Lattice QCD approach to baryon-baryon potentials

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in collaboration with

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Plan of the talk:

- Introduction and Backgroud
- Formalism
- Lattice QCD results
- Summary

See for detail
N.Ishii, S.Aoki, T.Hatsuda nucl-th/0611096, PRL in press.

See also

F.Wilzcek, Nature **445**(Jan.11, 2007) p156.

Introduction

The nuclear force is one of the most important building blocks in nuclear physics.

- ✓ Its attractive part in the medium to long range is responsible to the existence of bound nuclei.
- ✓ The repulsive core in the short range plays an important role for the stability of heavy nuclei.
- ✓ Proper knowledge of the nuclear force is important also for the astro-physics.
  - — It has an important influence on the maximum mass of the neutron stars and the Type II supernova explosions.
- ✓ Enormous efforts have been devoted to the theoretical studies of the nuclear force starting from Yukawa's original paper 70 years ago:
  - ➢ H. Yukawa, Proc. Math. Phys. Japan **17**, 48 (1935).

Introduction (cont'd)

The nature of the nuclear force is understood in the three regions separately.

✓ **The long distance region (r > 2fm)**
this region can be described in terms of the one pion exchange.

✓ **The medium distance region (1fm < r < 2fm)**
this region can be accessible with the meson-based theories

— heavier meson exchanges such as "σ", ρ , ω ,...

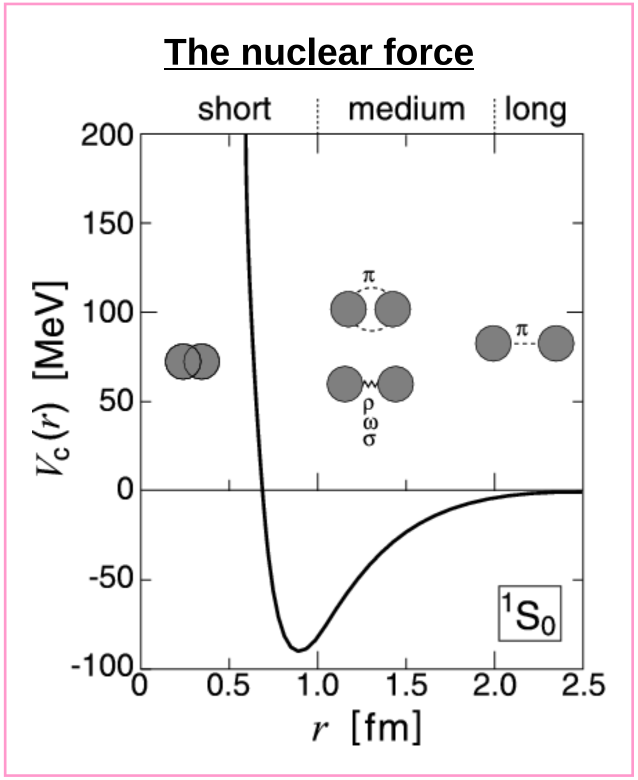
— multi pion exchange

✓ **The short distance region (r < 1fm)**
the understanding of this region is delayed.

— Phenomenological approaches are often used.
(1) phenomenological repulsive core model
(2) vector meson exchange model
(3) quark exchange (constituent quark model)

— This region is expected to reflect
(1) the internal structure of the nucleon
(2) the existence of the quark and the gluon degrees of freedom

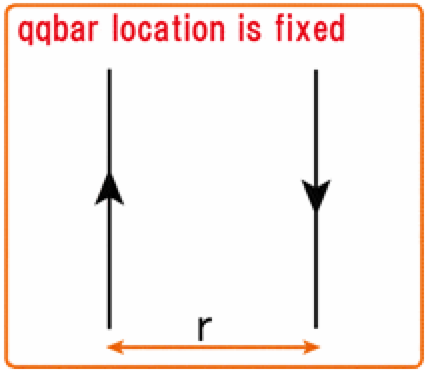
— One has desired the QCD understanding of the nuclear force for a long time.

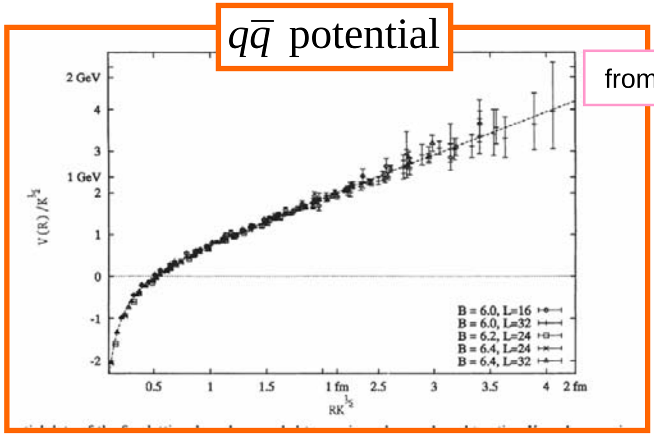


Conventional lattice QCD approach to various potentials

- **static qqbar potential** (one of the most successful formalism in lattice QCD)
  - qqbar potential is calculated as the total energy for the two static (anti-)quarks with infinity mass.

$$\Delta x \, \Delta v \approx \hbar / m \approx 0 \quad \text{as } m \to \infty$$

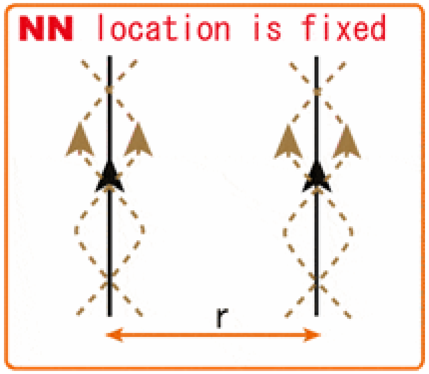


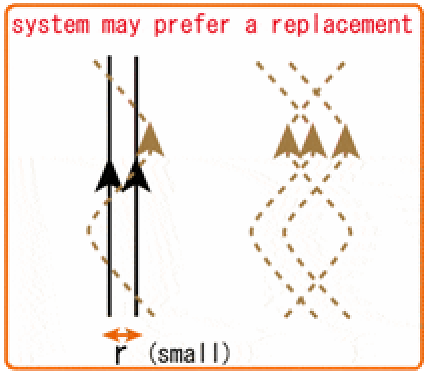


from G.S.Bali et al., PRD**46**,2636('92)

- One has attempted to extend this method to the NN potential.
  - A static quark is introduced in each nucleon to fix the locations of the two necleons.

cf) T.T.Takahashi et al., AIP. Conf. Proc.**842**, 246 (2006)





★ If we count the meson-meson potential and color SU(2) calculations, there are quite many articles ! (published ones onlly)

- ✓ D.G.Richards et al., PRD42, 3191 (1990).
- ✓ A.Mihaly et la., PRD55, 3077 (1997).
- ✓ C.Stewart et al., PRD57, 5581 (1998).
- ✓ C.Michael et al., PRD60, 054012 (1999).
- ✓ P.Pennanen et al, NPPS83, 200 (2000).
- ✓ A.M.Green et al., PRD61, 014014 (2000).
- ✓ H.R Fiebig, NPPS106, 344 (2002); 109A, 207 (2002).
- ✓ T.Doi et al., AIP Conf. Proc. 842, 246 (2006).

- This is an elaborate method. However, it does not work out so far.
- This method does not provide a NN potential, which is faithful to the realistic NN scattering data.

We use a totally different method.

We will extend the method recently proposed by CP-PACS collaboration,

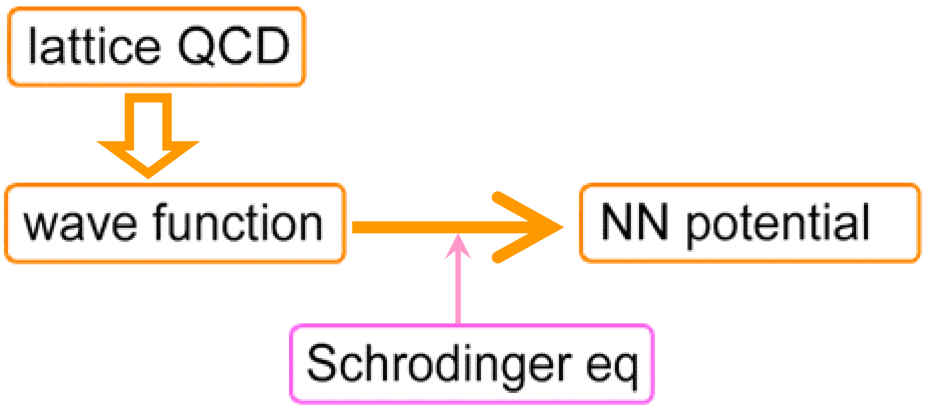
CP-PACS collab., S. Aoki et al., PRD**71**,094504(2005)

in studying $\pi\,\pi$ scattering length.

Sketch of our method:

(1) The NN wave function is constructed by using lattice QCD.

(2) The NN potential is reconstructed from the wave function by demanding that the wave function should satisfy the Schrodinger equation.



Symbolic expression

$$V(r) = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\psi(\vec{x})}{\psi(\vec{x})}$$

GOOD FEATURE

Methods which employ static quarks never lead to the realistic NN potential.

Our method is completely different.

It can provide the NN potential faithful to the experimental data in the near future.

The Formalism

- ✓ We begin with **the (effective) non-relativistic Schroedinger eq.** for NN system.

$$-\frac{\vec{\nabla}}{2\mu}\phi(\vec{r})+\int d^3r'\,U(\vec{r}-\vec{r}')\,\phi(\vec{r}')=E\phi(\vec{r})$$

  - ✓ For derivation, see S.Aoki et al., PRD**71**, 094504 (2005).
  - ✓ In general, the interaction kernel U(r-r') can **be non-local** and **depend on the E.**

- ✓ Possible forms of U are constrained by various symmetries.
Derivative expansion of U(r-r') at low energy leads to the parameterization as

$$V_{NN}(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}')\equiv U(\vec{r}-\vec{r}').$$

$$V_{NN}=V_C(r)+V_T(r)\,S_{12}+V_{LS}(r)\,\vec{L}\cdot\vec{S}+O(\vec{\nabla}^2).$$

which leads to the familiar NN Schroedinger equation:

$$-\frac{\vec{\nabla}}{2\mu}\phi(\vec{r})+V_{NN}\,\phi(\vec{r})=E\phi(\vec{r})$$

- ✓ If we have the wave function ϕ (r), the potential may be **schematically** expressed as

$$V_{NN}(r)=E+\frac{1}{2\mu}\frac{\vec{\nabla}\phi(\vec{r})}{\phi(\vec{r})}$$

← only **schematical** sense !
∵ V_{NN} involves **derivative** and **matrix structure** !

$^1S_0$ channel (The schematical expression becomes mathematically sound in this channel.)

★ L=0, S=0 ⇒ **Only the central force survive !**

$$V_{NN} = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + O(p^2)$$
$$\cong V_C(r)$$

★ We are left with the conventional Schrodinger equation for NN

$$-\frac{\vec{\nabla}^2}{2\mu}\phi(\vec{r}) + V_C(r)\phi(\vec{r}) = E\,\phi(\vec{r})$$

★ $V_C(r)$ is an ordinary function.

⇒ **$V_C(r)$ can be expressed as**

$$V_C(r) = E + \frac{1}{2\mu}\frac{\vec{\nabla}^2\phi(\vec{r})}{\phi(\vec{r})}$$

Mathematically sound expression

We will use this to calculate NN potential.

The wave function

- In QCD, the non-rela. NN wave function is an approximate concept.
- The closest concept is provided by

the Bethe-Salpeter(BS) wave function.

$$\phi_{\alpha\beta}(\vec{x}-\vec{y}) \equiv \langle 0 | p_\alpha(\vec{x})\, n_\beta(\vec{y}) | pn \rangle$$

✓ $|pn\rangle$ The lowest energy state in the baryon # = 2 sector (i.e, the pn state)

Most naively, it corresponds to the following object:

$$|pn\rangle \cong \int d^3x\, d^3y\, |p(\vec{x})\, n(\vec{y})\rangle\, \phi(\vec{x}-\vec{y}) + \cdots$$

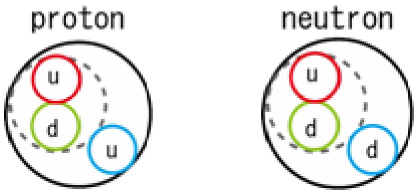
Degrees of freedom not attributed to NN

non-rela wave function

the state,
where proton is located at x
and neutron is located at y.

✓ standard proton and neutron operators

$$p_\alpha(x) \equiv \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_{c,\alpha} \qquad n_\alpha(y) \equiv \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_{c,\alpha}$$



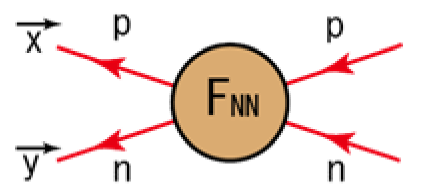
These operators are used to probe a proton at x and a neutron at y.

✓ **BS amplitude can be used to pick up the non-rela wave function φ(x,y) from |pn〉**

BS wave function from 4 point nucleon amplitude:

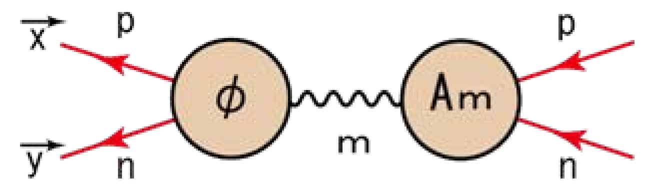
$$F_{NN}(\vec{x},\vec{y},t;\,t_0)$$

$$\equiv \left\langle 0 \middle| p(\vec{x},t)\,n(\vec{y},t)\;\;\overline{p}(\vec{0},t_0)\,\overline{n}(\vec{0},t_0) \middle| 0 \right\rangle$$



$$\mathbf{1} = \sum_m |m\rangle\langle m|$$

$$= \sum_{\vec{k}} A_{pn(k)}\; e^{-E_{pn}(\vec{k}^2)\,t}\;\; \phi(\vec{x}-\vec{y};\; pn(k)) + \cdots$$



energies of
the intermediate NN states

contributions from
non-NN intermediate states

★ At sufficiently **large t** (imaginary time),
contributions from all excited states are exponentially suppressed.
We can single out **the BS wave function of the lowest energy NN state**.

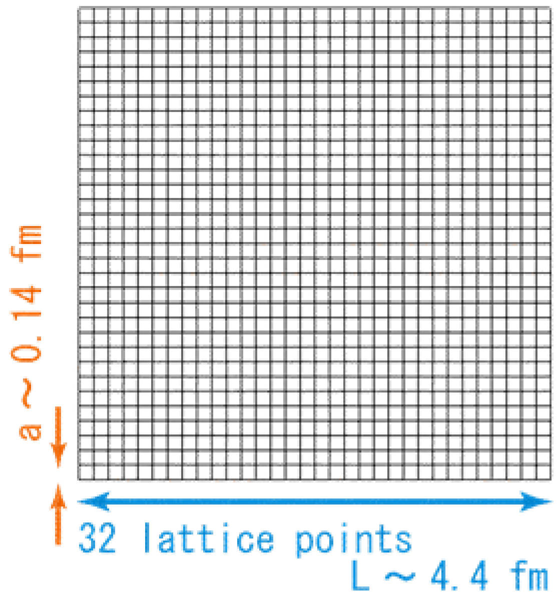
$$\phi(\vec{x}-\vec{y};\, pn(\text{ground})) \equiv \left\langle 0 \middle| p(\vec{x})\,n(\vec{y}) \middle| pn(\text{ground}) \right\rangle$$

Lattice QCD parameters

1. **Quenched QCD is used.**

2. **Standard Wilson gauge action.**

   - ✓ **$\beta = 2 N_c/g^2 = 5.7$ (⇔gauge coupling)**
   - ✓ the lattice spacing: **a ∼0.14 fm**
     (from ρ meson mass in the chiral limit)
   - ✓ the volume: 32^4 lattice (**L∼4.4 fm**)
   - ✓ 1900 gauge configs are used.
     - ✓ 3000 sweeps for thermalization.
       The gauge configs are separated by 200 sweeps.

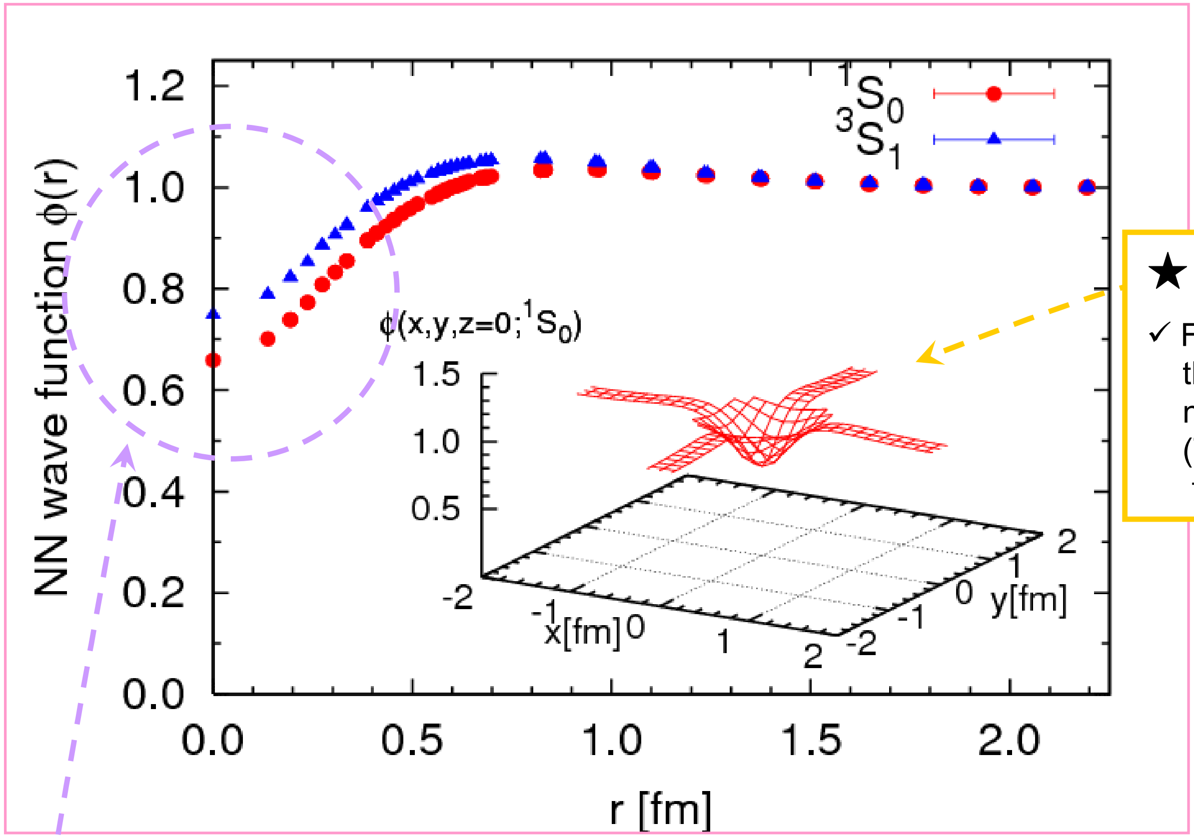


3. **Standard Wilson quark action**

   - ✓ **κ =0.1665 (⇔quark mass)**
   - ✓ **m_π ∼0.53 GeV**, m_ρ∼0.88 GeV, m_N∼1.34 GeV
     (Monte Carlo calculation becomes **hard** in **the light quark mass region**.)
   - ✓ Dirichlet BC along the temporal direction.
     The **wall source** on the time slice t_0=5.
     (The initial NN state is created on the time-slice t_0=5)
   - ✓ **NN wave function** is measured on the **time-slice t-t_0=6**.

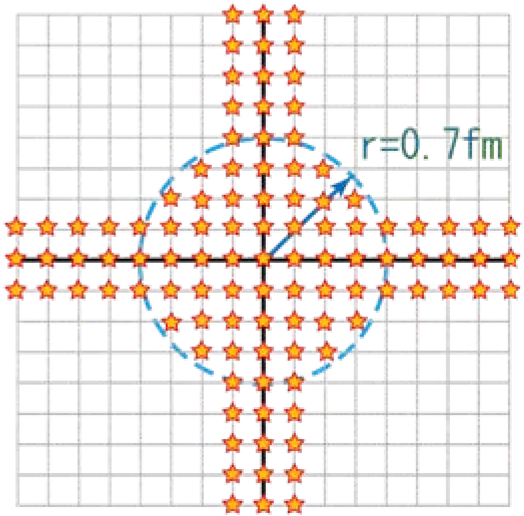
4. **Blue Gene/L at KEK** has been used for the Monte Carlo calculations.

The Lattice QCD result for NN wave function



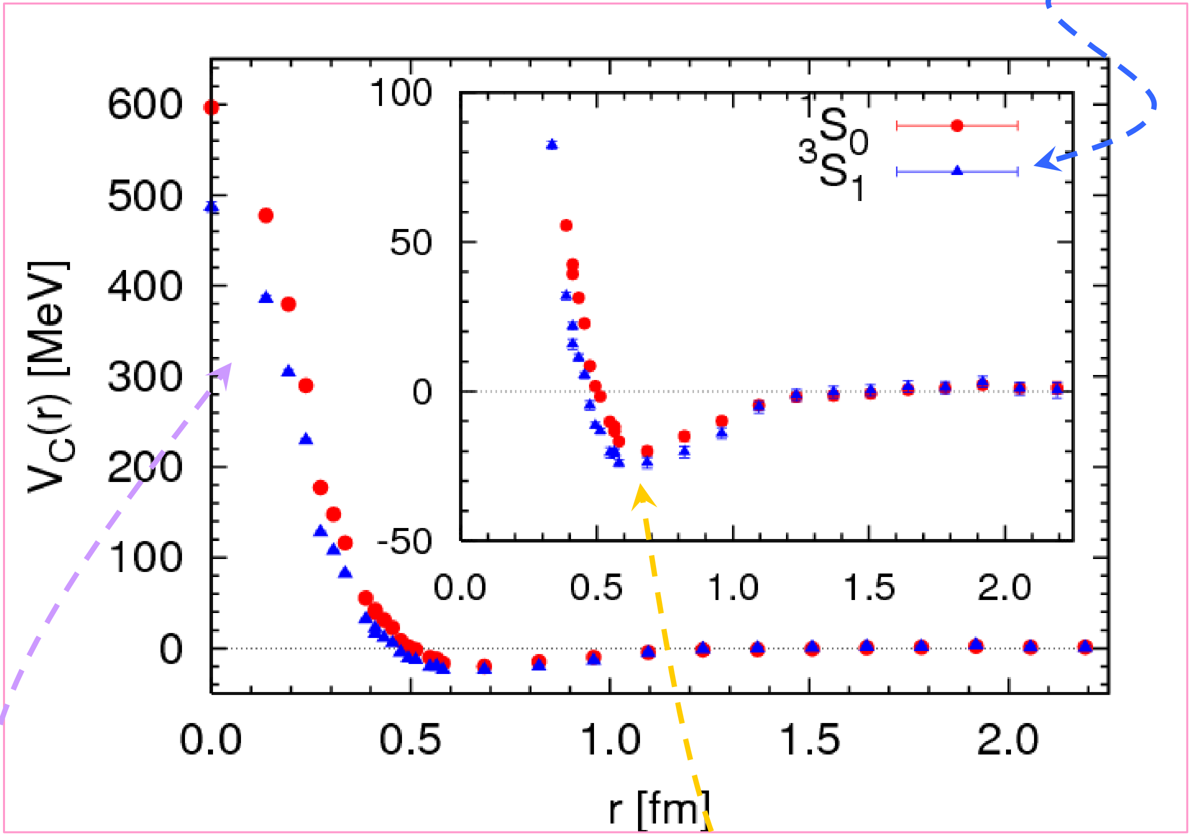
★ 3D plot of the wave function

✓ For r > 0.7 fm,
the calculation is performed only in the neighborhood of the coordinate axis.
(To obtain full wave function,
the calculation becomes ~60 times as tough !)



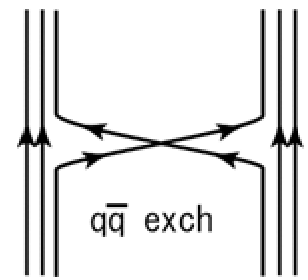
★ **The shrink at the short distance** suggests the existence of repulsion.

The Lattice QCD result of NN potential



The **effective central force** for $^3S_1$, which takes into account $^3D_1$ coupled via tensor force.

★ Our calculation includes the following diagram, which leads to the **OPEP** at long distance.



★ A quenched artifacts appears in the iso-scalar channel.

cf) S.R.Beane et al., PLB535,177(2002).

repulsive core ~ 500–600MeV

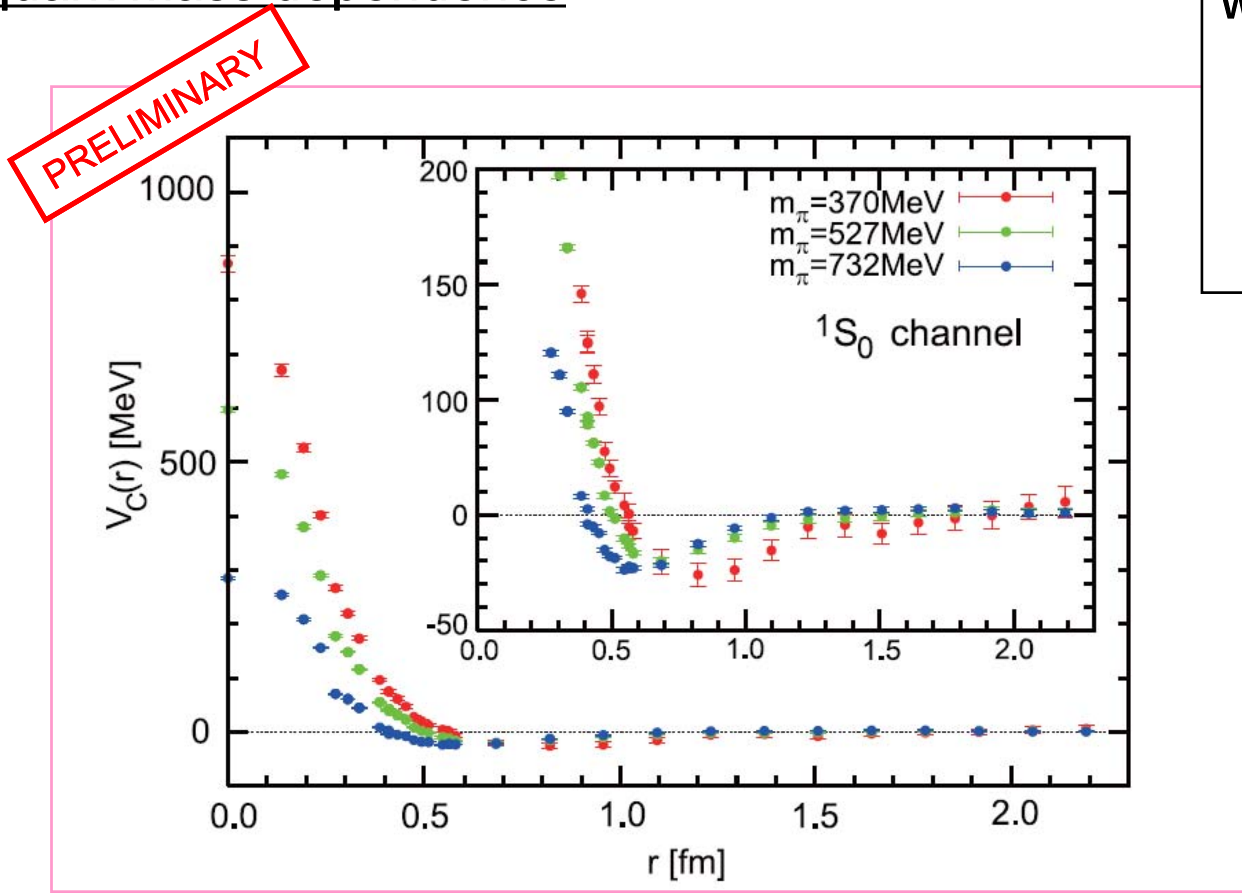
constituent quark model sugests
it is enhanced in light quark mass region.

medium range attraction ~ 30 MeV

✓ heavy m_π makes it weaker.
--- virtual pion cannot propagate long distance.
For lighter m_π, the attraction is enhanced.

✓ The effective central force in $^3S_1$ tends to be stronger than the central force in $^1S_0$.
(This tendency is good for deuteron)

quark mass dependence



We are accumulating statistics.

(1) m_π=370MeV: N_{conf}=1100

(2) m_π=527MeV: N_{conf}=1900

(3) m_π=732MeV: N_{conf}=1000

(4) m_π=990MeV: N_{conf}=1000

✓ The midium range attraction tends to be enhanced.

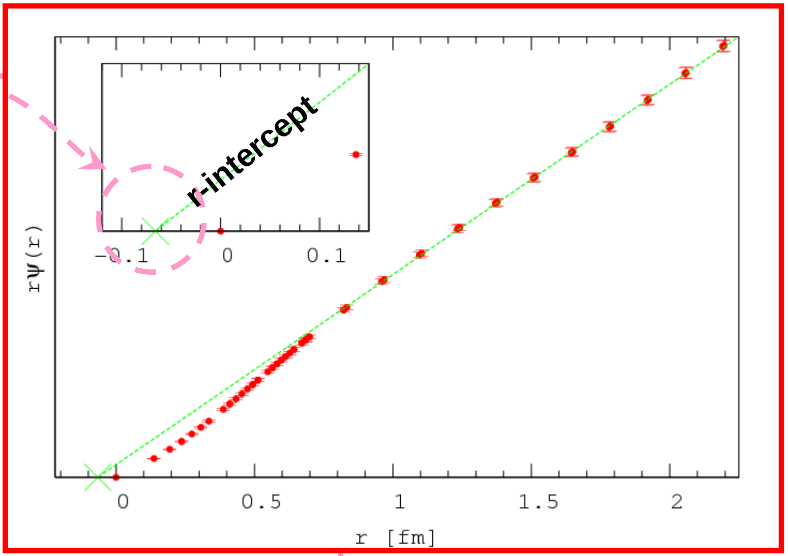
- less significant in maginitude
- range of the attraction tends to become wider.

Convergence is not good in this region. More statistics is needed.

✓ The repulsive core grows rapidly in the light quark mass region.

✓ It is necessary to perform a Monte Carlo calculation in the light quark mass region.
(This is left for the future)

This NN potential is attractive.



s-wave wave function outside the range of the potential is approximated in the low energy(E → 0) as

$$\phi_k^{asy}(\vec{r}) = \frac{\sin(kr + \delta_0(k))}{kr} \approx \frac{r + a_0}{r} \quad (k \to 0)$$

$$a_0 = \lim_{k\to 0} \delta_0(k)/k \quad \text{(scattering length)}$$

The scattering length a_0 can be easily obtained as the **r-intercept** of the **linear fit of r ϕ^{asy}(r).**

The **r-intercept** appears on the **negative side**:

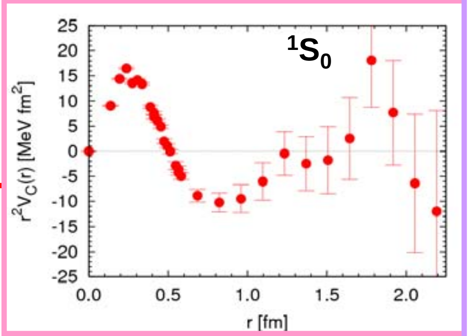
a_0 = 0.065(3) fm **(attractive)**

which indicates that

the net interaction is attraction.

★ This is understood by the formula of the **Born approx**:

$$a_0 \approx -m_N \int_0^\infty V_C(r)\, r^2 dr$$



This relation tells us

✓ The volume factor **r²dr** hides the repulsive core at short distance

✓ a_0 is a subtle quantity obtained as a result of a big cancellation between the repulston and the attraction.

※ comments:

✓ Our NN potential gives a **"correct" scattering length by construction.**
It is possible to prove **the equivalence**
(a) scattering length obtained by applying the standard scattering theory to our potential.
(b) scattering length obtained by applying Luescher's finite volume method
at L→∞limit.

✓ a_0～0.065 fm is significantly smaller than the empirical one a_0～20 fm.
This is due to the heavy pion mass m_π～530 MeV.

Uncertainties of our potential

★ Interpolating field dependence:

We employed the standard nucleon operators:

$$p_\alpha(x) \equiv \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right)u_{c,\alpha}, \quad n_\alpha(y) \equiv \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right)d_{c,\alpha}$$

These operators can couple with unwanted excited states of nucleons, generating the "**inelastic contribution**", which may spoil the relation to the non-rela wave function.

$$\langle 0|p_\alpha(x)n_\beta(y)|NN\rangle = \sum_m \langle 0|p_\alpha(x)|m\rangle\langle m|n_\beta(y)|NN\rangle$$
$$= \sum_{\vec{p}} \langle 0|p_\alpha(x)|N(\vec{p})\rangle\langle N(\vec{p})|n_\beta(y)|NN\rangle + I(x\text{-}y)$$

Dependence on choice of operators has to be checked in the future.

"inelastic" contribution

- ✓ it is exponentially suppressed at large distance
- ✓ Rough estimate suggests that it does not contribute at low energy

$$R \equiv \frac{\text{Inelastic}}{\text{elastic}} < \frac{\vec{p}^2/(2m)}{\delta m} \approx \frac{(2\text{MeV})^2/(2\times 940\text{MeV})}{140\text{MeV}}$$

However, its size has to be explicitly estimated in the near future.

★ Locality of our NN potential:

We have employed an Ansatz on the NN potential:

$$V_{NN} = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S} + O(\vec{\nabla}^2).$$

The locality of the NN potential is just an assumption,
This is an inevitable assumption so far, because we use a single wave function to construct the potential.

To avoid this, we have to use multiple wave functions associated with excited states.

Summary

1. We have presented our result on
   the lattice QCD calcuation of NN potential.

2. We have extended the method recently proposed by CP-PACS collaboration
   in the studies of $\pi\pi$ scattering phase shift.

3. Essential features of the nuclear force have been reproduced

   ✓ **a repulsive core of about 600 MeV** at short distance r < 0.5 fm.

   ✓ **weak attraction of about 30 MeV** in the medium distance 0.5 < r < 1.2 fm.
   (The attraction is weak due to the heavy pion ($m_\pi \sim$ 530 MeV).)

4. Preliminary results on the quark mass dependence have suggested:

   ✓ the repulsive core is enhanced in the light quark mass region.

   ✓ the attraction in the medium range tends to be enhanced in the light quark mass region.
   (more statistics is needed.)

5. Future plans:

   ✓ **the physical origin of the repulsive core**.
   (dependences on the quark mass, the flavor structure, ...)

   ✓ hyperon interactions (YN and YY) and meson-baryon system
   Dr.Nemura(the next speaker) will tell you about our preliminary attempt at this problem.

   ✓ LS-force, tensor force (and 3-body force)

   ✓ Non-locality, energy dependence of NN potential

   ✓ unquenched QCD, **physical quark mass**, large spatial volume,
   finer discretization, chiral quark actions, ...