Glasma

– From the Color Glass Condensate to a Quark-Gluon Plasma –

– Gluon Production in pA and AA –

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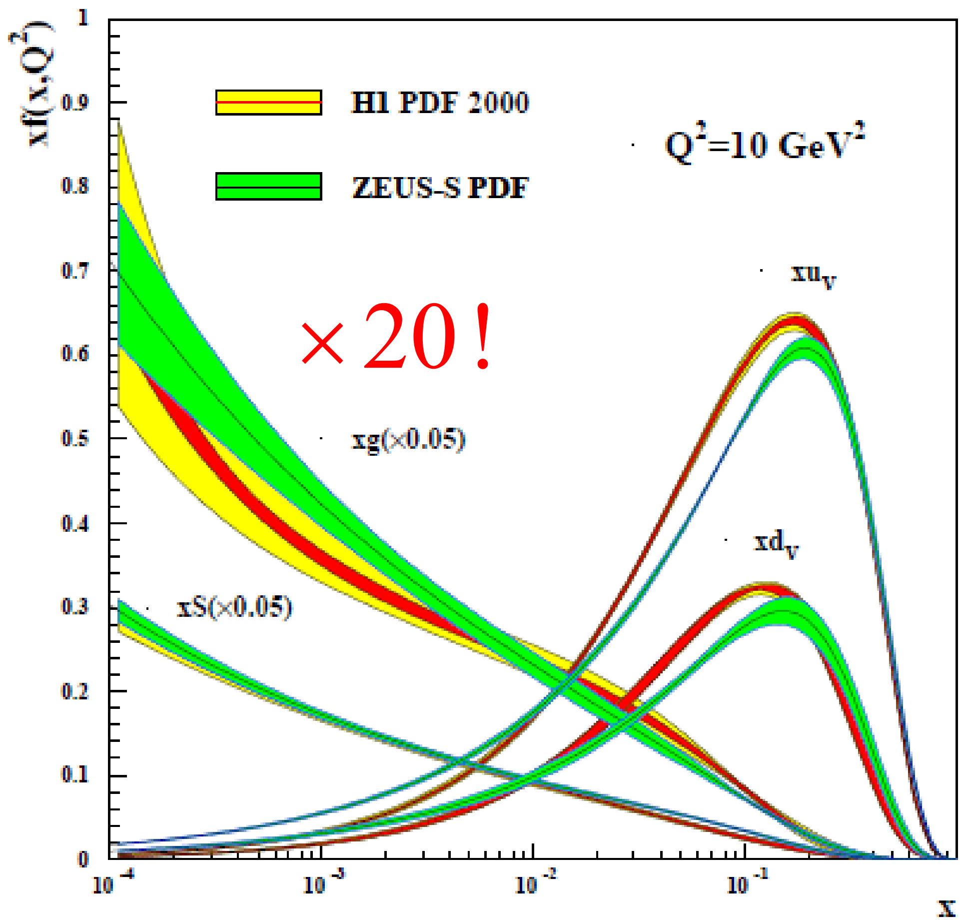
Talk Agenda

- Saturation Model in the Small-x Region
  - Saturation scale as a function of x
  - McLerran-Venugopalan (MV) model
- Particle Production in a Dilute-Dense Collision
  - One-gluon production
  - Two-gluon production
- Particle Production in a Dense-Dense Collision
  - Gluon distribution → Gluon production

HERA (ep collider) – x Evolution

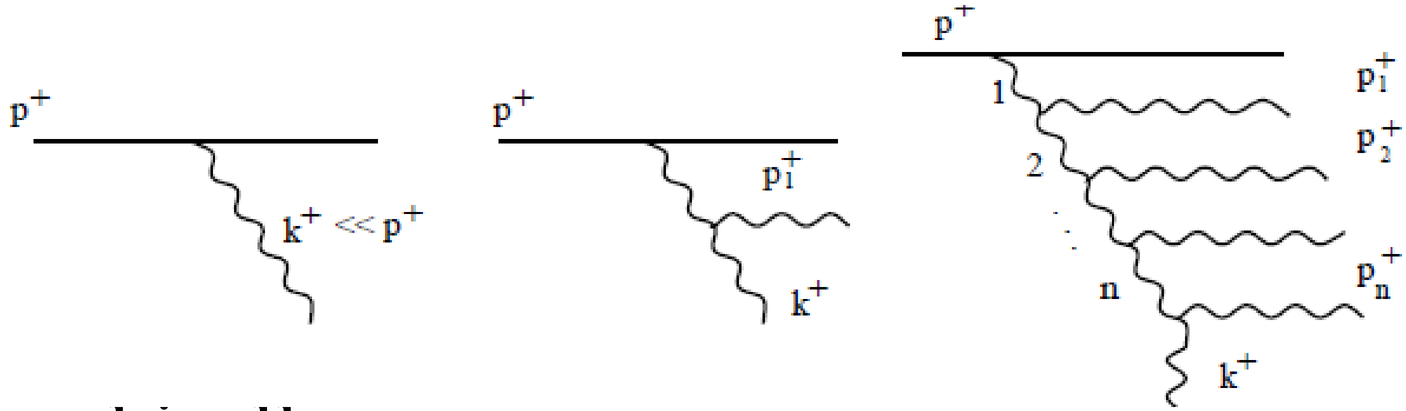
Quantum Evolution of PDFs



As x goes smaller than ~ 10^{-2} **gluon** is dominant.

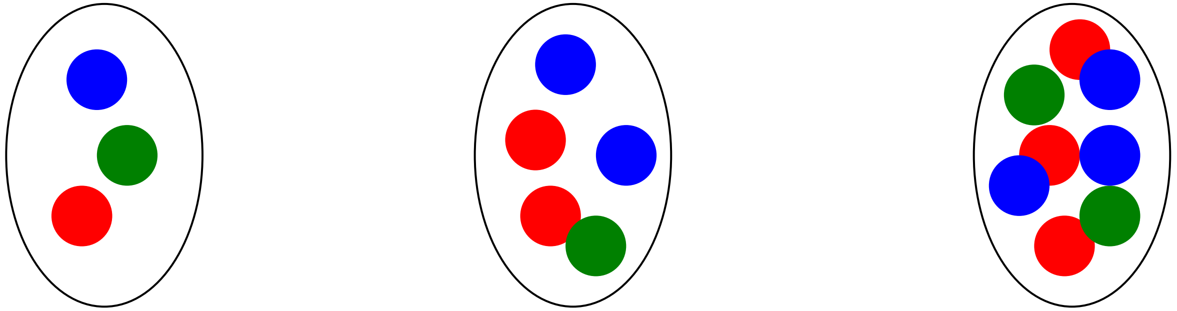
Going to Smaller x with Fixed Q^2

- Gluon increases with a fixed transverse area



BFKL
Iancu

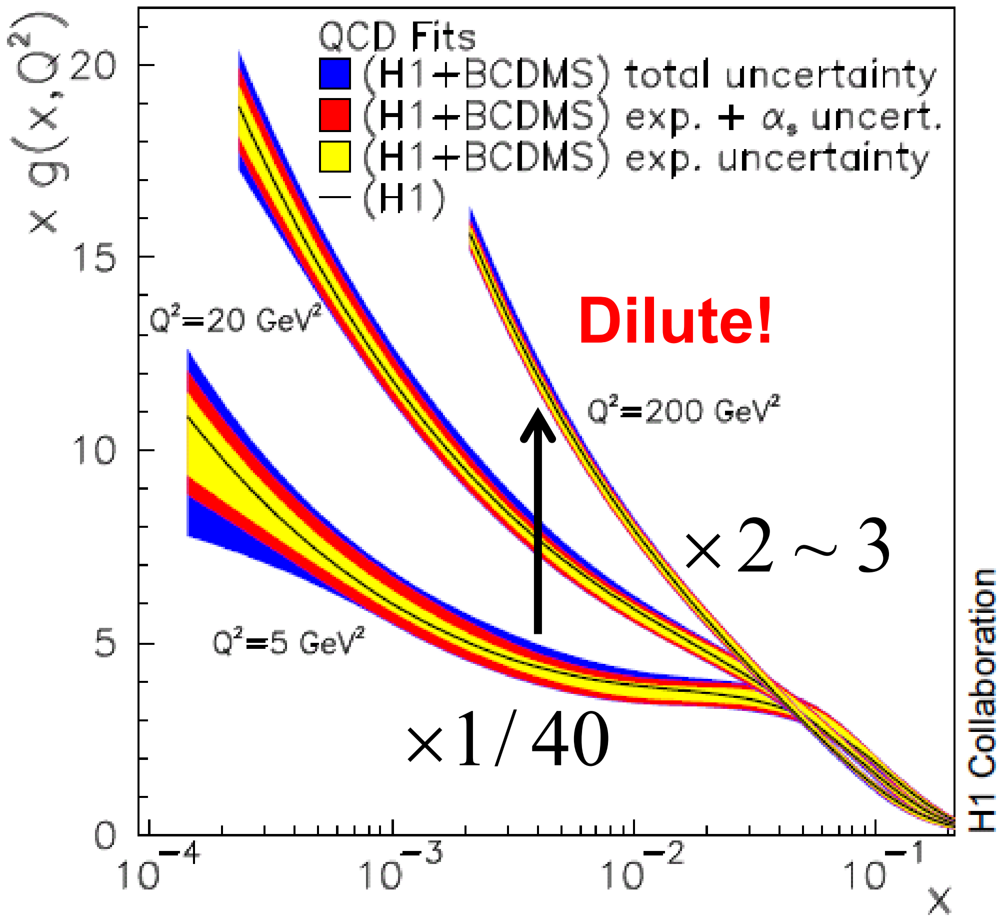
- Graphically



small-*x* → Dense Gluon Matter

Q^2 Dependence

Quantum Evolution of PDFs

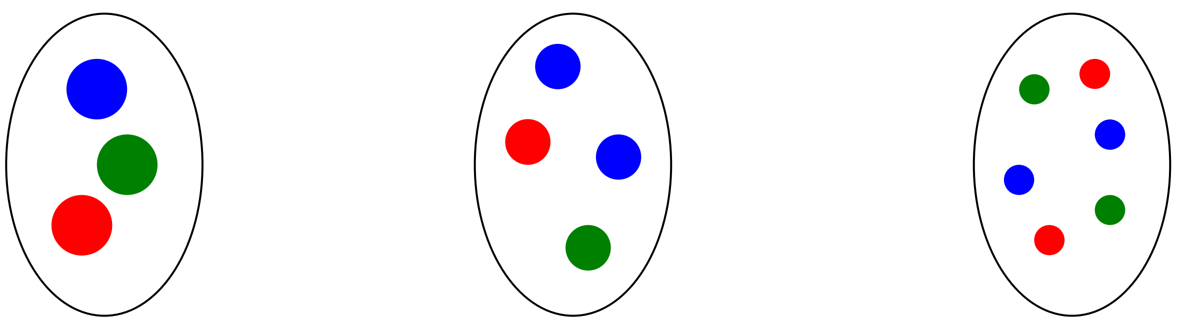


As Q^2 goes larger gluon grows slowly.

Going to Larger Q^2 with Fixed x

- Gluon slowly increases with a decreasing area

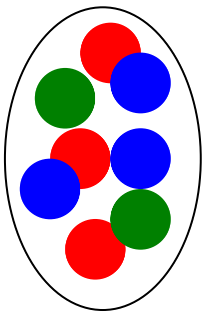
- Graphically, in the same way,



large Q → Dilute Gluon Matter

Saturation

- Gluons eventually cover the transverse area at small x and small Q^2



$$\text{Area} \quad \sim \pi R^2 \sim 75\,\text{GeV}^{-2} \quad \text{(proton)}$$

$$\text{Crammed density} \quad \sim (N_c^2 - 1) \cdot Q^2 \cdot \pi R^2$$

$$\sim 600 \quad (\text{for } Q^2 = 1\,\text{GeV}^2)$$

- Condition for saturation:

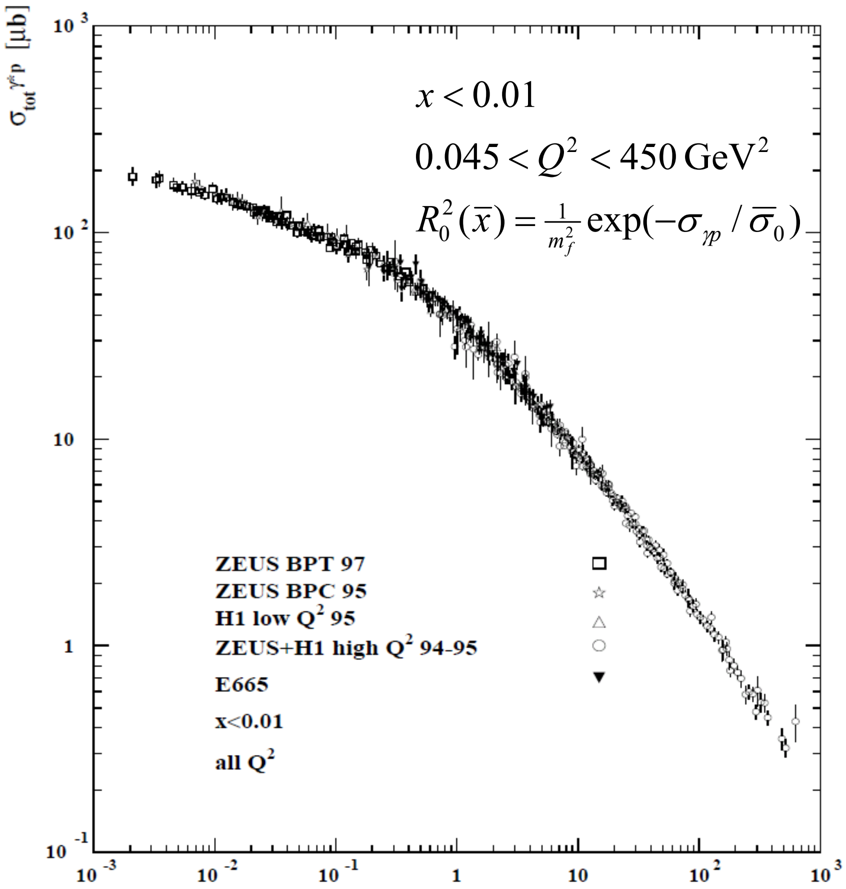
$$xg(x, Q_s) \Big/ (N_c^2 - 1) \cdot Q_s^2 \cdot \pi R^2 \quad \sim \frac{1}{\alpha_s N_c} \sim 1$$

Overlapping Factor

How can we expect the saturation effect in reality?

Scaling Behavior

Dipole Cross Section in a Saturation Model



Stasto-Golec-Biernat-Kwiecinski Plot

$$\hat{\sigma}(x,r) = \sigma_0\left(1 - \exp\left[-r^2/4R_0^2(x)\right]\right)$$

$$\sigma_{\gamma^* p}(x,Q^2) \to \sigma_{\gamma^* p}(Q^2 R_0^2(x))$$

Called the "Geometric Scaling"

Dias de Deus

$$R_0(x) = \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\lambda/2}$$

$$Q_0 = 1\,\mathrm{GeV}$$

$$x_0 = 3.04 \times 10^{-4}$$

$$\lambda = 0.288$$

Iancu-Itakura-McLerran, Iancu-Italura-Munier

2009 at KEK

Scaling Behavior Extended

- Let us put some numbers... $x = 10^{-4} \rightarrow Q_s^2 = 1.38\,\mathrm{GeV}^2$

$$xg(x,Q) \Big/ (N_c^2 - 1) \cdot Q^2 \cdot \pi R^2 \sim \frac{10}{8 \cdot 1.38 \cdot 75} \sim 0.01?$$

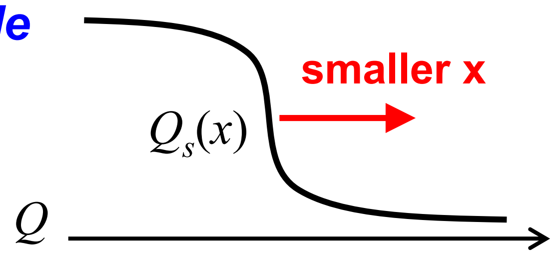
$$\times A^{1/3} (\sim 5.8 \text{ for Au})$$

- Scaling is consistent with pQCD **Iancu-Itakura-McLerran**

"Extended Geometric Scaling"

- BFKL (dilute regime) can fix the parameters

Dipole Amplitude



No need to realize saturation!

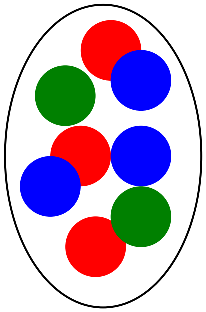
Message

There are some examples in which
a saturation-model description can work well
even when saturation is not yet reached.

Saturation Model

- Parametrized by a scaling variable
  - $Q_s(x)$ encompasses the scaling property.
  - Wave-function is characterized by $Q_s(x)$

- McLerran-Venugopalan model

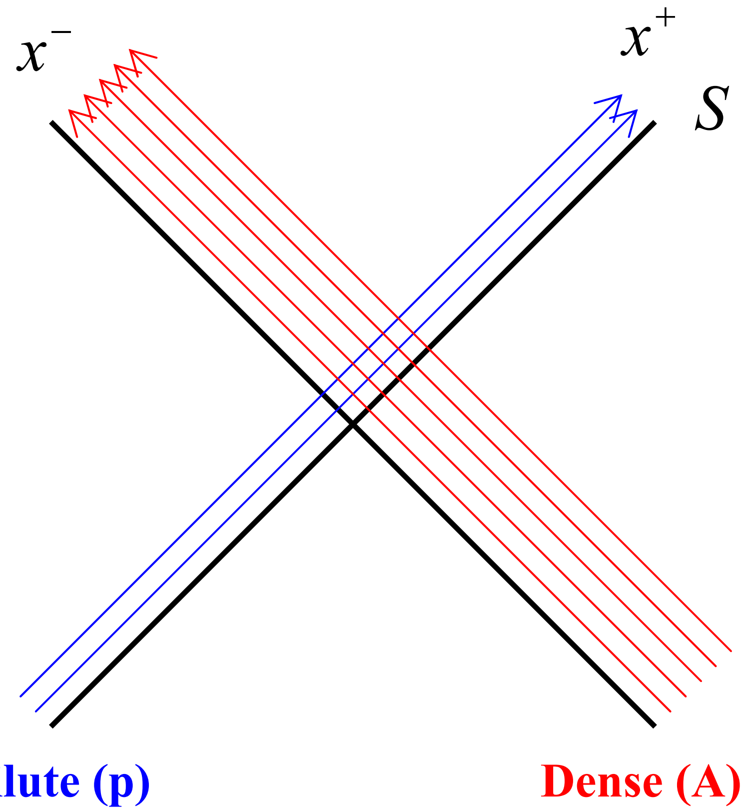


$$\mathcal{W}_x[\rho] = \exp\left[-\int d^3x \frac{|\rho(x)|^2}{2g^2\mu_x^2}\right]$$

$$g\mu_x \sim Q_s(x)$$

Scattering Problem

Scattering Amplitude in the Eikonal Approx.



$$S \sim \left\langle \sum_{\{\rho_p\}} \mathcal{W}_x[\rho_p] \prod_{\{\rho_p\}} V \cdot \sum_{\{\rho_A\}} \mathcal{W}_{x'}[\rho_A] \prod_{\{\rho_A\}} W \right\rangle$$

$$V(x_\perp) = \exp\left[ig \int dz^+ A^-(z^+, x_\perp) \right]$$

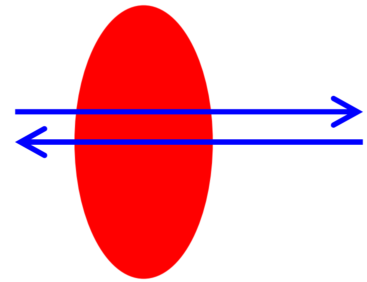
$$W(x_\perp) = \exp\left[ig \int dz^- A^+(z^-, x_\perp) \right]$$

In case of Dipole-CGC scattering

$$S \sim \left\langle \left\langle V(x_\perp) V^+(y_\perp) \right\rangle \right\rangle_{\rho_A}$$

Stationary-Point Approximation

Dipole Scattering Amplitude



$$S \sim \left\langle \sum_{\{\rho_A\}} \mathcal{W}_x[\rho_A] \prod_{\{\rho_A\}} W \cdot V(x_\perp) V^+(y_\perp) \right\rangle$$

$$= \sum_{\{\rho_A\}} \mathcal{W}_x[\rho_A] \int_{p^- < xP^-} [\mathcal{D}A]\, V(x_\perp) V^+(y_\perp) \exp[\, iS_{\mathrm{YM}}[A] + iS_{\mathrm{source}}[\rho_A, W]]$$

$$= \left\langle \left\langle V(x_\perp) V^+(y_\perp) \right\rangle \right\rangle_{\rho_A}$$

$$S_{\mathrm{source}} = \frac{i}{gN_c} \int d^4x \,\mathrm{tr}\big[\rho_A \ln W\big] \sim -\int d^4x\, \rho_A^a A_a^+$$

Easily solvable ↓

$$\left. \frac{\partial S_{\mathrm{YM}}}{\partial A_a^\mu} \right|_{A=\mathcal{A}} = \delta^{\mu -} \rho_A$$

Large enough ρ_A → Stationary-point approx.

Physical Observable

- Dipole scattering amplitude

$$\langle\langle V(x_\perp)V^+(y_\perp)\rangle\rangle_{\rho_A}$$

$$= \sum_{\{\rho_A\}} \mathcal{W}_x[\rho_A] \int_{p^+<xP^+} [\mathcal{D}A]\, V(x_\perp)V^+(y_\perp)\exp[\,iS_{\mathrm{YM}}[A]+iS_{\mathrm{source}}[\rho_A,W]]$$

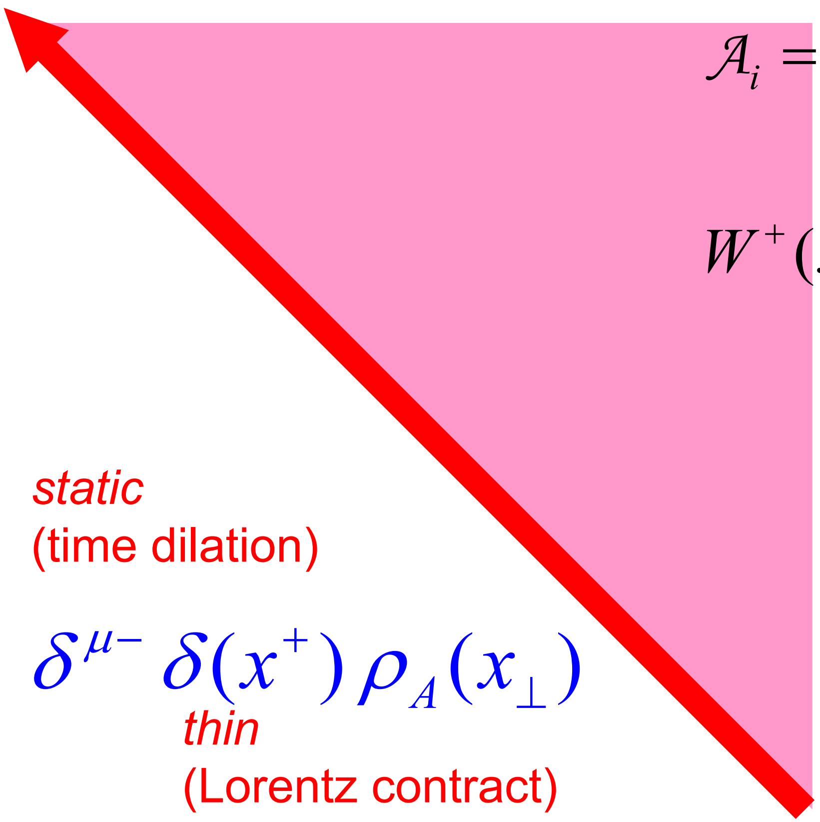
$$\sim \sum_{\{\rho_A\}} \mathcal{W}_x[\rho_A]\, V(x_\perp)V^+(y_\perp)\Big|_{A=\mathcal{A}[\rho_A]} \quad \text{(function of } g\mu\sim Q_s\text{)}$$

- In general

$$\boxed{\langle\langle O[A]\rangle\rangle_{\rho_A} \sim \int \mathcal{D}\rho_A \mathcal{W}_x[\rho_A]\, O[\mathcal{A}[\rho_A]]}$$

Quantum corrections lead to $\mathcal{W}_x \rightarrow \mathcal{W}_{x+\delta x}$ i.e. small-x evolution

Classical Solution



$$\mathcal{A}^+ = \mathcal{A}^- = 0$$

$$\mathcal{A}_i = \alpha_i^{(2)} - \frac{1}{ig} W(x_\perp)\partial_i W^+(x_\perp)$$

$$W^+(x_\perp) = P\exp\left[-ig\int dz^+ \frac{1}{\partial_\perp^2}\rho_A(x_\perp)\delta(z^+)\right]$$

c.f. in EM

$\partial_\perp^2 \phi = -\rho$ (Gauss law)

$\rightarrow \phi' = 0$

$\rightarrow A_i = \frac{1}{ie} e^{ie\phi}\partial_i e^{-ie\phi} \quad (= -\partial_i \phi)$

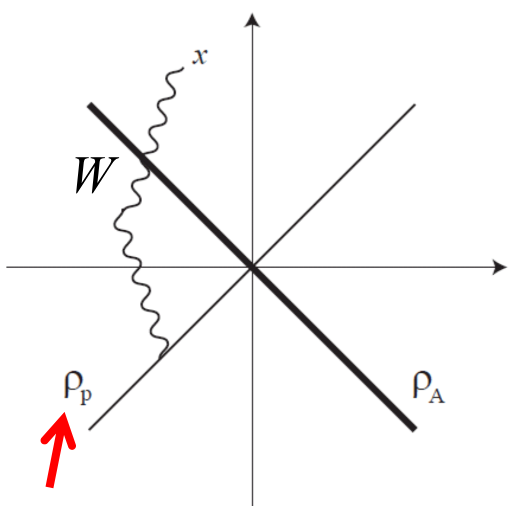
One-Gluon Production in pA

LSZ reduction formula

$$\langle \boldsymbol{p}; \lambda, a | \Omega \rangle = \mathrm{i}\, \epsilon_\mu^{(\lambda)}(\boldsymbol{p}) \int \mathrm{d}^4 x\, \mathrm{e}^{\mathrm{i}p\cdot x}\, \Box_x \langle \Omega | A_a^\mu(x) | \Omega \rangle$$

In LC gauge

$$\frac{\mathrm{d}N_\mathrm{g}}{\mathrm{d}y} = \frac{1}{4\pi} \int \frac{\mathrm{d}^2 \boldsymbol{p}_\perp}{(2\pi)^2}\, p^2 \mathcal{A}_a^{i*}(p)\, p^2 \mathcal{A}_a^i(p)$$



perturbation

Final expression is given in terms of

$\langle \rho_p \rho_p \rangle \to \varphi_p \quad \langle WW \rangle \to \varphi_A$ K_t-factorization

which is calculable as a function of $g\mu$ *(higher-twist)*

Dumitru-McLerran, Blaizot-Gelis-Venugopalan

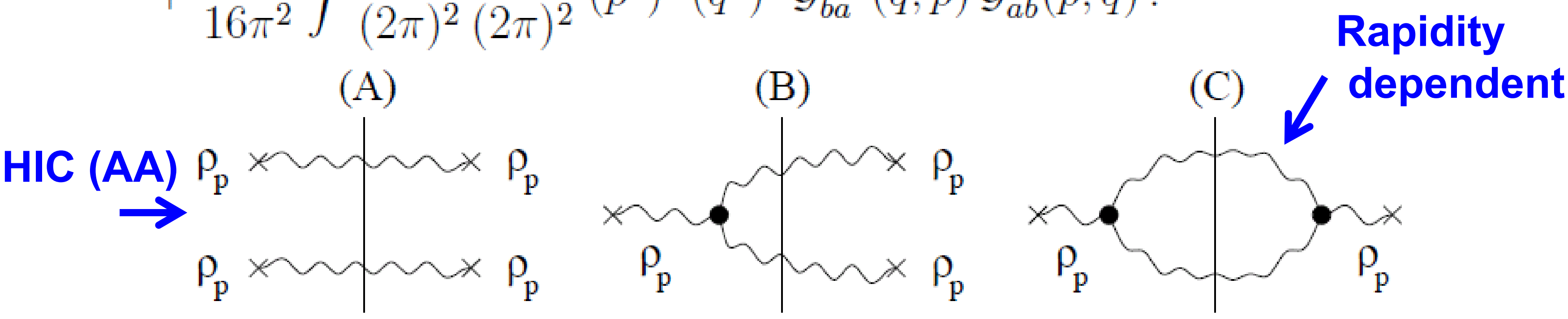
Two-Gluon Production in pA

LSZ reduction formula

$$\langle \boldsymbol{p}, \lambda, a; \boldsymbol{q}, \sigma, b | \Omega \rangle$$
$$= \mathrm{i}\, \epsilon_\mu^{(\lambda)}(\boldsymbol{p}) \int \mathrm{d}^4 x\, \mathrm{e}^{\mathrm{i}p\cdot x}\, \Box_x\, \mathrm{i}\, \epsilon_\nu^{(\sigma)}(\boldsymbol{q}) \int \mathrm{d}^4 y\, \mathrm{e}^{\mathrm{i}q\cdot y}\, \Box_y\, \langle \Omega | \mathrm{T}[A_a^\mu(x) A_b^\nu(y)] | \Omega \rangle$$

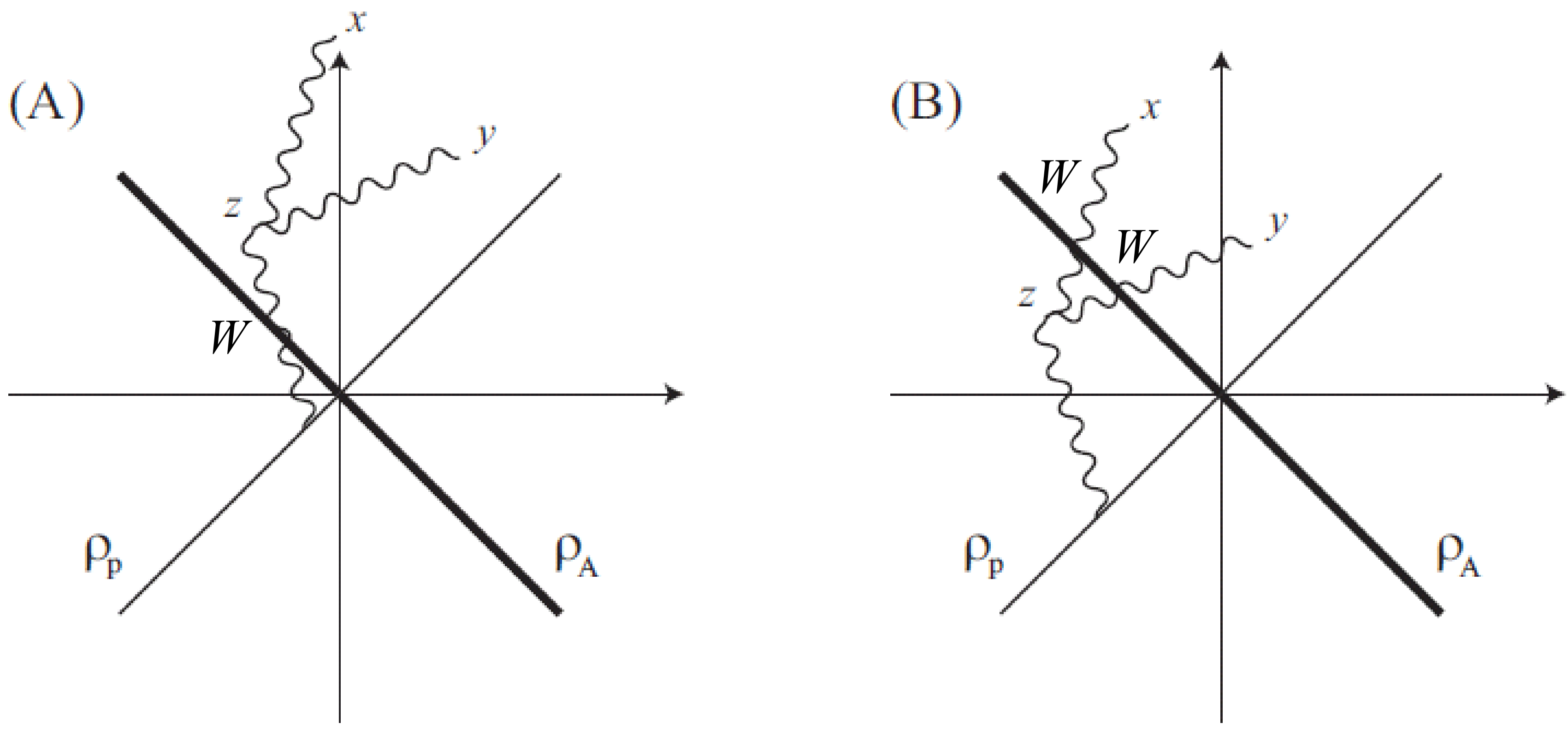
In LC gauge

$$\frac{\mathrm{d}N_{\mathrm{gg}}}{\mathrm{d}y_1 \mathrm{d}y_2} = \frac{1}{16\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2} \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} \sum_{\lambda,\sigma,a,b} \Big\langle \Big| \langle \boldsymbol{p}, \lambda, a; \boldsymbol{q}, \sigma, b | \Omega \rangle \Big|^2 \Big\rangle$$
$$= \frac{\mathrm{d}N_{\mathrm{g}}}{\mathrm{d}y_1} \frac{\mathrm{d}N_{\mathrm{g}}}{\mathrm{d}y_2} + \frac{1}{16\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2} \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} (p^2)^2 (q^2)^2\, 2\mathrm{Re}\Big[\mathcal{A}_a^{*i}(p)\, \mathcal{G}_{ab}^{ij}(p,q)\, \mathcal{A}_b^{*j}(q)\Big]$$
$$+ \frac{1}{16\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{p}_\perp}{(2\pi)^2} \frac{\mathrm{d}^2\boldsymbol{q}_\perp}{(2\pi)^2} (p^2)^2 (q^2)^2\, \mathcal{G}_{ba}^{\dagger\, ji}(q,p)\, \mathcal{G}_{ab}^{ij}(p,q)\,.$$



Two-Gluon Production in pA

Diagrams for the Connected Part



Fukushima-Hidaka

Dense-Dense (AA) Collision

Glasma = Glass + Plasma

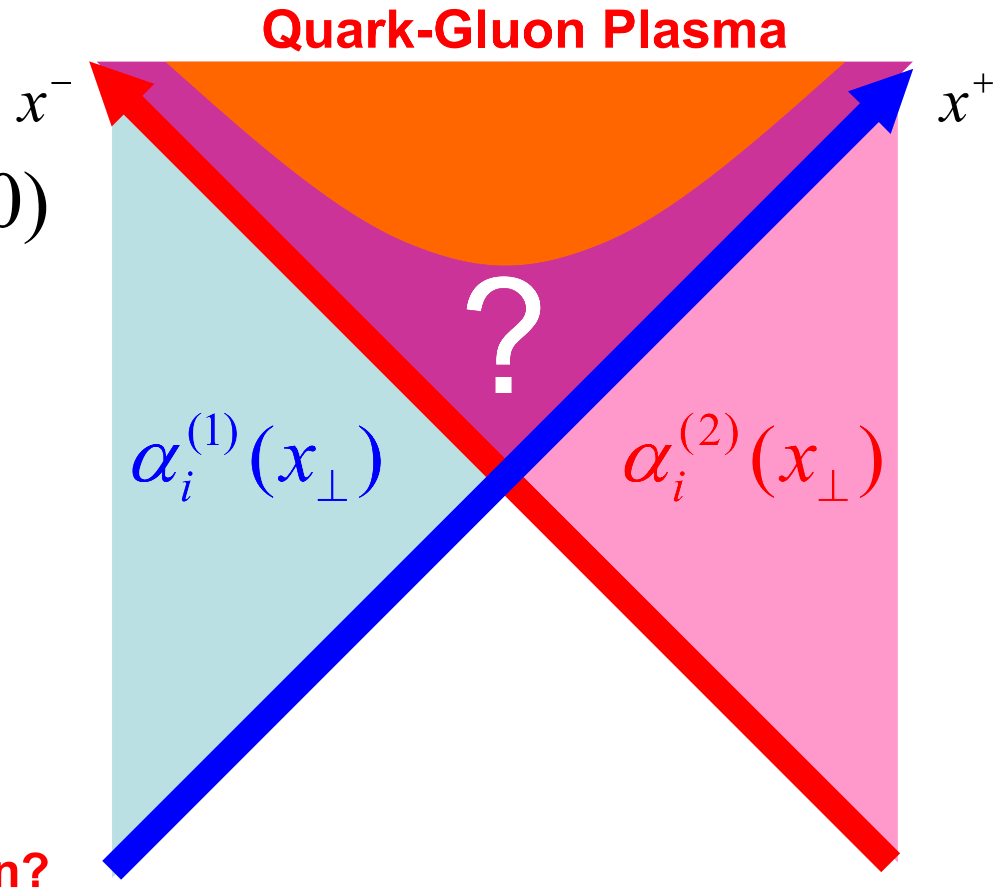
On the light cone ($\tau = 0$)

$$\mathcal{A}_i = \alpha_i^{(1)} + \alpha_i^{(2)}$$

$$\mathcal{A}_\eta = 0$$

$$\mathcal{E}^i = 0$$

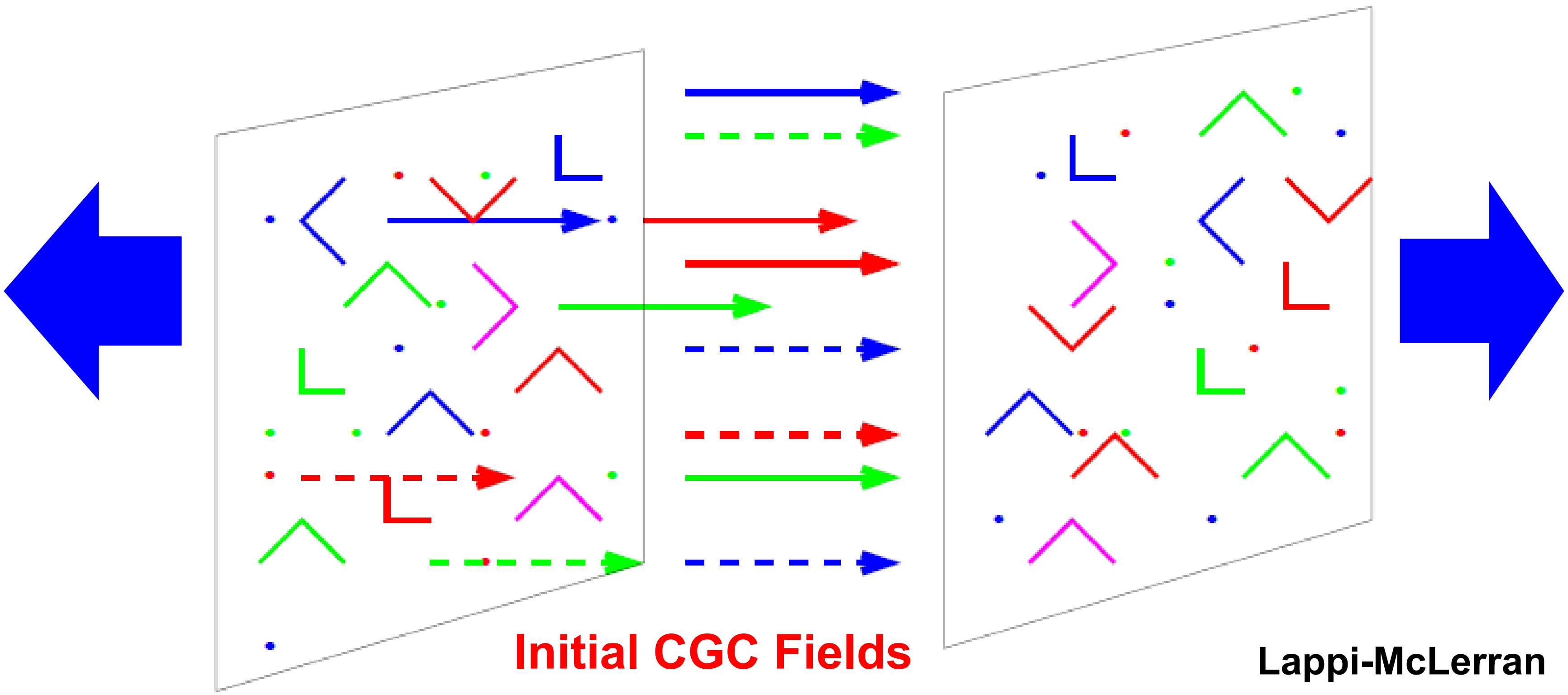
$$\mathcal{E}^\eta = ig[\alpha_i^{(1)}, \alpha_i^{(2)}]$$



How to compute gluon production?

Intuitive Picture at $\tau = 0$

Longitudinal Fields between Nucleus Sheets



Lappi-McLerran

Gaussian Average at τ^o

- Energy density

UV div.
(unphysical)

$$\varepsilon_{(0)} = \left\langle \mathrm{tr}\left[E^{\eta}_{(0)}E^{\eta}_{(0)} + B^{\eta}_{(0)}B^{\eta}_{(0)}\right]\right\rangle$$

$$= 2N_c(N_c^2-1)g^2\langle\alpha\alpha\rangle^2 = g^6\mu_A^4\cdot\frac{3}{\pi^2}\left[\ln\frac{\Lambda}{m}\right]^2$$

IR div.
(physical)

- Momentum decomposition

$$a \equiv \frac{N_c(g^2\mu_A)^2}{2m^2}$$

$$\varepsilon_{(0)}(k_\perp) = \frac{1}{V}\int d^2\boldsymbol{x}_\perp d^2\boldsymbol{y}_\perp\, e^{-i\boldsymbol{k}_\perp(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)}$$

$$\times\left\langle \mathrm{tr}\left[E^{\eta}_{(0)}(\boldsymbol{x}_\perp)E^{\eta}_{(0)}(\boldsymbol{y}_\perp) + B^{\eta}_{(0)}(\boldsymbol{x}_\perp)B^{\eta}_{(0)}(\boldsymbol{y}_\perp)\right]\right\rangle$$

$$= \frac{1}{2}N_c(N_c^2-1)g^6\mu_A^4$$

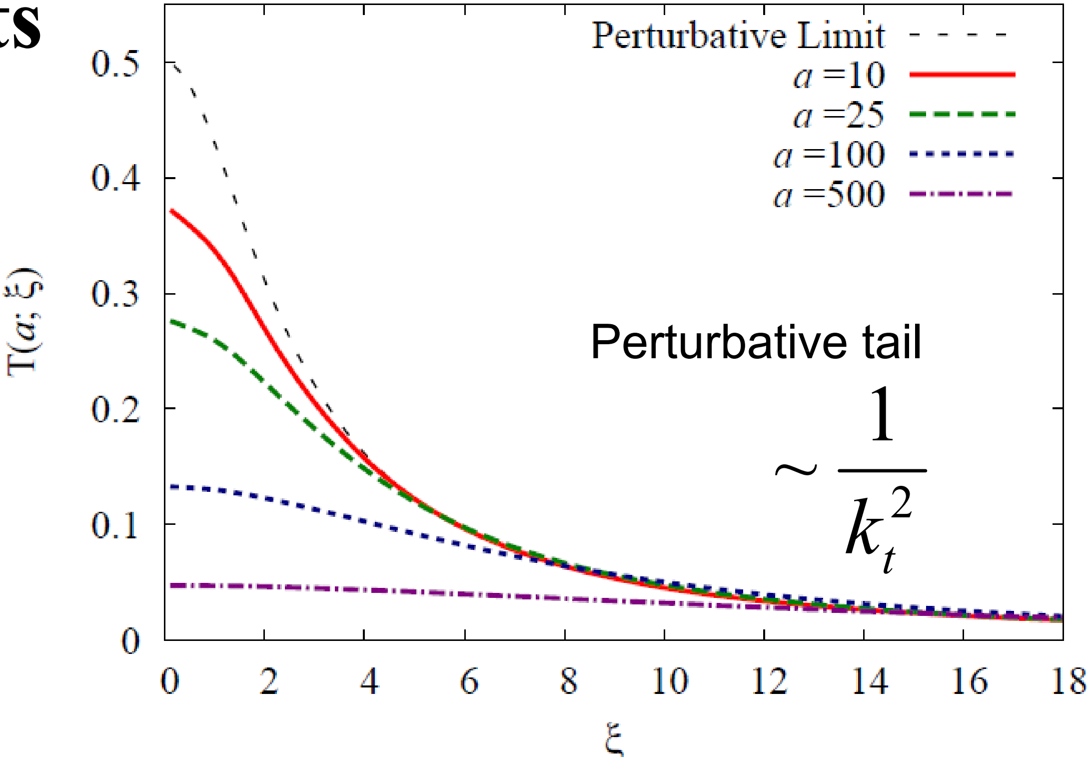
Expressed by $K_1(x)$

$$\times\int\frac{d^2\boldsymbol{q}_{1\perp}}{(2\pi)^2}\frac{d^2\boldsymbol{q}_{2\perp}}{(2\pi)^2}\frac{d^2\boldsymbol{q}_{3\perp}}{(2\pi)^2}\frac{\bar{C}_{\mathrm{adj}}(\boldsymbol{q}_{1\perp})\bar{C}_{\mathrm{adj}}(\boldsymbol{q}_{2\perp})}{(\boldsymbol{q}_{3\perp}^2+m^2)[(\boldsymbol{k}'_\perp-\boldsymbol{q}_{3\perp})^2+m^2]}$$

Spectral Intensity

Numerical Results

Physical choice is
***a* =10 ~ 40 at RHIC**



$$\varepsilon_{(0)}(k_\perp) = \frac{1}{4\pi m^2} N_c (N_c^2 - 1) g^6 \mu_A^4 \, \mathcal{T}(a; k_\perp / m)$$

UV Dominance

UV Contribution Dominance

→ **UV divergences come from derivatives.**

→ **Time evolution from derivative terms in EoM.**

$$E^\eta(\tau, \boldsymbol{k}_\perp) = E^\eta_{(0)}(\boldsymbol{k}_\perp) J_0(k_\perp \tau)\,,$$

$$E^i(\tau, \boldsymbol{k}_\perp) = E^i_{(2)}(\boldsymbol{k}_\perp) \frac{2\tau}{k_\perp} J_1(k_\perp \tau)\,,$$

$$B^\eta(\tau, \boldsymbol{k}_\perp) = B^\eta_{(0)}(\boldsymbol{k}_\perp) J_0(k_\perp \tau)\,,$$

$$B^i(\tau, \boldsymbol{k}_\perp) = B^i_{(2)}(\boldsymbol{k}_\perp) \frac{2\tau}{k_\perp} J_1(k_\perp \tau)\,.$$

Decreasing functions as k goes larger.

UV div is tamed by expansion.

Kovchegov
c.f. Fujii-Itakura

Gluon Distribution

Energy and Gluon Distribution (Multiplicity)

$$\varepsilon = \langle \mathcal{H} \rangle \qquad n(\tau, \boldsymbol{k}_\perp) = \frac{1}{\sqrt{k_\perp^2 + m^2}} \varepsilon(\tau, \boldsymbol{k}_\perp)$$

$$\frac{dE(\tau)}{d\eta}$$

Fourier decomposition

$$\frac{dN(\tau)}{d\eta}$$

Result

Fujii-Fukushima-
-Hidaka

$$\frac{dN(\tau)}{d\eta} = \pi R_A^2 \, \tau n(\tau) = \frac{3\pi R_A^2}{\pi^2 m^2} \cdot \frac{1}{g^2} (g^2 \mu_A)^4 (m\tau) I_N(a; m\tau)$$

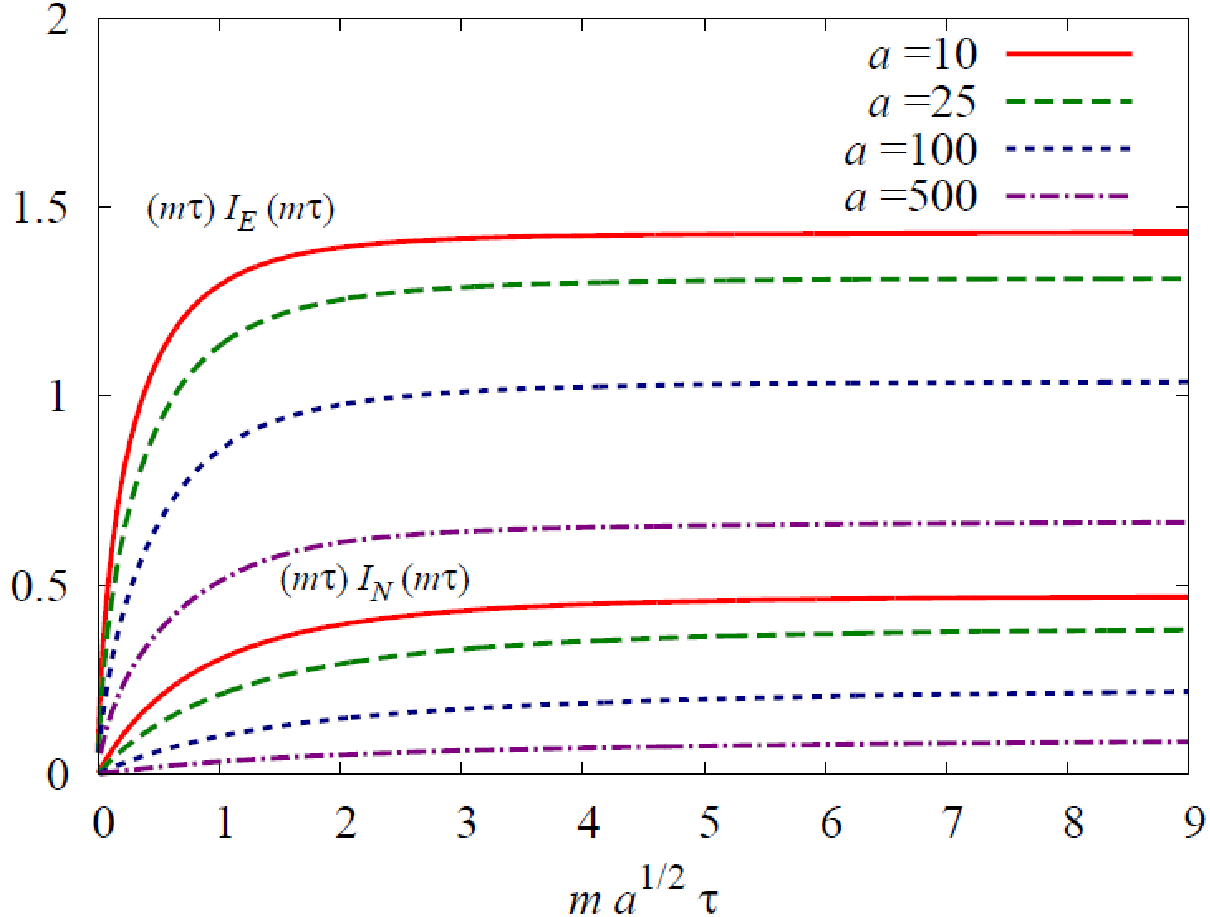
$$I_N(a; m\tau) = \int_0^\infty d\xi \, \frac{\xi \, \mathcal{T}(\xi)}{\sqrt{\xi^2 + 1}} \left\{ \left[J_0(\xi m\tau) \right]^2 + \left[J_1(\xi m\tau) \right]^2 \right\}$$

c.f. Gunion-Bertsch

Consistent with the LSZ reduction formula! (Gelis-Venugopalan)

Well-Defined Asymptotics

Numerical Results (~$dN/d\eta$)



Numerics

Results (with non-linear corrections) **for $Q_s^2 = 1\sim2\text{GeV}^2$**

a	$dE/d\eta$ (GeV)	$dN/d\eta$	c
10	$(2.8 - 7.8) \times 10^3$	$(0.7 - 1.4) \times 10^3$	0.86
25	$(2.5 - 7.0) \times 10^3$	$(0.9 - 1.8) \times 10^3$	1.10
100	$(1.6 - 4.5) \times 10^3$	$(1.0 - 1.9) \times 10^3$	1.18
500	$(1.0 - 2.9) \times 10^3$	$(1.0 - 1.9) \times 10^3$	1.17

$$\frac{1}{\pi R_A^2}\frac{dN}{d\eta} = c\,\frac{N_c^2 - 1}{\pi g^2 N_c}\,Q_s^2$$

Consistent with the empirical value at a =25 and Q_s^2 =1GeV^2
Hadron multiplicity at mid rapidity ~ 1100

Summary

- Saturation models enable us to carry out economical resummation of multiple scattering with dense color source.
- Gluon multiplicity can be computed.
  - One-gluon production in pA
  - Two-gluon production in pA, ...etc
- Gluon multiplicy in the expanding geometry in AA can be computed.