AMD and Statistical Model Study of Ξ Hypernuclear Physics

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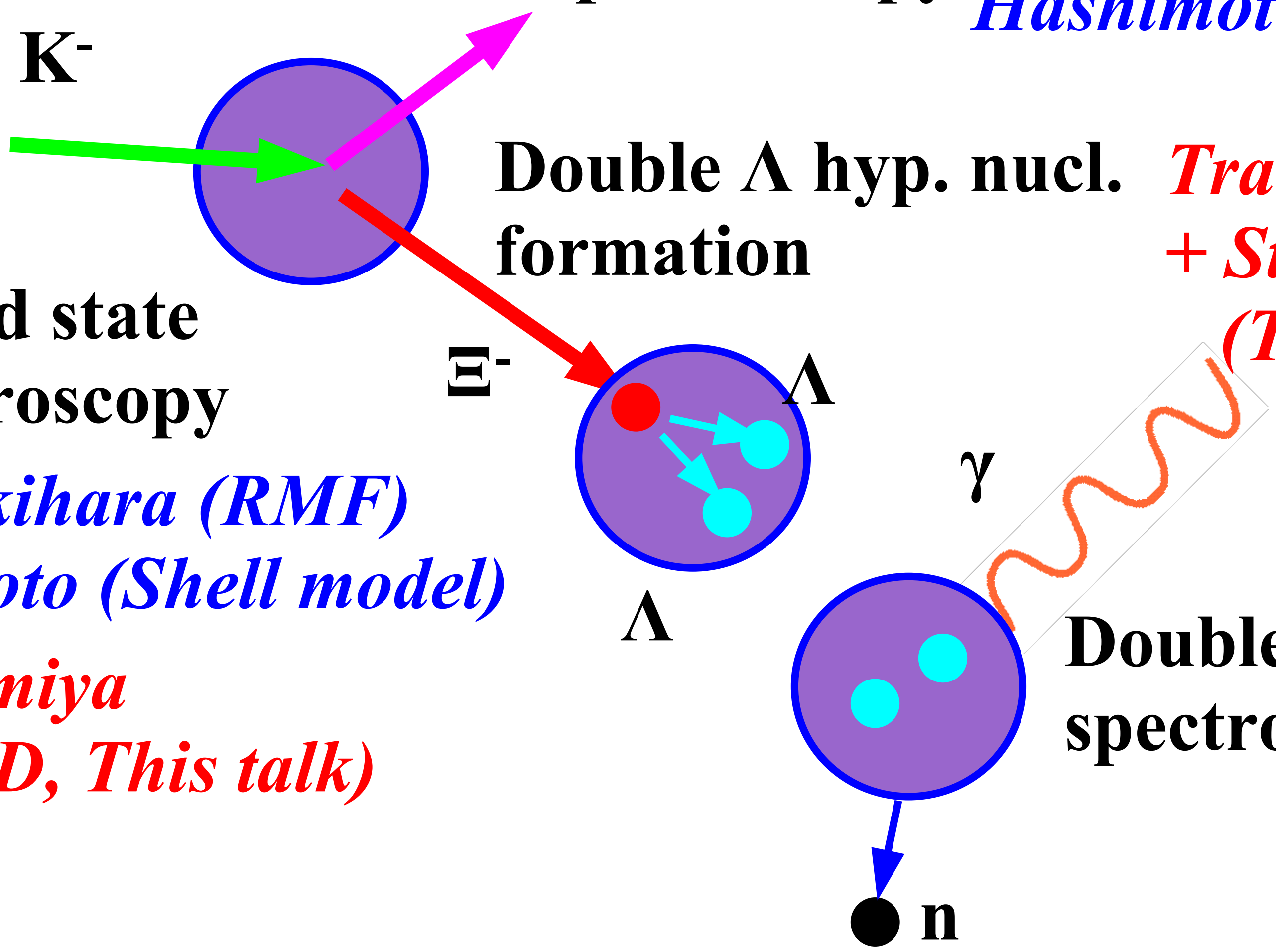
- **Introduction**
- **Spectroscopic Study of Ξ Hypernuclei in Coupled-Channel Antisymmetrized Molecular Dynamics**
- **Statistical Model Study of ΛΛ Hypernuclear Formation from Ξ- Absorption at Rest**
- **Summary**

Ξ Hypernuclear Physics

- **Ξ hypernuclei → Doorway to Multi Strangeness Systems**



Reaction spectroscopy

Maekawa (GFM-DWIA)
Hashimoto (SCDW)

Double Λ hyp. nucl. formation

Transport
+ Statistical Decay
(This talk)

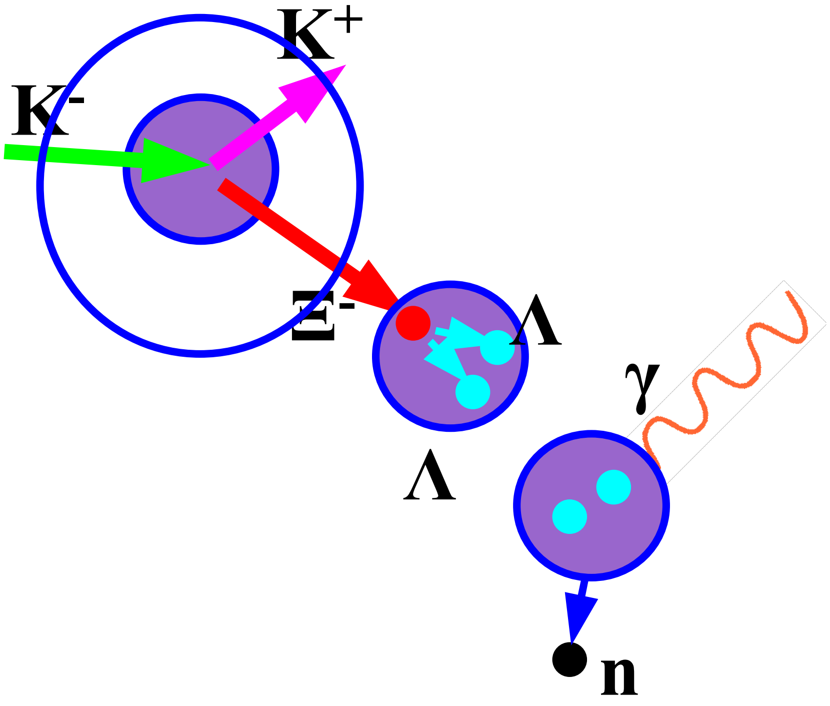
Bound state spectroscopy

Tsubakihara (RMF)
Sugimoto (Shell model)

Matsumiya
(ccAMD, This talk)

Double Λ hyp. nucl. spectroscopy

Ξ hypernuclear structure study in coupled channel AMD



**In producing Ξ hypernuclei,
we need to know
the excited levels
and the transition rate**

Antisymmetrized Molecular Dynamics (AMD)

- Microscopic Model for *Structure and Reaction* Studies

  Ono, Horiuchi, Maruyama, AO, 1992 / Kanada-En'yo, Horiuchi, 1995

  - Slater determinant of Gaussian wave packets
  - Capable of describing Shell / Cluster states
  - Variation of parameters determines the shape of nuclei
  - Good description of transition matrix element (B(E2), ..)
  - *Problems in structure studies:*

    High CPU cost for heavy nuclei → Wait for Faster CPU

    Nodes are generated by Antisym. → No Node for one particle

    → Multigauss AMD (*Dote,Akaishi,Horiuchi,Yamazaki, 2004*)

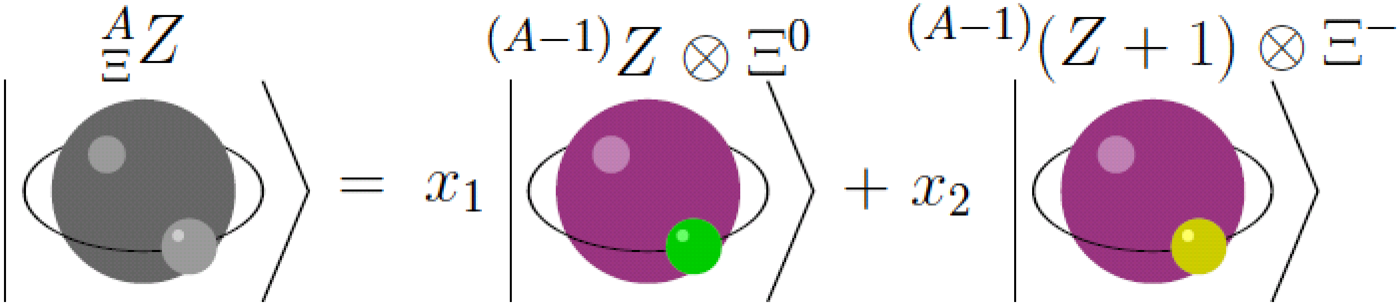
    Inverse matrix elements of s.p. overlap → No particle mixing

    → CoFactor rather than Inverse Matrix (*This work*)

Coupled Channel AMD (ccAMD)

- **Wave function = superposition of Channel AMD w.f.**

$$|\Psi\rangle = \sum_a x_a |\Phi^a\rangle \qquad (a : \text{channel}) \qquad |\Phi^a\rangle = \frac{1}{\sqrt{A!}}\det\left[|\varphi_j^a(i)\rangle\right]$$



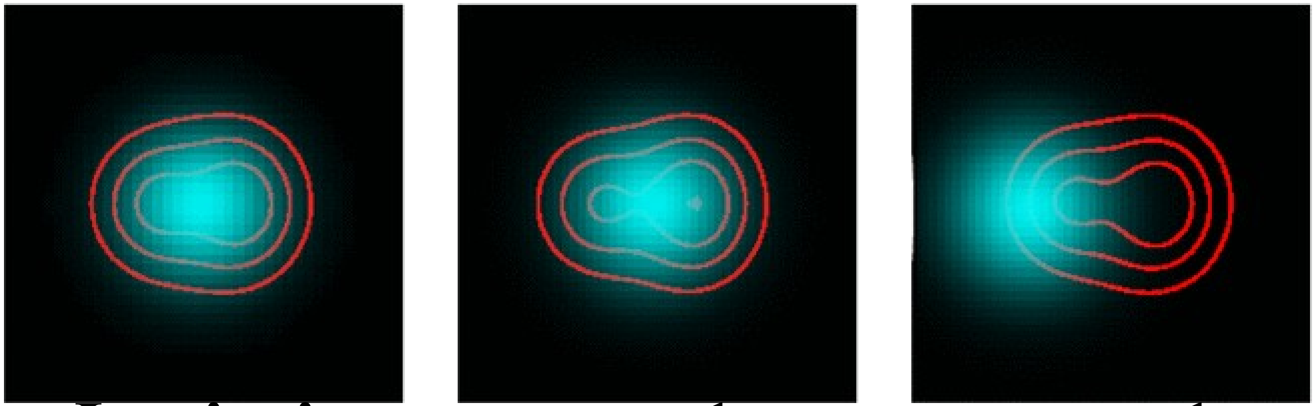
- **Hamiltonian =T + V$_{NN}$ + V$_{YN}$ + Mass diff.**

$$\hat{H} = \hat{T} - \hat{T}_{\mathrm{cm}} + \hat{V} + \Delta mc^2$$

  - **VNN : Brink-Boeker-Okabe (BBO1)**
  - **VYN : G-matrix of Nijmegen Extended Soft Core (ESC04d, *Rijken, Yamamoto, 2006*)**
    Consistency VYN (ρ) ↔ ρ =< ρ> (Dote-Akaishi prescription)

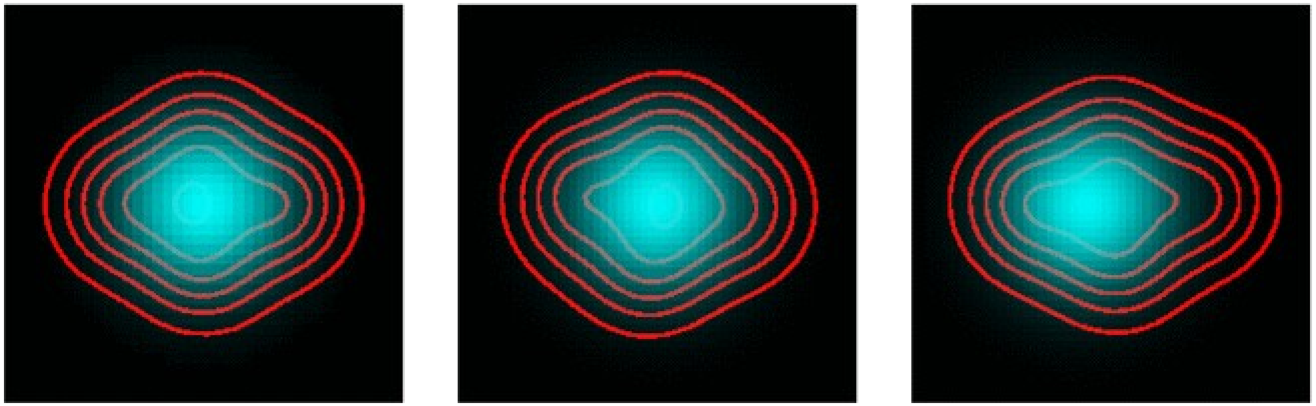
Density Distribution

- $^{12}_{\Xi}$Be



Intrinsic π=-1 π =+1

- $^{28}_{\Xi}$Mg

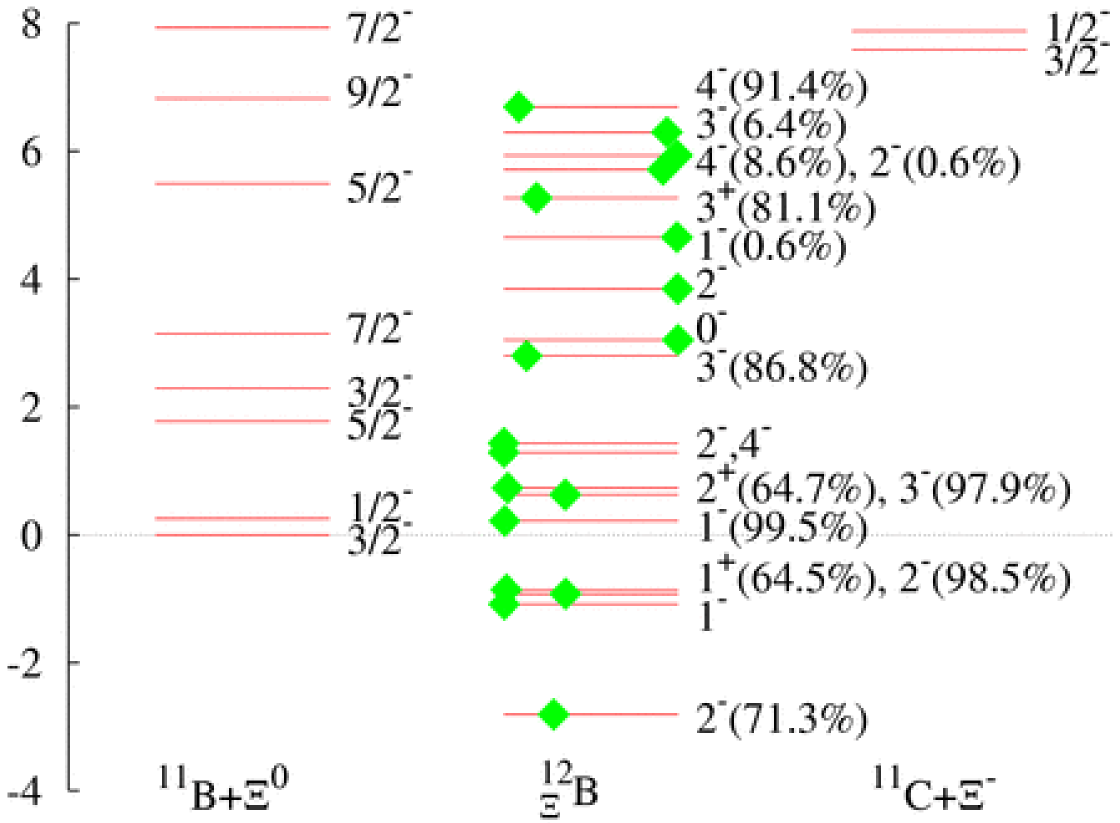


Intrinsic π=+1 π =-1

- Nuclei are not necessarily spherical !
- Core and Ξ Parities are mixed !

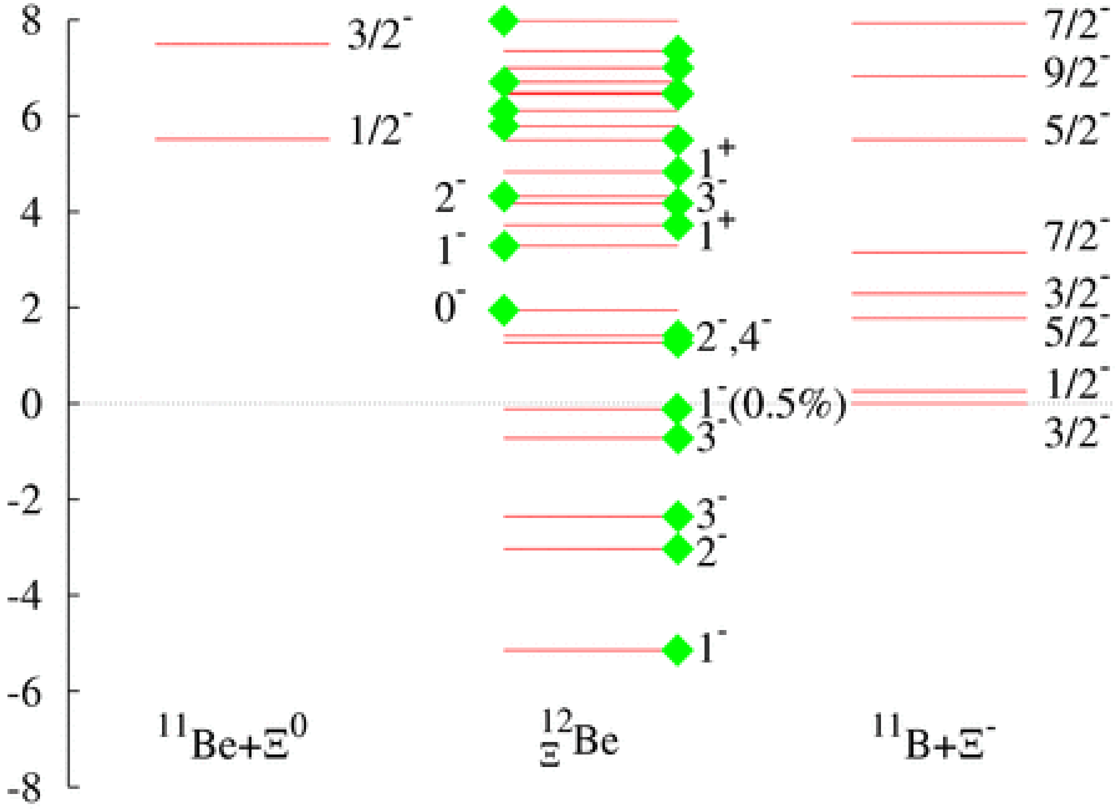
Level Structure: ${}^{12}_{\Xi}$B (Coherent mixing ?)

- 12C(K$^-$,K^0)${}^{12}_{\Xi}$B, ${}^{12}_{\Xi}$B =(^{11}B+Ξ^0)+(^{11}C+Ξ^-)
  - (Mirror Core)$\otimes$ Ξ, T=0, 1 → Ξ^0 : Ξ^- = 1:1 without isospin breaking
  - Mass diff. (M(Ξ^-) ~ M(Ξ^0)+7 MeV) & Coulomb break isospin sym. → We need Charge base !



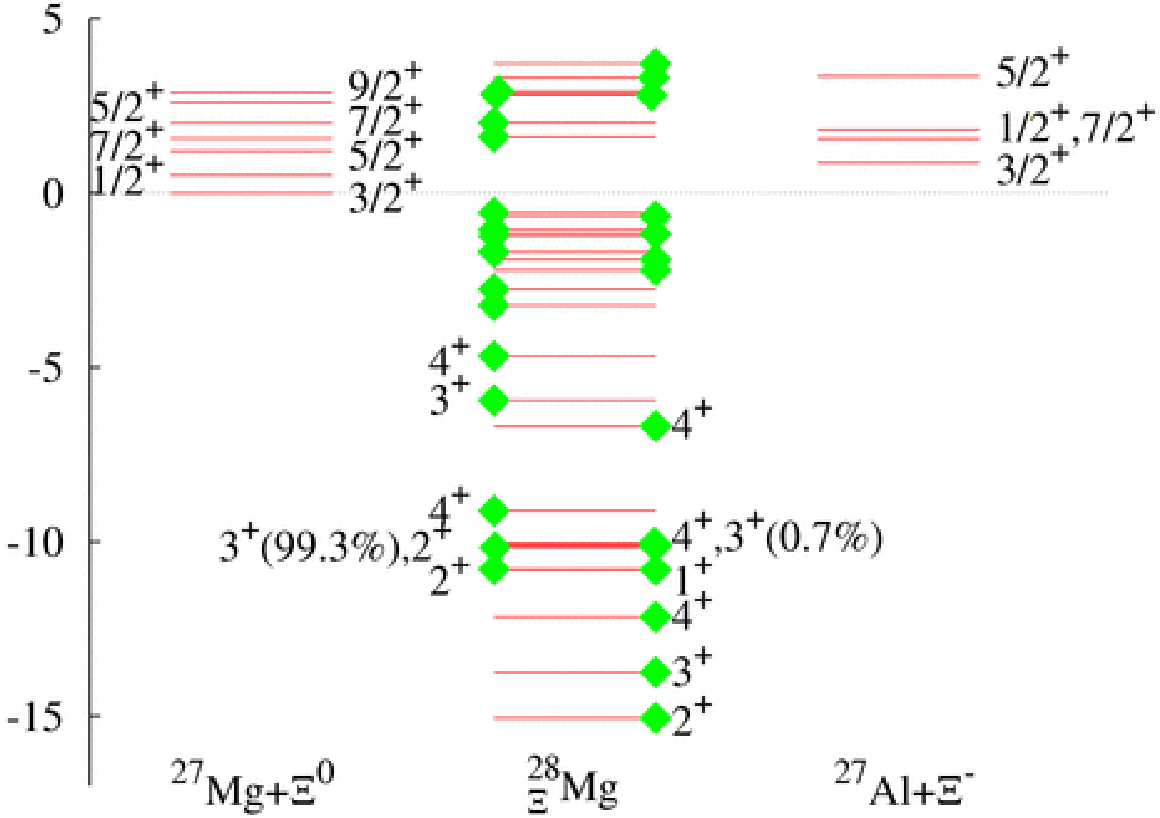
Level Structure: ${}^{12}_{\Xi}$Be (Day-One Experiment Nucleus)

- 12C(K^-,K^+)${}^{12}_{\Xi}$Be, ${}^{12}_{\Xi}$Be =(^{11}Be(T=3/2)+Ξ^0)+(^{11}B(T=1/2)+Ξ^-)
  - "Mass diff. of Core > Mass diff. of Ξ" + "Core T diff."
    → Almost No Coupling Effects
    → Single Channel (potential) description would be good enough !

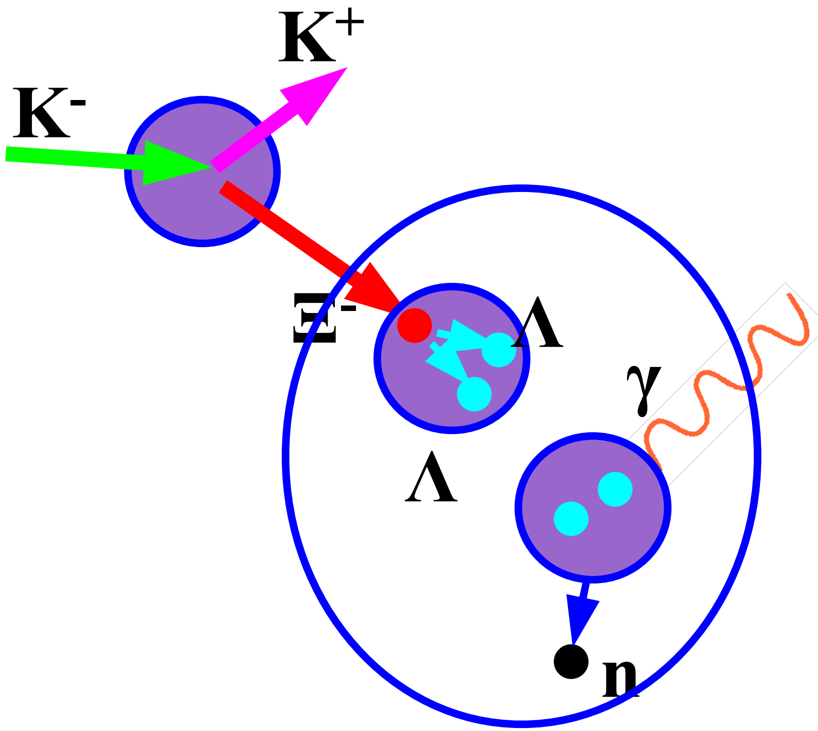


Level Structure: ${}^{28}_{\Xi}$Mg (Day-Two Experiment ?)

- ^{28}Si(K$^-$,K$^+$)${}^{28}_{\Xi}$Mg, ${}^{28}_{\Xi}$Mg =(^{27}Mg(T=3/2)+Ξ^0)+(^{27}Al(T=1/2)+Ξ^-)
  - **"Masses of Core+Ξ" are Comparable, but Core T are different.**
  - **Almost no mixture of Ξ^- and Ξ^0 channels**
    → Ξ^- states will be selectively populated in (K$^-$,K$^+$) reaction



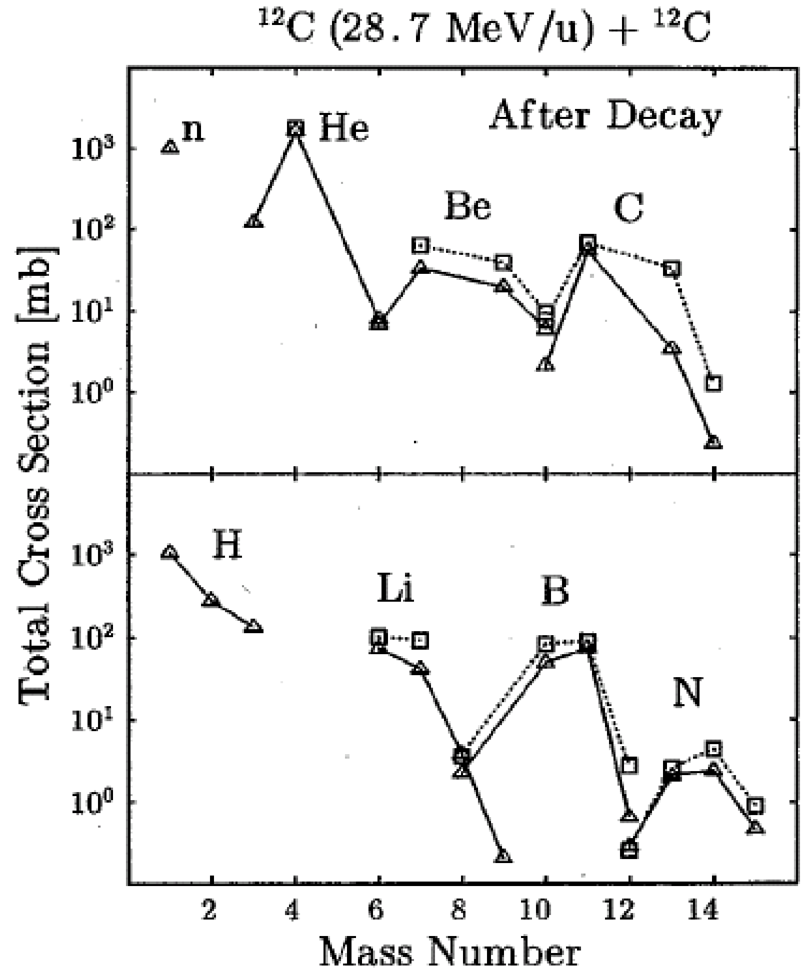
Statistical decay model study of Ξ hypernuclei



For double Λ hyp. nucl. (DΛHN) γ ray spectroscopy, it is necessary to know which DΛHN are produced !

Transport + Statistical Decay Model (1)

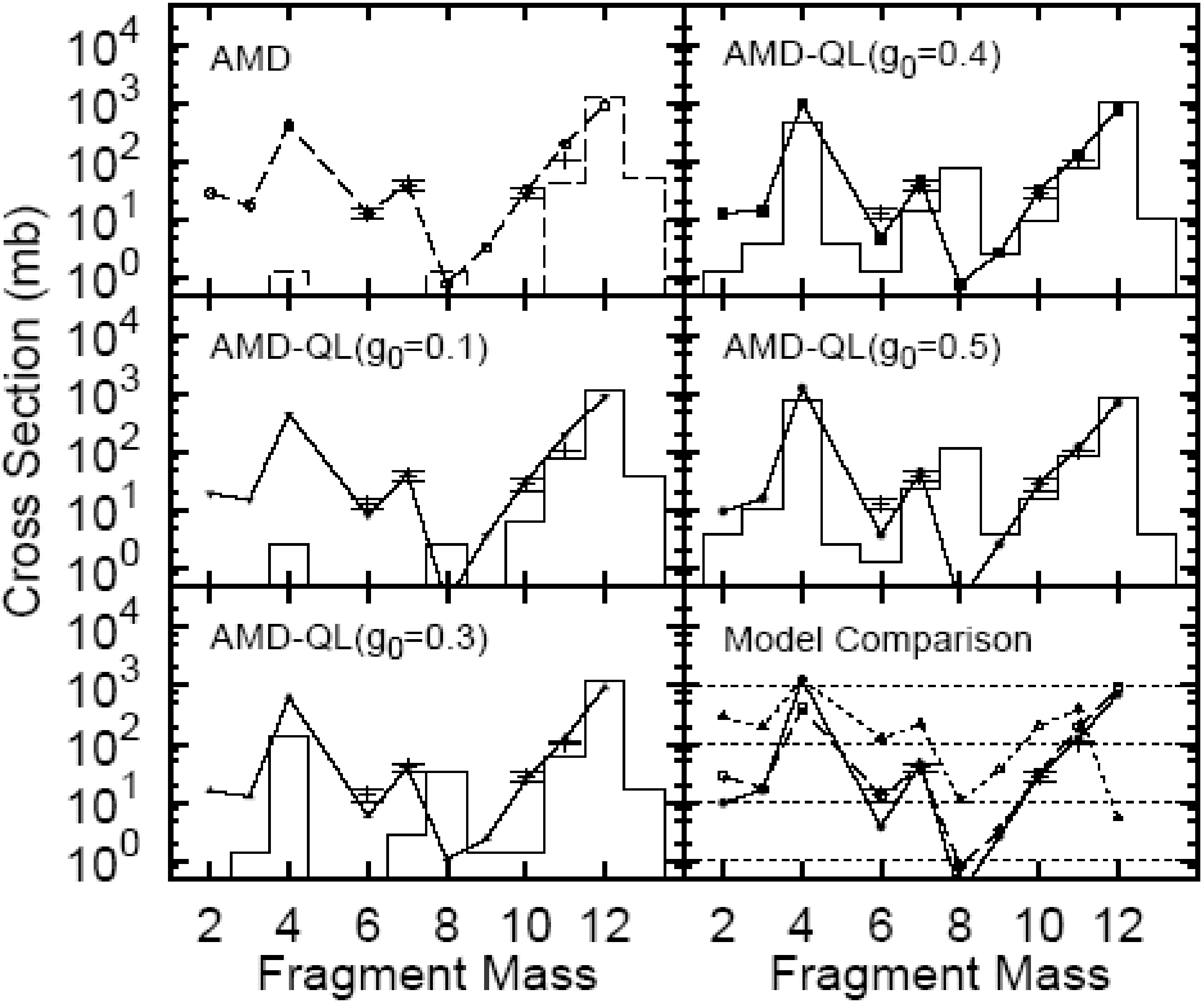
- **Multistep evap. of nucleons and α from Excited Nuclei (Statistical Cascade Decay Process)**
  - **Established decay process**
  - **Combined with transport model, Cascade gives reliable results !**
  - **AMD + Cascade (Δ)**
    → Describes frag. form. data (□) within a factor of two !



Ono, Horiuchi, Maruyama, AO, PRL and PTP, 1992.

Transport + Statistical Decay Model (2)

- **AMD-QL (with Quantum Statistical Fluctuation) + Cascade decay**
  - **Incl. quantum fluc. simulats stat. decay dynamically.**



Hirata, Nara, AO, Harada, Randrup, 1999

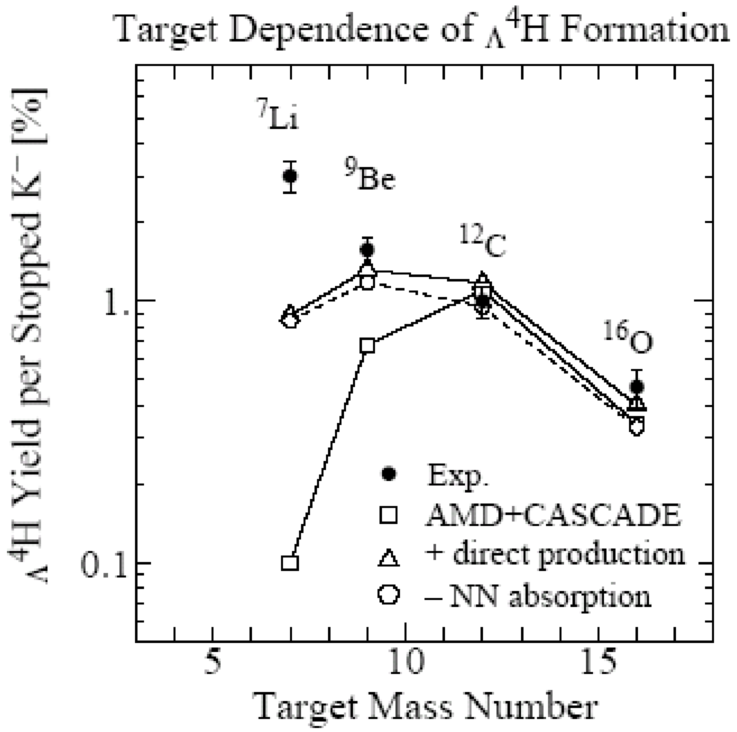
Hyperfragment Formation

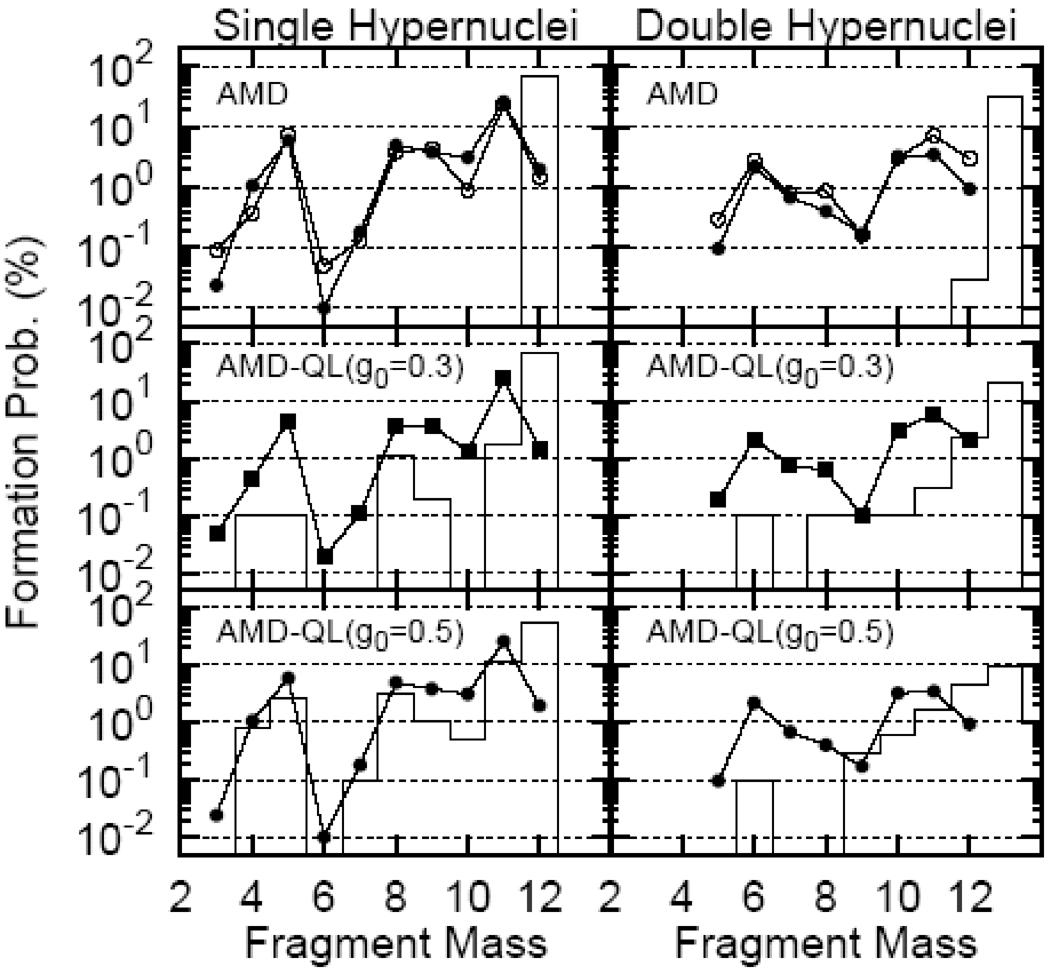
- **Stopped K- + (Li, C, O) → $^{4}_{\Lambda}\mathrm{H}$**

  Tamura et al. 1989 / Nara,AO,Harada, 1995

- **Stopped Ξ^- + ^{12}C → Double, Twin, Single hypernuclei**

  KEK-E176 / Hirata et al. 1999





Double Hypernuclear Formation from Stopped Ξ^-

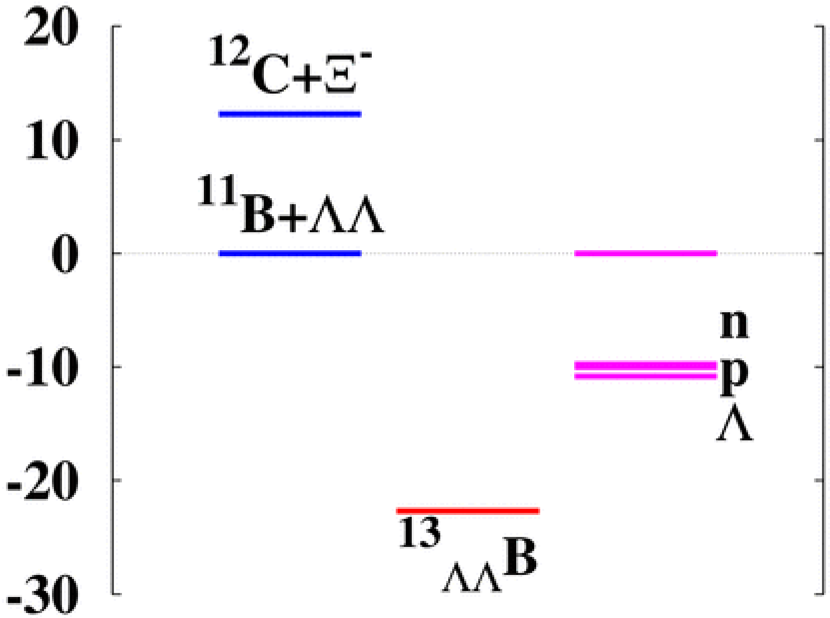
- **Theoretical Models**
  - AMD/AMD-QL + Cascade (*Hirata et al., 1999*)
  - Direct Reaction (*Yamada, Ikeda, 1997*)
    Two-Cluster Res. dominance in Twin hypernuclear form.
  - Statistical Decay (*Yamamoto, Sano, Wakai, 1994*)
    Canonical dist. model (~ Copenhagen model, *c.f. Botvina's talk*)

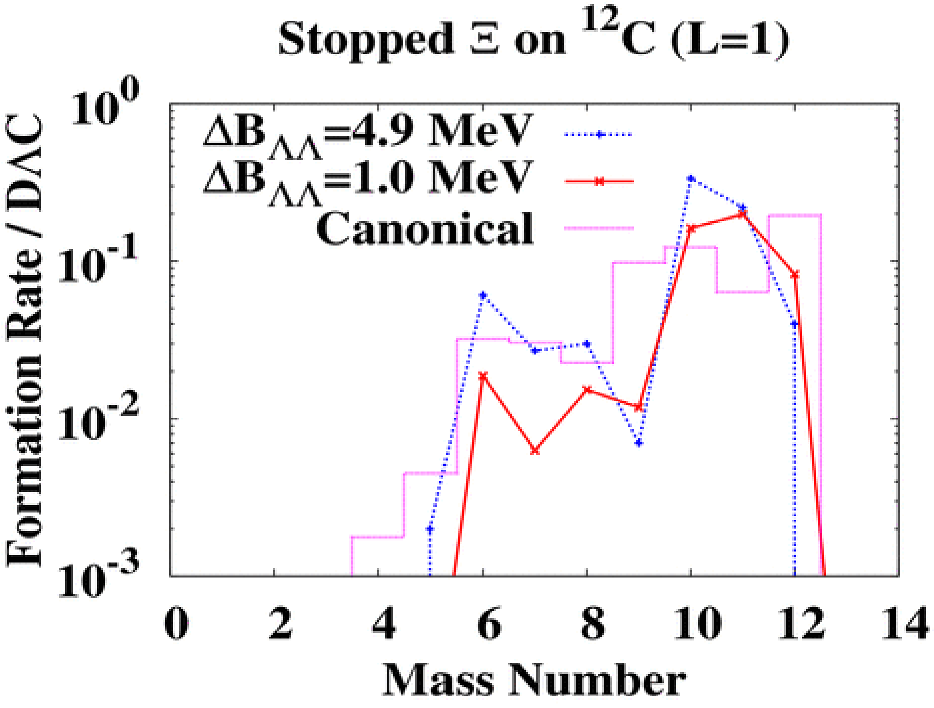
→ Dominant Double Λ hypernuclear formation Prob.
= (Double Λ Compound Nucleus (DΛC) formation prob.
w/o quantum fluc.)
× (Double Λ hypernucleus (DΛHN) survival prob.)

Let's evaluate DΛHN survival prob.
in Cascade Decay Model
in Stopped Ξ^- on ^{56}Fe

Stopped Ξ^- on ^{12}C

- **Double Λ Compound Nucleus (DΛC) Formation Prob.**
  ~ 30 % (AMD), 16 %, (AMD-QL (with quantum fluc.))
- **Double Λ Hyp. Nucl. (DΛHN) survival prob.**
  ~ 72 % ($\Delta B_{\Lambda\Lambda} = 4.9$ *MeV*), 49 % ($\Delta B_{\Lambda\Lambda} = 1.0$ *MeV*)
- **Main products($\Delta B_{\Lambda\Lambda} = 1.0$ *MeV*)**
  $^{11}_{\Lambda\Lambda}$Be (15 %), $^{10}_{\Lambda\Lambda}$Be (16 %), $^{12}_{\Lambda\Lambda}$B (6 %), $^{6}_{\Lambda\Lambda}$He (1.8 %)




Stopped Ξ on ^{12}C (L=1)

Stopped Ξ^- on ^{12}C

- **Our Previous Estimate**

  → Mass Table based on old data, $\Delta B_{\Lambda\Lambda}$ ~ 4.9 MeV

  - **DΛC form. prob. ~ 30 % (AMD), 16 % (AMD-QL)**
  - **DΛHN ($^{6}_{\Lambda\Lambda}$He) form. prob.**

    ~ 18 (2.7) % (AMD+Casc), 11 (1.9) % (AMD-QL+Casc.)

- **Present Estimate**

  → Mass Table based on new data (Nagara), $\Delta B_{\Lambda\Lambda}$ ~ 1.0 MeV

  - **DΛHN ($^{6}_{\Lambda\Lambda}$He) *survival* prob. ~ 49 (1.8) %**
  - **DΛHN ($^{6}_{\Lambda\Lambda}$He) *formation* prob.**

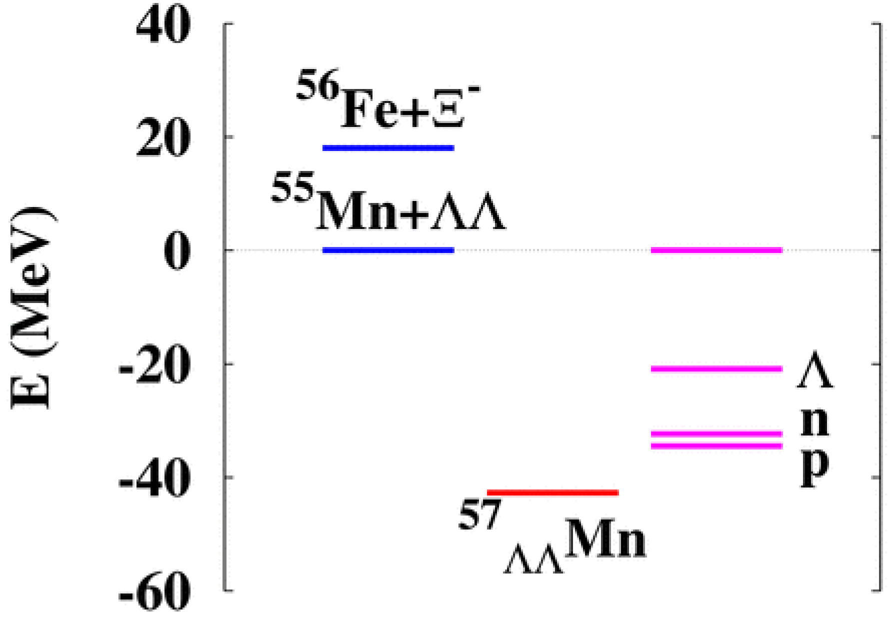
    ~ (DΛC form. ~ 30 %) × (DΛHN surv. prob.) ~ 15 (0.5) %

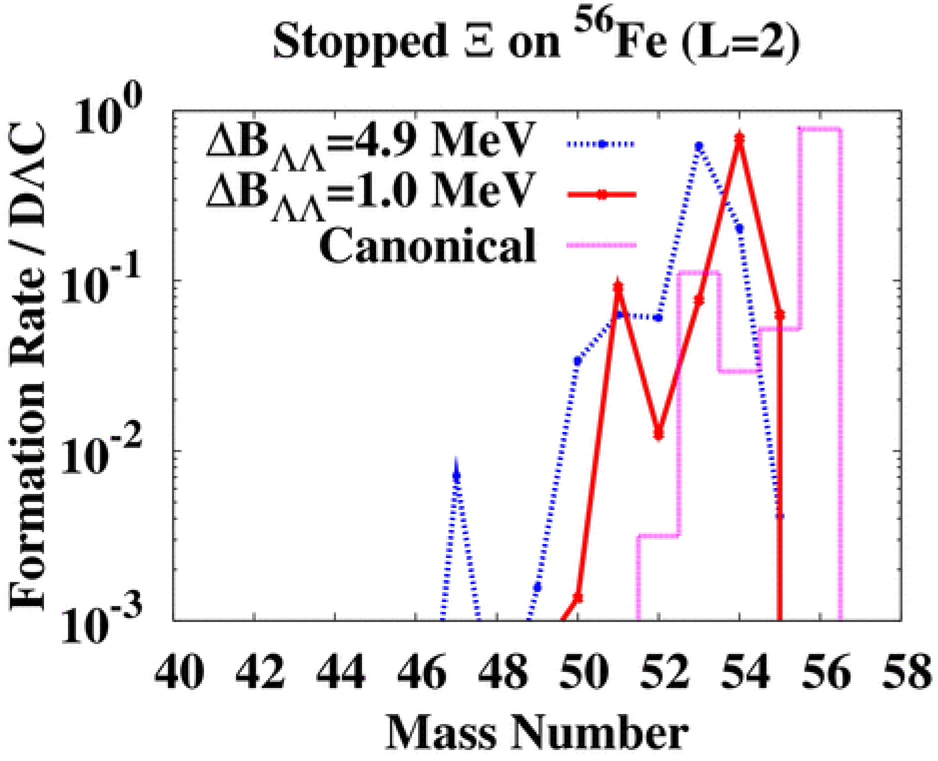
    → Explains why we cannot observe $^{6}_{\Lambda\Lambda}$He frequently !

    (E176, E373)

Stopped Ξ^- on ^{56}Fe

- DΛC formation prob. ~ (5-30) % ?
- Double Λ hyp. nucl. survival prob.
  ~ 99 % ($\Delta B_{\Lambda\Lambda}$ = 4.9 MeV), 93 % ($\Delta B_{\Lambda\Lambda}$ = 1.0 MeV)
- Main products
  $^{54}_{\Lambda\Lambda}$Mn (3n emission, 52 %), $^{54}_{\Lambda\Lambda}$Cr (p2n emission, 16 %)





Stopped Ξ^- on ^{56}Fe

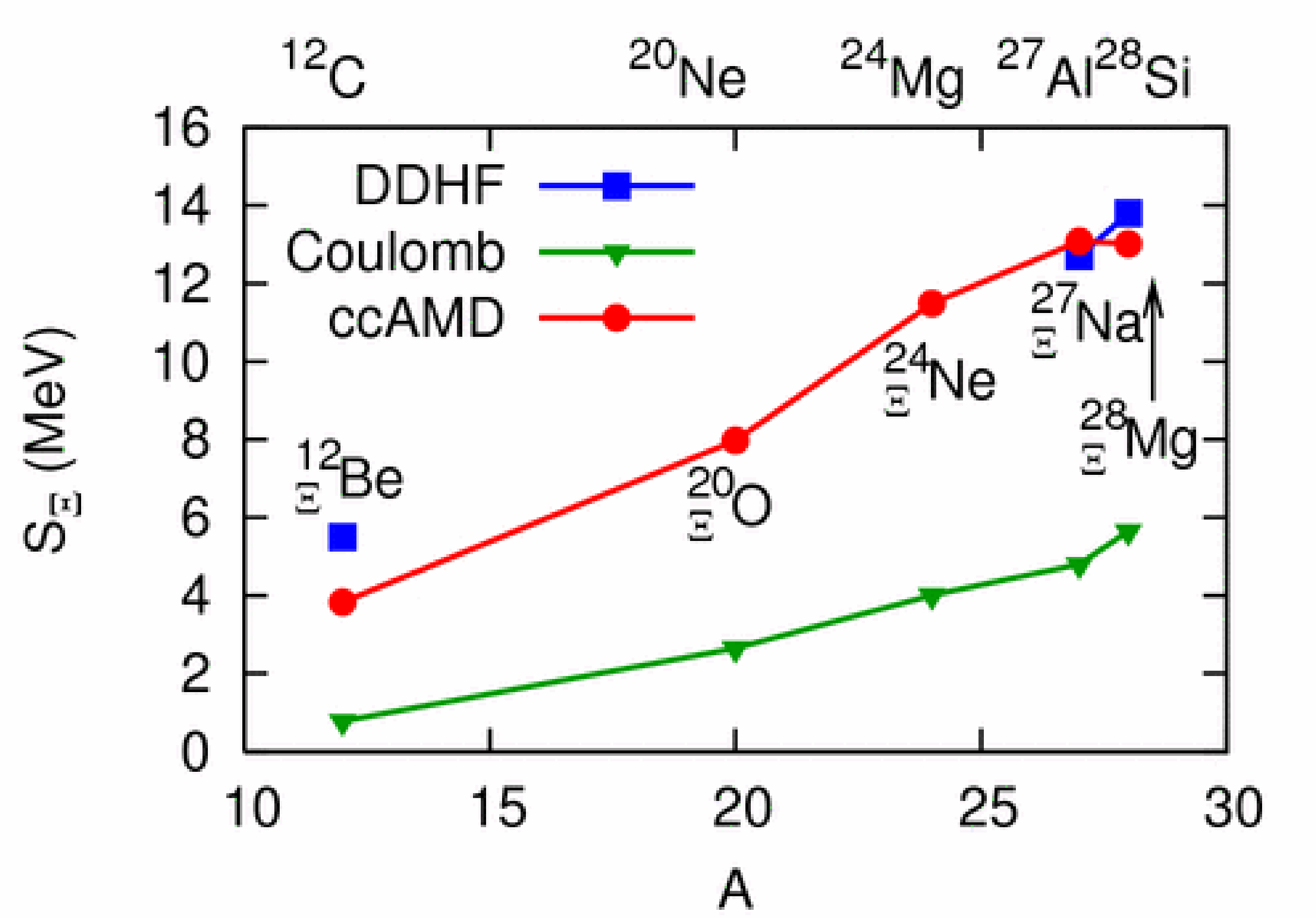
- **Why do we have high DΛHN survival prob. ?**
  - **Total Excitation E ~ 60 MeV → E*/A ~ 1 MeV**
  - **$E^*/A = T^2/8$ → T ~ 3 MeV**
  - **$S_p \sim 8$ MeV, $S_n \sim 10$ MeV, $S_\Lambda \sim 22$ MeV**
    Emission prob. ~ exp(-S/T)
    → One Λ emission / One n emission ~ 1.4 %

Summary

- ***Structure study of Ξ hypernuclei in ccAMD***
  - **Coupled Channel AMD is also applicable to NKbar-Yπ and Nπ/N coupling system.**
    c.f. AMD with neutral π coherent state, Isshiki,Naito,AO, 2005
  - **Core Isospin & Core/Ξ mass diff. & Coulomb pot. suppresses coupling of Ξ^- and Ξ^0 states.**
  - *To do: Effective number / Transition density / Multigauss AMD*
- ***Statistical model study of stopped Ξ^- reactions***
  - **Double Λ form. prob.**
    ~ (Double Λ comp. form.) × (Double Λ surv.) ~ (5-40 %) × (90 %)
  - **^{56}Fe target → $^{54}_{\Lambda\Lambda}$Mn will be abundantly formed.**
  - *To do: Estimate of L-dependent DΛC form. prob.*
- ***Ξ physics requires us to understand all of Production, Structure and Decay.***

Binding Energies

- **Binding Energies in ccAMD**
  ~ DDHF Results in sd-shell nuclei



AMD

- **w.f.**

$$|\Phi\rangle = \frac{1}{\sqrt{A!}}\det\left[|\varphi_j(i)\rangle\right] \qquad |\varphi_j\rangle = |\boldsymbol{z}_j\rangle \underbrace{\left|\chi^\sigma_j\right\rangle}_{\text{spin}} \overbrace{\left|\chi^\tau_j\right\rangle |f_j\rangle}^{\text{isospin,flavor}}$$

$$\left|\chi^\sigma_j\right\rangle = \left\{ \begin{array}{l} |\uparrow\rangle \\ |\downarrow\rangle \end{array} \right. , \qquad \begin{aligned} \left|\chi^\tau_j\right\rangle \otimes |f_j\rangle &= |T, T_z\rangle \otimes \left\{ \begin{array}{c} |N\rangle \\ |\Xi\rangle \\ \vdots \end{array} \right\} \\ &= |n\rangle, |p\rangle, \left|\Xi^0\right\rangle, \left|\Xi^-\right\rangle, \cdots \end{aligned}$$

$$|\Phi\rangle = \frac{1}{\sqrt{A!}}\det\left[|\varphi_j(i)\rangle\right] \qquad |\varphi_j\rangle = \overset{\text{空間部分}}{|\boldsymbol{z}_j\rangle} \left|\chi^\sigma_j\right\rangle \left|\chi^\tau_j\right\rangle |f_j\rangle$$

$$\boldsymbol{z}_j = \sqrt{\nu}\boldsymbol{d}_j + \frac{i\boldsymbol{k}_j}{2\hbar\sqrt{\nu}}$$

$$\begin{aligned} \langle \boldsymbol{r} | \boldsymbol{z}_j \rangle &= \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left[-\nu\left(\boldsymbol{r} - \frac{\boldsymbol{z}_j}{\sqrt{\nu}}\right)^2 + \frac{\boldsymbol{z}_j^2}{2}\right] \\ &\propto \exp\left[-\nu\left(\boldsymbol{r} - \boldsymbol{d}_j\right)^2 + i\boldsymbol{k}_j \cdot \boldsymbol{r}/\hbar\right] \end{aligned}$$