

Strange sector of baryon-baryon interactions from lattice QCD

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Introduction:

- ⊗ Study of **hyperon-nucleon (YN)** and **hyperon-hyperon (YY)** interactions is one of the important subjects in the nuclear physics.
 - ⊗ Structure of the neutron-star core,
 - ⊗ Hyperon mixing, softning of EOS, inevitable strong repulsive force,
 - ⊗ H-dibaryon problem,
 - ⊗ To be, or not to be,
- ⊗ The project at J-PARC:
 - ⊗ Explore the multistrange world,
- ⊗ However, the phenomenological description of YN and YY interactions has **large uncertainties**, which is in sharp contrast to the nice description of phenomenological NN potential.

Extension from NN to YN and YY:

- ⊗ If we take only non-strange sector, there are only 2 representations for isospin space.

$$\begin{array}{ccccccc}
 2 & & 2 & & 3 & & 1 \\
 \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & \otimes & \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & = & \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \\
 I=\frac{1}{2} & & I=\frac{1}{2} & & I=1 & & I=0
 \end{array}$$

- ⊗ On the other hand, if we take account of strange degree of freedom, other representations should be included.

$$\begin{array}{cccccccccccc}
 8 & & 8 & & 27 & & 10^* & & 1 & & 8 & & 10 & & 8 \\
 \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & \otimes & \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & = & \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \square & & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \square & & \square \\ \hline \square & & \square \\ \hline \end{array} & \oplus & \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \square & & \square \\ \hline \square & & \square \\ \hline \end{array}
 \end{array}$$

- ⊗ This means that the YN and YY interactions cannot be determined from the precise NN experimental data even if we assume the flavor SU(3) symmetry.
- ⊗ **Lattice QCD** is desirable for the study of the YN and YY interaction, because this is *ab initio* numerical simulation.

Recent impressive works of lattice QCD:

⊗ S. Aoki, *et al.*, PRD71, 094504 (2005);

π - π scattering length from the wave function.

⊗ N. Ishii, *et al.*, nucl-th/0611096, PRL in press;

NN potential from the wave function.

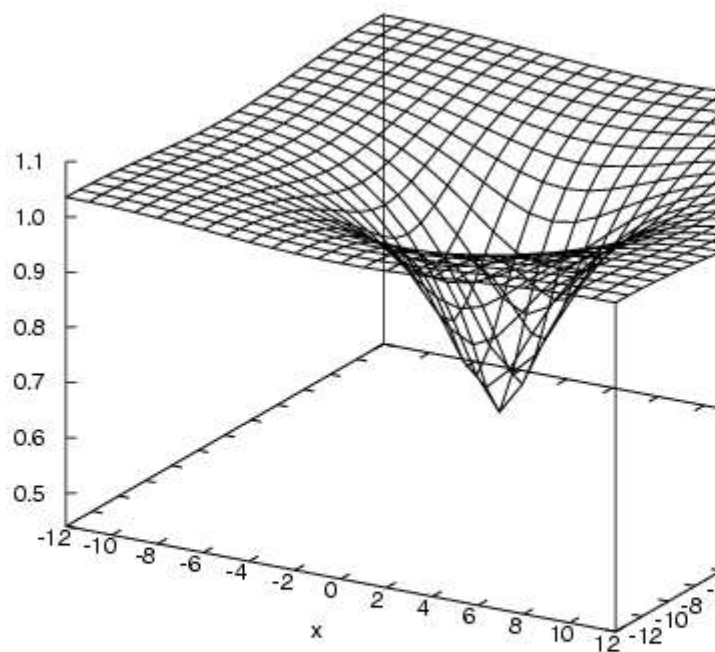
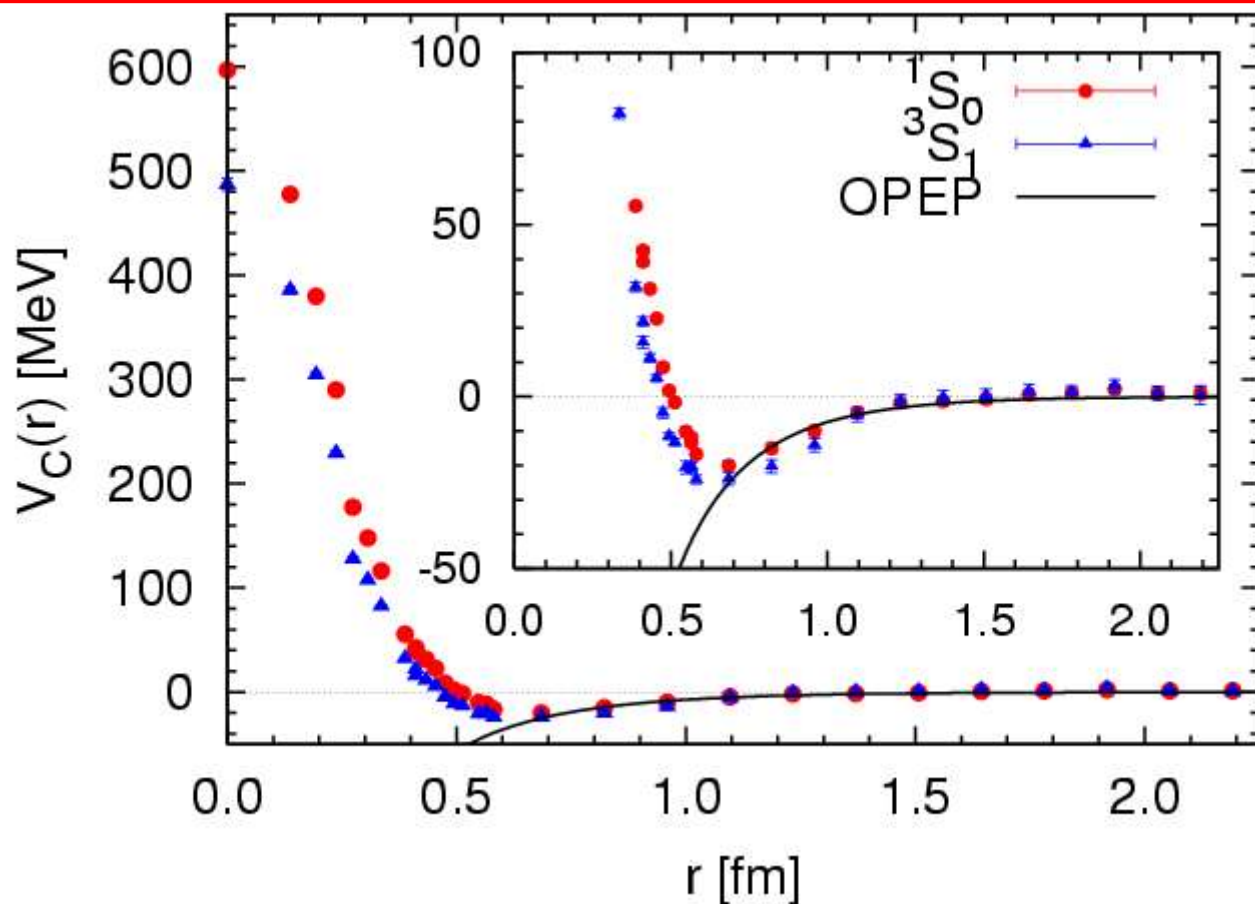


FIG. 1. Two-pion wave function $\phi(\vec{x}; k)$ on the $(t, z) = (52, 0)$ plane for $m_\pi^2 = 0.273 \text{ GeV}^2$. The vector is set at $\vec{x}_0 = (7, 5, 2)$ ($x_0 = |\vec{x}_0| = 8.832$).



⊗ This work;

YN and YY potentials by applying these techniques.

The purpose of this work

- ⊗ YN and YY potentials from lattice QCD
 - ⊗ $N\Lambda$, $N\Sigma$, $\Lambda\Lambda$, $N\Xi$, ...
- ⊗ $N\Xi$ potential as a first step
 - ⊗ Main target of the J-PARC DAY-1 experiment
 - ⊗ Few experimental information, so far
- ⊗ Focus on the $I=1$ channel, 1S_0 , 3S_1
 - ⊗ $I=1$; $N\Xi$ - $\Lambda\Sigma$ - $\Sigma\Sigma$: $N\Xi$ is the lowest state.
 - ⊗ $I=0$; $\Lambda\Lambda$ - $N\Xi$ - $\Sigma\Sigma$: $N\Xi$ is not the lowest state.
- ⊗ $I=0$ channel will be studied in the future.

A recipe for $N\Xi$ potential:

⊗ More accurate explanation, for NN , was given by Ishii-san.

⊗ Calculate the **4-point $N\Xi$ correlator** on the lattice,

$$\phi_{N\Xi}(x-y)e^{-E(t-t_0)} \simeq \langle p_\alpha(x,t) \Xi_\beta^0(y,t) \overline{\Xi_{\beta'}^0(0,t_0)} \overline{p_{\alpha'}(0,t_0)} \rangle$$

⊗ Which has the physical meanings of,

⊗ Create a $N\Xi$ state and making imaginary time evolution, in order to have the lowest state of the $N\Xi$ system.

⊗ Take the **amplitude $\phi(x-y)$** , which can be understood as a wave function of the non-relativistic quantum mechanics.

⊗ Obtain the **effective central potential** by assuming that the WF is a solution of **effective Schroedinger equation**.

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + V(r) \right) \phi(r) = E \phi(r)$$



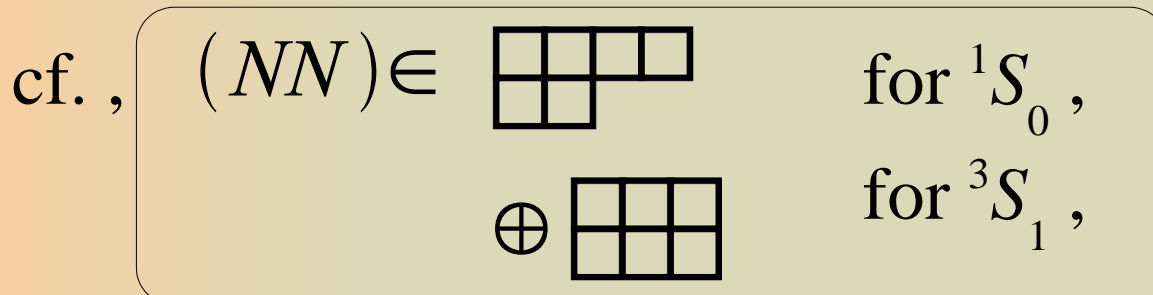
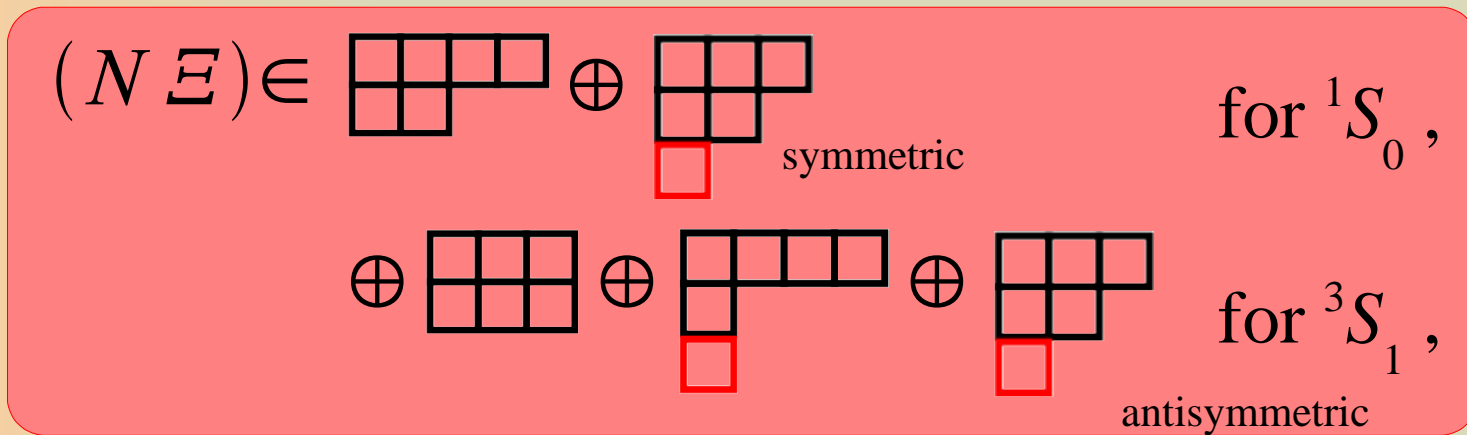
$$V(r) = E + \frac{\hbar^2}{2\mu} \frac{\nabla^2 \phi(r)}{\phi(r)}$$

My turn in this work:

- Calculate the **4-point $N\Xi$ correlator** on the lattice,

$$\phi_{N\Xi}(x-y)e^{-E(t-t_0)} \simeq \langle p_\alpha(x,t) \Xi_\beta^0(y,t) \overline{\Xi_{\beta'}^0(0,t_0)} \overline{p_{\alpha'}(0,t_0)} \rangle$$

- This gives the different pattern of the Wick contraction from the NN ,



- Calculate the **2-point correlators for N and Ξ ,**

$$\sum_y \langle \Xi_\beta^0(y,t) \overline{\Xi_{\beta'}^0(0,t_0)} \rangle$$

$$\sum_x \langle p_\alpha(x,t) \overline{p_{\alpha'}(0,t_0)} \rangle$$

We need the reduced mass to construct the potential.

$$V(r) = E + \frac{\hbar^2}{2\mu} \frac{\nabla^2 \phi(r)}{\phi(r)}$$


Interpolating fields and parameters:

- ⊗ Interpolating fields:

$$p_\alpha(x) = \varepsilon_{abc} (u_a(x) C \gamma_5 d_b(x)) u_{c\alpha}(x),$$

$$\Xi_\beta^0(y) = \varepsilon_{abc} (u_a(y) C \gamma_5 s_b(y)) s_{c\beta}(y),$$

- ⊗ The lattice calculations were performed by using **KEK Blue Gene/L** supercomputer.


- ⊗ The C++ code reached **1.3GFlops/processor**, which is almost a half of the peak value. 

1.3TFlops at
512node que

- ⊗ Volume: $32^3 \times 32$ lattice (**$L \sim 4.4$ fm**).

- ⊗ Lattice spacing: $a \sim 0.14$ fm.

- ⊗ Standard Wilson action:

- ⊗ $\kappa_{ud} = 0.1678$ for u and d quarks, and 

- ⊗ $\kappa_s = 0.1665$ for s quark.

- ⊗ For more details, see Ishii-san's talk.

Meson masses:

$$m_\pi \sim 0.377(3) \text{ GeV}$$

$$m_\rho \sim 0.844(6) \text{ GeV}$$

$$m_K \sim 0.463(1) \text{ GeV}$$

$$m_{K^*} \sim 0.868(3) \text{ GeV}$$

Results — hadron masses

⊗ Path integrals for the correlators are performed by using 491 gauge configurations, so far:

⊗ Calculated baryon masses (in units of GeV):

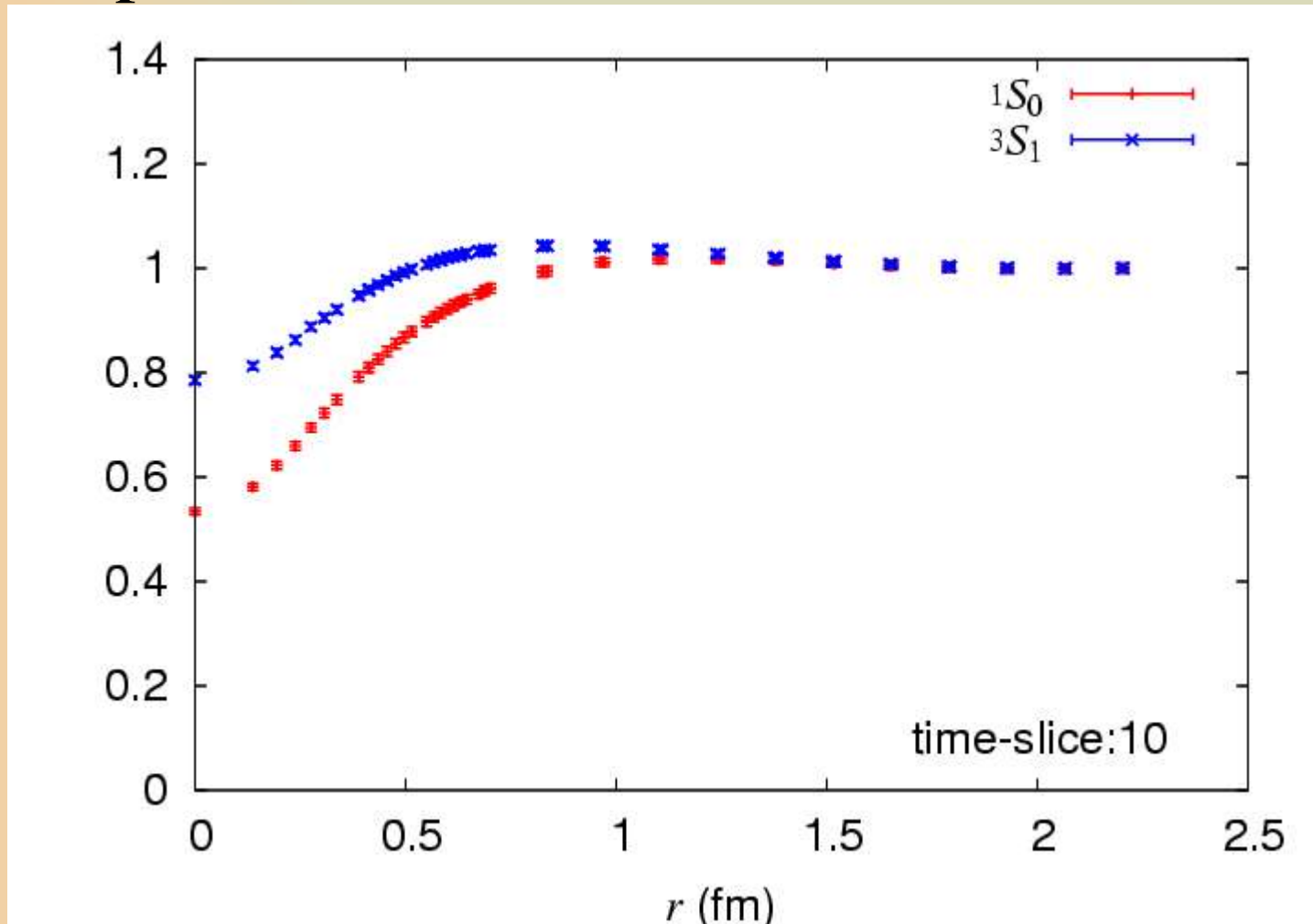
m_p	m_{Ξ}	m_{Λ}	m_{Σ}
1.210(11)	1.291(5)	1.244(8)	1.271(7)

⊗ Interpolating fields for Λ and Σ^+ :

$$\Lambda_{\alpha}(x) = \frac{1}{\sqrt{3}} \varepsilon_{abc} \left\{ (d_a C \gamma_5 s_b) u_{c\alpha} + (s_a C \gamma_5 u_b) d_{c\alpha} - 2(u_a C \gamma_5 d_b) s_{c\alpha} \right\}$$
$$\Sigma_{\beta}^{+}(y) = -\varepsilon_{abc} (u_a(y) C \gamma_5 s_b(y)) u_{c\beta}(y),$$

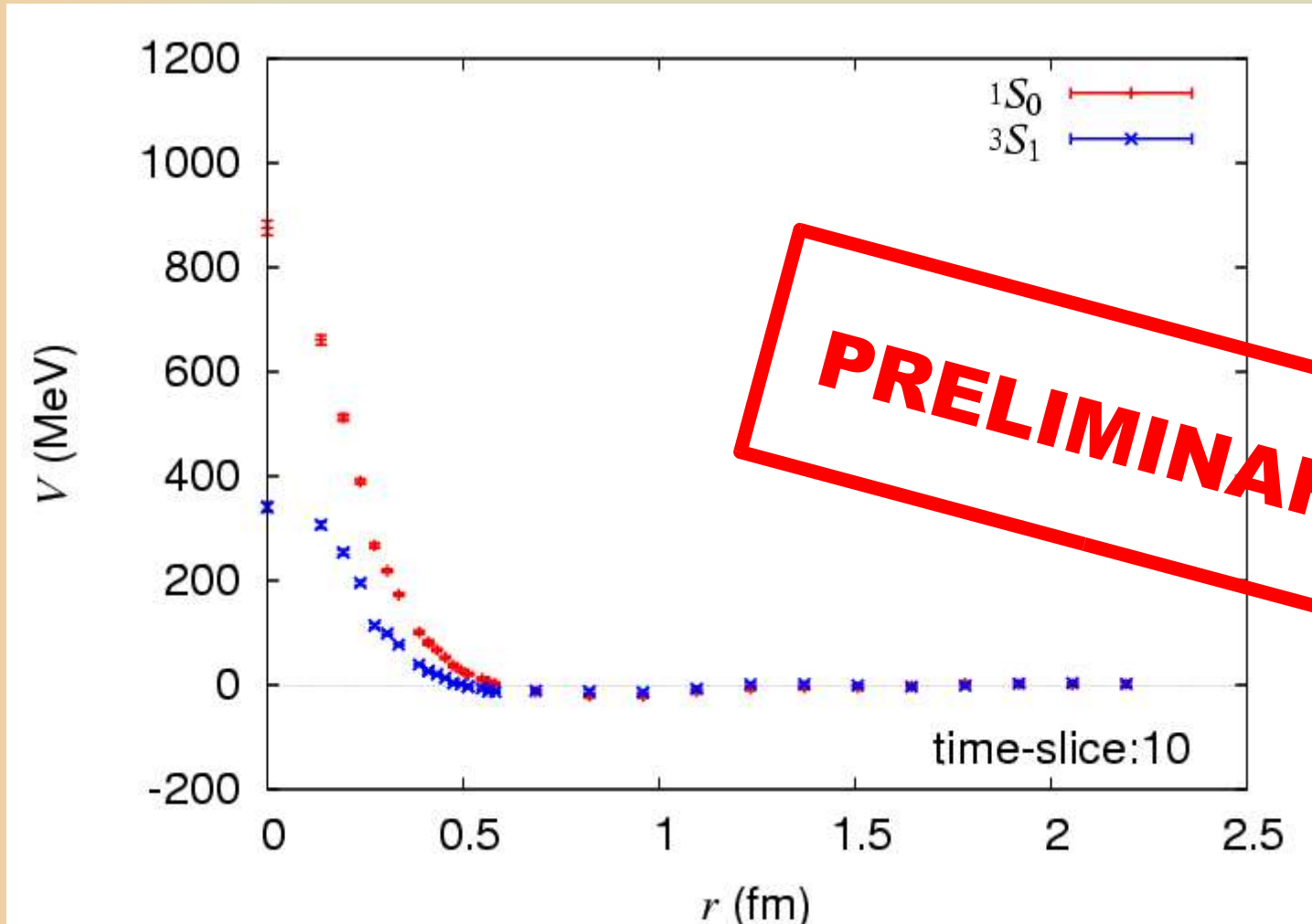
Results — wave function

- ⊗ Suggests the **repulsive core** in short range and **attractive force** in medium range ($0.5\text{fm} < r < 1\text{fm}$) for both spin $S=0$ and 1.



Results — potential

⊗ $N\bar{E}$ potential ($I=1$), from lattice QCD for the first time.

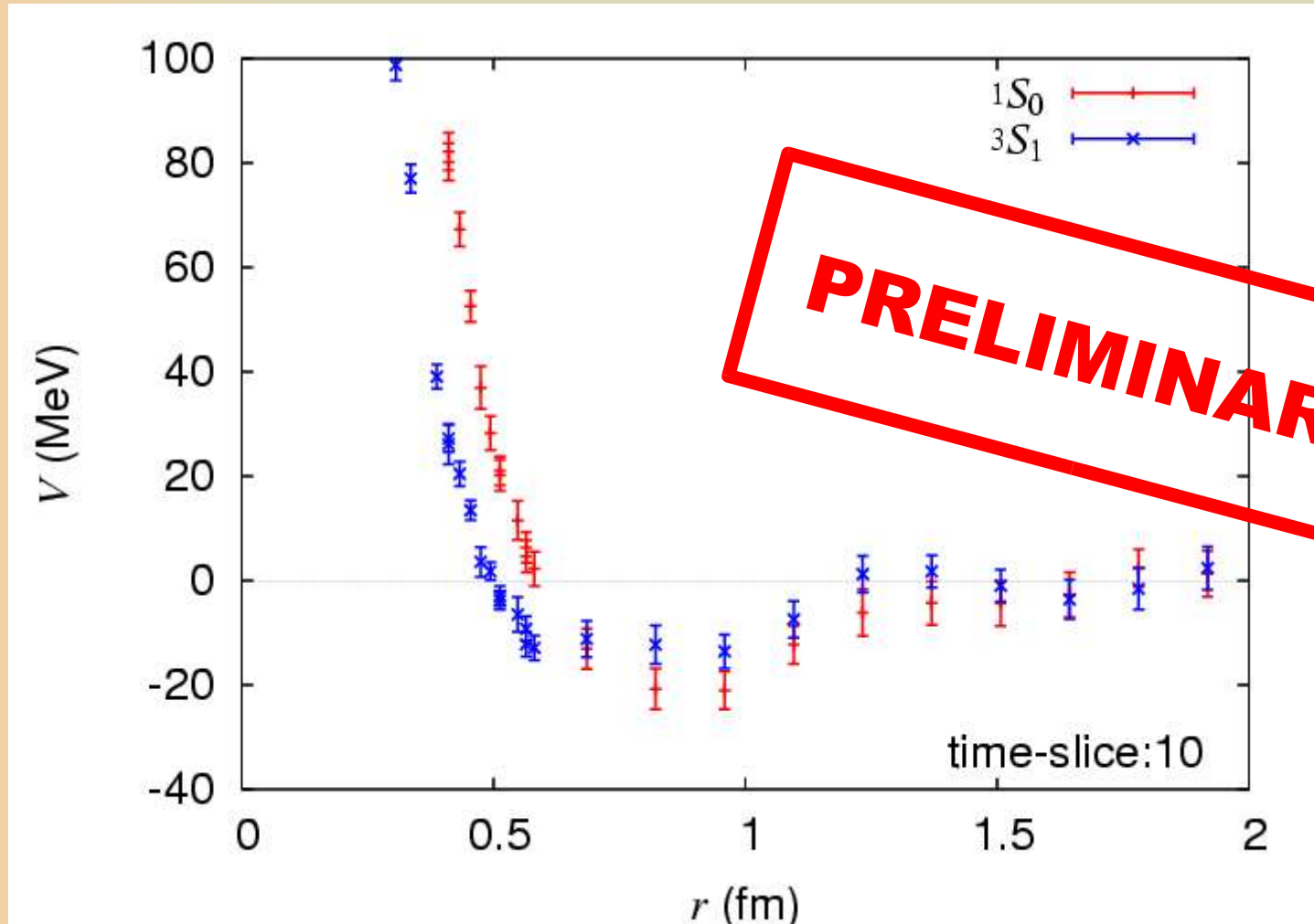


⊗ Strong repulsive core in spin $S=0$ channel.

⊗ Strong spin dependence.

Results — potential

⊗ $N\bar{E}$ potential ($I=1$), from lattice QCD for the first time.



⊗ **Attractive force** in medium and long-range region for both spin $S=0$ and 1.

Summary:

- ⊗ The first lattice QCD results for hyperon-nucleon potentials.
- ⊗ $N\Xi$ potential in isospin $I=1$ channel.
 - ⊗ Which will be studied by DAY-1 experiment at J-PARC.
- ⊗ Strong spin dependence:
 - ⊗ Strong repulsive core in spin $S=0$ channel and
 - ⊗ Relatively weak repulsive core in spin $S=1$ channel.
- ⊗ The present preliminary results suggest that $N\Xi$ potential ($I=1$) is **attractive** for both spin $S=0$ and 1 channels.
- ⊗ We will study further with
 - ⊗ **More statistics.**
 - ⊗ Physical quark mass for u , d and s quarks.
 - ⊗ Beyond the quenched approximation.
 - ⊗ **Other baryon-baryon pairs.**