格子QCDによる核子の一般化形状因子とクォークの角運動量の解析

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for UKQCD-QCDSF Collab.

- Introduction
- Form Factors and Physical Quantities
- Numerical results of lattice simulation
  - Axial FFs and quark spin
  - Moments of GPD(vector) and total angular momentum
- Summary

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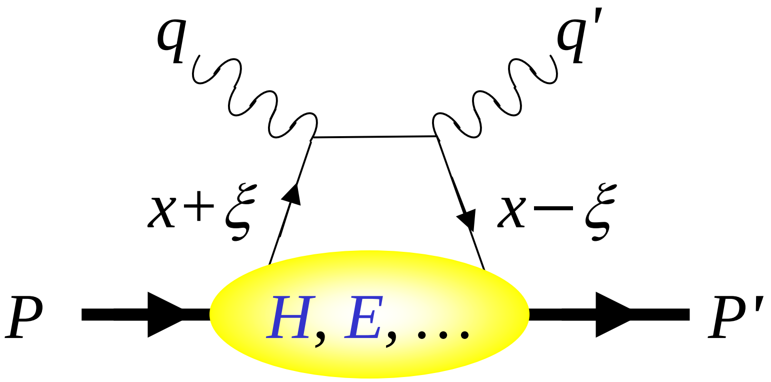
10 Jan. 核子構造研究の新展開2009@ KEK

Introduction

- Generalized Parton Distributions of Nucleon



momentum transfer squared:

$$t = (\Delta \equiv P' - P)^2$$

longitudinal mt. transfer:

$$\xi = -n \cdot \Delta / 2$$

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\tfrac{\eta n}{2}) \gamma^\mu \begin{bmatrix} 1 \\ \gamma_5 \end{bmatrix} \mathcal{U} q(\tfrac{\eta n}{2}) | P \rangle = \bar{N}(P') \Bigg(\gamma^\mu \begin{bmatrix} H(x,\xi,t) \\ \gamma_5 \tilde{H}(x,\xi,t) \end{bmatrix} + i \frac{\sigma^{\mu\nu}\Delta_\nu}{2M} \begin{bmatrix} E(x,\xi,t) \\ \gamma_5 \tilde{E}(x,\xi,t) \end{bmatrix} \Bigg) N(P)$$

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \bar{q}(-\tfrac{\eta n}{2}) \sigma^{\mu\nu} \mathcal{U} q(\tfrac{\eta n}{2}) | P \rangle = \bar{N}(P') \Bigg(i\sigma^{\mu\nu} H_{\rm T}(x,\xi,t) + \frac{\bar{P}^{[\mu}\Delta^{\nu]}}{M^2} \tilde{H}_{\rm T}(x,\xi,t)$$
$$+ \frac{\gamma^{[\mu}\Delta^{\nu]}}{2M} E_{\rm T}(x,\xi,t) + \frac{\gamma^{[\mu}\bar{P}^{\nu]}}{M} \tilde{E}_{\rm T}(x,\xi,t) \Bigg) N(P)$$

GPDs encode important information on Nucleon structure !

Generalized Parton Distributions

PDF

$$\begin{cases} q(x) = H(x,0,0) \\ \Delta q(x) = \widetilde{H}(x,0,0) \\ \delta q(x) = H_T(x,0,0) \end{cases}$$

Form Factors

$$\begin{cases} F_1(t) = \int dx\, H(x,\xi,t) \\ G_A(t) = \int dx\, \widetilde{H}(x,\xi,t) \\ G_T(t) = \int dx\, H_T(x,\xi,t) \end{cases}$$

forward limit

local limit

GPDs

Fourier transf.

as moments in the forward limit

Quark density in $b_\perp$ plane

$$q(x, b_\perp) = \int d^2\Delta_\perp\, e^{-i\,\Delta_\perp b_\perp}\, H(x, \xi{=}0, \Delta^2)$$

Angular momentum X.Ji, PRL**78**(1997)

$$\begin{cases} \langle J\rangle^q = 1/2 \int dx\, x\,(H(x,\xi,0) + E(x,\xi,0))^{u+d} \equiv 1/2\,(A_{20} + B_{20}) \\ \langle s\rangle^q = 1/2 \int dx\, \widetilde{H}(x,\xi,0)^{u+d} \equiv 1/2\, \widetilde{A}_{10} \end{cases}$$

Moments of GPD: Generalized Form Factors

- Polynomiality

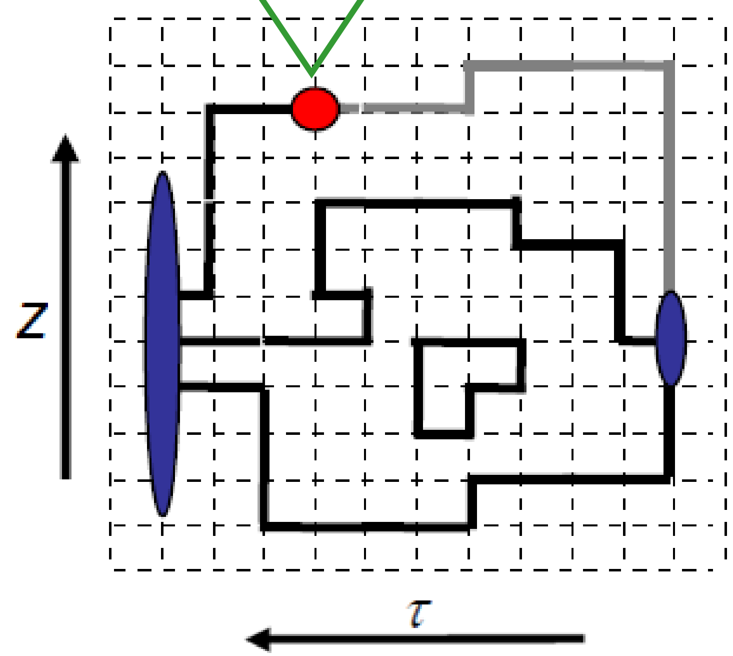
X.Ji, J.Phys.**G24**(1998)1181

$$\int dx\, x^{n-1} \begin{bmatrix} H(x,\xi,t) \\ E(x,\xi,t) \end{bmatrix} = \sum_{k=0}^{[n-1/2]} (2\xi)^{2k} \begin{bmatrix} A_{n,2k}(t) \\ B_{n,2k}(t) \end{bmatrix} \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

$A_{n,2k}$, $B_{n,2k}$, C_n are related to $\langle P \,|\, \bar{\boldsymbol{q}}\, \gamma^{\{\mu_1} \boldsymbol{D}^{\mu_2} \cdots \boldsymbol{D}^{\mu_n\}} \boldsymbol{q} \,|P'\rangle$

Calculate ratio of 2pt & 3pt correlation functions on lattice

Extract GFF



LHPC, PRD**68**(2003)034505

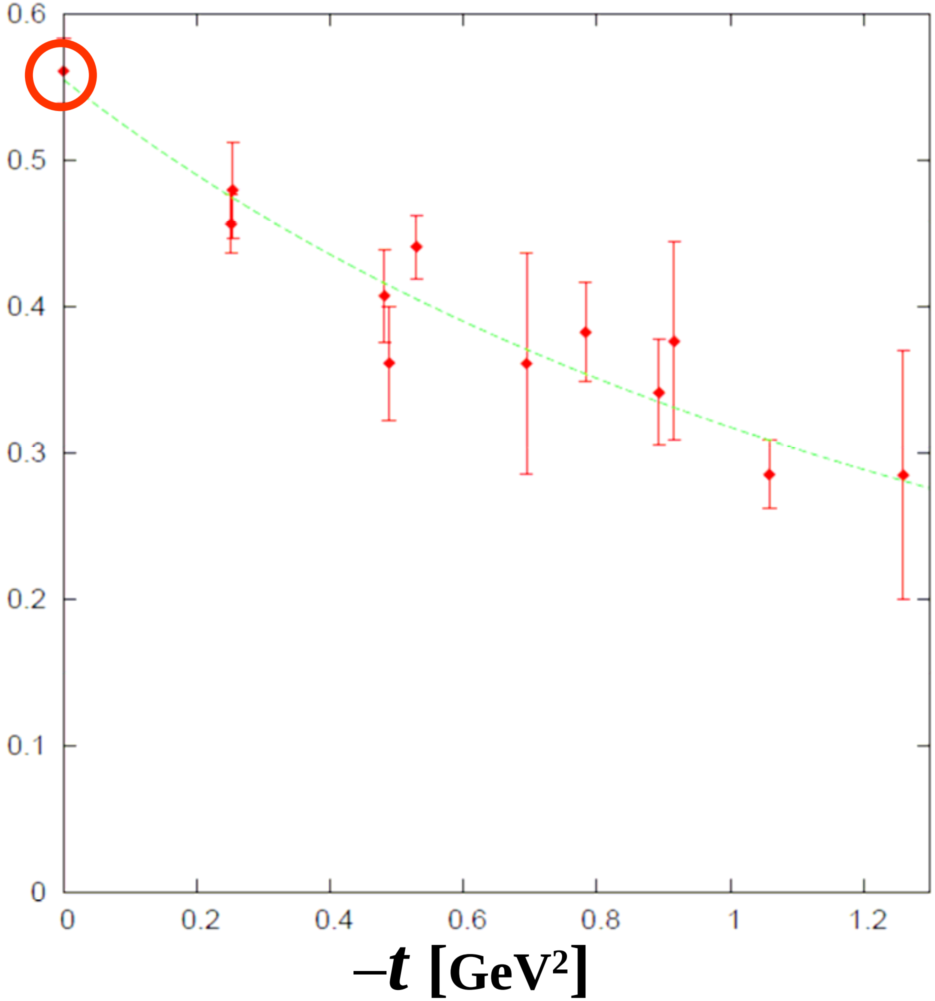
Simulation parameters

β	κ	volume	a [fm]	m_π[GeV]
5.20	0.13420	$16^3 \times 32$	0.0856	1.347
	0.13500	"		0.956
	0.13550	"		0.670
5.25	0.13460	"	0.0794	1.225
	0.13520	"		0.949
	0.13575	$24^3 \times 48$		0.635
	0.13600	"		0.457
5.29	0.13400	$16^3 \times 32$	0.0753	1.511
	0.13500	"		1.102
	0.13550	$24^3 \times 48$		0.857
	0.13590	"		0.629
	0.13620	"		0.414
	0.13632	$32^3 \times 64$		0.279
5.40	0.13500	$24^3 \times 48$	0.0672	1.183
	0.13560	"		0.917
	0.13610	"		0.648
	0.13625	"		0.559
	0.13640	"		0.451
	0.13660	$32^3 \times 64$		0.255

- $N_f = 2$ Wilson fermions w/ clover improvement
- # of config: 400-2200 for each (β, κ)
- Physical unit translated by r_0^c / a
- $O(a)$ improved operators
- non-perturbative renormalization into $\overline{\text{MS}}$ @ $\mu = 2$ GeV

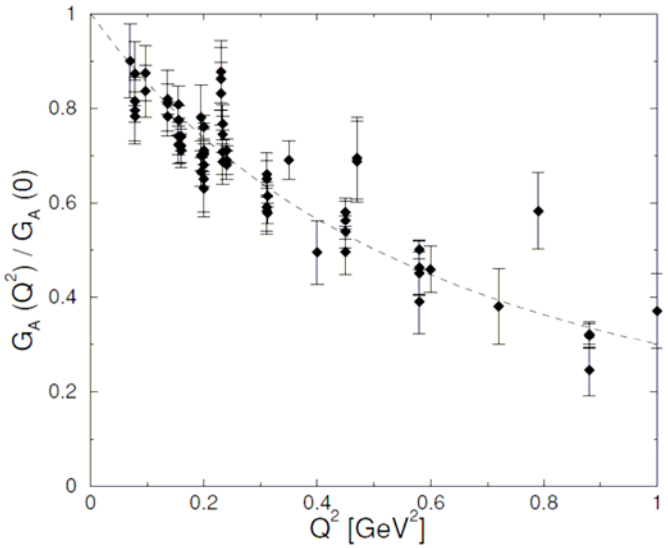
t dependence of Axial Form Factor

$\tilde{A}_{10}^{u+d}(t)$ with $\beta = 5.29, \kappa = 1.3632$



cf. $\tilde{A}_{10}^{u\text{-}d}$ from expt: $p + e \to e' + \pi^+ + n$

V.Bernard *et.al.* J.Phys.G **28**(2002)R1



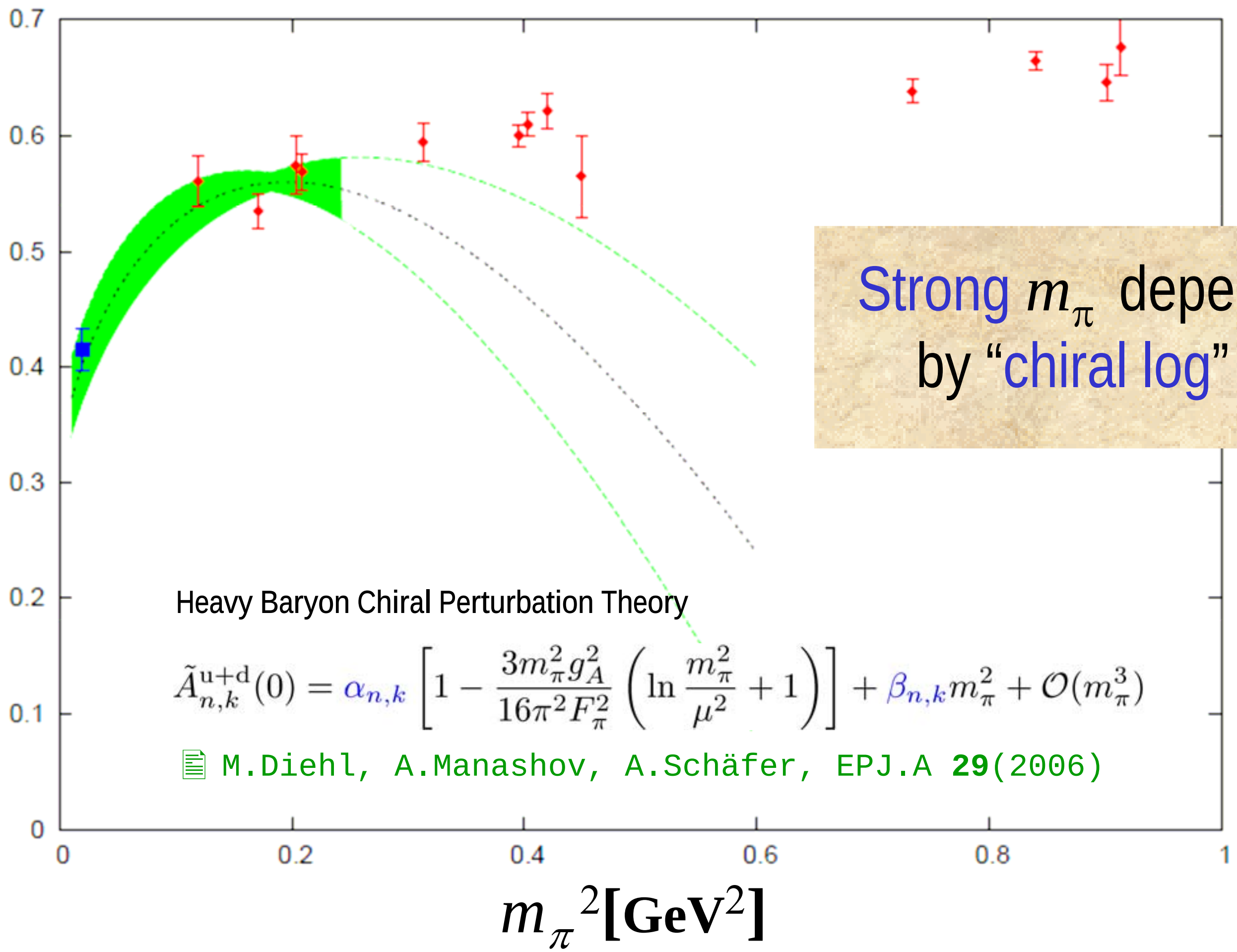
Dipole form:

$$\tilde{A}_{10} \approx \frac{\#}{(1 - t / m_A^2)^2}$$

Chiral extrapolation and quark spin

$\tilde{A}_{10}^{u+d}(t=0)$: $2\langle s \rangle^{u+d} = \Delta\Sigma^{u+d} \sim$ **0.402 ±0.024** ($@\, m_\pi = .14$GeV)

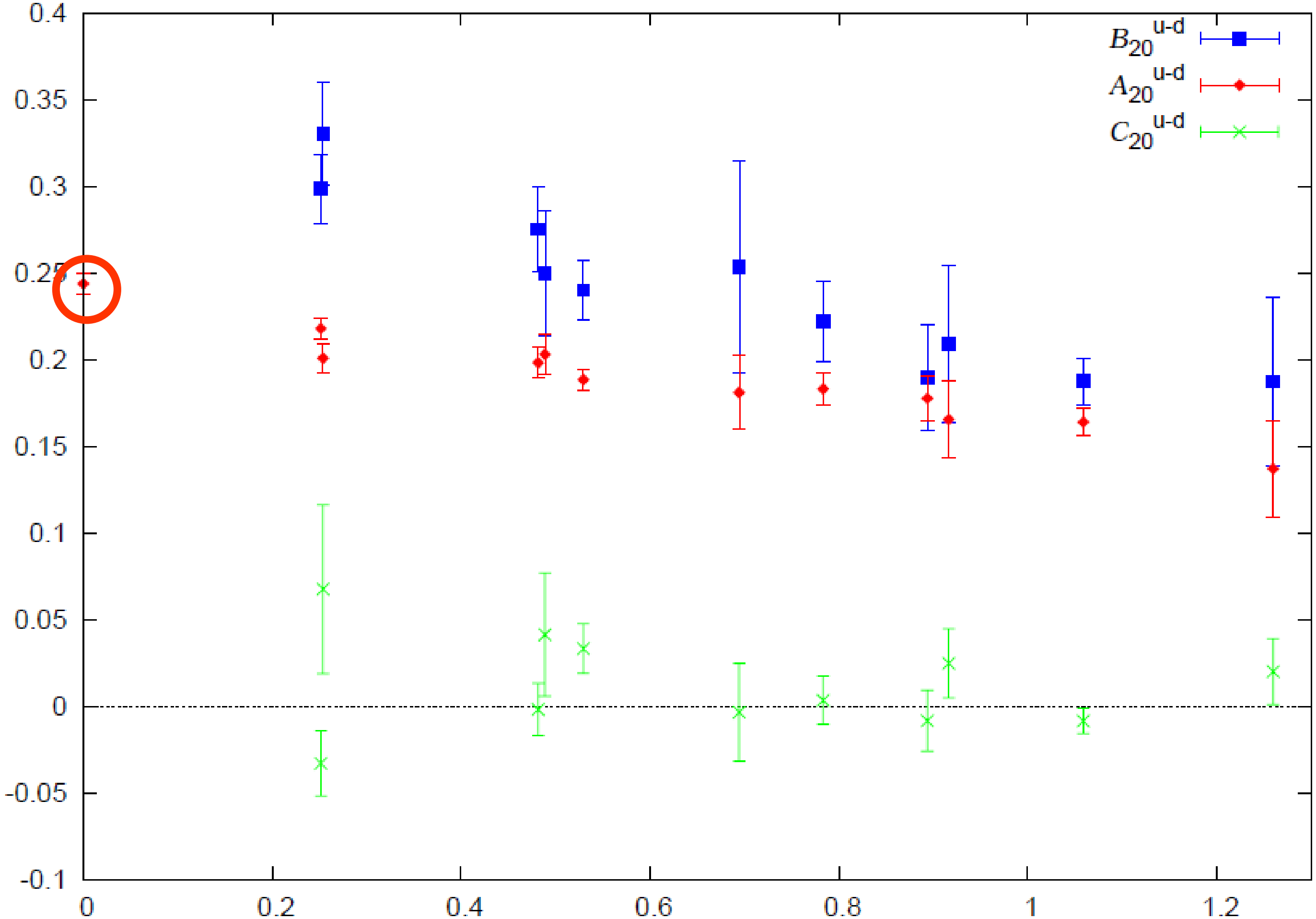
HERMES, PRD**75**(2007)012007 ⇒



Strong m_π dependence by "chiral log" term

t dependence of the 2nd Moments

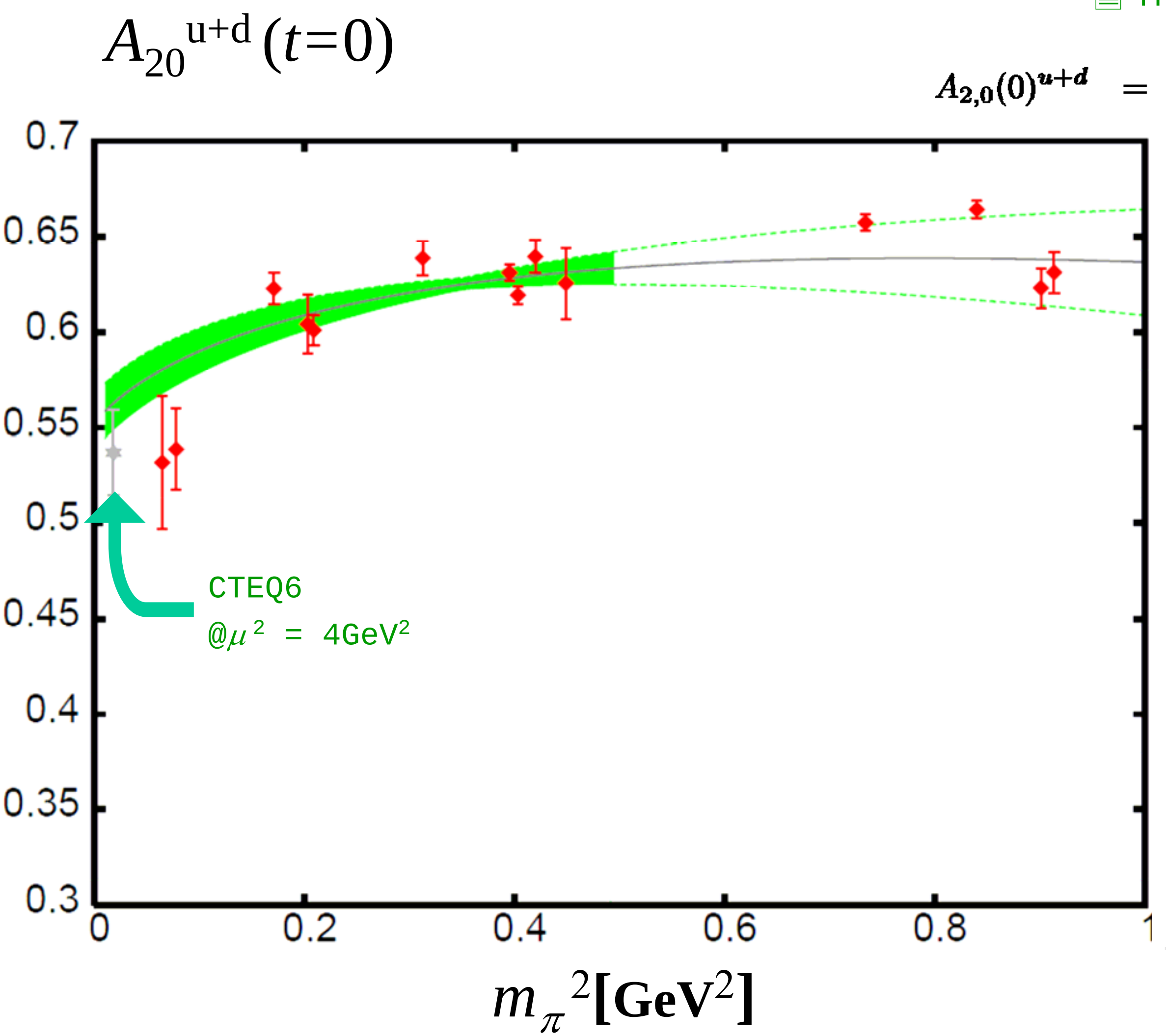
$A_{20}^{u\text{-}d}(t)$, $B_{20}^{u\text{-}d}(t)$ and $C_{20}^{u\text{-}d}(t)$ with $\beta = 5.29, \kappa = 1.3632$



$-t$ [GeV²]

Chiral extrapolation of $A_{20}^{u+d}(t=0)$

M.Dorati *et.al.* nucl-th/0703073



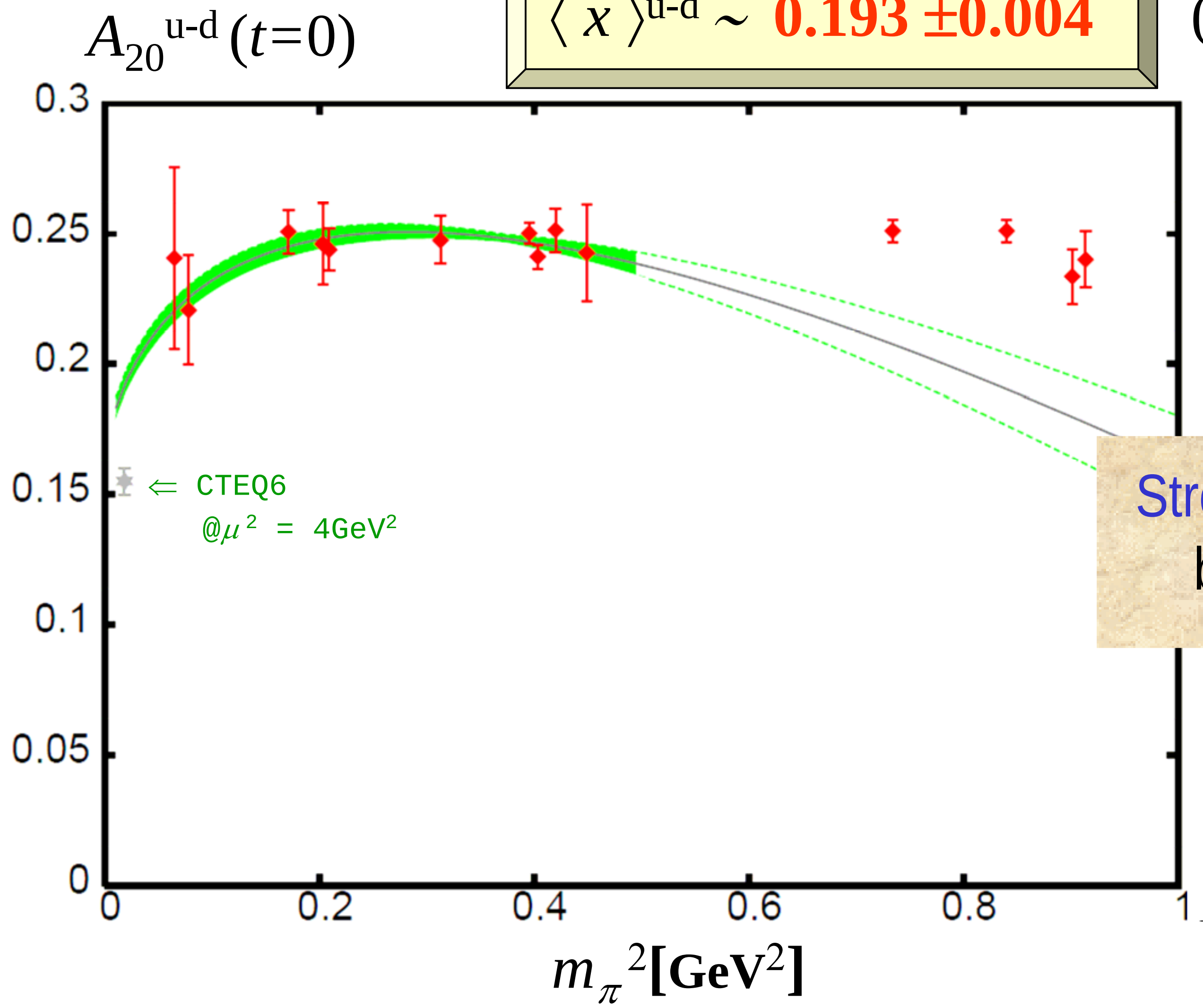
$$A_{2,0}(0)^{u+d} = a^s_{2,0} + c_9 \frac{4m_\pi^2}{M_0^2} + \mathcal{O}(p^3)$$
$$- a^s_{2,0}\frac{3g_A^2 m_\pi^2}{16\pi^2 F_\pi^2}\left[\frac{m_\pi^2}{M_0^2} + \frac{m_\pi^2}{M_0^2}\left(2-\frac{m_\pi^2}{M_0^2}\right)\ln\frac{m_\pi}{M_0}\right.$$
$$\left. + \frac{m_\pi}{\sqrt{4M_0^2-m_\pi^2}}\left(2-4\frac{m_\pi^2}{M_0^2}+\frac{m_\pi^4}{M_0^4}\right)\arccos\frac{m_\pi}{2M_0}\right]$$

$\langle x \rangle^{u+d} \sim$ **0.563 ±0.014**

$(@\ m_\pi = .14\mathrm{GeV})$

Chiral extrapolation of $A_{20}^{u\text{-}d}(t=0)$

$\langle x \rangle^{u\text{-}d} \sim$ **0.193 ±0.004** (@ m_π = .14GeV)



Strong m_π dependence
by "chiral log" term

Generalized Form Factors in Chiral Perturbation

M.Dorati, T.A.Gail and T.R.Hemmert, nucl-th/0703073

$$B_{2,0}^{v}(t) = (b_{2,0}^{v} + \hat{\delta}_B\, m_\pi^2 + \hat{\delta}_B^t\, t)\, \frac{M_N(m_\pi)}{M_0} + \frac{a_{2,0}^{v} g_A^2 M_0^2}{48\pi^2 F_\pi^2} G(t)$$

$$B_{2,0}^{s}(t) = (b_{2,0}^{s} + \hat{B}_{33}\, m_\pi^2 + \hat{B}_{34}\, t)\, \frac{M_N(m_\pi)}{M_0} - \frac{a_{2,0}^{s} g_A^2 M_0^2}{16\pi^2 F_\pi^2} G(t)$$

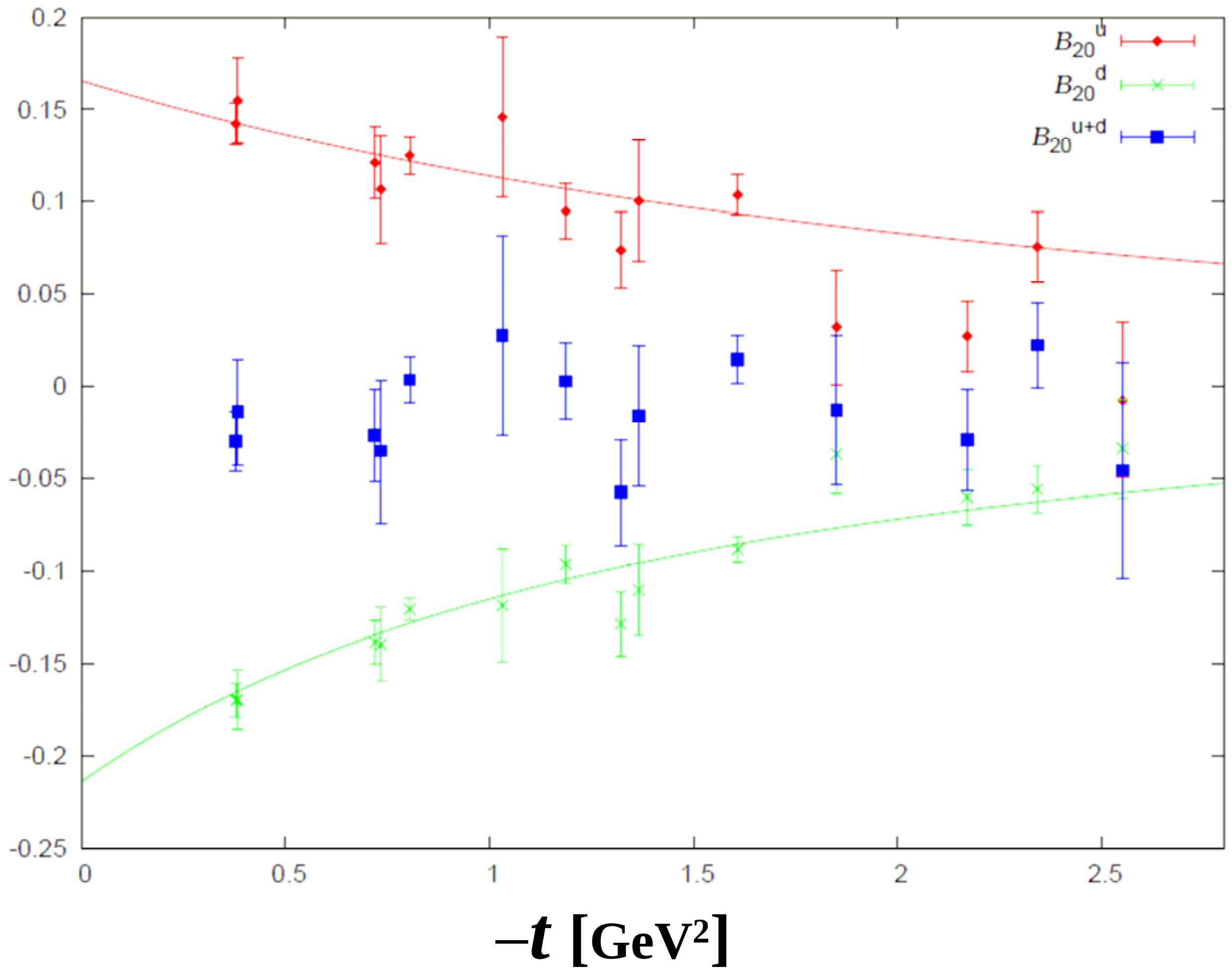
$$G(t) = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{du}{\tilde{M}^8} \left[\left(M_0^2 - \tilde{M}^2\right) \tilde{M}^6 + 9 m_\pi^2 M_0^2 \tilde{M}^4 - 6 m_\pi^4 M_0^2 \tilde{M}^2 + 6 m_\pi^2 M_0^2 \left(m_\pi^4 - 3 m_\pi^2 \tilde{M}^2 + \tilde{M}^4\right) \log \frac{m_\pi}{\tilde{M}} - \frac{6 m_\pi^3 M_0^2}{\sqrt{4\tilde{M}^2 - m_\pi^2}} \left(m_\pi^4 - 5 m_\pi^2 \tilde{M}^2 + 5\tilde{M}^4\right) \arccos \frac{m_\pi}{2\tilde{M}} \right]$$

- 3 param. in each GFFs
- t dependence via

$$\tilde{M}^2 = M_0^2 + \left(u^2 - \tfrac{1}{4}\right) t$$

Dipole fit and forward limit of $B_{20}^{u,d}(t)$

$$B_{20}^q(t) \overset{t\to 0}{\longrightarrow} 2\langle J \rangle^q - \langle x \rangle^q$$



in CQSM

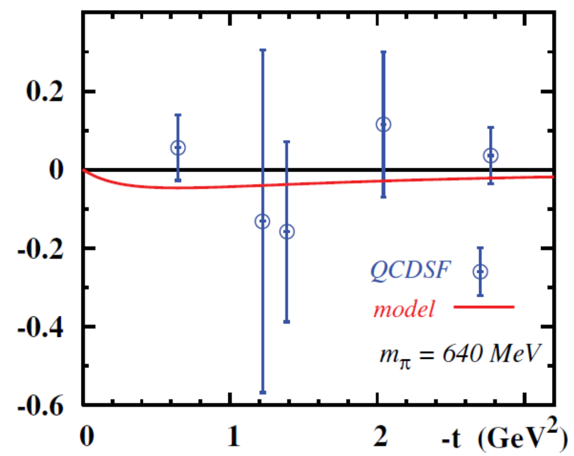
$\langle J \rangle^{u+d} = 1/2$

$\langle x \rangle^{u+d} = 1$

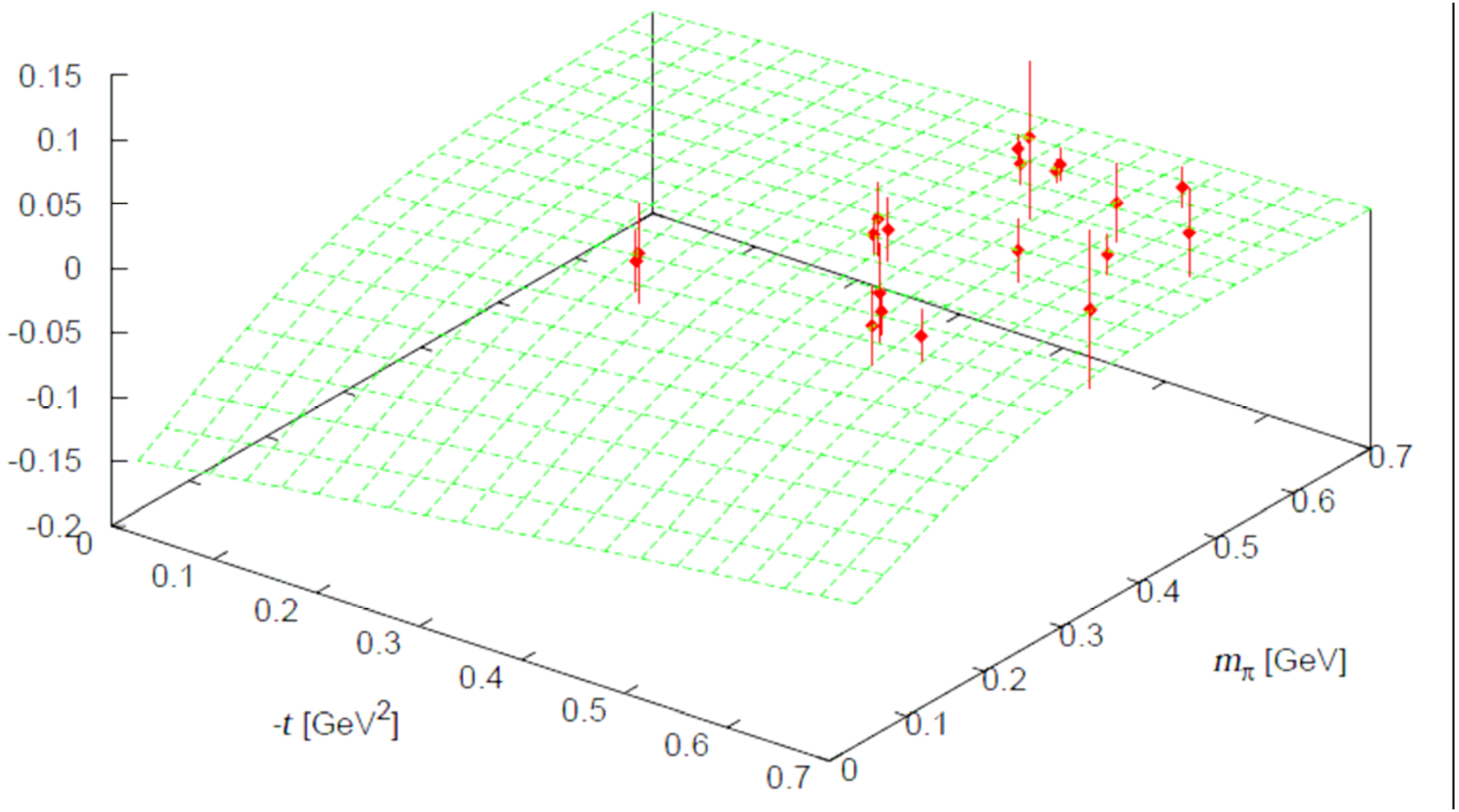
$\because$) no dynamical gluon

K.Goeke *et. al.*, PRC**75**(2007)

$B_2^{u+d}(t)$ in CQSM for comparison



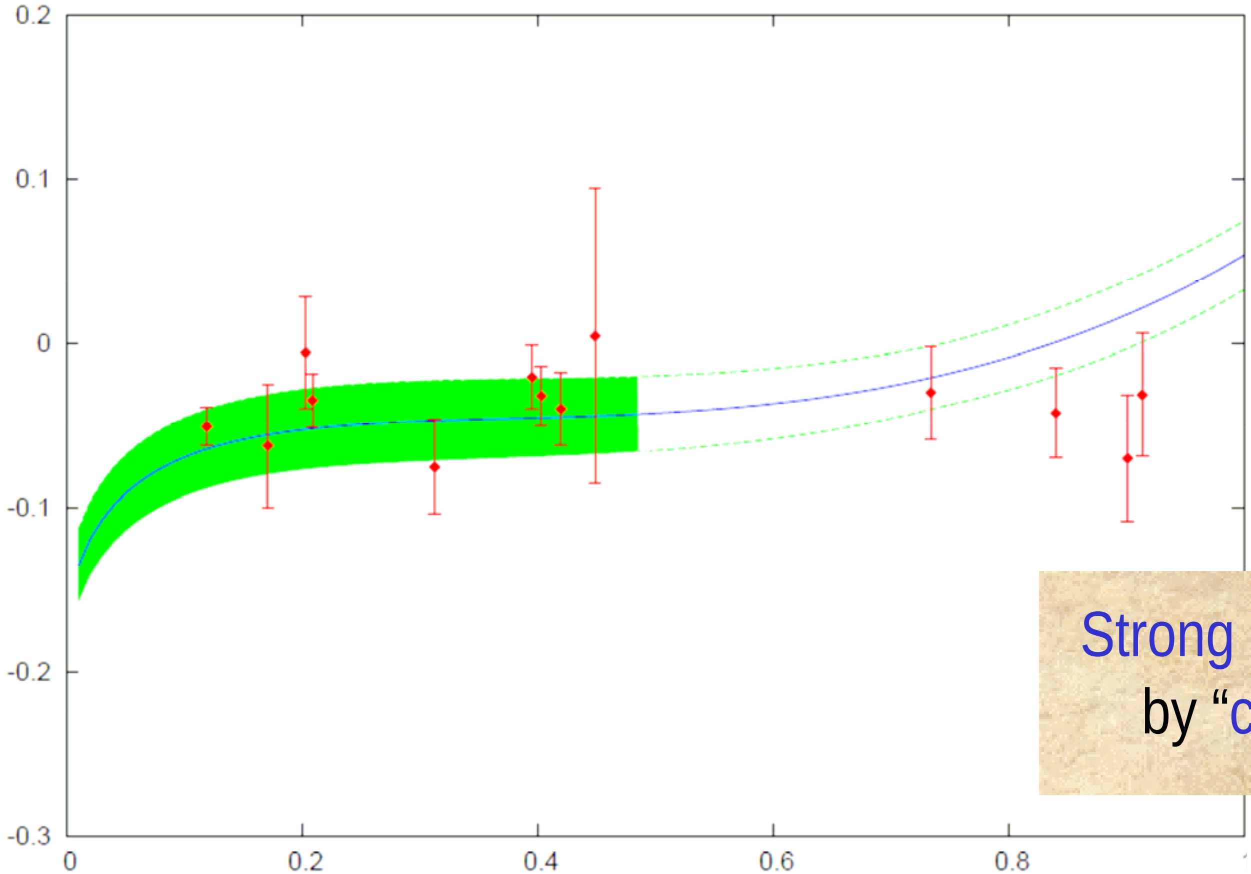
$B_{20}^{u+d}(t)$ and covariantized Baryon ChPT



Chiral extrapolation of $B_{20}^{u+d}(0)$

$B_{20}^{u+d}(0) \sim -0.120 \pm 0.023$ ($@\ m_\pi = .14\text{GeV}$)

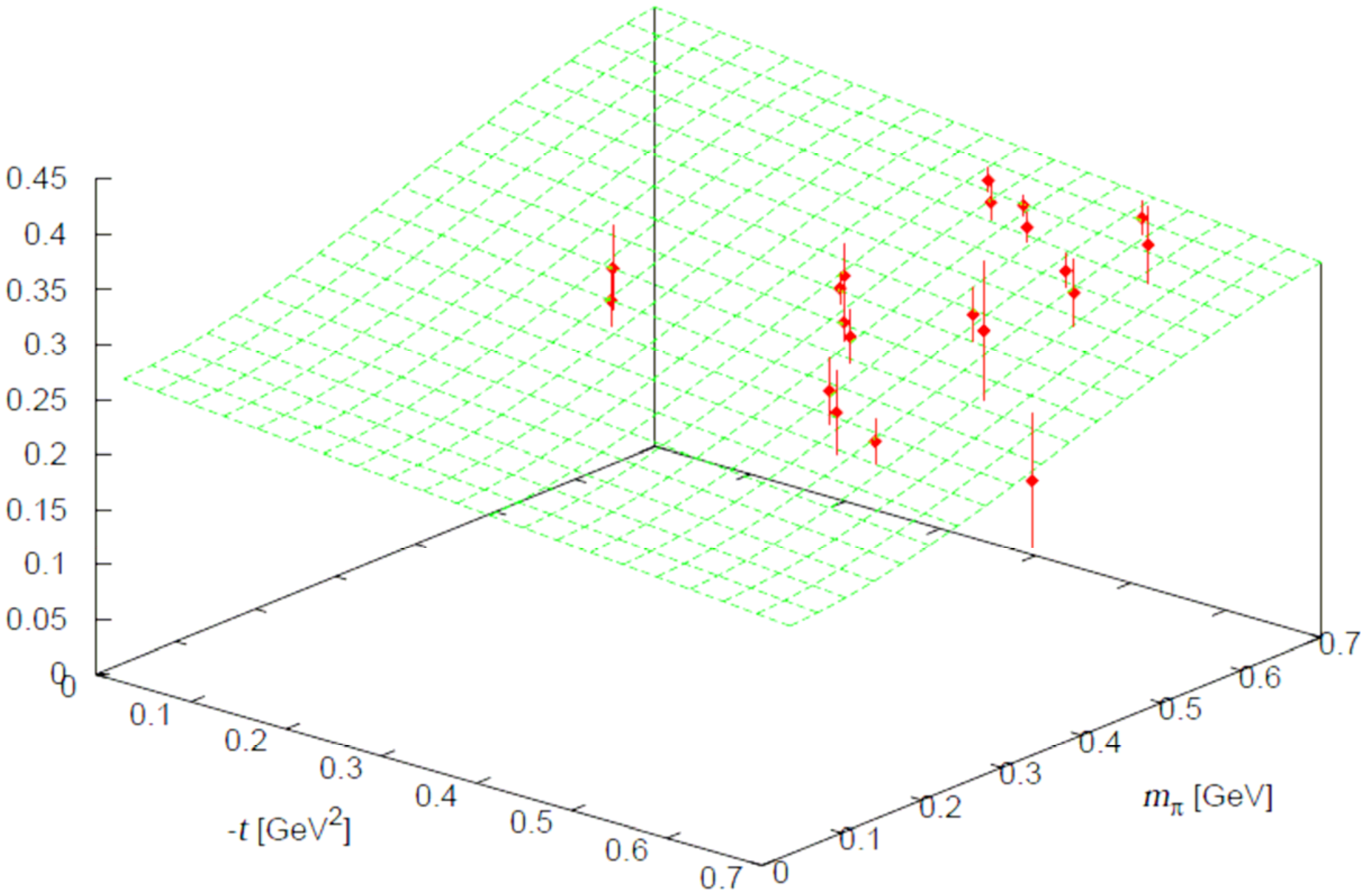
$B_{20}^{u+d}(0)$



m_π^2[GeV^2]

Strong m_π dependence
by "chiral log" term

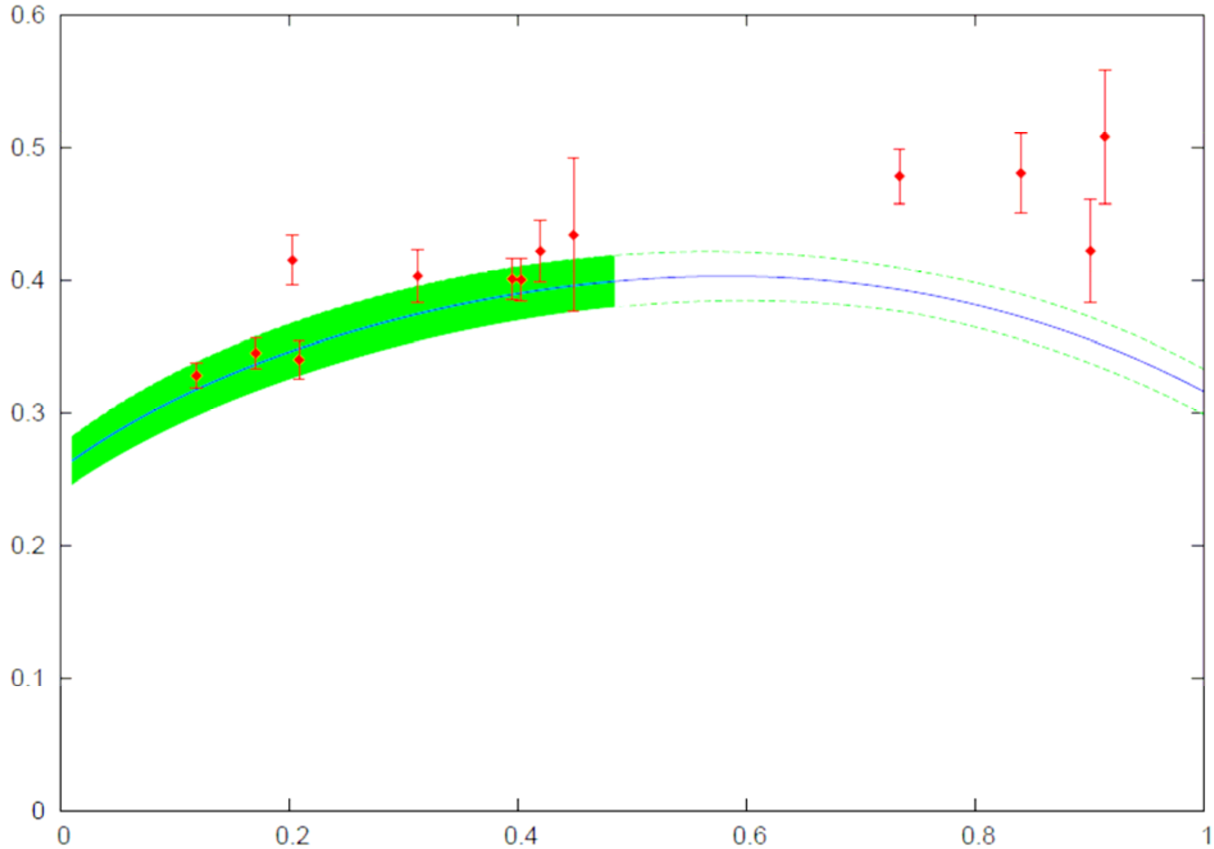
$B_{20}^{u-d}(t)$ and covariantized Baryon ChPT



Chiral extrapolation of $B_{20}^{u-d}(0)$

$B_{20}^{u-d}(0)$

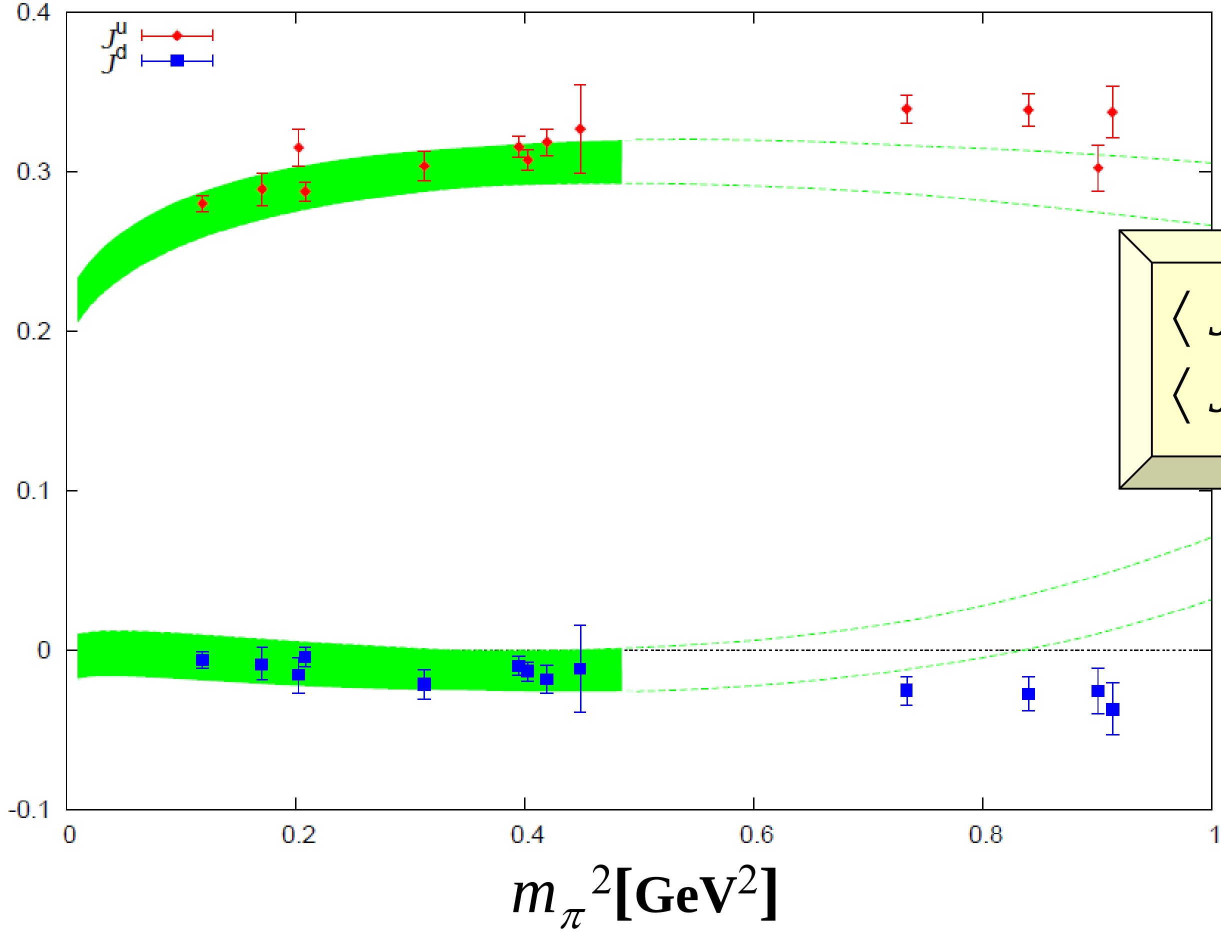
$B_{20}^{u-d}(0) \sim$ **0.269 ±0.020** ($@\ m_\pi = .14\text{GeV}$)



m_π^2**[GeV^2]**

Chiral extrapolation of J^u , J^d

Ji's sum rule : $\langle J \rangle^q = ½ [A_{20}^q(0) + B_{20}^q(0)]$



$\langle J \rangle^u \sim$ **0.226 ±0.009**

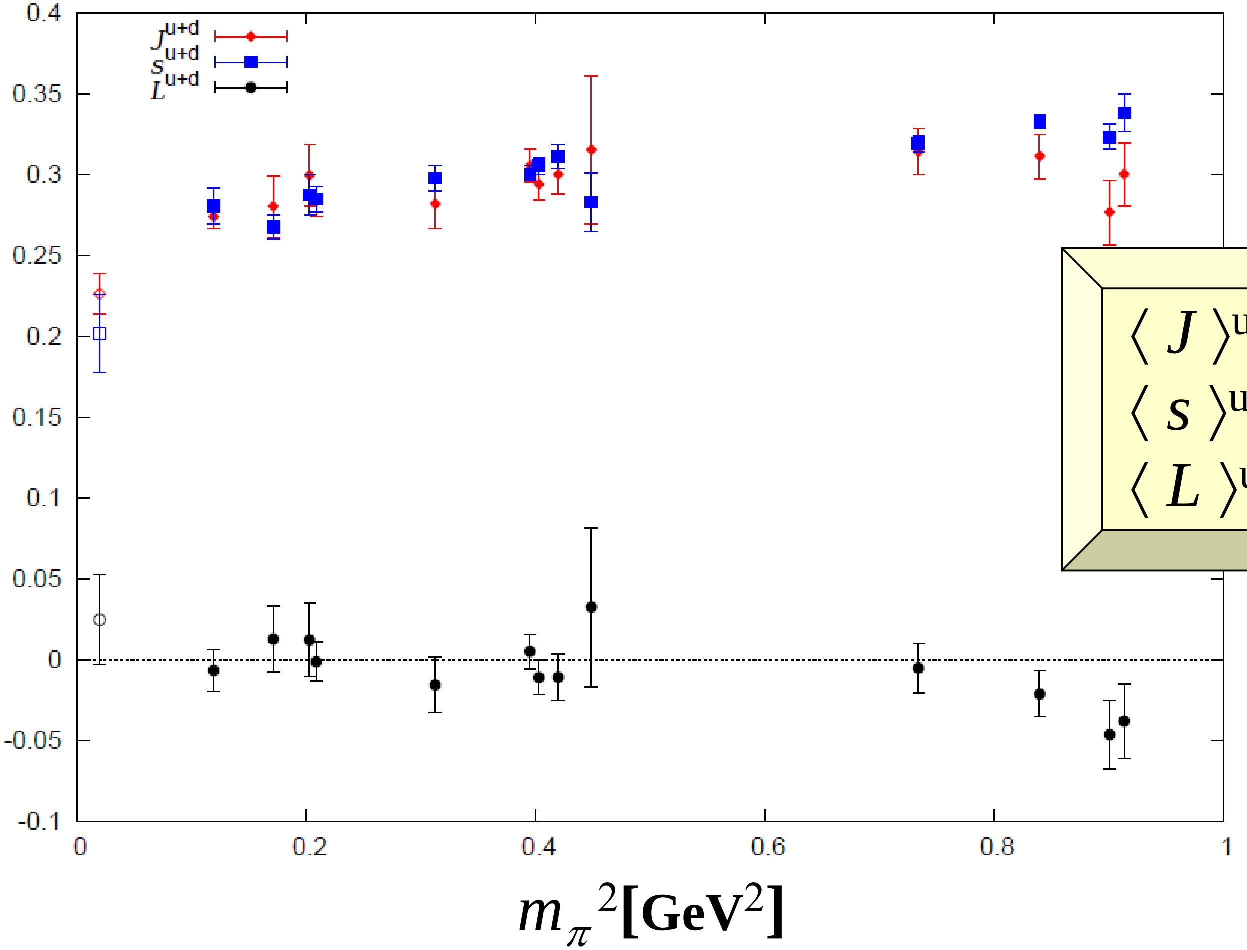
$\langle J \rangle^d \sim$ **−0.005 ±0.009**

($@\ m_\pi = .14$GeV)

decomposition of quark angular momentum

$$\begin{cases} \langle J \rangle^{u+d} = \tfrac{1}{2}\,[\, A_{20}^{u+d}(0) + B_{20}^{u+d}(0)\,] \\ \langle s \rangle^{u+d} = \tfrac{1}{2}\, \widetilde{A}_{10}^{u+d}(0) \end{cases}$$

; $L^{u+d} = \langle J \rangle^{u+d} - \langle s \rangle^{u+d}$



$\langle J \rangle^{u+d} \sim$ **0.222 ±0.014**

$\langle s \rangle^{u+d} \sim$ **0.201 ±0.024**

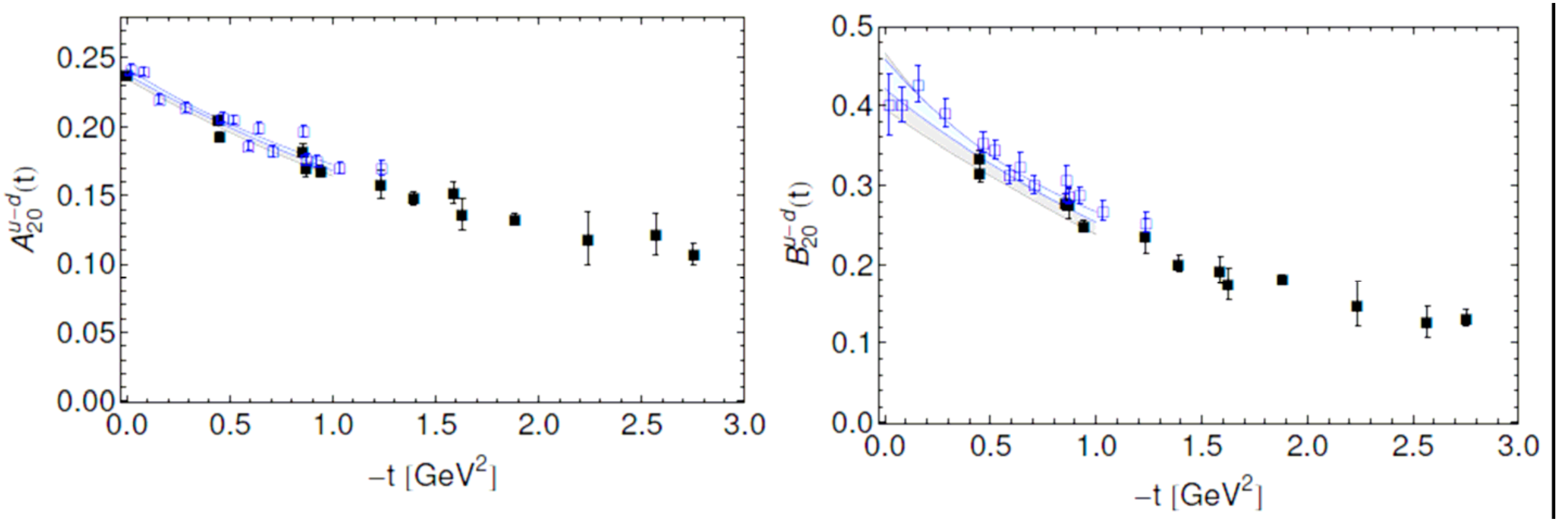
$\langle L \rangle^{u+d} \sim$ **0.021 ±0.028**

(@ m_π = .14GeV)

Twisted boundary condition

$$q(x_k + L) = e^{i\theta_k} q(x_k) \quad \Rightarrow \quad -t = (\Delta E)^2 - [(\vec{p}_f + \vec{\theta}_f / L) - (\vec{p}_i + \vec{\theta}_i / L)]^2$$

$$\vec{p}_i \in 2\pi/L \times Z$$



Summary and outlook

- Generaized Parton Distribution

$\Longrightarrow$ spin content, transverse quark distribution, Form factors,…

- accessible experimentaly via DVCS
- theoretical calculations in CQSM, Skyrme model,…

- moments of GPD in lattice QCD

- $A_{20}^{u\text{-}d}$ and B_{20}^{u+d} have strong "chiral log" corrections.
- Chiral extrapolation of $A_{20}(0)$ & $B_{20}(0)$ via BChPT nucl-th/0703073 leads

$\langle J \rangle^{u} \sim$ **0.226 ±0.009**

$\langle J \rangle^{d} \sim$ **–0.005 ±0.009**

$\langle J \rangle^{u+d} \sim$ **0.222 ±0.014**

$\langle s \rangle^{u+d} \sim$ **0.201 ±0.024** ($@\ m_\pi = .14\text{GeV}$)

$\langle L \rangle^{u+d} \sim$ **0.021 ±0.028**

- lighter m_π, larger volume (for $t \to 0$), Finite size corrections, Continuum limit, disconnected diagram, $N_F = 2+1$…