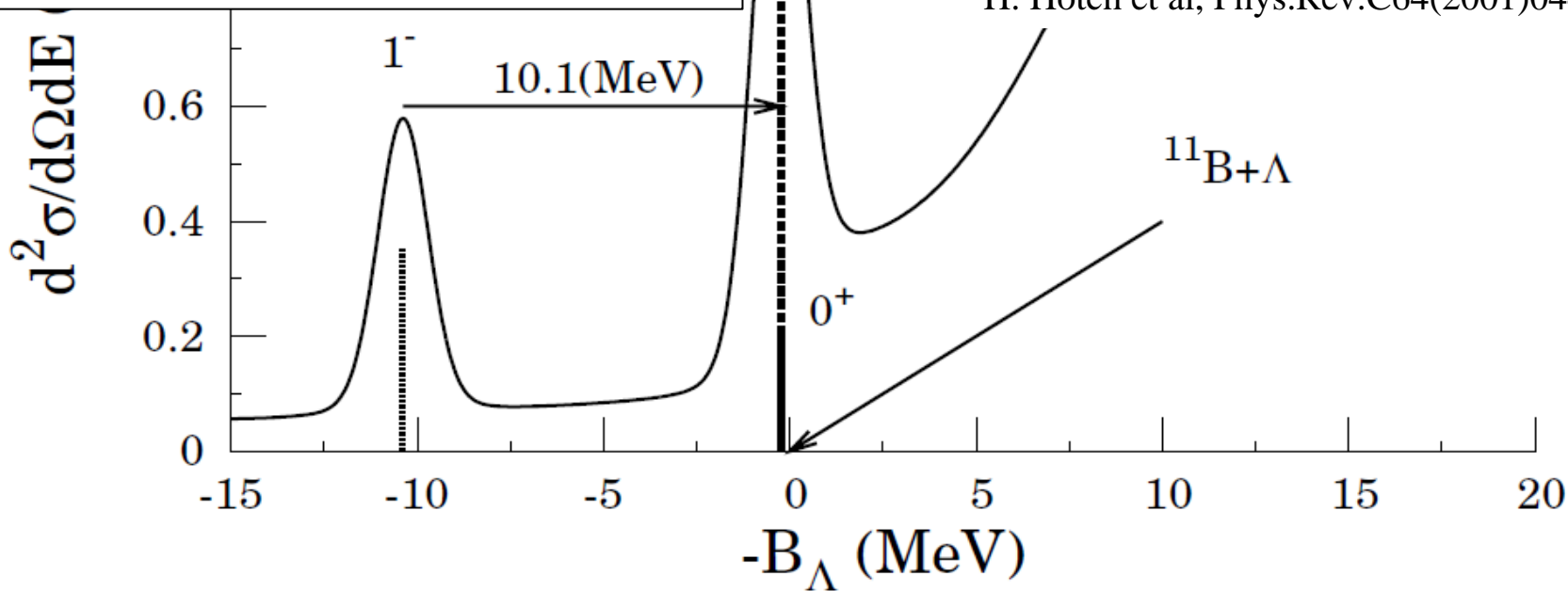
Λ production spectra on ^{89}Y



Λ production spectra on ^{12}C

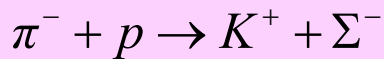


H. Hotch et al, Phys.Rev.C64(2001)044302

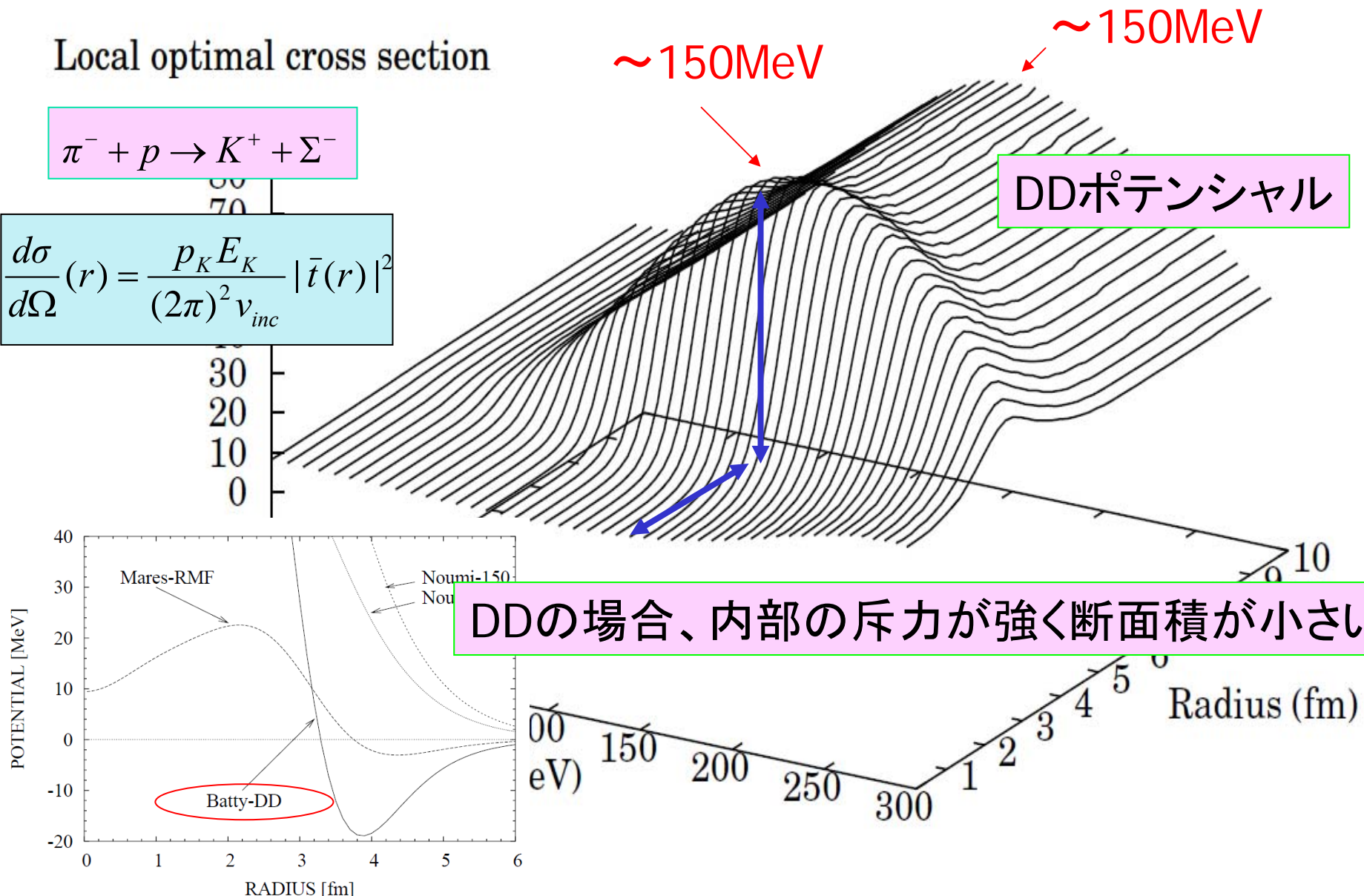


局所最適フェルミ平均した素過程断面積 (t行列)

Local optimal cross section



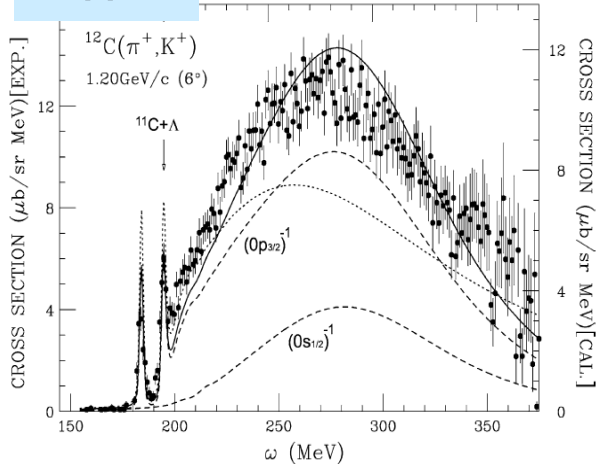
$$\frac{d\sigma}{d\Omega}(r) = \frac{p_K E_K}{(2\pi)^2 v_{inc}} |\bar{t}(r)|^2$$



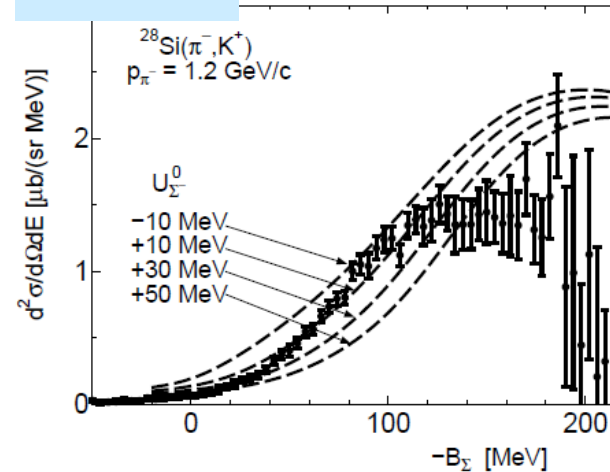
DDの場合、内部の斥力が強く断面積が小さい

Fermi averaging with the on-shell condition

DWIA



SCDW



Several authors pointed out.

In order to describe the Quasi-Free spectrum,

On-shell condition is very important under the averaging t-matrix of elementary process.

T. Harada, Y. Hirabayashi, Nucl. Phys. A 744 (2004) 323.

M. Kohno, Y. Fujiwara, M. Kawai et al., Phys. Rev. C 74:064613, 2006.

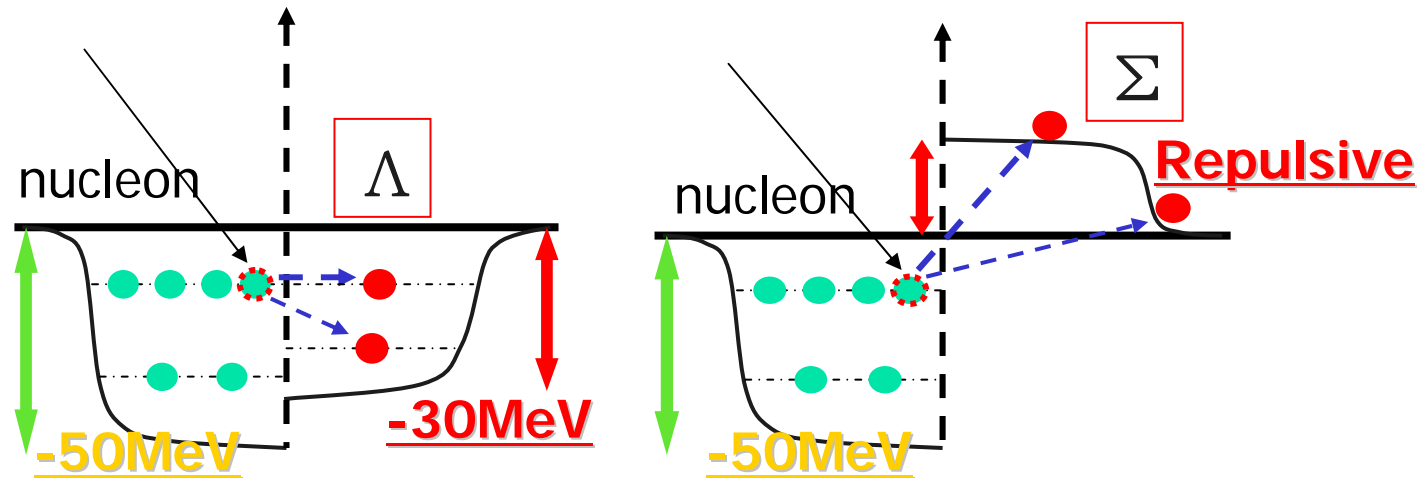
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H. Maekawa, K. Tsubakihara, A. Ohnishi, nucl-th/0701066.

DWIA+最適Fermi平均(自由空間での平均)

→QFの記述に重要

→素過程におけるポテンシャル効果は？



研究目的

- ・最適Fermi平均にポテンシャル効果を取り入れ、Fermi平均の取り扱いによる核内での素過程のエネルギー依存性の変化について議論する
- ・反応データとの比較からのポテンシャル情報を引き出す

Cascade-Nucleus interaction

A different sign of cascade-Nucleus potential:

Nijmegen model D: $V \sim -23$ MeV

Nijmegen model F: $V \sim 28$ MeV

→ Knowledge of the cascade-nucleus potential would give a valuable information for the YN interaction models.

Old emulsion data analysis

$${}^8_{\Xi}\text{He}: B_{\Xi} = 5.9 \pm 1.2 \text{ MeV},$$

$${}^{11}_{\Xi}\text{B}: B_{\Xi} = 9.2 \pm 2.2 \text{ MeV},$$

$${}^{13}_{\Xi}\text{C}: B_{\Xi} = 18.1 \pm 3.2 \text{ MeV},$$

$${}^{15}_{\Xi}\text{C}: B_{\Xi} = 16.0 \pm 4.7 \text{ MeV},$$

$${}^{17}_{\Xi}\text{O}: B_{\Xi} = 16.0 \pm 5.5 \text{ MeV},$$

$${}^{28}_{\Xi}\text{Al}: B_{\Xi} = 23.2 \pm 6.8 \text{ MeV},$$

$${}^{29,30}_{\Xi}\text{Mg}: B_{\Xi} = 2.4 \pm 6.3 \text{ MeV}.$$

Woods-Saxon type:

Potential depth

$$\mathbf{-24 < V < -20 \text{ (MeV)}}$$

C. B. Dover and A. Gal, Annals of Phys, 146(1983)309.