QCD correction to the Drell-Yan process at J-PARC energy

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「J-PARCにおける高エネルギーハドロン物理」

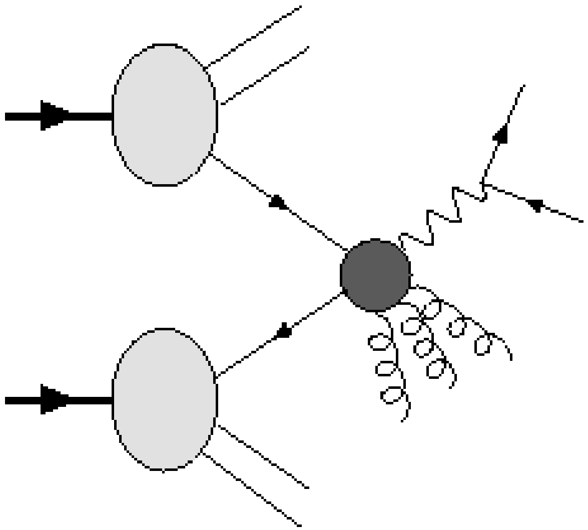
第1回 8月29日(水) @ KEK 4号館

- Introduction : Leading order, general structure
- Next-to-leading order : collinear singularity, factorization
- Threshold resummation :
- Summary

Introduction

Drell-Yan process : $N_1 N_2 \to \ell^- \ell^+ + X$

$$\sigma \sim \sum_{a,b} f_{a/N_1} \otimes f_{b/N_2} \otimes \hat{\sigma}_{ab}$$



$f_{a/N}$: parton distribution functions

$\hat{\sigma}_{ab}$: partonic cross section ← perturbatively calculable

J-PARC & GSI-FAIR experiment

Recently, new experiments in moderate energy have been proposed

Polarized Antiproton EXperiments

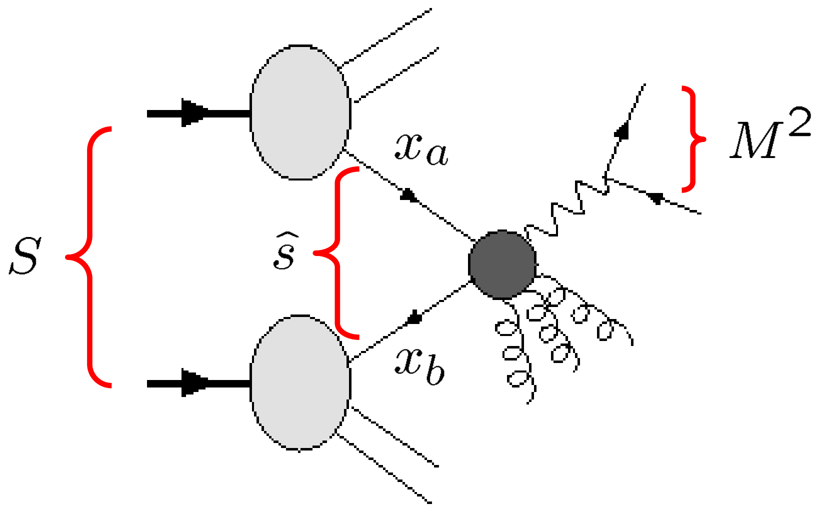
- J-PARC : $pp,\ \sqrt{S} = 10$ GeV
- GSI-FAIR (PAX) : $\bar{p}p,\ \sqrt{S} = 14.5$ GeV
- COMPASS : $\pi p,\ \sqrt{S} = 14$ GeV

- Large QCD correction
- PDFs at large-x
- Power correction,,,

Drell-Yan cross section formula

- Cross Section Formula :

$$\frac{d\sigma}{dM^2} = \sum_{a,b} \int dx_a\, q_a(x_a,\mu) \int dx_b\, q_b(x_b,\mu)\, \frac{d\hat{\sigma}_{ab}}{dM^2}(z, M, \alpha_s, M/\mu) + \mathcal{O}\left(\frac{\lambda}{M}\right)^p$$

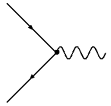


$$\tau = M^2/S$$

$$z = M^2/\hat{s}$$

$$\hat{s} = x_a x_b S$$

$$\tau \le z = \frac{\tau}{x_a x_b} \le 1$$

LO = parton model :  $$\frac{d\hat{\sigma}_{q\bar{q}}}{dM^2} = \frac{4\pi\alpha^2}{3N_c\hat{s}} e_q^2\,\delta(\hat{s}-M^2)$$

Hadronic CS :

$$\begin{aligned}\frac{d\sigma}{dM^2} &= \sum_q \int dx_1 f_q(x_1) \int dx_2 f_{\bar{q}}(x_2) \frac{d\hat{\sigma}_{q\bar{q}}}{dM^2} \\ &= \frac{4\pi\alpha^2}{3N_cSM^2} \sum_q e_q^2 \int dx_1 f_q(x_1) \int dx_2 f_{\bar{q}}(x_2) \delta(\tau - x_1x_2) \\ &= \frac{\sigma_0}{M^2}\,\mathcal{L}(\tau)\end{aligned}$$

$$\boxed{M^2 \frac{d\sigma}{dM^2} = \tau \frac{d\sigma}{d\tau} = \sigma_0\,\mathcal{L}(\tau)}$$

$\sigma_0 = \frac{4\pi\alpha^2}{3N_cS}$ $\mathcal{L}(\tau)$: parton luminosity

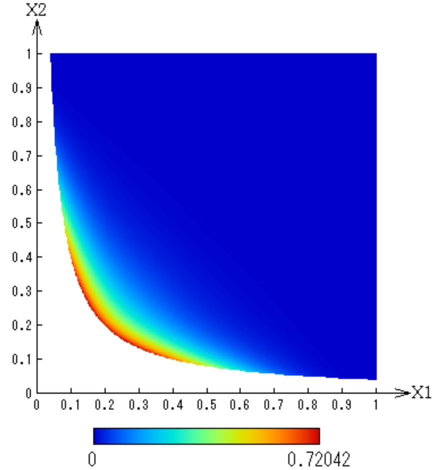
scaling law of the DY cross section

- $\sigma(\text{JPARC}) > \sigma(\text{RHIC})$?

τ	$\sqrt{S} = 10,$	200 GeV
0.01	$M = 1\text{GeV}$	20
0.1	3	60
0.3	5.5	110

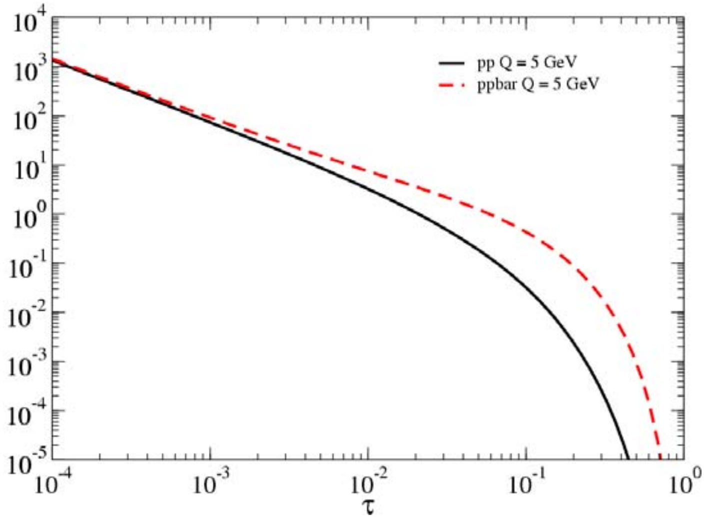
Parton Luminosity :

$$\mathcal{L}(\tau) = \sum_{q,\bar{q}} e_a^2 \int dx_1 f_a(x_1) \int dx_2 f_{\bar{a}}(x_2) \delta(\tau - x_1 x_2)$$
$$= \sum_{q,\bar{q}} e_a^2 \int \frac{dx_1}{x_1} f_a(x_1) \int dx_2 f_{\bar{a}}(\tau / x_1)$$



- $\mathcal{L}(\tau) \propto \tau^{-b} \quad (b > 1)$
- 実際には、scaleにも依存 (DGLAP発展)

$$\mathcal{L}(\tau) \to \mathcal{L}(\tau, \mu)$$

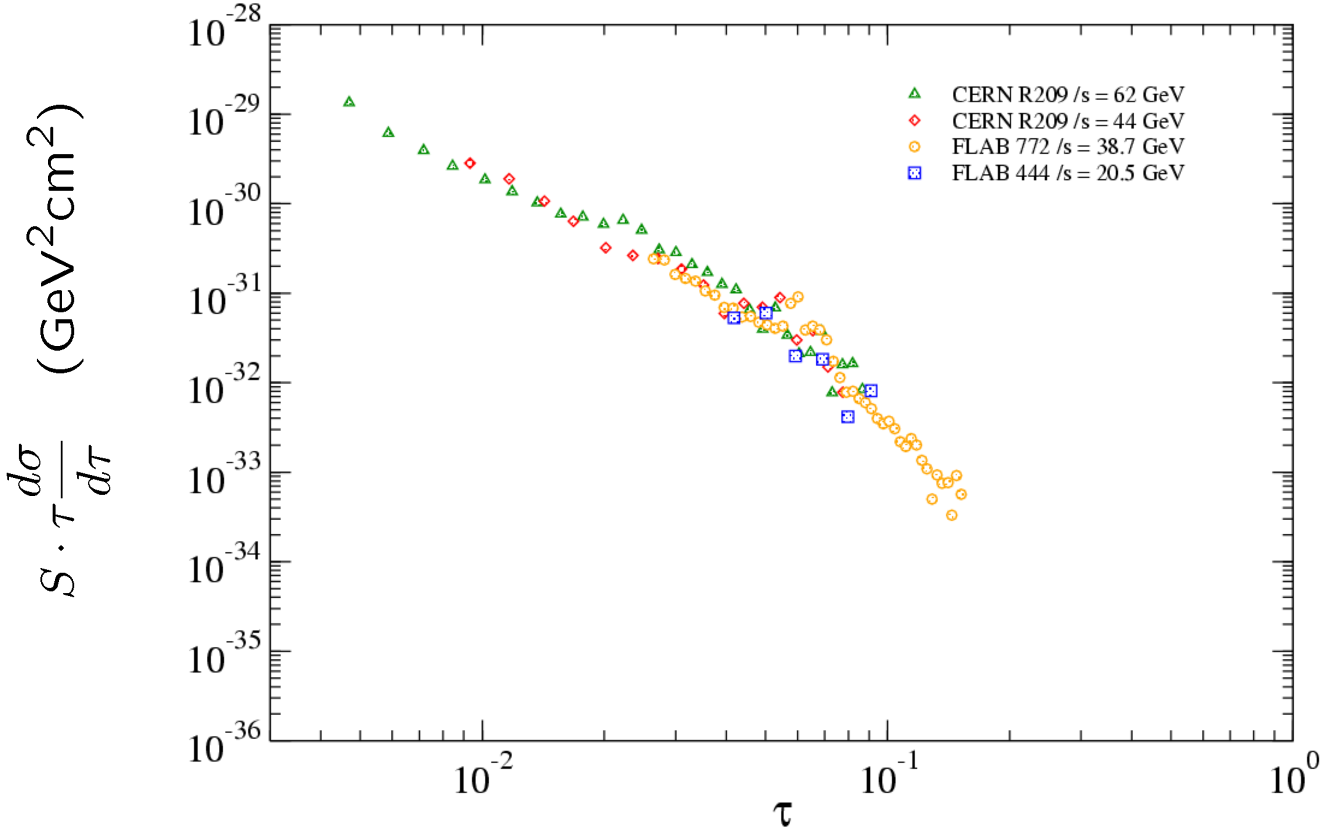


http://durpdg.dur.ac.uk/hepdata/

Proton-Proton collision data

W.J.Stirling and M.R.Whalley ('93)

$(\sqrt{S} = 20.5,\ 38.7,\ 44,\ 62\ \text{GeV})$



Fixed-Order Corrections

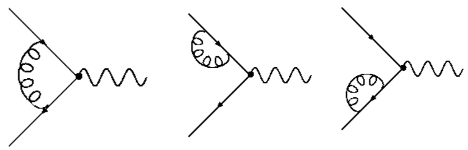
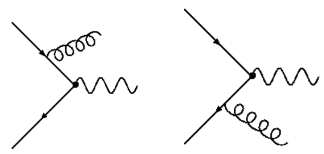
- QCD corrections

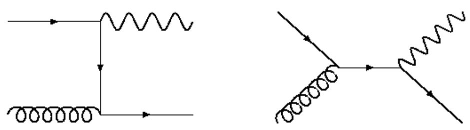
$$M^2 \frac{d\hat{\sigma}_{ab}}{dM^2} = \hat{\sigma}_0 K_{ab}(z, \alpha_s, r)$$
$$= \hat{\sigma}_0 \left[K_{ab}^{(0)}(z) + \frac{\alpha_s}{2\pi} K_{ab}^{(1)}(z, r) + \cdots \right]$$

$$\hat{\sigma}_0 = \frac{4\pi\alpha^2}{3N_c\hat{s}}$$

$$K_{q\bar{q}}^{(0)} = e_q^2 \delta(1-z)$$

NLO : $K_{q\bar{q}}^{(1)}(z, r)\,,\, K_{qg}^{(1)}(z, r)$

$q\bar{q} \to \gamma^*(g)$ virtual  real 

$qg \to \gamma^* q$ 

NLO correction

Nuclear Physics B157 (1979) 461–497

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

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The total cross section $d\sigma/dQ^2$ for the production of a muon pair of invariant mass Q^2 *via* the Drell-Yan mechanism and the Feynman x_F differential cross section $d^2\sigma/dQ^2dx_F$ are calculated in QCD retaining all terms up to order $\alpha_s(Q^2)$. The calculations are performed using dimensional regularisation of the intermediary infrared and collinear singularities, but we present our results in a form independent of such details. The corrections to both these cross sections coming from radiative corrections to the lowest-order $q\bar{q}$ annihilation diagram are found to be large at present values of Q^2 and S when the cross section is expressed in terms of parton densities derived from leptoproduction, for all Drell-Yan processes of practical interest. Numerical calculations are presented which show, for any reasonable parametrisation of the parton densities, that the neglect of higher-order terms in $\alpha_s(Q^2)$ is not justifiable. The quark-gluon diagrams on the other hand give small corrections in this order and are only important for PP scattering.

G.Altarelli,R.Ellis,G.Martinelli,
NP**B**157, 461 (1979)

pQCDの古典的論文

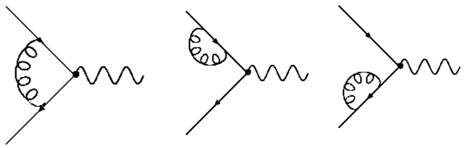
- Drell-YanのNLO Correction (次元正則化)
- (一つ計算間違いがある)
- Also in gluon-mass 正則化

  J.Kubar-Andre, F.Paige ('79)

  Harada,Kaneko,Sakai ('78)

$q\bar{q} \to \gamma^*(g)$

virtual

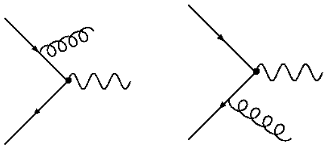


UV pole was renormalized

$$K^{(1),v}_{q\bar{q}}(z) = C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{2\pi^2}{3}\right] \delta(1-z)$$

IR double pole cancel between virtual and real corrections

real



$$K^{(1),r}_{q\bar{q}}(z) = C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[\frac{2}{\epsilon^2}\delta(1-z) - \frac{2}{\epsilon}\frac{1+z^2}{(1-z)_+} + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z\right]$$

- plus function : $\int_0^1 dx\, [f(x)]_+ \, g(x) \equiv \int_0^1 dx\, f(x)\, [g(x) - g(1)]$

$$K^{(1)}_{q\bar{q}}(z) = C_F \left[4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z + \left(\frac{2}{3}\pi^2 - 8\right)\delta(1-z)\right]$$

$$+ 2P^{(0)}_{qq}(z)\left\{-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi + \ln\left(\frac{M^2}{\mu^2}\right)\right\}$$

collinear singularity

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

Altarelli-Paris kernel

- Collinear singularity is factorized into the (renormalized) parton distribution functions

$$\sigma = q_0 \otimes q_0 \otimes \hat{\sigma}_0 = q \otimes q \otimes \hat{\sigma}$$

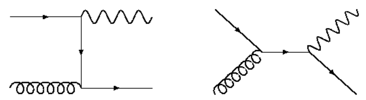
as for the DIS structure function, $F_2 = q_0 \otimes C_{2,0} = q \otimes C_2$

$$q_0(x) \to q_{\overline{\text{MS}}}(x,\mu^2) = q_0(x) \otimes \left[\delta(1-z) + \frac{\alpha_s}{2\pi}P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right)\right]$$

$\overline{\text{MS}}$ scheme

$$K^{(1),\overline{\text{MS}}}_{q\bar{q}}(z) = D_q(z) + 2P^{(0)}_{qq}(z)\ln\left(\frac{M^2}{\mu^2}\right)$$

- Also for qg subprocess,



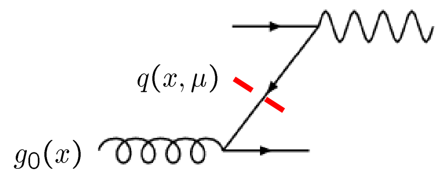
$$K_{qg}^{(1)}(z) = T_R\left[(z^2+(1-z)^2)\ln\left(\frac{(1-z)^2}{z}\right)-\frac{7}{2}z^2+3z+\frac{1}{2}\right]$$
$$+P_{qg}^{(0)}(z)\left\{-\frac{1}{\epsilon}+\gamma_E-\ln 4\pi+\ln\left(\frac{M^2}{\mu^2}\right)\right\}$$

$$P_{qg}(z) = T_R\left[z^2+(1-z)^2\right]$$

factorize collinear singularity! and push onto quark distribution!!

$$q_0(x)\to q_{\overline{\mathrm{MS}}}(x,\mu^2) = q_0(x)\otimes\left[\delta(1-z)+\frac{\alpha_s}{2\pi}\left\{P_{qq}(z)\left(-\frac{1}{\epsilon}+\gamma_E-\ln 4\pi\right)\right\}\right]$$
$$+g_0(x)\otimes\frac{\alpha_s}{2\pi}\left\{P_{qg}(z)\left(-\frac{1}{\epsilon}+\gamma_E-\ln 4\pi\right)\right\}$$

$$K_{qg}^{(1),\overline{\mathrm{MS}}}(z) = D_g(z) + P_{qg}^{(0)}(z)\ln\left(\frac{M^2}{\mu^2}\right)$$



- quark and gluon contributions mix by factorization (evolution).

- Factorization scheme :

we can subtract arbitrary function with the collinear singularity

$$q_{sch}(x,\mu^2) = q_0(x) \otimes \left[\delta(1-z) + \frac{\alpha_s}{2\pi}\left\{P_{qq}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \underline{C_q^{sch}(z)}\right\}\right]$$

$$+ g_0(x) \otimes \frac{\alpha_s}{2\pi}\left\{P_{qg}(z)\left(-\frac{1}{\epsilon} + \gamma_E - \ln 4\pi\right) + \underline{C_g^{sch}(z)}\right\}$$

$$K_{q\bar{q}}^{(1),sch}(z) = D_q(z) - 2\underline{C_q^{sch}(z)} + 2P_{qq}^{(0)}(z)\ln\left(\frac{M^2}{\mu^2}\right)$$

$$K_{qg}^{(1),sch}(z) = D_g(z) - \underline{C_g^{sch}(z)} + P_{qg}^{(0)}(z)\ln\left(\frac{M^2}{\mu^2}\right)$$

example : DIS scheme

$$C_q^{\mathrm{DIS}}(z) = C_{2,q}^{\overline{\mathrm{MS}}}(z) = C_F \left[(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \frac{1}{(1-z)_+} - \frac{1+z^2}{1-z} \ln z + 3 + 2z - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(1-z) \right]$$

$$C_g^{\mathrm{DIS}}(z) = C_{2,g}^{\overline{\mathrm{MS}}}(z) = T_R \left[((1-z)^2 + z^2) \ln \left(\frac{1^z}{z} \right) - 8z^2 + 8z - 1 \right]$$

so that no NLO term remains in F_2 : $F_2(x, Q^2) = \sum_q e_q^2 f_q(x, Q^2)$

(absorb all DIS $O(a_s)$ correction into PDFs)

$$K_{q\bar{q}}^{\mathrm{DIS}}(z) = C_F \left[2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ + \frac{3}{(1-z)_+} - 6 - 4z + \delta(1-z) \left(1 + \frac{4\pi^2}{3} \right) \right]$$

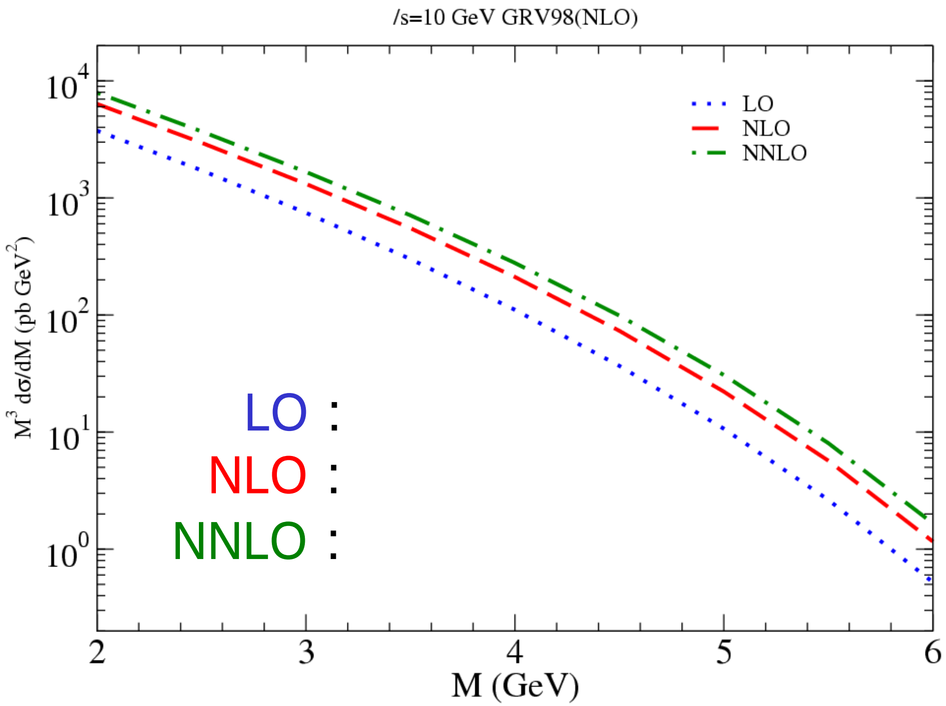
$$K_{qg}^{\mathrm{DIS}}(z) = T_R \left[(z^2 + (1-z)^2) \ln(1-z) + \frac{3}{2} - 5z + \frac{9}{2} z^2 \right]$$

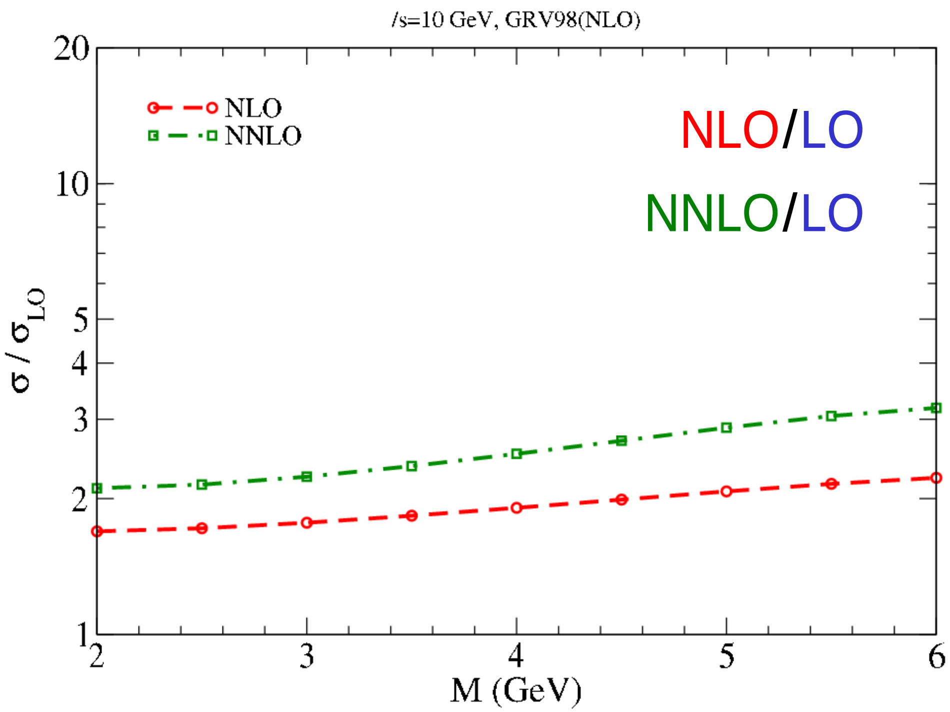
NNLO calculation has been already done

$K_{q\bar{q}}^{(2)}(z,r), K_{qg}^{(2)}(z,r), K_{gg}^{(2)}(z,r), K_{qq}^{(2)}(z,r)$

Hamberg,van Neerven,Matsuura('91,'02); Harlander,Kilgore('02)

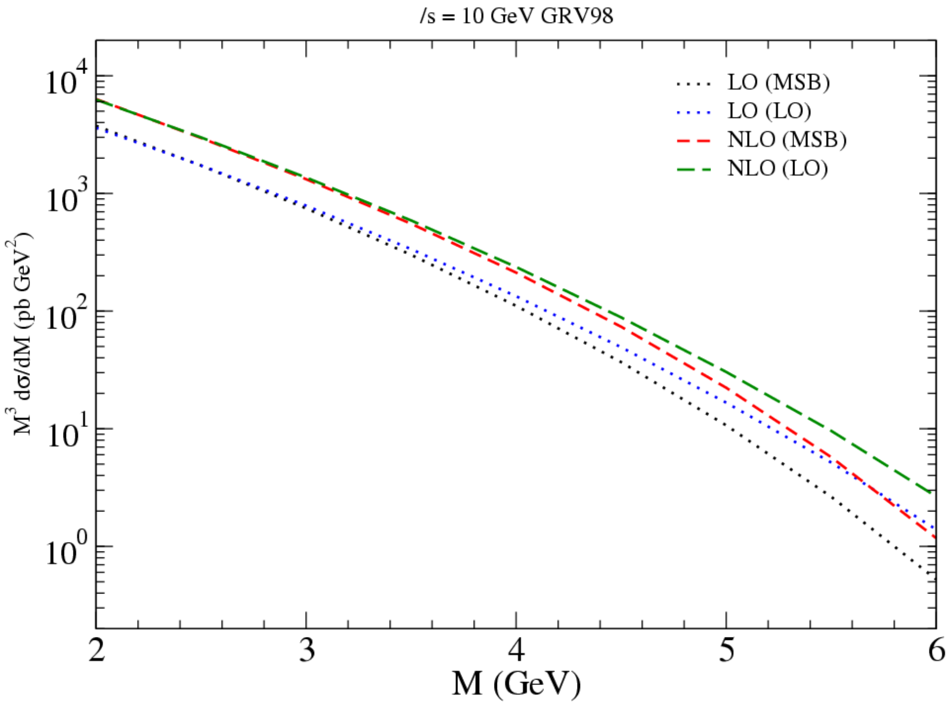
J-PARC pp $\sqrt{S} = 10\text{GeV}$



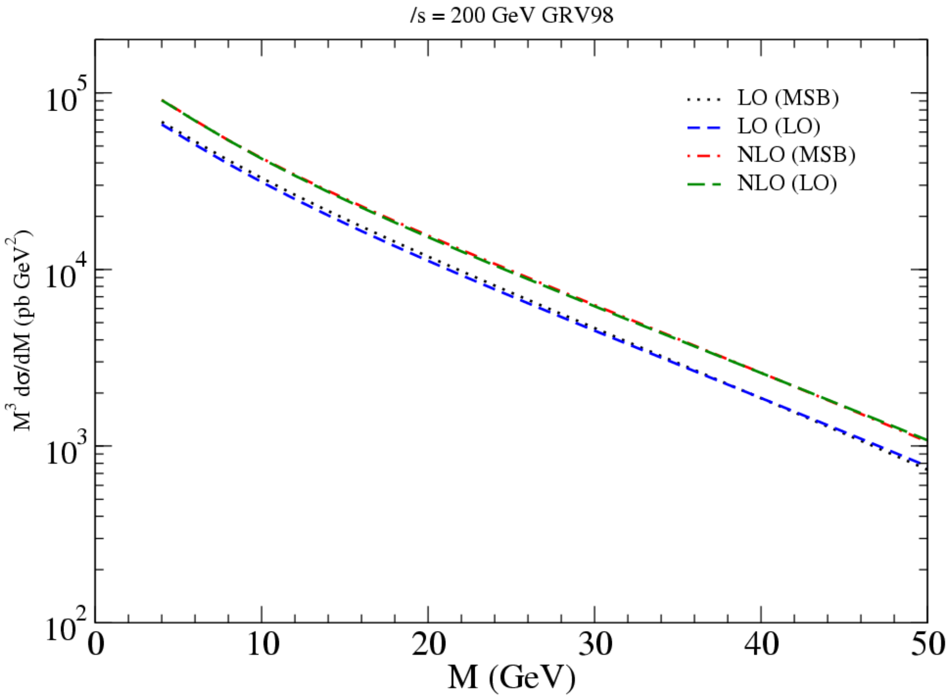


factorization scheme dependence : $\overline{\text{MS}}$ scheme vs. DIS scheme @ NLO

J-PARC



RHIC $\sqrt{S} = 200$ GeV



- Note, qg subprocess contribution is negative in $\overline{\text{MS}}$ scheme and also in DIS scheme

  due to the subtraction of the collinear singularity

RHIC $\sqrt{s} = 200$ GeV, $M = 40$ GeV

$\overline{\text{MS}}$ scheme :
$$M^3 \frac{d\sigma}{dM}\Big|_{\text{NLO}} = \underset{\mathcal{O}(\alpha_s^0)}{1870} \underset{\mathcal{O}(\alpha_s)q\bar{q}}{+900} \underset{\mathcal{O}(\alpha_s)qg}{-170} \ (\text{GeV}^2 \text{ pb})$$

DIS scheme :
$$M^3 \frac{d\sigma}{dM}\Big|_{\text{NLO}} = \underset{\mathcal{O}(\alpha_s^0)}{1870} \underset{\mathcal{O}(\alpha_s)q\bar{q}}{+800} \underset{\mathcal{O}(\alpha_s)qg}{-70} \ (\text{GeV}^2 \text{ pb})$$

Mellin moment : $f_N = \int_0^1 dx x^{N-1} f(x)$ $\sigma(\tau) = \int_\tau^1 dx_1 f(x_1) \int_{\tau/x_1}^1 dx_2 f(x_2) \hat{\sigma}(\tau/x_1 x_2)$

$$\to \sigma_N = f_{N+1} \cdot f_{N+1} \cdot \hat{\sigma}_N$$

$$K_{q\bar{q},N}^{(1),\overline{\mathrm{MS}}} = C_F \left[4S_1^2 - \frac{4}{N(N+1)} S_1 + \frac{2(2N^2+2N^2+1)}{N^2(N+1)^2} + \left(\frac{4\pi^2}{3} - 8 \right) \right] + 4\gamma_{qq}(N) \ln r$$

$$K_{qg,N}^{(1),\overline{\mathrm{MS}}} = T_R \left[-\frac{2(N^2+N+2)}{N(N+1)(N+2)} S_1 + \frac{N^4+11N^3+22N^2+14N+4}{N^2(N+1)^2(N+2)^2} \right] + 2\gamma_{qg}(N) \ln r$$

- large-N limit : $S_1 \to \psi(N+1) + \gamma_E \sim \ln N + \gamma_E$

$$K_{q\bar{q},N}^{(1),\overline{\mathrm{MS}}} \sim C_F \left[4 \ln \bar{N}^2 + \frac{4\pi^2}{3} - 8 \right] + 4\gamma_{qq}(N) \ln r$$

$$\left(K_{q\bar{q},N}^{(1),\mathrm{DIS}} \sim C_F \left[2 \ln \bar{N}^2 - 3 \ln \bar{N} + \frac{4\pi^2}{3} + 1 \right] + 4\gamma_{qq}(N) \ln r \right)$$

Harmonic Sum :

$$S_a(N) = \sum_{k=1}^{N} \frac{1}{k^a},$$
$$S_{ab}(N) = \sum_{k=1}^{N} \frac{1}{k^a} \sum_{j=1}^{k} \frac{1}{j^b}$$
$$S_{aa}(N) = \frac{1}{2} \left(S_a^2 + S_{2a} \right)$$

J.Blumlein,, J.Vermaseren,,

Threshold Resummation :

Large corrections come from the partonic threshold region (z~1)

✓ real emission suppressed by the phase space restriction

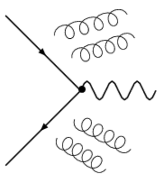
✓ imbalance between real and virtual gluon corrections

threshold logs : $\frac{\alpha_s}{\pi}\left(\frac{\ln(1-z)}{1-z}\right)_+ \Rightarrow \left(\frac{\alpha_s}{\pi}\right)^k\left(\frac{\ln^{2k-1}(1-z)}{1-z}\right)_+$

$z = \frac{M^2}{\hat{s}} \sim 1$ → only soft gluon can be emitted

→ soft gluon (eikonal) approximation

to treat these logs up to all orders



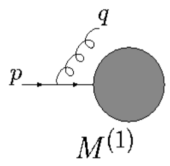
= threshold resummation Sterman('87);Catani,Trentadue('89)

Keypoint : ① factorization of soft-gluon radiation amplitude

② factorization of phase-space in moment-space

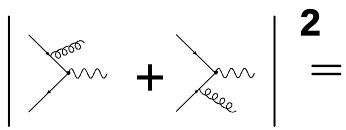
(Multiple) soft-gluon radiation amplitude :

- eikonal approximation :



$$
\begin{aligned}
&= \cdots \frac{(\not{p}-\not{q})}{(p-q)^2}\left(g_s T^a \gamma^\mu\right) u(p)\epsilon^*_\mu(q) \\
&\approx \cdots g_s T^a \frac{\not{p}}{-2p\cdot q}\gamma^\mu u(p)\epsilon^*_\mu(q) \\
&= M^{(0)}\left(-g_s T^a \frac{p\cdot\epsilon^*}{p\cdot q}\right)
\end{aligned}
$$

- One-gluon radiation case :



$$
\begin{aligned}
&= \left|M^{(1)}\right|^2 \\
&\approx \left|M^{(0)}\right|^2 \left(-g_s^2 C_F\right) \left|\frac{p_1\cdot\epsilon^*}{p_1\cdot q} - \frac{p_2\cdot\epsilon^*}{p_2\cdot q}\right|^2 \\
&= \left|M^{(0)}\right|^2 \left(g_s^2 C_F\right) \frac{2p_1\cdot p_2}{p_1\cdot q p_2\cdot q} \equiv \left|M^{(0)}\right|^2 J_1(q)
\end{aligned}
$$

Multiple gluon radiation phase-space :

$$d\Phi(\gamma^*, g^n) = (2\pi)^4\delta^4(P - q - \sum q_i)\frac{d^3\mathbf{q}}{(2\pi)^3 2q^0}\prod_{k=1}^{n}\frac{d^3\mathbf{q}_k}{(2\pi)^3 2q_k^0}$$

$$= \frac{2\pi}{\hat{s}}\delta\left(1 - z - \sum\frac{q_k^0}{E}\right)\prod_{k=1}^{n}\frac{d^3\mathbf{q}_k}{(2\pi)^3 2q_k^0}$$

energy-conservation delta-function

$$\delta\left(1 - z - \sum\frac{q_k^0}{E}\right) \longrightarrow \int dz z^{N-1}\delta\left(1 - z - \sum\frac{q_k^0}{E}\right) = \left(1 - \sum\frac{q_k^0}{E}\right)^{N-1} \approx \prod_{k=1}^{n}\left(1 - \frac{q_k^0}{E}\right)^{N-1}$$

therefore, phase-space factorize in the moment-space in soft-gluon limit

$$d\Phi(\gamma^*, g^n) \longrightarrow \prod_{k=1}^{n}\left(1 - \frac{q_k^0}{E}\right)^{N-1}\frac{d^3\mathbf{q}_k}{(2\pi)^3 2q_k^0}$$

Multi-gluon radiation cross section (moment-space) :

$$\sigma_N = \sum_{n=0}^{\infty} \sigma_N^{(n)} = \frac{1}{2s} \sum_{n=0}^{\infty} \frac{1}{n!} \left| M^{(n)} \right|^2 d\Phi_n$$

$$\approx \sigma_0 \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{k=1}^{n} \frac{d^3\mathbf{q}_k}{(2\pi)^3 2q_k^0} \left(1 - \frac{q_k^0}{E}\right)^{N-1} J_1(q_k)$$

$$= \sigma_0 \exp\left[\int \frac{d^3\mathbf{q}_k}{(2\pi)^3 2q_k^0} \left(1 - \frac{q_k^0}{E}\right)^{N-1} J_1(q_k) \right]$$

exponentiation of one-gluon radiation factor !

$$= \sigma_0 \exp\left[2 \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \int_0^{M^2(1-z)^2} \frac{dk_T^2}{k_T^2} \left(\frac{\alpha_s}{\pi} C_F \right) \right]$$

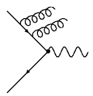
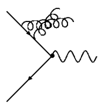
virtual correction

collinear singularity

cut at μ_f^2 : factorization scale

- $\alpha_s \to \alpha_s(k_T^2)$: rescaling takes in the higher order logs

- gluon correlation effect :

 and  have different color charge, but still exponentiate with modified color charge

Gatheral('83),Frenkel,Talyor('84)

as a result, perturbative series in the exponent

$$\frac{\alpha_s}{\pi}C_F \;\to\; A_q(\alpha_s) = \frac{\alpha_s}{\pi}A_q^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_q^{(2)} + \cdots$$

$$A_q^{(1)} = C_F$$

$$A_q^{(2)} = \frac{C_F}{2}\left(C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n_f\right)$$

Kodaira,Trentadue('82)

Threshold resummation

Sterman('87);Catani,Trentadue('89)

- General Formula : Sudakov Exponent

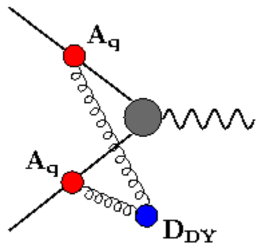
$$K_{q\bar{q}}^{\text{res}}(N,\alpha_s,r) = C_{DY}(\alpha_s,r)\cdot\exp\left[\int_0^1 dz\frac{z^{N-1}-1}{1-z}\left(2\int_{\mu_f^2}^{M^2(1-z)^2}\frac{dk_T^2}{k_T^2}A_q(\alpha_s(k_T^2)) + D_{DY}\left(\alpha_s(M^2(1-z)^2)\right)\right)\right]$$

$A_q(\alpha_s)$: soft collinear gluon

$C_{DY}(\alpha_s,r)$: coefficient function

$D_{DY}(\alpha_s)$: large-angle soft gluon

μ_f : factorization scale



- Integral contains the Landau pole singularity through the scale of strong coupling

but, logarithms arise only from $z \leq 1 - \frac{e^{-\gamma_E}}{N}$

approximation : $z^{N-1}-1 \sim -\Theta(1-z-1/\bar{N})$ $\qquad \bar{N} = Ne^{\gamma_E}$

$$K_{q\bar{q}}^{\text{res}} = C_{DY} \exp\left[2\int_{M^2/\bar{N}^2}^{M^2} \frac{dk_T^2}{k_T^2} \ln\left(\frac{\bar{N}k_T}{M}\right) A_q(\alpha_s(k_T^2)) - 2\int_{\mu_f^2}^{M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T^2)) \ln \bar{N}\right]$$

perform integral with keeping $\lambda = b_0 \alpha_s \ln \bar{N}$ finite

$$= C_{DY} \exp\left[\frac{1}{\alpha_s} h_q^{(1)}(\lambda) + h_q^{(2)}(\lambda, r) + \alpha_s h_q^{(3)}(\lambda, r) + \cdots\right]$$

LL $\alpha_s^k L^{2k}$

NLL $\alpha_s^k L^{2k-1}$

NNLL

$$h_q^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$h_q^{(2)}\left(\lambda, \frac{M^2}{\mu^2}\right) = \frac{A_q^{(1)} b_1}{2\pi b_0^3}\left[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\right] + \left(\frac{A_q^{(1)}}{2\pi b_0}\ln\frac{M^2}{\mu_r^2} - \frac{A_q^{(2)}}{2\pi^2 b_0^2}\right)\left[2\lambda + \ln(1-2\lambda)\right] - \frac{A_q^{(1)}}{\pi b_0}\lambda \ln\frac{M^2}{\mu_f^2}$$

still singularity at $\lambda = \frac{1}{2} \rightarrow \bar{N} = e^{1/2\alpha_s b_0} = M/\Lambda_{QCD}$

- NNLL accuracy : 3-loop split. func. gives $A_q^{(3)}$ Moch,Vermaseren,Vogt('04)

$D_{\rm DY}^{(2)}$, $C_{\rm DY}^{(2)}(r)$ given by NNLO

	LL	NLL	NNLL	$\cdots$
LO	1			
NLO	$\alpha_s L^2$	$\alpha_s L$		
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^{\{3,2\}}$	$\alpha_s^2 L$	
$\cdots$	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1,2}$	$\alpha_s^k L^{2k-3,4}$	

Kramer,Laenen,Spira('98);
Catani,de Florian,Grazzini('02);
Kulesza,Sterman,Vogelsang('02,'04)

- Collinear improvement :

Taking into account the universal collinear (non-soft) gluon radiation

- re-arrange the exponent

$$\ln\left(K^{\mathrm{res}}\right) \sim -\int_{M^2/N^2}^{M^2} \frac{dk_T^2}{k_T^2} A_q(\alpha_s(k_T^2)) \ln\left(\frac{M^2}{k_T^2}\right) + \int_{\mu_F^2}^{M^2/N^2} \frac{dk_T^2}{k_T^2} \left[-2A_q(\alpha_s(k_T^2)) \ln N\right]$$

- re-cover the full evolution kernel by : $-2A_q^{(1)}(\alpha_s) \ln N \to \gamma_{qq}^{(1)}(N, \alpha_s)$
- correctly re-produce the $\alpha_s^k L^{2k-1}/N$ terms to all orders
- numerically sizable effects

Catani,Mangano,Nason,Trentadue('96)

- Minimal prescription :

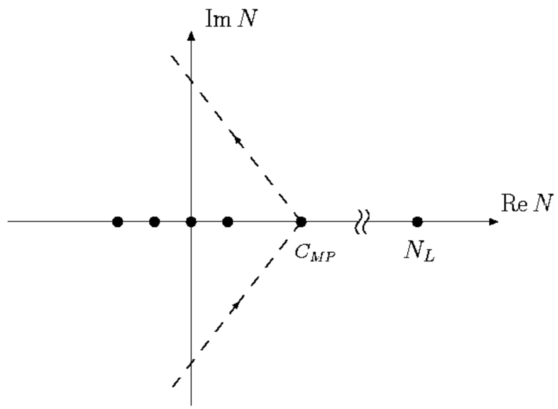
Inverse Mellin integral $$\sigma^{\text{res}}(\tau) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dN \tau^{-N}\, \hat{\sigma}_N^{\text{res}} f_N^2$$

how to treat Landau pole singularity at $\lambda = \frac{1}{2} \to \bar{N} = e^{1/2\alpha_s b_0} = M/\Lambda_{QCD}$?

- Answer : don't care! define the inverse Mellin contour as the left of the Landau pole

Landau pole is not related with the summation of logarithmic corrections.

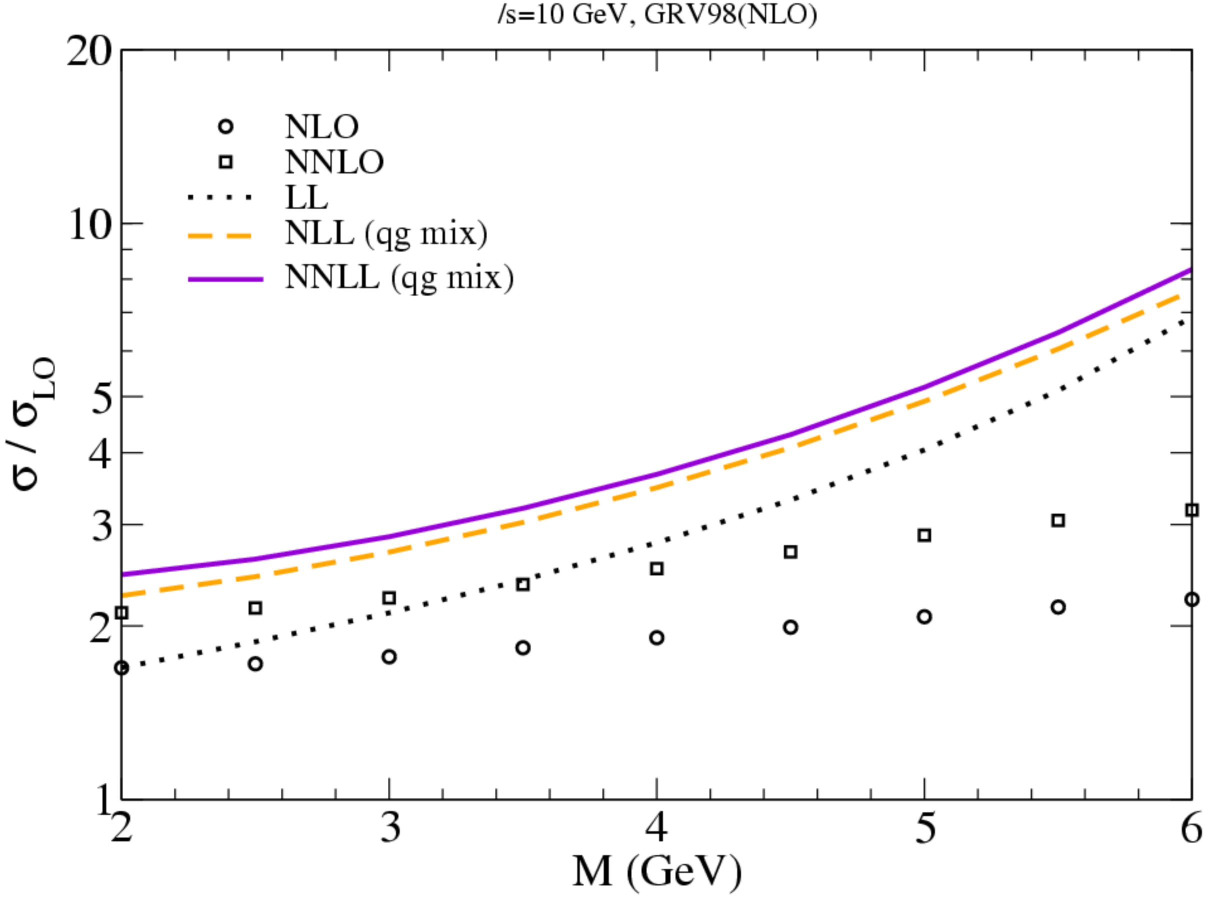
- reproduce all logarithmic terms, *of course* !!
- no factorial growth
- definition of the pert. content of the resum.



Threshold resummation

$$K_{q\bar{q}}^{\text{res}} = C_{DY} \cdot \exp\left[\frac{1}{\alpha_s} h_q^{(1)}(\alpha_s L) + h_q^{(2)}(\alpha_s L, r) + \alpha_s\, h_q^{(3)}(\alpha_s L, r) + \cdots\right]$$

LL : NLL : NNLL :

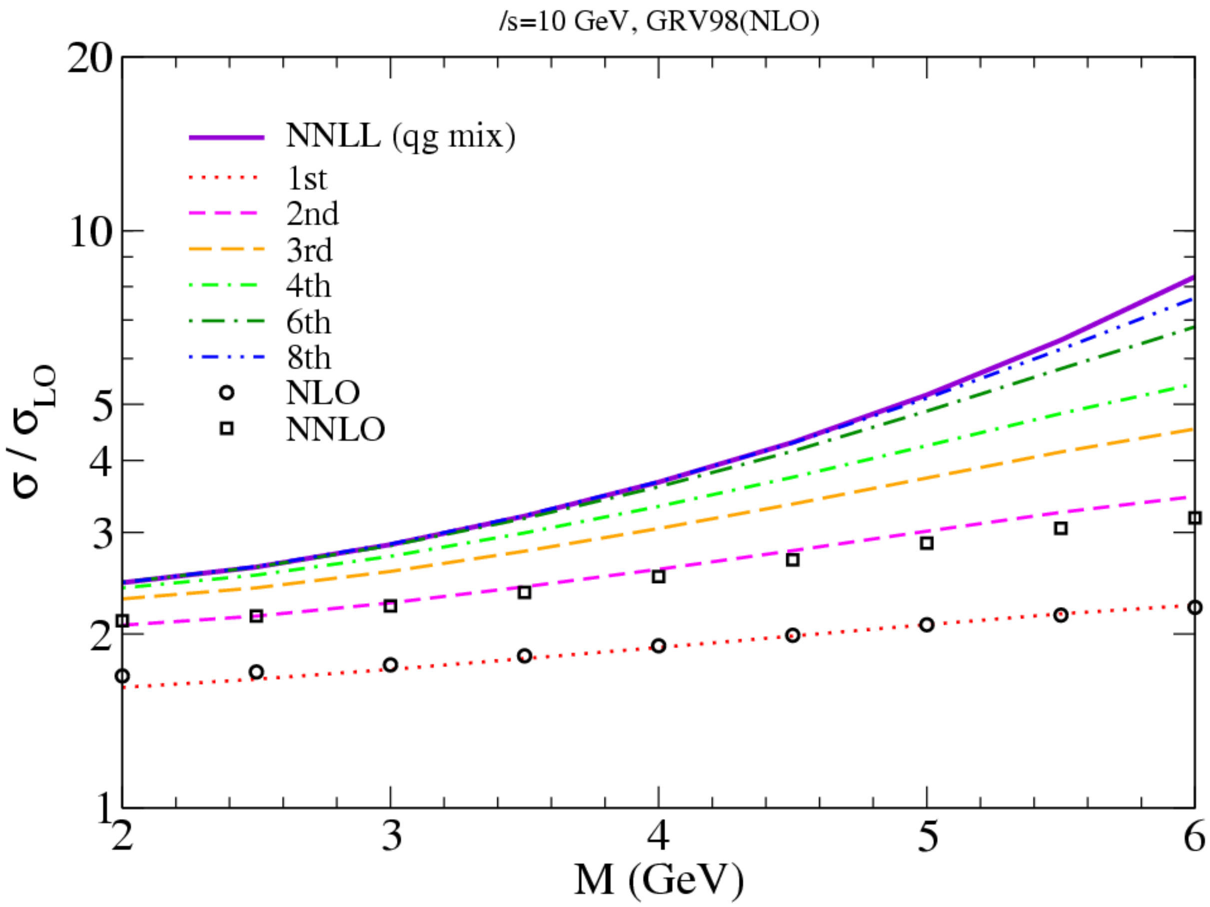


J-PARC pp

$\sqrt{S} = 10\text{GeV}$

Convergence

not only the convergence of logarithmic accuracy (N^{n}LL),
but also the convergence of the power expansion of Sudakov exponent
to $\mathcal{O}(\alpha_s^n)$



J-PARC pp

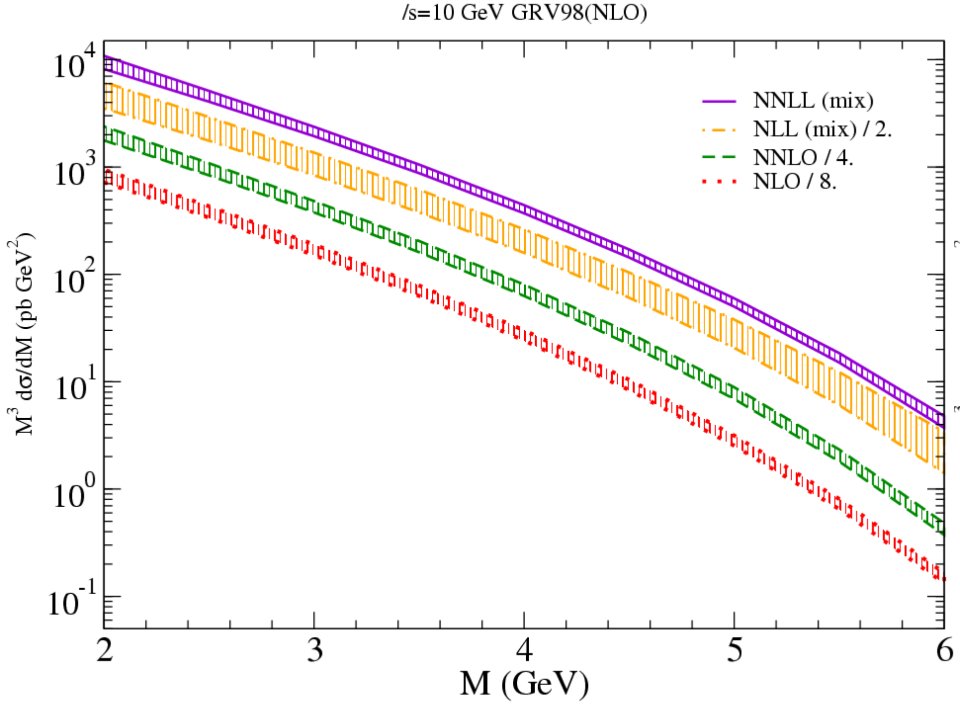
$\sqrt{S} = 10\text{GeV}$

Scale ambiguity

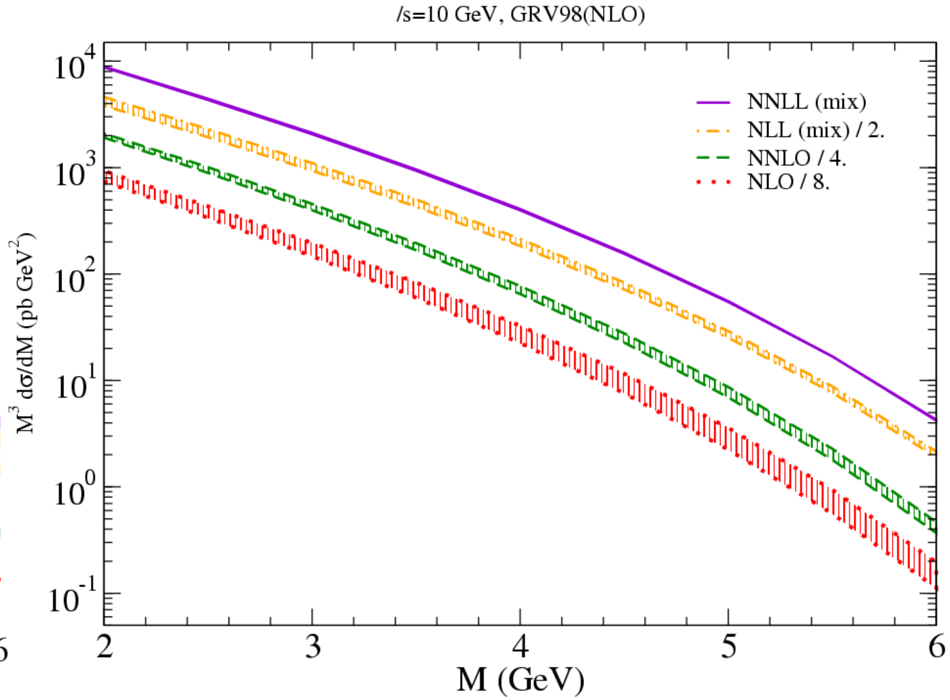
Renormalization/Factorization scale dependence

$$M/2 \leq \mu \leq 2M$$

繰り込みスケール



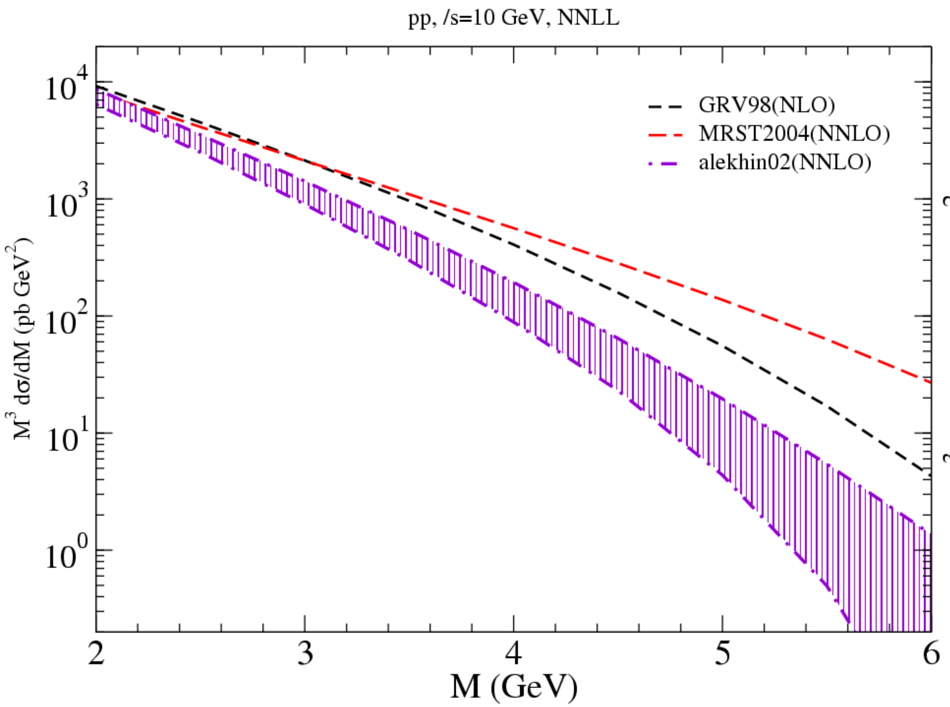
因子化スケール



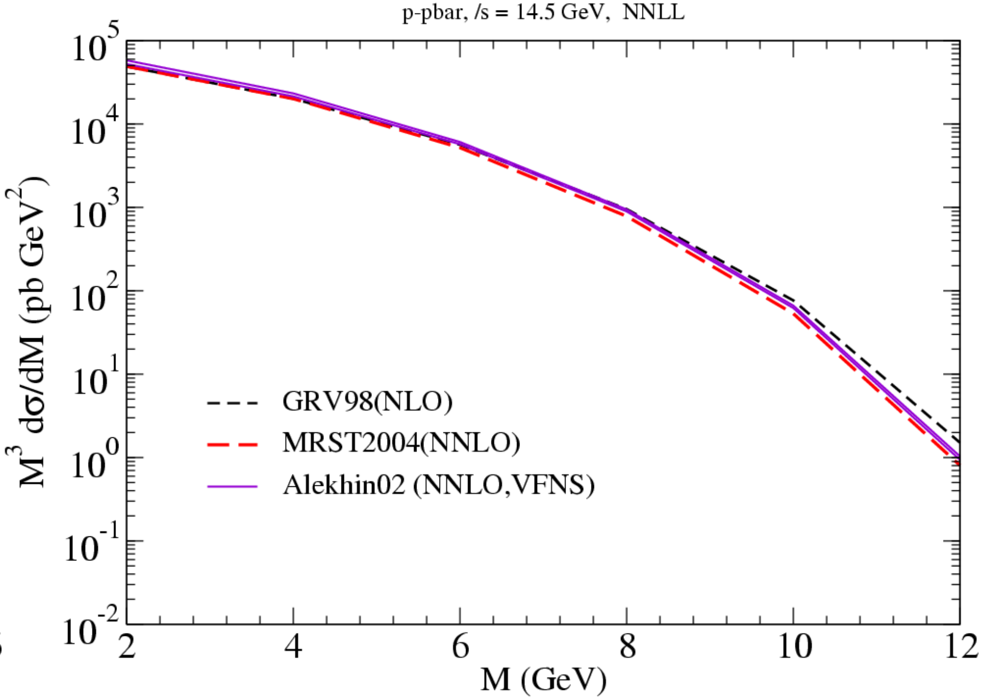
PDF uncertainty

Unpolarized sea-quark and gluon distributions are still uncertain !

$pp,\ \sqrt{s} = 10\ \text{GeV}$



$p\bar{p},\ \sqrt{s} = 14.5\ \text{GeV}$



Matching with fixed order

$$K^{\mathsf{match}} = K^{\mathsf{res}} - K^{\mathsf{res}}|_{\mathcal{O}(\alpha_s^n)} + K^{\mathsf{f.o.}\,\mathcal{O}(\alpha_s^n)}$$

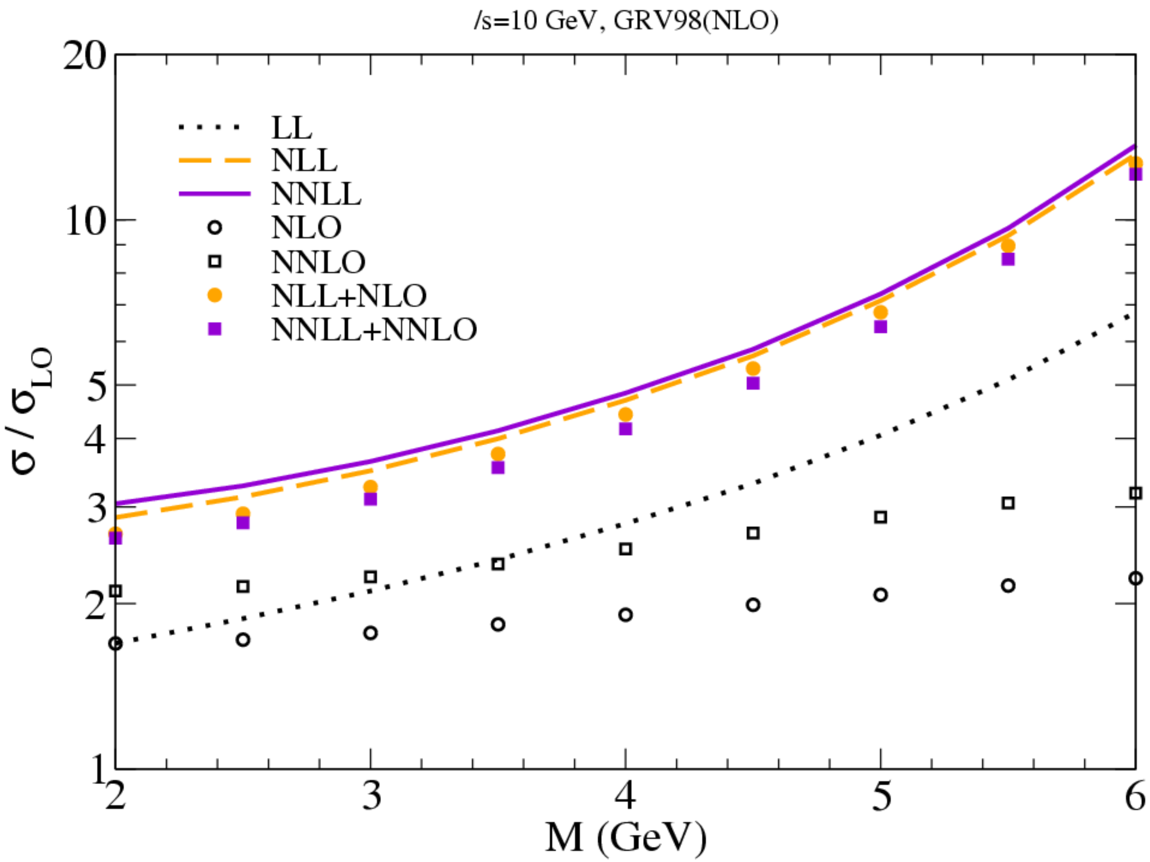
✓ relevant for all phase space regions

✓ qg sub-process contributions

LL :

NLL+NLO :

NNLL+NNLO :

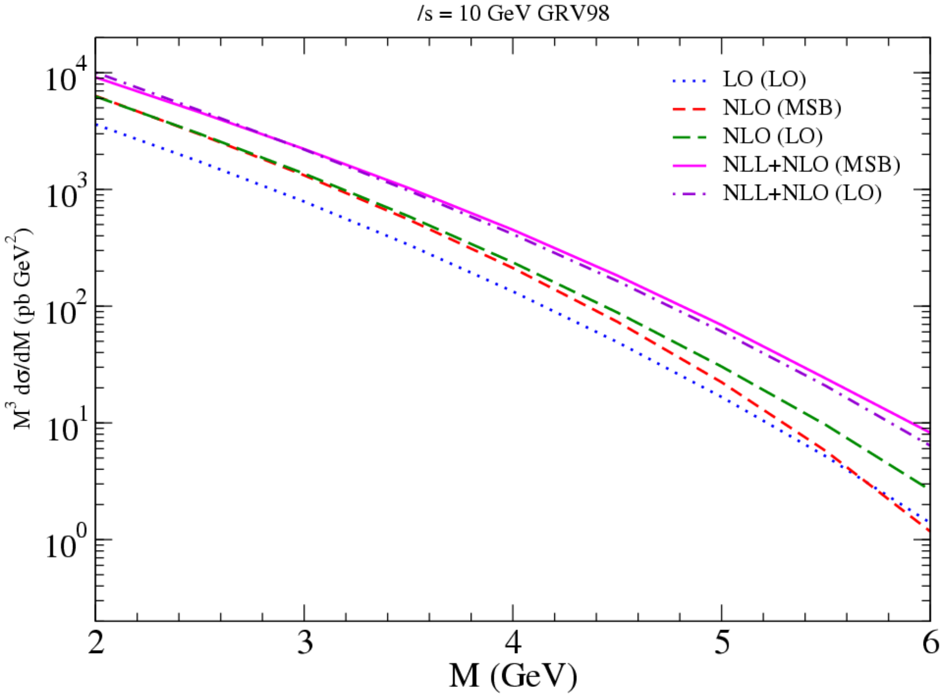


Resummation in DIS scheme

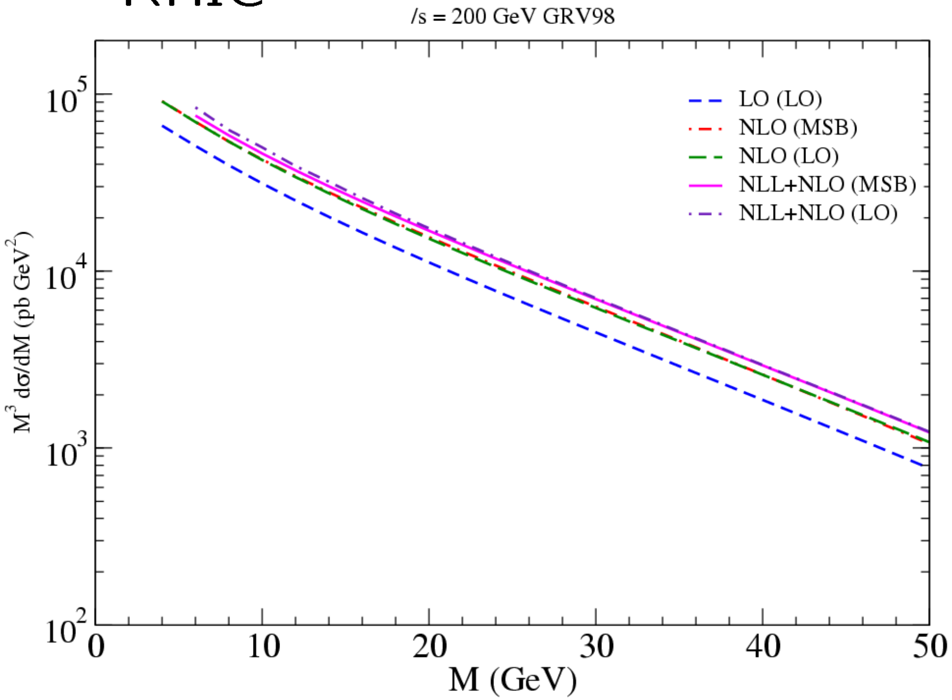
Sudakov exponent in DIS scheme : $K_N^{\mathrm{DIS}} = K_N^{\overline{\mathrm{MS}}} / (C_{2,N}^{\overline{\mathrm{MS}}})^2$

$$\frac{\exp\left[\frac{\alpha_s}{\pi} C_F \left(2L^2\right)\right]}{\left(\exp\left[\frac{\alpha_s}{\pi} C_F \left(\frac{1}{2}L^2 + \frac{3}{4}L\right)\right]\right)^2} = \exp\left[\frac{\alpha_s}{\pi} C_F \left(L^2 - \frac{3}{2}L\right)\right]$$

J-PARC



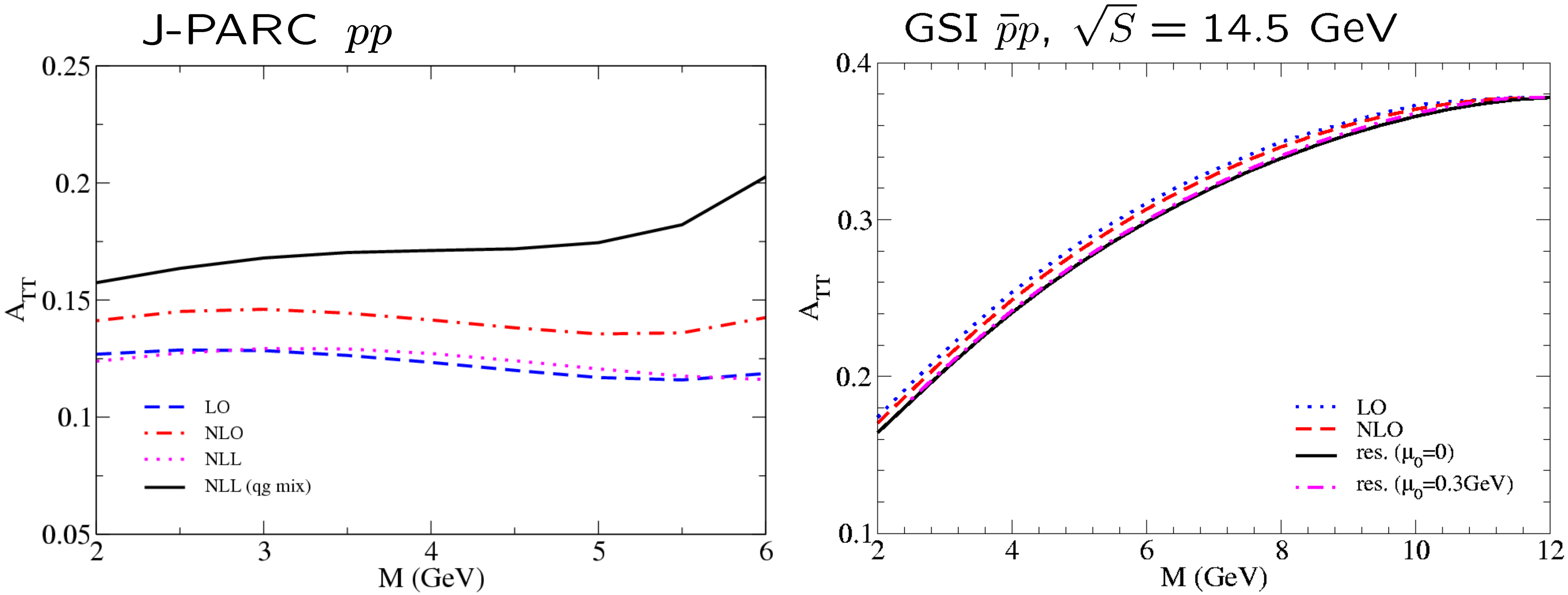
RHIC



Double Transverse Spin Asymmetry

H.Shimizu,G.sterman,
W.Vogelsang,HY('05);
W.Vogelsang,HY('06)

$$A_{TT} \equiv \frac{d\delta\sigma/dM^2 d\phi}{d\sigma/dM^2 d\phi} \sim \frac{\cos(2\phi)}{2} \frac{\sum e_q^2\, \delta q \otimes \delta q}{\sum e_q^2\, f_q \otimes f_q}$$



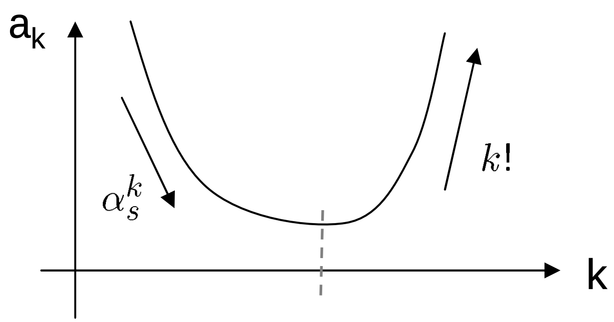
same Trans. pol. PDFs model, for all lines

Power-suppressed correction

M.Beneke, Phys.Rept. 317 ('99);
M.Beneke,V.Braun, NPB454,253 (95); hep-ph/0010208

how to treat divergent series,

like $\sum_{k=0}^{\infty} \left(\frac{2b_0}{p}\right)^k \alpha_s^k \, k!$ (factorial growth)



the best : define the sum by cutting the series,

at where the correction become minimum : $\frac{2b_0}{p}\alpha_s\, k \sim 1 \longrightarrow k = k_c \sim \frac{p}{2b_0\alpha_s}$

then, the error of the sum is about the order of the last term :

$$k_c^{-k_c}\, k_c! \sim e^{-k_c} = e^{-p/2b_0\alpha_s} = \left(\frac{\Lambda_{QCD}}{M}\right)^p$$

Power-suppressed correction

- power correction is defined by defining the sum of the divergent (asymptotic) series

- Landau pole is a consequence of the divergent series in a_s

$$\alpha_s(k_T) = \frac{\alpha_s}{1 + b_0 \alpha_s \ln\left(\frac{k_T^2}{\mu^2}\right)} = \alpha_s \sum_{n=0}^{\infty} \left(-b_0 \alpha_s \ln\left(\frac{k_T^2}{\mu^2}\right)\right)^n$$

- Landau pole in the resummed exponent may tell us

  the structure of the power-suppressed correction

$$\ln(K_{q\bar{q}}^{\text{Res}}) = 4 \int_{M/\bar{N}}^{M} \frac{dk_T}{k_T} A_q(\alpha_s(k_T)) \ln\left(\frac{\bar{N} k_T}{M}\right)$$

$k_{T,\text{min}} < \Lambda_{QCD}$, when $N \to$ large

- Cut-off :

$$k_{T,\text{min}} = \max(M/\bar{N}, \mu_0) \sim \left((M/\bar{N})^2 + \mu_0^2\right)^{1/2}$$

gives $\lambda \to \lambda' = \lambda - \frac{1}{2} b_0 \alpha_s \ln\left(1 + \frac{\bar{N}^2 \mu_0^2}{M^2}\right)$

order is consistent with Beneke,Braun('95)

$$\delta\left(\ln\left(K_{q\bar{q}}^{\text{res}}\right)\right) \sim \mathcal{O}\left(\frac{\mu_0^2 \bar{N}^2}{M^2}\right)$$

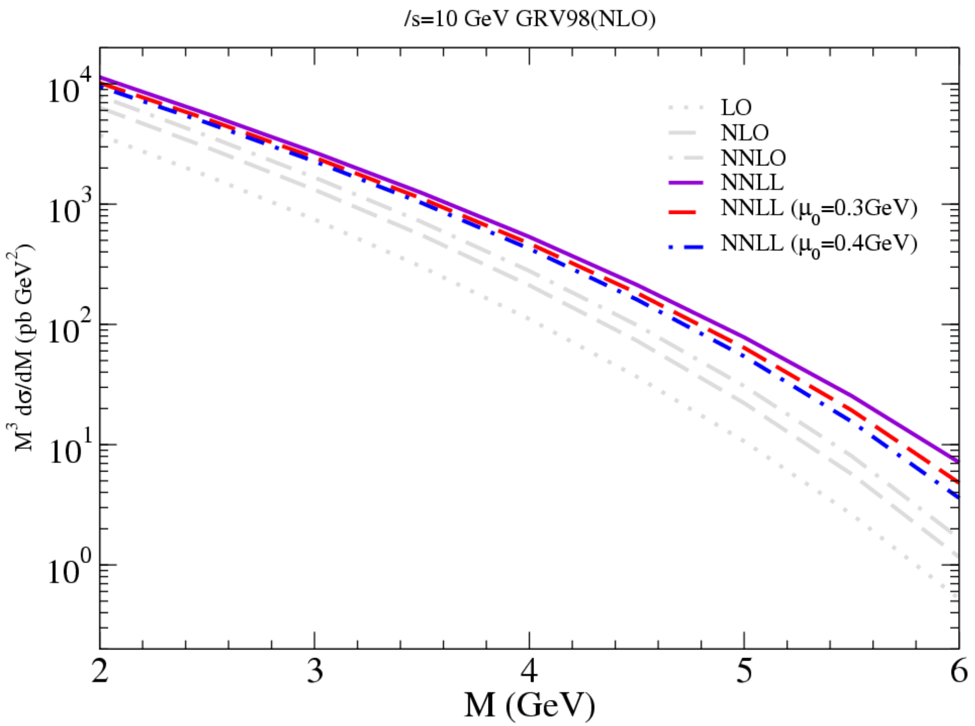
- Size of the cut-off effect :

  as a hint of the size of the power-suppressed correction

cut-off scale ~ hadronic scale

set $\mu_0 = 0.3$ GeV, 0.4 GeV

reduce the Sudakov enhancement
by few-ten percent,
but not spoil all the enhancment

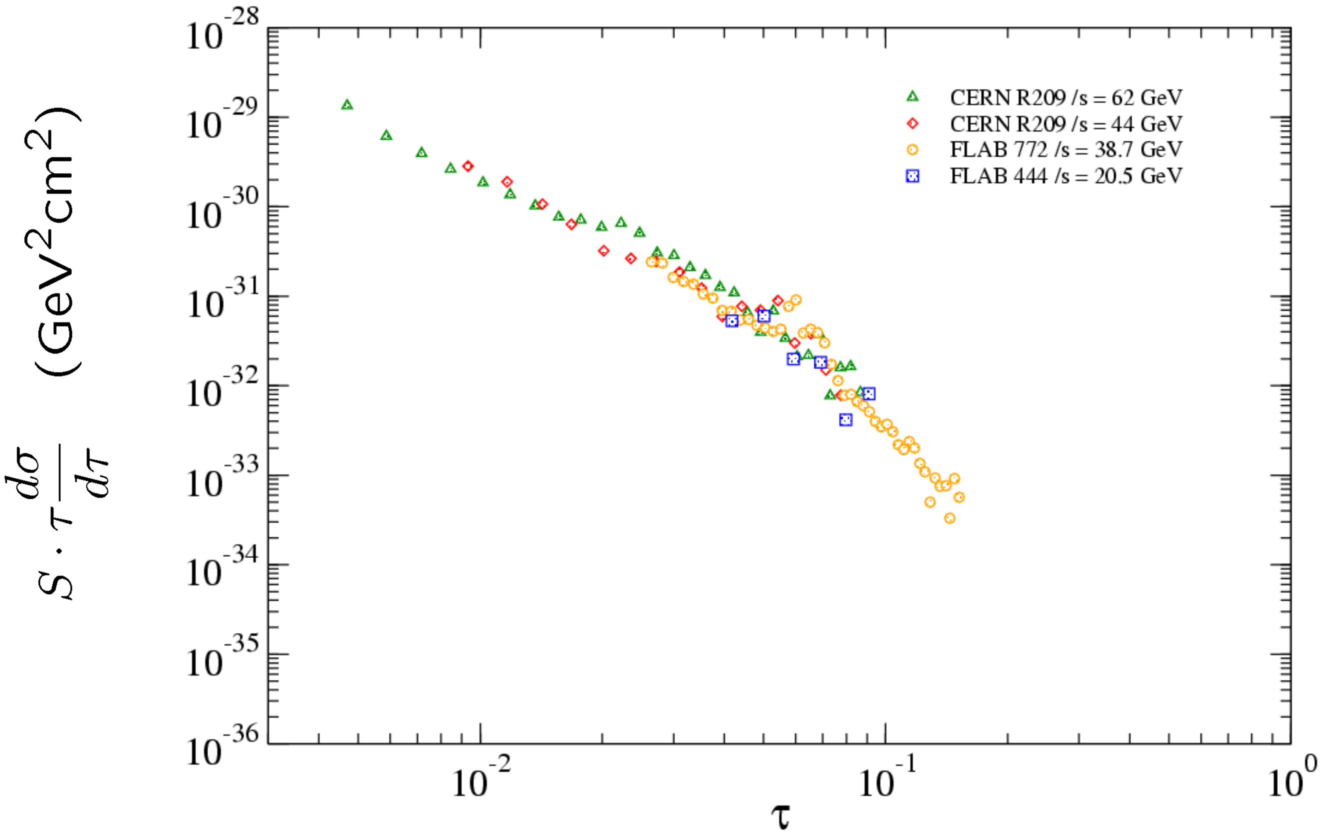


- need more developments, in
  theoretically and phenomenologically.
- desire experimental suggestion !

Comparison with past DY data

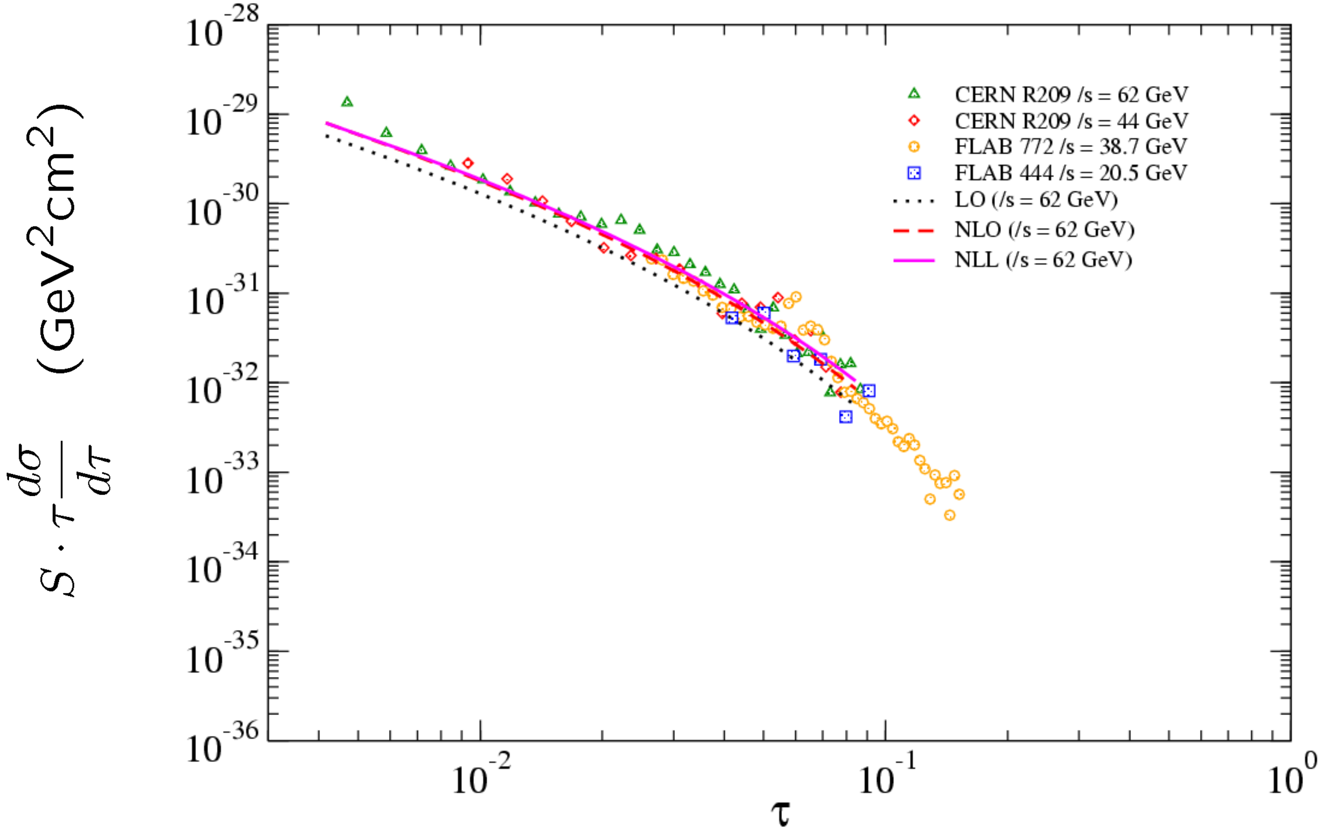
Proton-Proton collision

$$S \cdot \tau \frac{d\sigma}{d\tau} \text{ depends on } \left\{ \tau, \mu, \alpha_s(\mu), \frac{M}{\mu} \right\}$$



Proton-Proton collision

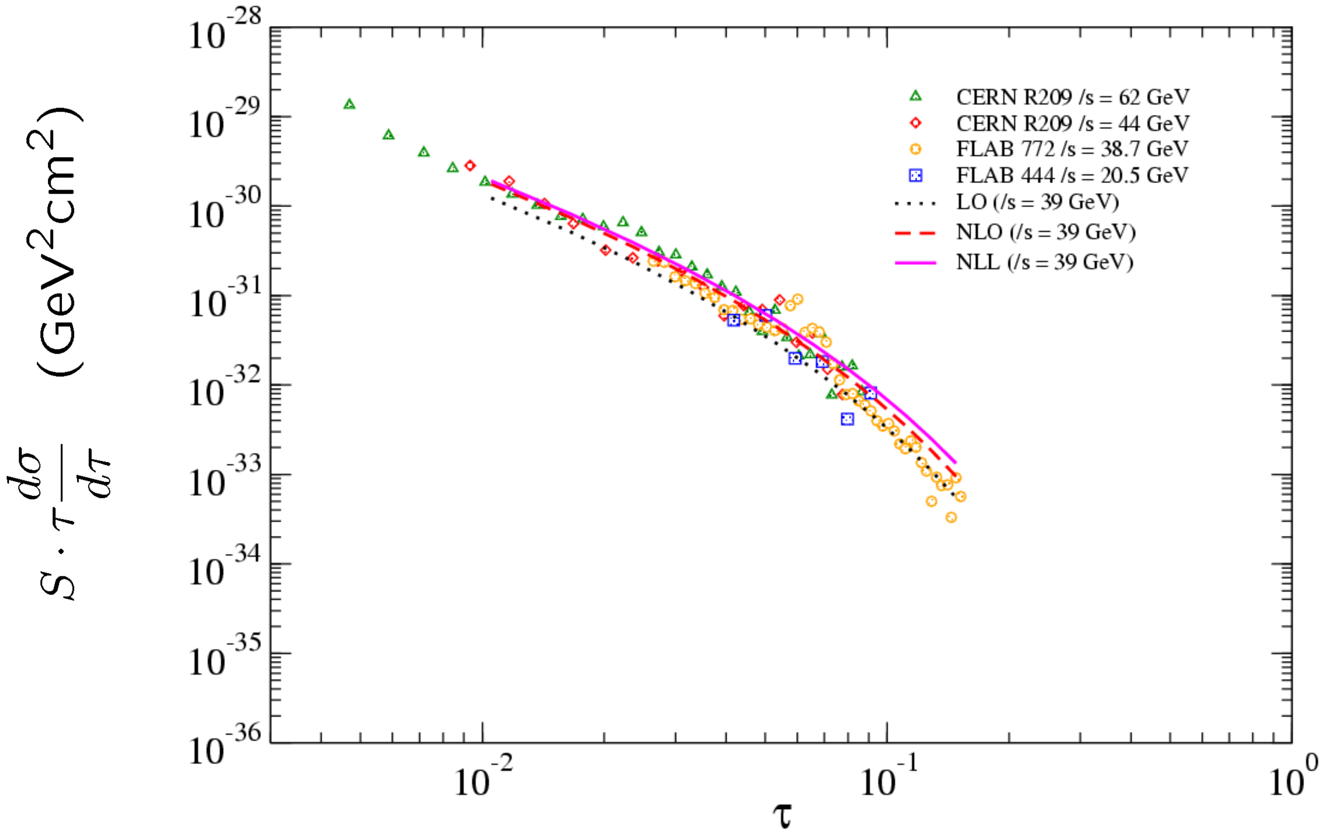
Prediction for /s = 62 GeV



with GRV98

Proton-Proton collision

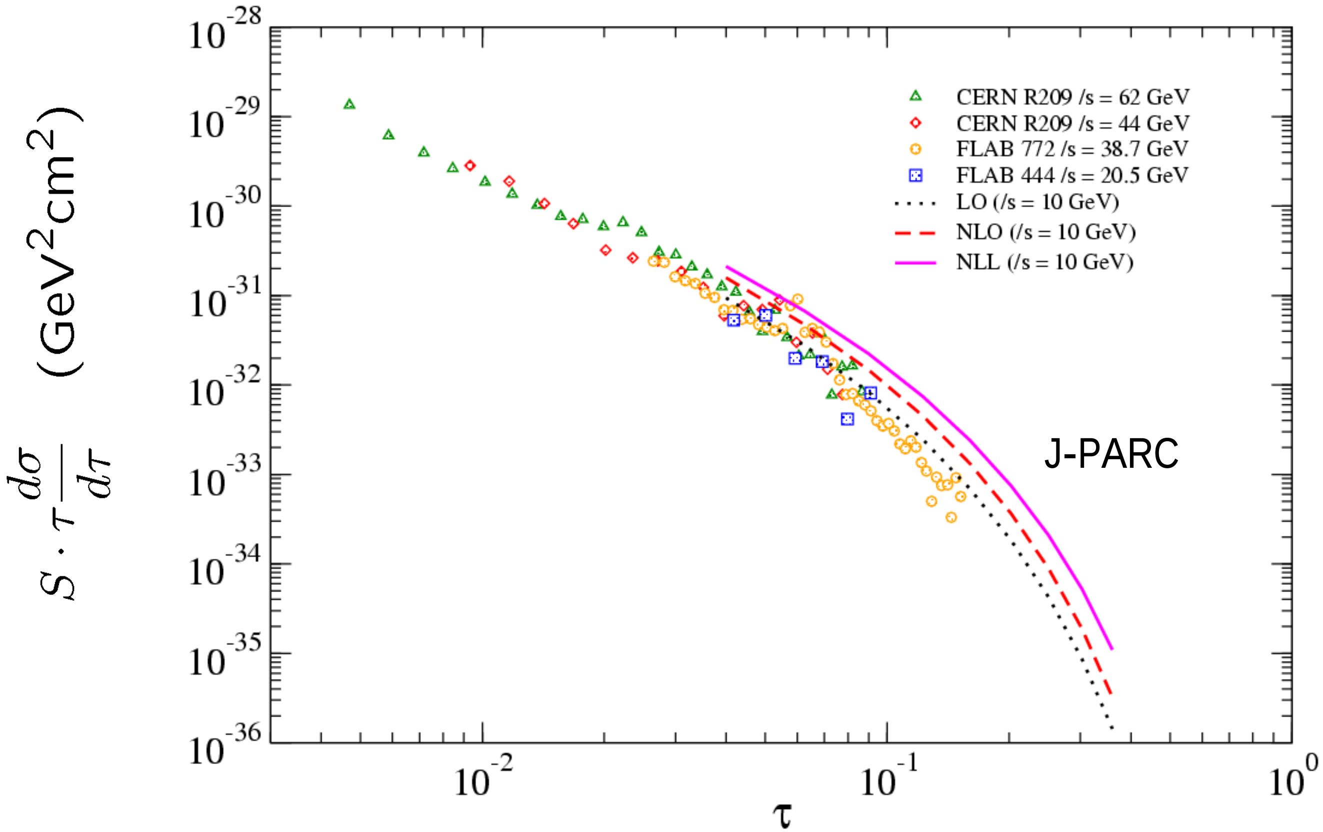
Prediction for /s = 39 GeV



with GRV98

Proton-Proton collision

Prediction for /s = 10 GeV



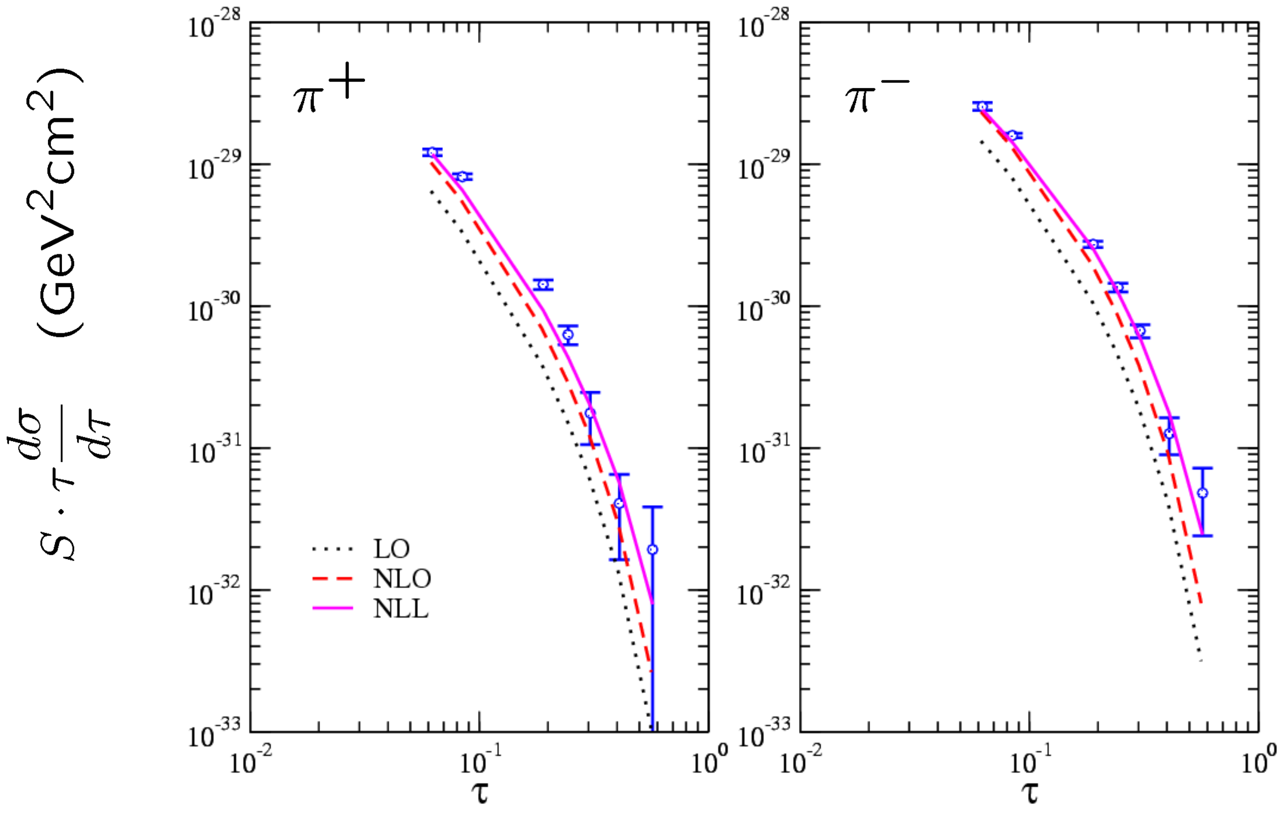
with GRV98

M = 2 ~ 6 GeV

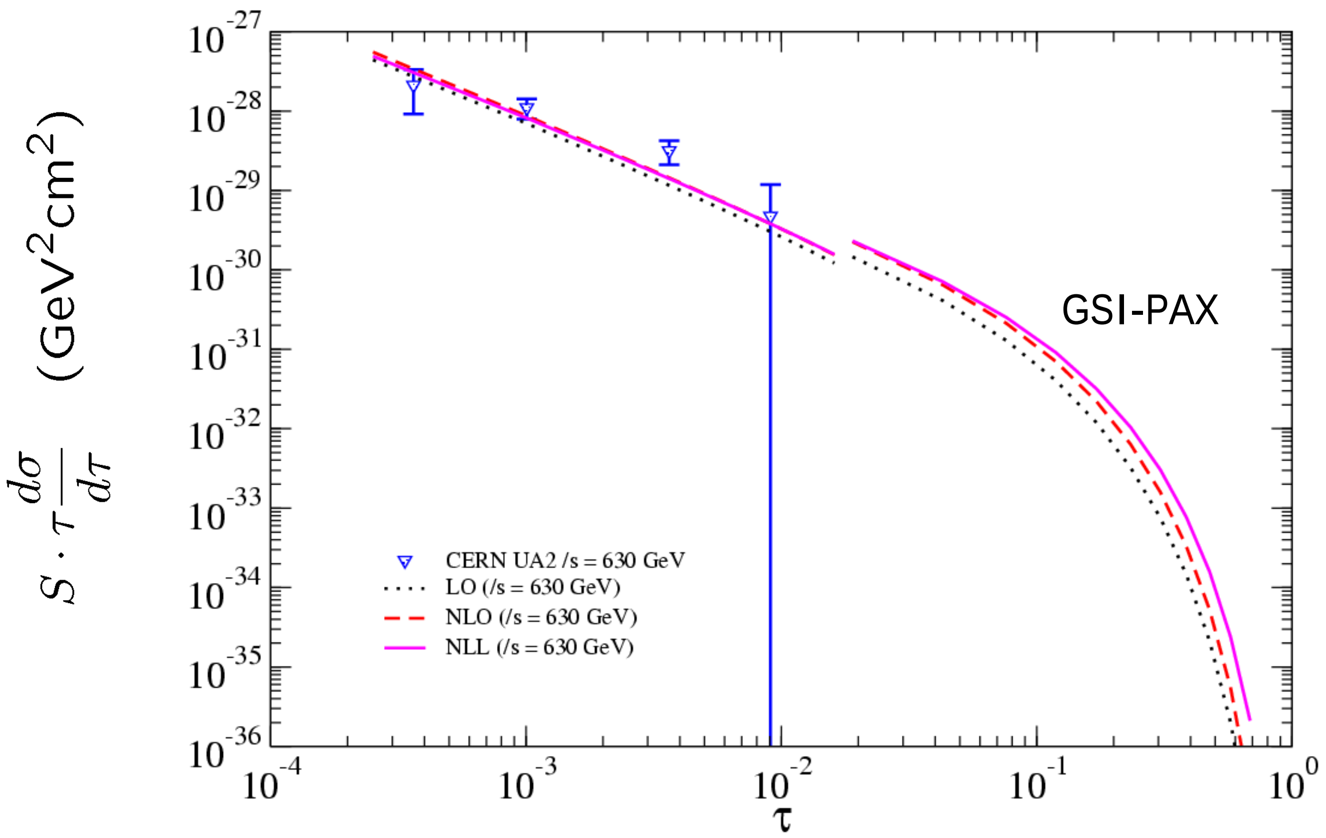
Pion-Nucleus collision

GRV π-PDF, isospin symmetry,
no nuclear effect are assumed

$\sqrt{S} = 8.6\text{GeV}$ **CERN-WA39 ('80)** (Tungsten target)



Proton-AntiProton collision



with GRV98

Summary

Drell-Yan process at the J-PARC energy,

- QCD correction is very important, and higher-order corrections beyond NLO may be required.
- Resummation studies tell us, however, pQCD correction can be controlled by summing the large log terms.
- Power-corrections must become relevant, and needs more studies.
- Unpolarized PDFs (sea-quark, gluon) is still unknown, and have to be measured at the J-PARC experiments.

- Rapidity distribution :

NNLO : Anastasiou,Dixon,Melnikov,Petriello('04)
Resummation : Mukerjee,Vogelsang('06),
Bolzoni('06),Vogelsang,HY('06)

- Transverse-momentum distribution : Kawamura,Kodaira,Tanaka('07)

Physics Interest :

Proposal for J-PARC, P.Reimer, arXiv:0704.3612

Unpolarized : sea-quark distribution, Boer-Mulders function,
nuclear effects, power correction,,,

Single-spin asymmetry, Double-spin asymmetry
: Transversity, Sivers function, Higher-twist,,,

- Other processes : J/Ψ, direct photon, hadron production,,,