Heterotic Type-II Duality

♣ Heterotic string vs Type-II string

Heterotic string

perturbative gauge symmetry

No D-branes, RR fluxes

$$g_s^2 = g_{YM}^2$$

Type II string

D-branes

gauge symmetry appears non-perturbatively

$$g_s = g_{YM}^2$$

How are they dual to each other?

Heterotic string on $T^4 \Longleftrightarrow$ Type IIA string on K_3 at 6-dimension

Hull-Townsend

Duality group: $SO(20,4)$

Moduli space: $\dfrac{SO(20,4)}{SO(20)SO(4)}$, moduli of K_3 surface: $\dfrac{SO(19,3)}{SO(19)SO(3)}$

20 hypermultiplets $H_2(K_3) = 22,\ 57 + 1 + 22 = 80 = 4 \times 20$

24 vectors

Further compactification on P^1 down to 4-dimension ($\mathcal{N} = 2$ susy)

$$T^4 \times \mathrm{P}^1 = T^2 \times \mathrm{P}^1 \times T^2 = K_3 \times T^2$$

$$K_3' \times \mathrm{P}^1 = \text{CY with } K_3 \text{ fibration}$$

Heterotic on $K_3 \times T^2 \Longleftrightarrow$ type-IIA on CY with K_3 fibration

gauge moduli of heterotic string is dual to geometric moduli of Calabi-Yau.

Kachru-Vafa

Consider standard embeddings of spin connection into gauge connection and series of breakings:

$$E_8 \Longrightarrow E_7 \times SU(2) \Longrightarrow E_6 \times SU(3) \Longrightarrow \cdots$$

Example

$$E_8 \times E_8 \times U(1)^4 \Longrightarrow E_7 \times E_8 \times U(1)^4$$

vector multiplts : $7 + 8 + 4 = 19 = 18 + 1$

hypermultiplets : $45 + 20 = 65$

Denote as $(65, 19) = (M, N)$

Type-IIA

$$b_{11} = N - 1, \qquad b_{12} = M - 1$$

Breaking of gauge symmetry K to G

$$K \supset G \times H$$

H denotes the part broken due to instanton.

Matter from gauge multiplet

$$adjoint\, K = \sum_i (M_i, R_i)$$

M_i, R_i are representations of G, H.

Index theorem

$$N_{M_i} = -\frac{1}{2}\int_{K_3} Tr_{R_i} F^2 + \frac{1}{48} dim\, R_i \int_{K_3} Tr R^2$$
$$= dim\, R_i - \frac{1}{2}\int_{K_3} c_2(V) \times index(R_i)$$

Case $G = E_7, H = SU(2)$

$$\int_{K_3} c_2(V) = 24$$

$$248 = (133, 1) + (56, 2) + (1, 3)$$
$$N_{56} = 2 - \frac{1}{2} \cdot 24 \cdot 1 = -10$$
$$N_1 = 3 - \frac{1}{2} \cdot 24 \cdot 4 = -45$$

45(gauge moduli) $+ 20$(K_3 moduli) $= 65$(CY moduli)

Case $G = E_6, H = SU(3)$

$$248 = (78, 1) + (27, \bar{3}) + (\overline{27}, 3) + (1, 8)$$

$$N_{27} = 3 - \frac{1}{2} \cdot 24 \cdot 1 = -9$$
$$N_1 = 8 - \frac{1}{2} \cdot 24 \cdot 6 = -64$$

$$(M, N) = (84, 18)$$

These examples have large values of vector multiplets. It is not known if CY manifolds with such Hodge numbers actually exist.

♣ Simpler cases with N=3,4

Let us find examples of CY manifolds with small values of N.

N=3 case

Choose $\tau = \rho$ so that T^2 has enhanced gauge symmetry $SU(2) \times U(1)^3$.

Then distribute instantons as

$$\begin{array}{ccccccc} E_8 & \times & E_8 & \times & SU(2) & \times & U(1)^3 \\ \uparrow & & \uparrow & & \uparrow & & \\ 10 & & 10 & & 4 & & = 24 \end{array}$$

$$E_8 \supset E_7 \times SU(2)$$
$$N_{56} = 2 - \frac{1}{2} \cdot 10 \cdot 1 = -3, \qquad N_1 = 3 - \frac{1}{2} \cdot 10 \cdot 4 = -17$$

$$SU(2) \supset SU(2)$$
$$N_1 = 3 - \frac{1}{2} \cdot 4 \cdot 4 = -5$$

Neutral hypermultiplets:

$$17 \times 2 + 5 + 20 = 59$$

Higgsing of $E_7 \times E_7$

$$2 \times (3 \times 56 - 133) = 70$$

Thus

$$M = 59 + 70 = 129, \quad N = 3.$$

There is a well-known CY manifold with Hodge numbers $b_{11} = 2$, $b_{12} = 128$:

$$X^*_{12}[1,1,2,2,6]:$$
$$W = z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - \psi z_1 z_2 z_3 z_4 z_5 - \phi z_1^6 z_2^6 = 0$$

N=4 case

$$\begin{array}{ccccc} E_8 & \times & E_8 & \times & U(1)^4 \\ \uparrow & & \uparrow & & \\ 12 & & 12 & & = 24 \end{array}$$

$$N_{56} = -4, \quad N_1 = -21$$

Neutral hypermultiplets:= 2x21 + 20 = 62

$E_7 \times E_7$ Higgsing

$$2 \times (4 \times 56 - 133) = 182$$

62+182=244.

Thus we look for CY with b_{11}=3 and b_{12}=243. Again there is a well-known example of CY manifold with these Hodge numbers.

$$X^*_{24}[1,1,2,8,12]:$$
$$W = \frac{B}{24}(x_1^{24}+x_2^{24}) - \frac{\psi_2}{12}(x_1x_2)^{12} + \frac{1}{12}x_3^{12} + \frac{1}{3}x_4^3 + \frac{1}{2}x_5^2 = 0$$
$$-\psi_0 x_1x_2x_3x_4x_5 - \frac{1}{6}\psi_1(x_1x_2x_3)^6$$

This space also has a K3 fibration. By a change of variable $x_0 = x_1x_2,\ \zeta = (x_1/x_2)^{12}$ it is rewritten as

$$W = \frac{B'}{12}x_0^{12} + \frac{1}{12}x_3^{12} + \frac{1}{3}x_4^3 + \frac{1}{2}x_5^2 - \psi_0 x_0x_3x_4x_5 - \frac{1}{6}\psi_1(x_0x_3)^6$$
$$B' = \frac{1}{2}(B\zeta + \frac{B}{\zeta}) - \psi_2$$

Discriminant is given by

$$\Delta_{CY} = B^2(B^2 - (\psi_1^2 + \psi_2)^2)(B^2 - ((\psi_1 + \psi_0^6)^2 + \psi_2)^2)(B^2 - \psi_2^2)$$

$\Downarrow$ Decoupling; $\searrow$ SU(3) $\swarrow$; $\Downarrow$ LCS

$SU(3)$ gauge theory limit is taken as

$$B = \epsilon\Lambda^3, \quad \psi_0^6\psi_1 = \epsilon u^{3/2}, \quad \psi_1^2 + \psi_2 = \epsilon(v - u^{3/2}), \qquad \epsilon \longrightarrow 0$$

By a suitable redefinition of variables we obtain an A_2 singularity fibered over P^1

$$W = \epsilon\left[\frac{1}{12}(\zeta + \frac{\Lambda^6}{\zeta}) + \frac{y_3^2}{2} + \frac{y_4^2}{2} + \frac{y_5^3}{3} - \frac{u}{4}y_5 - \frac{v}{12}\right]$$

K_3 fibration

Define heterotic (axion-)dilaton as

$$S = \frac{1}{g^2} - \frac{i\theta}{8\pi^2}$$

Heterotic prepotential is then given by

$$F = F_0(S, T^i) + F_1(T^i) + \cdots , \qquad F_0 = -\gamma_{ij} T^i T^j S$$

$$K = -\log(S + \bar{S}) - \log \gamma_{ij}(T^i + \bar{T}^i)(T^j + \bar{T}^j)$$

Type II side

$$B + iJ = \sum_{\alpha=1}^{h^{11}} (B + iJ)_\alpha e_\alpha$$

where $\{e_\alpha\}$ is a basis of $H_2(X; \mathrm{Z})$. Divisior D_α is Poincare dual to e_α.
Type-II prepotential has a form

$$F_0 = -\frac{i}{6} \sum_{\alpha,\beta,\gamma} \langle D_\alpha \cdot D_\beta \cdot D_\gamma \rangle t_{\alpha\beta\gamma} + \text{world-sheet instantons}$$

Heterotic dilaton maps to some divisor D_s and Kähler parameter $(B + iJ)_s = 4\pi i S$ of type IIA side. $B \to B + 1$ corresponds to $\theta \to \theta + 2\pi$. Then we have

$$D_s \cdot D_s \cdot D_s = 0, \qquad D_s \cdot D_s \cdot D_i = 0 \Longrightarrow D_s \cdot D_s = 0$$

Then according to a mathematical theorem CY has a K_3 fibration over P^1. Size of P^1 is $\sim 1/g^2$.

Problems

More example of dual pairs

Moduli stabilization

Understanding of gauge $\leftrightarrow$ geometry moduli

(Perturbative?) identification of type-II D-branes

Phenomenology