Orbifolding the Membrane Action

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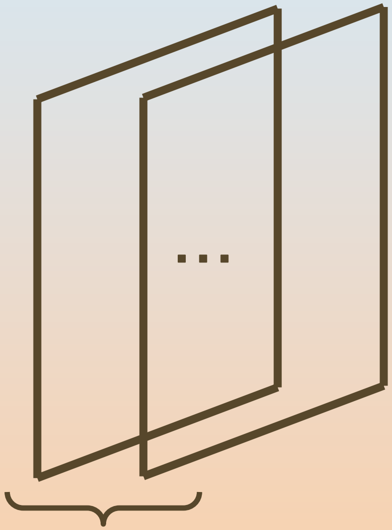
ArXiv: 0807.0368

Question

What is the low energy effective theory on multiple M2 branes (membranes) ?



***N* M2 branes**

$l_p \to 0$

World volume theory of Membranes

Cf. *N* D3 branes on flat space → 4dim $\mathcal{N}$=4 U(*N*) SYM

Hopeful Candidate

World volume theory of membranes suggested by ABJM

By Aharony, Bergman, Jaffris, Maldacena
ArXiv: 0806.1218 [hep-th]

- 3dim $\mathcal{N}$=6 Chern-Simons matter theory
- M2 branes probing $\mathbf{C}^4/\mathbf{Z}_k$ (k: level of Chern-Simons term)

Different from the method of
orbifolding the theory of D branes !!

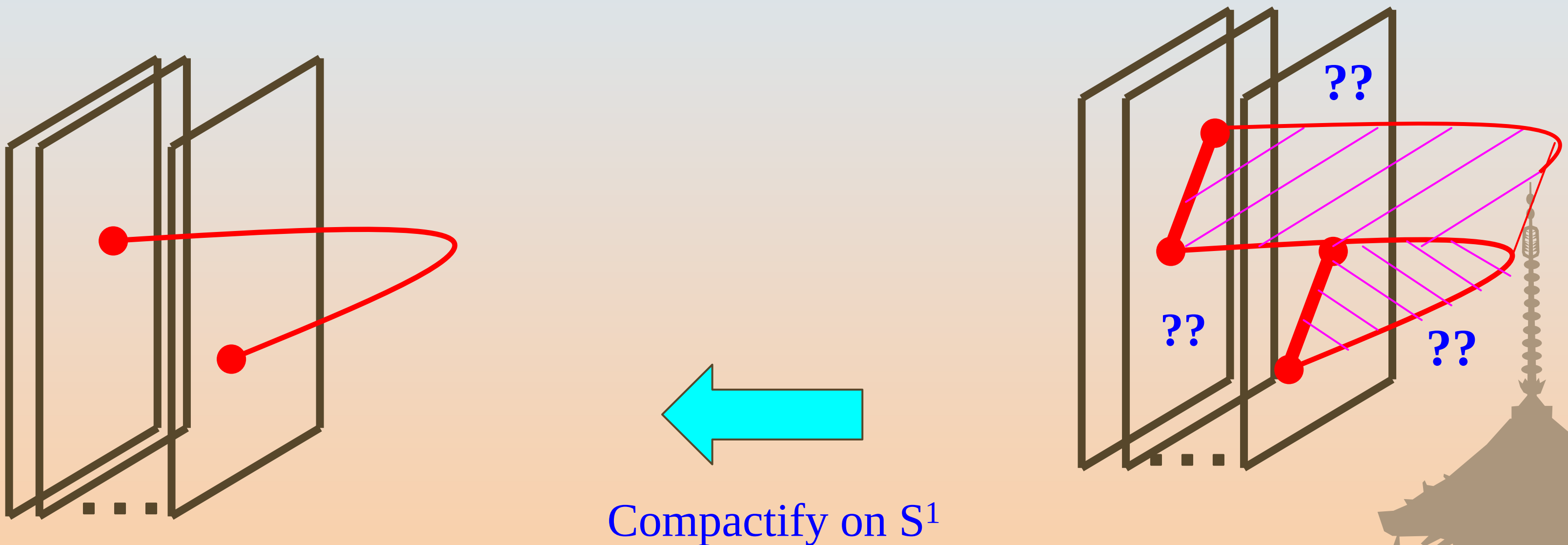
Douglas and Moore
(hep-th/9603167)

Orbifolding by the Z_n action → $U(N)^n$ Quiver gauge theory

Orbifolding is non-trivial for membrane theory !!

Chan-Paton factors
∼ Which D-branes the open string attached.
∼ Gauge indices

Structure of the countterpart of Chan-Paton factor is not clear



Method of orbifolding D branes is based on this picture.

Systematic method of orbifolding membranes is not clear *a-priori*

They should be somehow related to each other.

Theme of this talk

・What is the relationship between orbifold structure naturally encoded in the ABJM theory and the method of orbifolding for D-branes?

・They are actually equivalent?

・The orbifold action Z_k encoded in the ABJM theory can be reproduced from the method of orbifolding for D-branes? → **No!**

・Does the method for D-branes applicable for membranes for other orbifold actions?

・Always applicable (Actually equivalent ?)

・Method for D-branes are not applicable for membranes?

・**Methods of orbifolding depends on the orbifold action ?**

Plan of this talk

✓ **§ 1 Introduction**

§ 2 ABJM theory

（**Satisfying the properties expected for the world volume theory of M2 branes.**）

（**P**robing $\mathbf{C}^4/\mathbf{Z}_k$ ）

§ 3 Orbifolding the D branes

(Orbifold structure is)

§ 4 Orbifolding the membranes

（**Further orbifold of ABJM theory**）

§ 5 Summary and Discussion

§ 2 ABJM theory

By **A**harony, **B**ergman, **J**affris, **M**aldacena
ArXiv: 0806.1218 [hep-th]

		$U(N)$	$\times U(N)$
Chiral superfield	$Z_A (A=1,2)$	N	$\overline{N}$
	$W_B (B=1,2)$	$\overline{N}$	N
Vector superfield	$V^{(1)}$	(Adj)	1
	$V^{(2)}$	1	(Adj)

$$S = S_{\rm CS} + S_{\rm kin} + S_{\rm pot}$$

$$S_{\rm CS} = -\frac{k}{8\pi}\int d^3x\, d^4\theta \int_0^1 dt\, {\rm Tr}\left[V^{(1)}\, \overline{D}^{\alpha} e^{tV^{(1)}} D_{\alpha} e^{-tV^{(1)}}\right] - {\rm Tr}\left[V^{(2)}\, \overline{D}^{\alpha} e^{tV^{(2)}} D_{\alpha} e^{-tV^{(2)}}\right]$$

$$\Rightarrow \frac{k}{2\pi}\varepsilon^{\mu\nu\rho}\int d^3x\, {\rm Tr}\left(\frac{1}{2}A^{(1)}{}_{\mu}\partial_{\nu}A^{(1)}{}_{\rho} + \frac{1}{3}A^{(1)}{}_{\mu}A^{(1)}{}_{\nu}A^{(1)}{}_{\rho}\right) - {\rm Tr}\left(\frac{1}{2}A^{(2)}{}_{\mu}\partial_{\nu}A^{(2)}{}_{\rho} + \frac{1}{3}A^{(2)}{}_{\mu}A^{(2)}{}_{\nu}A^{(2)}{}_{\rho}\right)$$

$$S_{\rm kin} = \int d^3x\, d^4\theta\, {\rm Tr}\left(\overline{Z}_A e^{-V} Z^A e^{V} + \overline{W}_A e^{V} W^A e^{-V}\right)$$

k: integer

$$S_{\rm pot} = \frac{4\pi}{k}\int d^3x\, d^2\theta\, {\rm Tr}\left[Z^1W^1Z^2W^2 - Z^1W^2Z^2W^1\right] + h.c.$$

Moduli space is $(C^4/Z_k)^N/S_N$

Consider the case N=1. V=0. Divide by the gauge transformation

$$Z^A \to e^{i(\Lambda_{(1)}-\Lambda_{(2)})} Z^A \qquad W_A \to e^{-i(\Lambda_{(1)}-\Lambda_{(2)})} W_A$$

$$A_{(1)} \to A_{(1)} - d\Lambda_{(1)} \qquad A_{(2)} \to A_{(2)} - d\Lambda_{(2)}$$

It is sufficient to consider the transformation where Λ is constant.
The variation of CS term by the gauge transformation is

$$\delta S_{CS} = \frac{k}{2\pi}\int_{bounary}\left(\Lambda_{(1)}\wedge F_{(1)} - \Lambda_{(2)}\wedge F_{(2)}\right)$$

Here we consider the quantization condition

$$\int_{bounary} F_i \in 2\pi Z$$

When

$$\Lambda_i \in \frac{2\pi}{k} Z$$

CS term is invariant even at the boundary

It results in C^4/Z_k !

$$Z_k : (y^1, y^2, y^3, y^4) \to \left(e^{\frac{2\pi i}{k}} y^1, e^{\frac{2\pi i}{k}} y^2, e^{\frac{2\pi i}{k}} y^3, e^{\frac{2\pi i}{k}} y^4\right)$$

§ 3 Orbifolding the D branes

N D-branes on $C \times C^2/\Gamma$

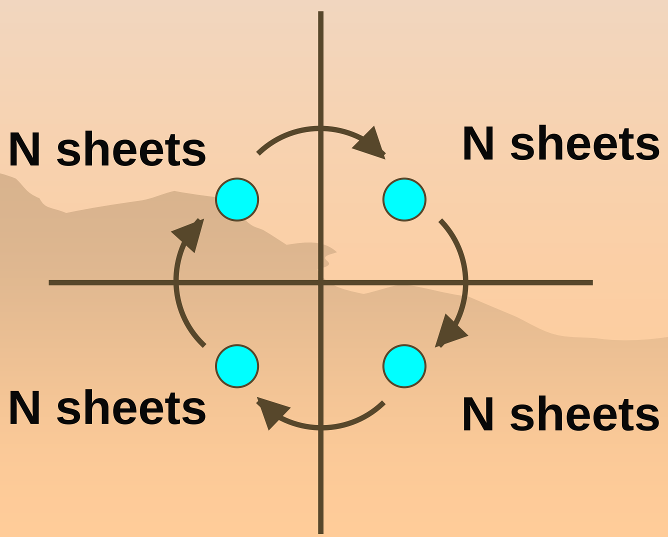
Γ : Discrete symmetry of C^2

Douglas and Moore (hep-th/9603167)

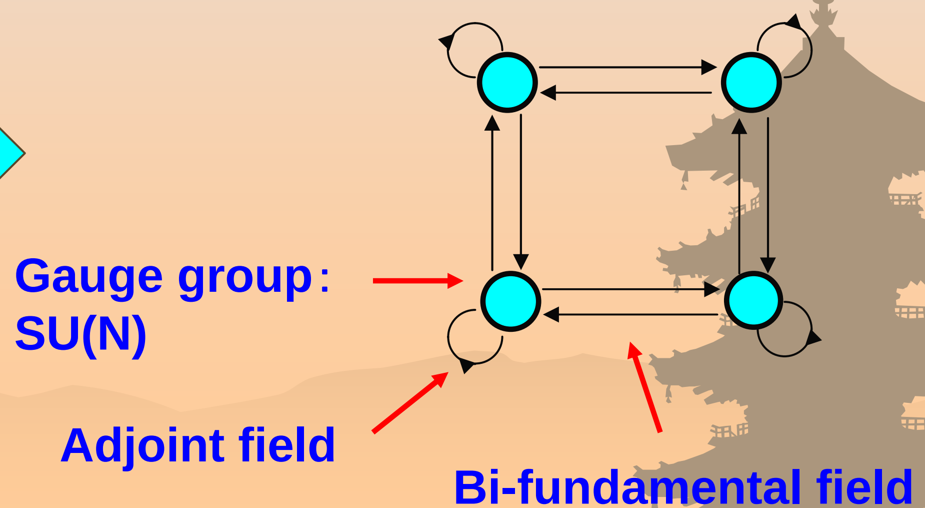
⇕

$N \times |\Gamma|$ D3 branes are put in a way that N sets of D3 branes are transformed each other by Γ

Eaxample: $\Gamma = Z_4$



Quiver gauge theory



Writing this condition,

X_i (i=1,2) ← correspond to C^2/Γ
X_3 ← correspond to C

U(4N) Adjoints

$$\Omega X_i \Omega^{-1} = e^{\frac{2\pi i}{4}} X_i$$

$$\Omega X_3 \Omega^{-1} = X_3$$

$$\Omega = \begin{pmatrix} & 1_{N\times N} & & \\ & & 1_{N\times N} & \\ & & & 1_{N\times N} \\ 1_{N\times N} & & & \end{pmatrix}$$

Ω : Regular rep. of Z_4 (| Γ | dim rep)
Shift from a set of N D-branes to another set

For Generators of U(4N)

$$\Omega T \Omega^{-1} = T$$

Note: In actual calculation,
Use the base diagonalize Ω

$$\Omega = \mathrm{diag}(1_{N\times N}, e^{\frac{2\pi i}{4}} 1_{N\times N}, e^{\frac{4\pi i}{4}} 1_{N\times N}, e^{\frac{6\pi i}{4}} 1_{N\times N})$$

To solve,

$$T = \mathrm{diag}(T_1, T_2, T_3, T_4)$$

U(4N) → U(N)^4

$$X_i = \begin{pmatrix} 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \\ * & 0 & 0 & 0 \end{pmatrix}, \quad X_3 = \begin{pmatrix} * & 0 & 0 & 0 \\ 0 & * & 0 & 0 \\ 0 & 0 & * & 0 \\ 0 & 0 & 0 & * \end{pmatrix}$$

Adjoint rep. of U(N) corresponding to T_4

Bi-fundamental rep. of U(N) corresponding to T_1 and T_4

Substitute these into the U(4N) action before orbifolding

Divide the action by 4 (of Z_4)

The effect of preparing 4N D-branes which are 4 times as many as the actual D-branes.

Action of the world volume theory of D3 branes probing $C \times C^2/Z_4$.

Theme

・What is the relationship between orbifold structure naturally encoded in the ABJM theory and the method of orbifolding for D-branes?

・They are actually equivalent?

・The orbifold action Z_k encoded in the ABJM theory can be reproduced from the method of orbifolding for D-branes? → **No!**

・Does the method for D-branes applicable for membranes for other orbifold actions?

・Always applicable (Actually equivalent ?)

・Method for D-branes are not applicable for membranes?

・**Methods of orbifolding depends on the orbifold action ?**

§ 4　Orbifolding the membranes

Construction of orbifolding by Z_n

0. Prepare theory of *nN* sheets of M2 branes
1. Solve the constraints for gauge group and fields
2. Divide action by *n*

Chern-Simons coupling must be integer even after dividing the action by *n*
→ *k* before orbifolding is a multiple of *n* !! (*k*=*k'n*)

・The orbifold action Z_k encoded in the ABJM theory cannot be reproduced from the method of orbifolding for D-branes.

Orbifolding ABJM theory by Z_n ($k=k'n$)

$$Z_k : (y^1, y^2, y^3, y^4) \to \left(e^{\frac{2\pi i}{k}} y^1, e^{\frac{2\pi i}{k}} y^2, e^{\frac{2\pi i}{k}} y^3, e^{\frac{2\pi i}{k}} y^4 \right)$$

← **Already incoded in the ABJM theory**

$$Z_n : (y^1, y^2, y^3, y^4) \to \left(e^{\frac{2\pi i}{n}} y^1, e^{\frac{2\pi i}{n}} y^2, e^{-\frac{2\pi i}{n}} y^3, e^{-\frac{2\pi i}{n}} y^4 \right)$$

← **Further orbifolding**

$$Z_1 = \begin{pmatrix} 0 & Z_1^{\ 1} & & & \\ & 0 & Z_1^{\ 2} & & \\ & & \ddots & \ddots & \\ & & & 0 & Z_1^{\ n-1} \\ Z_1^{\ n} & & & & 0 \end{pmatrix}, \quad Z_2 = \begin{pmatrix} 0 & & & & Z_2^{\ n} \\ Z_2^{\ 1} & 0 & & & \\ & Z_2^{\ 2} & \ddots & & \\ & & \ddots & 0 & \\ & & & Z_2^{\ n-1} & 0 \end{pmatrix},$$

$$W_1 = \begin{pmatrix} 0 & W_1^{\ 1} & & & \\ & 0 & W_1^{\ 2} & & \\ & & \ddots & \ddots & \\ & & & 0 & W_1^{\ n-1} \\ W_1^{\ n} & & & & 0 \end{pmatrix}, \quad W_2 = \begin{pmatrix} 0 & & & & W_2^{\ n} \\ W_2^{\ 1} & 0 & & & \\ & W_2^{\ 2} & \ddots & & \\ & & \ddots & 0 & \\ & & & W_2^{\ n-1} & 0 \end{pmatrix}$$

$$V_{(1)} = diag(V_{(1)}^{\ 1}, V_{(1)}^{\ 2}, \cdots, V_{(1)}^{\ n}), \qquad V_{(2)} = diag(V_{(2)}^{\ 1}, V_{(2)}^{\ 2}, \cdots, V_{(2)}^{\ n}),$$

$$S = S_{\rm CS} + S_{\rm kin} + S_{\rm pot}$$

$$S_{\rm CS} = -\frac{k'}{8\pi} \int d^3x\, d^4\theta \int_0^1 dt\, tr\left[V\, \overline{D}^{\alpha} e^{tV} D_{\alpha} e^{-tV} \right]$$

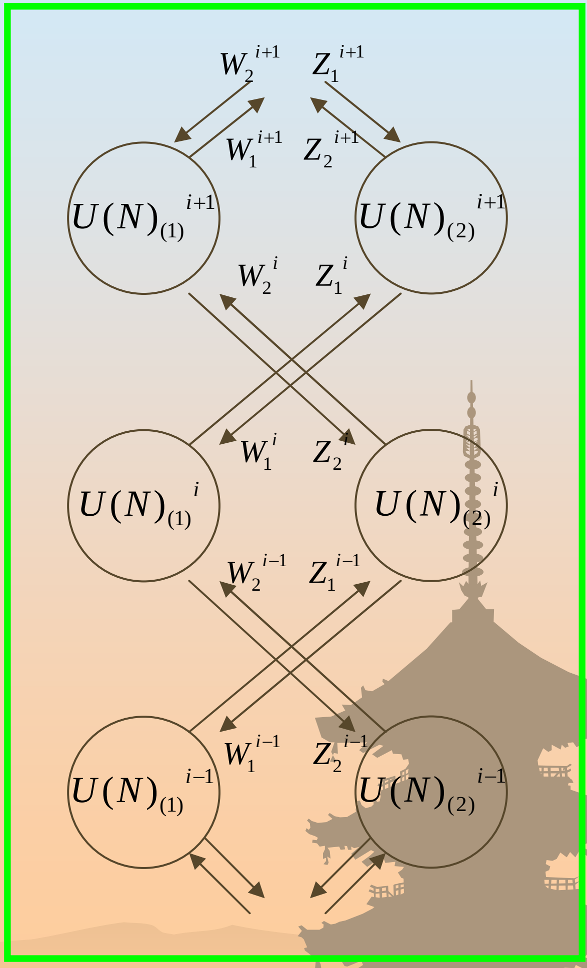
$$S_{\rm kin} = \frac{1}{n} \int d^3x\, d^4\theta\, tr\left(\overline{Z}^A e^{-V} Z_A e^{V} + \overline{W}^B e^{V} W_B e^{-V} \right)$$

$$S_{\rm pot} = \frac{4\pi n^2}{k'} \int d^3x\, d^2\theta\, tr\left[Z_1 W_1 Z_2 W_2 - Z_1 W_2 Z_2 W_1 \right]$$

- Solve V=0
- Divide by the gauge transformation

The moduli space is

$[(C^4/Z_k)/Z_n\ ]^N/S_N$



Obtained Quiver Gauge Theory

§ 5 Summary and Discussion

・We constructed the theory which we suggest to the world volume theory of M2 branes probing

$(C^4/Z_k)/Z_n$ $(k = nk')$

$$Z_k : (y^1, y^2, y^3, y^4) \to \left(e^{\frac{2\pi i}{k}} y^1, e^{\frac{2\pi i}{k}} y^2, e^{\frac{2\pi i}{k}} y^3, e^{\frac{2\pi i}{k}} y^4 \right)$$

$$Z_n : (y^1, y^2, y^3, y^4) \to \left(e^{\frac{2\pi i}{n}} y^1, e^{\frac{2\pi i}{n}} y^2, e^{-\frac{2\pi i}{n}} y^3, e^{-\frac{2\pi i}{n}} y^4 \right)$$

by using the method for D-branes.

・We checked that its moduli space is

$[(C^4/Z_k)/Z_n]^N/S_N$

Which is consistent with the picture that M2 branes are probing

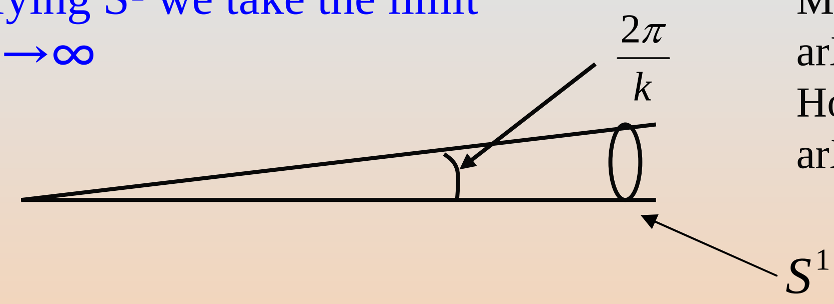
$(C^4/Z_k)/Z_n$

We discussed that we cannot use the method for D-branes for orbifold action to reproduce one which is encoded in ABJM theory.

Discussion

・Method of orbifolding similar to the case of D branes are applicable <u>for some cases</u>.

・When compactifying S^1 we take the limit
$k \to \infty$ $\langle Z \rangle, \langle W \rangle \to \infty$



Mhuki, Papageorgakis
arXiv: 0803.3218
Honma, Iso, Sumitomo, Zhang
arXiv; 0806.3498

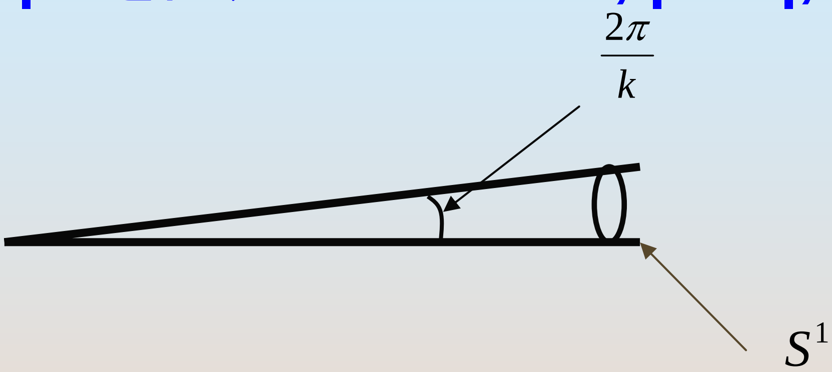
・It is not applicable if orbifolding structure vanishes when taking this limit.

・It is applicable only when $k=k'n$.
→ Need to explain in the M theory side!

どうもありがとうございました。

S^1コンパクト化でD2ブレーン上の理論になる

$|\langle Z\rangle|/k$, $|\langle W\rangle|/k$ を固定して $k\to\infty$, $|\langle Z\rangle|$, $|\langle W\rangle|\to\infty$



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細長いコーンの、先端から離れた場所を見ていることになる
→　そのあたりはほぼシリンダー！！　S^1コンパクト化に相当

ABJM理論にこの操作を施すと、確かにD2ブレーン上のゲージ理論が現れる。

Action: $S = S_{\mathrm{CS}} + S_{\mathrm{kin}} + S_{\mathrm{pot}}$

Chern-Simons 項:

$$S_{CS} = \int d^3x \frac{k}{2\pi} \varepsilon^{\mu\nu\rho} \mathrm{Tr}\left(\frac{1}{2} A^{(1)}{}_{\mu} \partial_{\nu} A^{(1)}{}_{\rho} + \frac{i}{3} A^{(1)}{}_{\mu} A^{(1)}{}_{\nu} A^{(1)}{}_{\rho} \right)$$

$$- \frac{k}{2\pi} \varepsilon^{\mu\nu\rho} \mathrm{Tr}\left(\frac{1}{2} A^{(2)}{}_{\mu} \partial_{\nu} A^{(2)}{}_{\rho} + \frac{i}{3} A^{(2)}{}_{\mu} A^{(2)}{}_{\nu} A^{(2)}{}_{\rho} \right)$$

運動項:

$$S_{\mathrm{kin}} = \int d^3x \Big[-\mathrm{Tr}(D_{\mu} Z^{A})^{\dagger} D^{\mu} Z_{A} - \mathrm{Tr}(D_{\mu} W^{A})^{\dagger} D^{\mu} W_{A}$$

$$+ i\mathrm{Tr}\zeta^{\dagger A} \gamma^{\mu} D_{\mu} \zeta_{A} + i\mathrm{Tr}^{\dagger} \omega^{A} \gamma^{\mu} D_{\mu} \omega_{A} \Big]$$

ポテンシャル項:

$$S_{\mathrm{pot}} = -\int d^3x\, V$$

Mass 次元

スカラー場: 1/2
スピノル場: 1

ポテンシャル: 3

$$V = V^{D}{}_{\text{bos}} + V^{F}{}_{\text{bos}} + V^{D}{}_{\text{ferm}} + V^{F}{}_{\text{ferm}}$$

$$V^{\text{bos}}{}_{D} = \frac{4\pi^2}{k} Tr\left|(Z^A Z^{\dagger}{}_A - W^{\dagger A} W_A) Z^B - Z^B (Z^{\dagger}{}_A Z^A - W_A W^{\dagger A})\right|^2$$

$$+ \frac{4\pi^2}{k} Tr\left|(W^A W^{\dagger}{}_A - Z^{\dagger A} Z_A) Z^B - W^B (W^{\dagger}{}_A W^A - Z^{\dagger}{}_A Z^A)\right|^2$$

$$V^{\text{bos}}{}_{F} = -\frac{16\pi^2}{k} \mathrm{Tr}\left|W_A Z^B W_C - W_C Z^B W_A\right|^2 + \frac{16\pi^2}{k} \mathrm{Tr}\left|Z_A W^B Z_C - Z_C W^B Z_A\right|^2$$

$$V^{\text{ferm}}{}_{D} = \frac{2\pi i}{k} \mathrm{Tr}\Big[(\zeta^{\dagger}{}_A \zeta^A - \omega_A \omega^{\dagger A})(Z^{\dagger}{}_B Z^B - W_B W^{\dagger B})$$

$$- (\zeta^A \zeta^{\dagger}{}_A - \omega^{\dagger A} \omega_A)(Z^B Z^{\dagger}{}_B - W^{\dagger B} W_B)\Big]$$

$$V^{\text{ferm}}{}_{F} = \frac{2\pi}{k} \varepsilon_{AC} \varepsilon^{BD} \mathrm{Tr}\Big[2\zeta^A W_B Z^C \omega_D + 2\zeta^A \omega_B Z^C W_Z$$

$$+ Z^A \omega_B Z^C \omega_D + \zeta^A W_B \zeta^C W_D\Big] + h.c.$$

CFTである

・パラメータがChern-Simons couplingの *k* しかなく、
これはMass次元を持たない。
→少なくとも古典的には作用はスケール不変

・*k*は整数
→連続性を仮定すると、繰り込みを受けない
→恐らく量子論的にもスケール不変

Chern-Simons coupling *k* が整数でないとならない理由

ゲージ変換 $A_\mu \to A_\mu + g^{-1} A_\mu g + g^{-1} \partial_\mu g$

の下でChern-Simons 項は

$$\delta L_{CS} = L_{CS} - \frac{k}{4\pi} \varepsilon^{\mu\nu\rho} \partial_\mu \mathrm{Tr}(\partial_\nu g\, g^{-1} A_\rho) - \frac{k}{12\pi} \varepsilon^{\mu\nu\rho} \mathrm{Tr}(g^{-1}\partial_\mu g\, g^{-1}\partial_\nu g\, g^{-1}\partial_\rho g)$$

のように変化する。ここで、

$$\frac{1}{12\pi} \int d^3x\, \varepsilon^{\mu\nu\rho}\, \mathrm{Tr}(g^{-1}\partial_\mu g\, g^{-1}\partial_\nu g\, g^{-1}\partial_\rho g) \in 2\pi Z$$

であることから、ゲージ不変性が保たれるためには *k* が整数でなければならない。