行列模型における有効相互作用とオリエンティフォールディング

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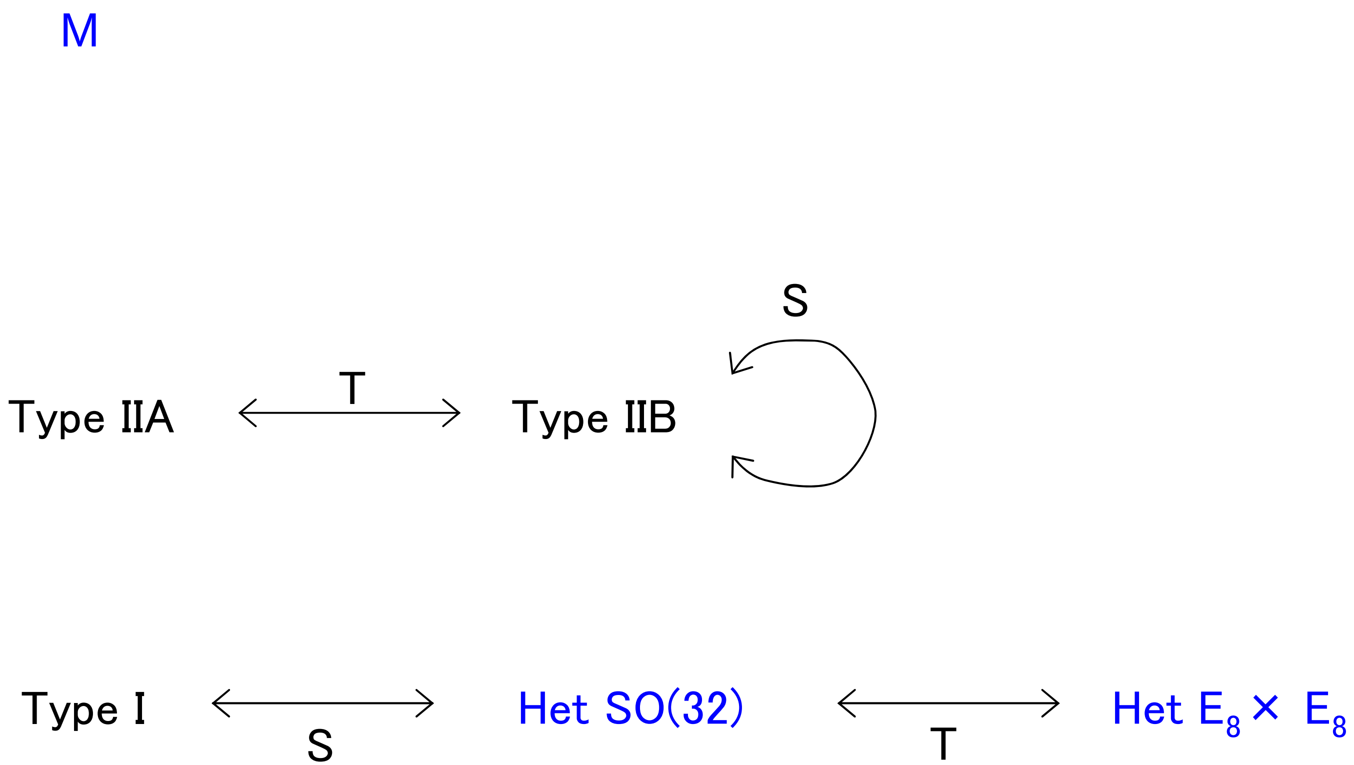
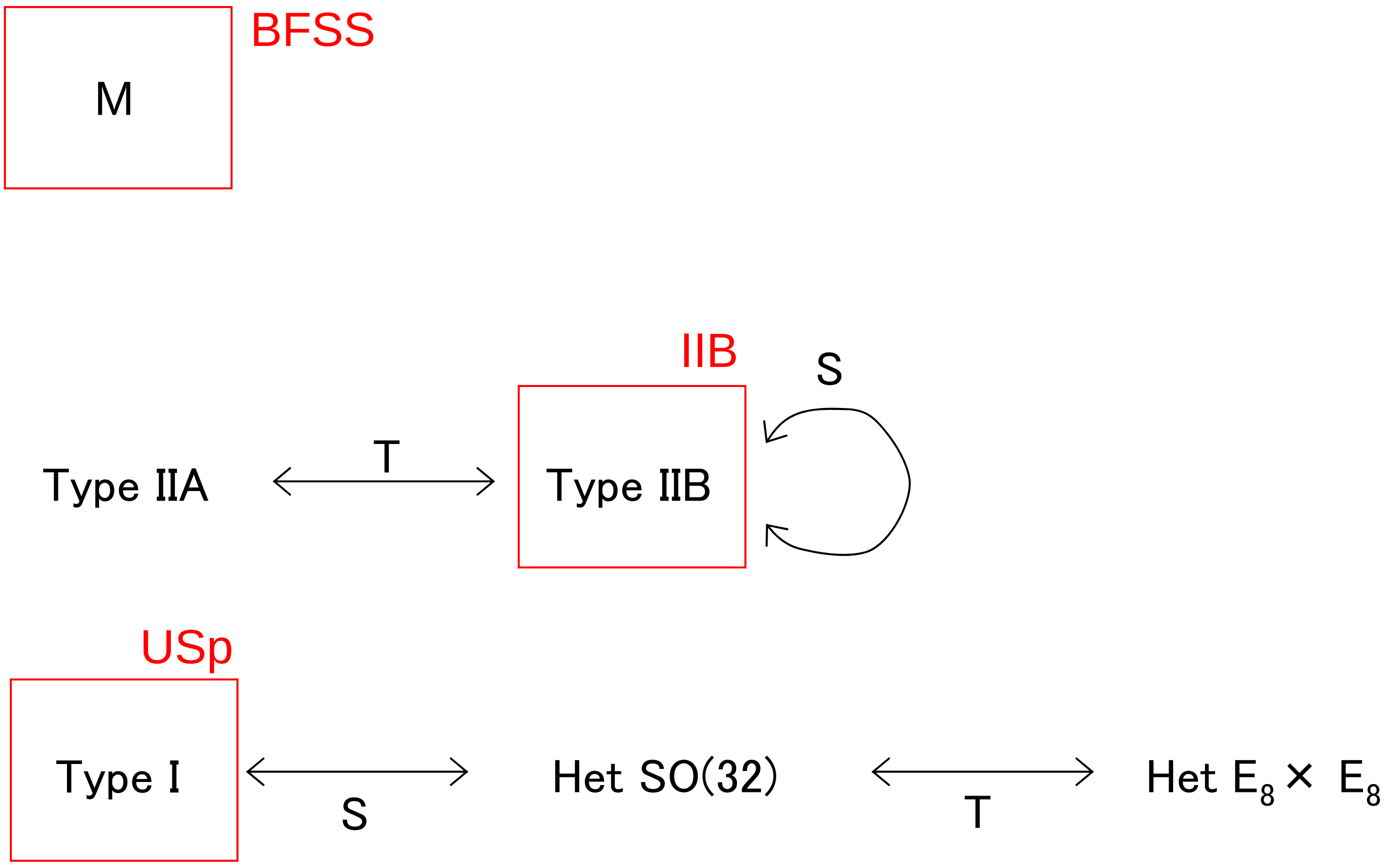
Introduction – string duality –



Introduction – string duality –



Type IIB superstrings

16+16 susy

closed

orientable

Type I superstrings

8+8 susy

open+closed

unorientable

Reduced model

IIB Matrix Model
[Ishibashi-Kawai-Kitazawa-Tsuchiya]

Matrix Orientifolding →

USp Matrix Model
[Itoyama-Tokura]

IIB Matrix Model

N=1, D=10 超対称Yang–Mills理論を0次元へ次元縮小して得られる

$$S_{\mathrm{IIB}} = -\frac{1}{g^2}\left(\frac{1}{4}\mathrm{tr}[\underline{v}^M,\underline{v}^N]^2 + \frac{1}{2}\mathrm{tr}(\underline{\bar{\Psi}}\Gamma^M[\underline{v}_M,\underline{\Psi}])\right)$$

$$\underline{v}_M,\ \underline{\Psi} \in \mathfrak{u}(N) \qquad (M=0,\ldots,9)$$

$\underline{\Psi}$: 10-d Majorana-Weyl spinor

この模型は16+16個の超対称性を持っている

$$\delta^{(1)}_{\mathrm{IIB}}\underline{\Psi} = \frac{i}{2}[\underline{v}_M,\underline{v}_N]\Gamma^{MN}\epsilon \qquad \delta^{(2)}_{\mathrm{IIB}}\underline{\Psi} = \xi$$

$$\delta^{(1)}_{\mathrm{IIB}}\underline{v}_M = i\bar{\epsilon}\Gamma^M\underline{\Psi} \qquad \delta^{(2)}_{\mathrm{IIB}}\underline{v}_M = 0$$

From U To USp

$$\mathfrak{u}(2k) \xrightarrow{\hat{\rho}_-} \mathfrak{adj}(2k) = \{X \in \mathfrak{u}(2k) | X^t F + F X = 0\} \qquad X_\mu = \begin{pmatrix} M_\mu & N_\mu \\ N_\mu^* & -M_\mu^t \end{pmatrix}$$

$$\mathfrak{u}(2k) \xrightarrow{\hat{\rho}_+} \mathfrak{asym}(2k) = \{X \in \mathfrak{u}(2k) | X^t F - F X = 0\} \qquad X_a = \begin{pmatrix} A_a & B_a \\ B_a^\dagger & A_a^t \end{pmatrix}$$

USp projector : $\hat{\rho}_\mp \bullet = \frac{1}{2}(\bullet \mp F^{-1} \bullet^t F)$ $\qquad F = \begin{pmatrix} 0 & -1_k \\ 1_k & 0 \end{pmatrix}$

$$\begin{cases} \underline{v}_M \\ \underline{\Psi}_A \end{cases} \to \begin{cases} v_M \equiv \hat{\rho}_{b\mp}^{(M)} \underline{v}_M \\ \Psi_A \equiv \hat{\rho}_{f\mp}^{(A)} \underline{\Psi}_A \end{cases}$$

$$\hat{\rho}_{b\mp}^{(M)} \equiv \Theta(M \in \mathcal{M}_-)\hat{\rho}_- + \Theta(M \in \mathcal{M}_+)\hat{\rho}_+$$

$$\hat{\rho}_{f\mp}^{(A)} \equiv \Theta(A \in \mathcal{A}_-)\hat{\rho}_- + \Theta(M \in \mathcal{A}_+)\hat{\rho}_+$$

$$M \in \mathcal{M} = \{0,1,2,3,4,5,6,7,8,9\}$$

$$A \in \mathcal{A} = \{1,2,5,6,9,10,13,14,19,20,23,24,27,28,31,32\}$$

$$\mathcal{M} \to \mathcal{M}_- \cup \mathcal{M}_+ \qquad \mathcal{M}_- \cap \mathcal{M}_+ = \emptyset$$

$$\mathcal{A} \to \mathcal{A}_- \cup \mathcal{A}_+ \qquad \mathcal{A}_- \cap \mathcal{A}_+ = \emptyset$$

$$[\hat{\rho}_{\mp}, \delta_{\rm susy}] = 0 \quad \Rightarrow$$ 8+8 susy

ϵを選ぶ

- $\epsilon = (\epsilon_0, 0, \epsilon_1, 0, 0, 0, 0, 0, 0, \bar{\epsilon}_0, 0, \bar{\epsilon}_1, 0, 0, 0, 0)^t$

$$\mathcal{A}_{adj} = \{1, 2, 5, 6, 19, 20, 23, 24\}$$

$$\mathcal{A}_{asym} = \{9, 10, 13, 14, 27, 28, 31, 32\}$$

$$\mathcal{M}_{adj} = \{0, 1, 2, 3, 4, 7\}$$

$$\mathcal{M}_{asym} = \{5, 6, 8, 9\}$$

$\underline{\Psi}$ は以下のようにとる:

$$\underline{\Psi} = (\lambda, 0, \psi_{(1)}, 0, \psi_{(2)}, 0, \psi_{(3)}, 0, 0, \bar{\lambda}, 0, \bar{\psi}_{(1)}, 0, \bar{\psi}_{(2)}, 0, \bar{\psi}_{(3)})^t$$

$$\lambda, \psi_{(1)}, \bar{\lambda}, \bar{\psi}_{(1)} \in \mathfrak{adj}(2k)$$

$$\psi_{(2)}, \psi_{(3)}, \bar{\psi}_{(2)}, \bar{\psi}_{(3)} \in \mathfrak{asym}(2k)$$

$\Downarrow$ + open string (基本表現)

USp Matrix Model

1-loop effective action for matrix model

[Aoki-Iso-Kawai-Kitazawa-Tada]

$$v_M = x_M + \tilde{v}_M, \qquad \Psi = \xi + \tilde{\Psi}.$$

$$[x_M, x_N] = 0, \qquad \{\xi_a, \xi_b\} = 0$$

ゲージ固定項とゴーストを加える:

$$S_{g.f.} = -\frac{1}{2g^2}\mathrm{tr}[x_M, v^M]^2, \qquad S_{\mathrm{ghost}} = -\frac{1}{g^2}\mathrm{tr}[x_M, b][x_M, c]$$

c : ghost, b : anti-ghost

$$S_b^{(2)} + S_{g.f.} = -\frac{1}{2g^2}\mathrm{tr}[x_M, \tilde{v}_N][x^M, \tilde{v}^N]$$

$$S_f^{(2)} = -\frac{1}{2g^2}\mathrm{tr}\bar{\tilde{\Psi}}\Gamma^M[x_M, \tilde{\Psi}] - \frac{1}{2g^2}\mathrm{tr}[\bar{\xi}, \tilde{v}_M]\Gamma^M\tilde{\Psi} - \frac{1}{2g^2}\mathrm{tr}\bar{\tilde{\Psi}}\Gamma^M[\tilde{v}_M, \xi],$$

$$\Rightarrow \quad e^{-S_{\mathrm{eff}}^{\text{1-loop}}[x,\xi]} = \int d\tilde{v}d\tilde{\Psi}dbdc\exp\{-(S^{(2)} + S_{g.f.} + S_{\mathrm{ghost}})\}$$

IIB [Aoki-Iso-Kawai-Kitazawa-Tada]

$$x_M = \begin{pmatrix} x_M^1 & & & \\ & x_M^2 & & \\ & & \ddots & \\ & & & x_M^k \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^1 & & & \\ & \xi^2 & & \\ & & \ddots & \\ & & & \xi^k \end{pmatrix}$$

1-loop 有効作用

$$e^{-S_{\rm eff}^{\text{1-loop}}[x,\xi]} = \prod_{i,j} \det(\eta^{MN} + S_{(ij)}^{MN})^{-1}$$

$$S_{(ij)}^{MN} = \bar{\xi}^{ij}\Gamma^{MPN}\xi^{ij}\frac{x_P^{ij}}{(x^{ij})^4}$$

$$x_M^{ij} = x_M^i - x_M^j$$

$$\xi^{ij} = \xi^i - \xi^j$$

$$\Rightarrow \quad S_{\rm eff}^{\text{1-loop}}[x,\xi] = \sum_{i,j} \mathrm{tr}\log(\eta^{\mu\nu} + S_{(ij)}^{\mu\nu})$$

$$= -\sum_{i,j} \mathrm{tr}\left(\frac{S_{(ij)}^4}{4} + \frac{S_{ij}^8}{8}\right) \qquad ((x^i - x^j)^2 \gg g)$$

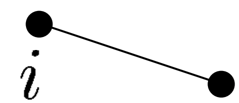
IIB

$$e^{-S_{\text{eff}}^{\text{1-loop}}[x]} = \int d\xi \prod \left[1 + \frac{\text{tr}(S^4_{(ij)})}{4} + \left(\frac{1}{2} \left(\frac{\text{tr}(S^4_{(ij)})}{4} \right)^2 + \frac{\text{tr}(S^8_{(ij)})}{8} \right) \right]$$

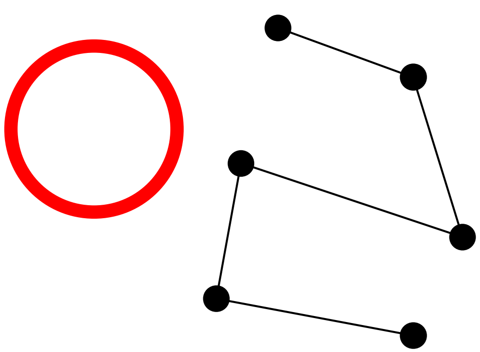
$S_{(ij)}$ は ξ^i に $\xi^{ij} = \xi^i - \xi^j$ してのみ依存する。

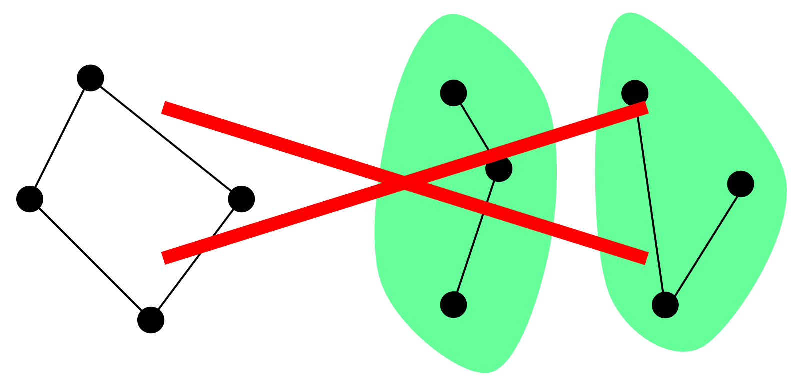
$$S^{MN}_{(ij)} = \bar{\xi}^{ij} \Gamma^{MPN} \xi^{ij} \frac{x^{ij}_P}{(x^{ij})^4}$$

そこで

ξ^i の1成分の積分 $\Leftrightarrow$ 

ξ・ − 積分で“maximal tree”のみが生き残る。 $\Rightarrow$ branched polymer, complex phase, …





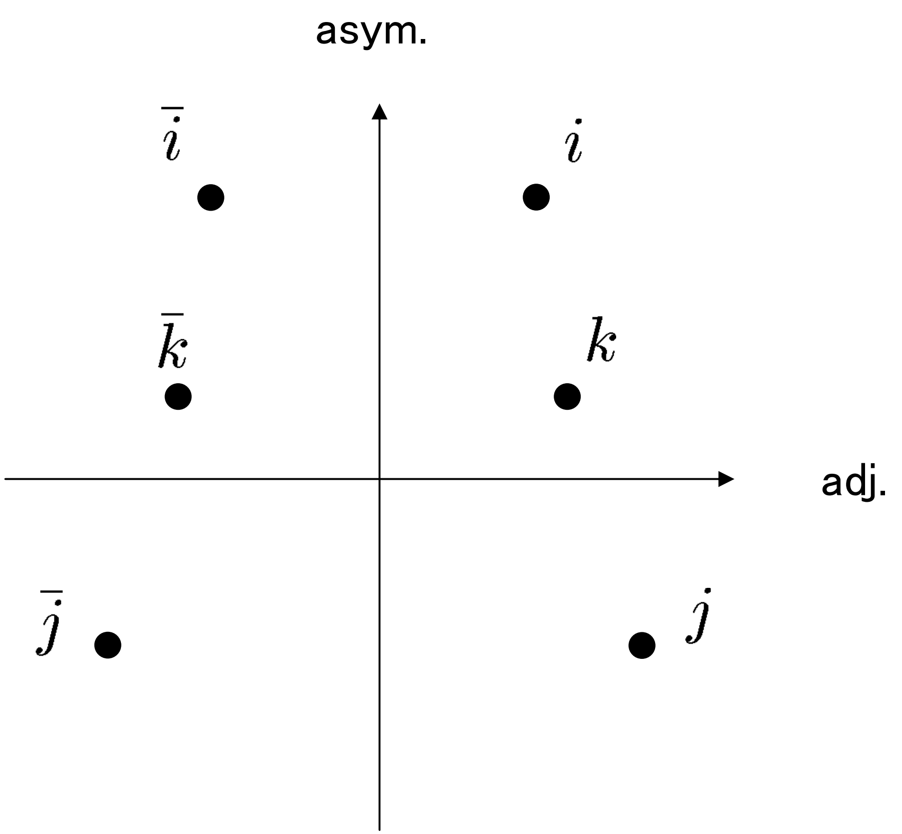
USp

adj. : $X = \begin{pmatrix} M_\mu & N_\mu \\ N_\mu^* & -M_\mu^t \end{pmatrix}$ $\Rightarrow$ $\mathrm{diag}\{x^1, x^2 \cdots, x^k, -x^1, -x^2, \cdots, -x^k\}$

asym. : $X = \begin{pmatrix} A_a & B_a \\ B_a^\dagger & A_a^t \end{pmatrix}$ $\Rightarrow$ $\mathrm{diag}\{x^1, x^2 \cdots, x^k, x^1, x^2, \cdots, x^k\}$

行列の固有値分布 ＝ 時空

鏡像点を明記するためにバー付きで表わす

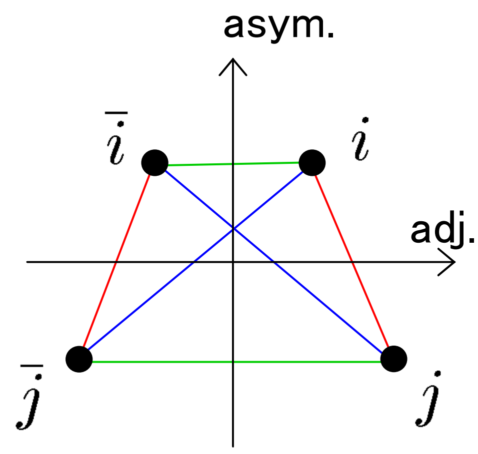


USp

$$x_\mu = \begin{pmatrix} X_\mu & 0 \\ 0 & -X_\mu \end{pmatrix} \quad \xi_\alpha = \begin{pmatrix} \Xi_\alpha & 0 \\ 0 & -\Xi_\alpha \end{pmatrix}$$

$$x_a = \begin{pmatrix} X_a & 0 \\ 0 & X_a \end{pmatrix} \quad \xi_{\alpha'} = \begin{pmatrix} \Xi_{\alpha'} & 0 \\ 0 & \Xi_{\alpha'} \end{pmatrix}$$

$$X_M = \begin{pmatrix} x_M^1 & & & \\ & x_M^2 & & \\ & & \ddots & \\ & & & x_M^k \end{pmatrix}, \quad \Xi = \begin{pmatrix} \xi^1 & & & \\ & \xi^2 & & \\ & & \ddots & \\ & & & \xi^k \end{pmatrix}$$



1-loop 有効作用

$$e^{-S_{\rm eff}^{\text{1-loop}}[x,\xi]} = \prod_{i,j} \det(\eta^{MN} + T_{(ij)}^{MN})^{-1} \det(\eta^{MN} + T_{(i\bar{j})}^{MN})^{-1} \det(\eta^{MN} + T_{(i\bar{i})}^{MN})^{-1}$$

$$T_{(ij)} = \begin{pmatrix} T_0^{\mu\nu} & T_1^{\mu b} \\ T_1^{a\nu} & T_0^{ab} \end{pmatrix} = S_{(ij)} - \begin{pmatrix} T_1^{\mu\nu} & T_0^{\mu b} \\ T_0^{a\nu} & T_1^{ab} \end{pmatrix}$$

$$T_0^{MN} = \frac{1}{(x^{ij})^4} \bar{\xi}^{ij} \Gamma^M \begin{pmatrix} \Gamma^\mu x_\mu^{ij} & \Gamma^a x_a^{ij} \\ \Gamma^a x_a^{ij} & \Gamma^\mu x_\mu^{ij} \end{pmatrix} \Gamma^N \xi^{ij}$$

$$T_1^{MN} = \frac{1}{(x^{ij})^4} \bar{\xi}^{ij} \Gamma^M \begin{pmatrix} \Gamma^\mu x_\mu^{ij} & \Gamma^a x_a^{ij} \\ \Gamma^a x_a^{ij} & \Gamma^\mu x_\mu^{ij} \end{pmatrix} \Gamma^N \xi^{ij}$$

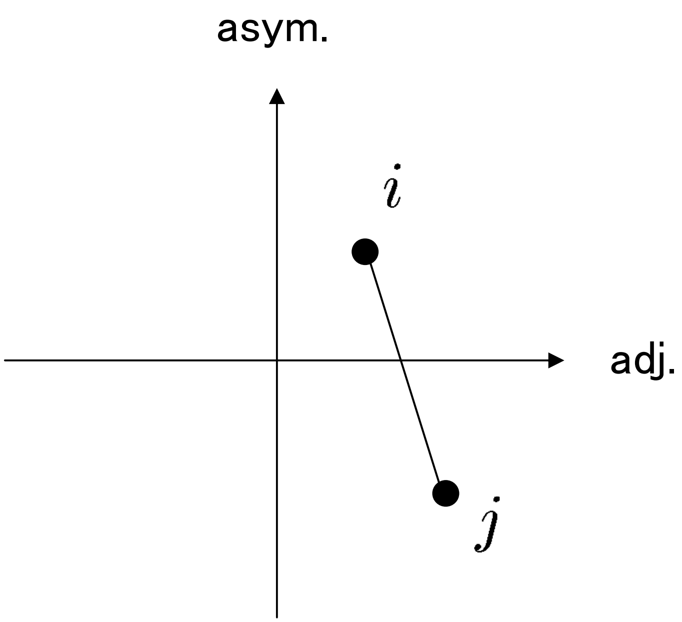
ある点と鏡像点との間の相互作用項は

$$T_{(i\bar{j})} = \tilde{T}_{(ij)}$$

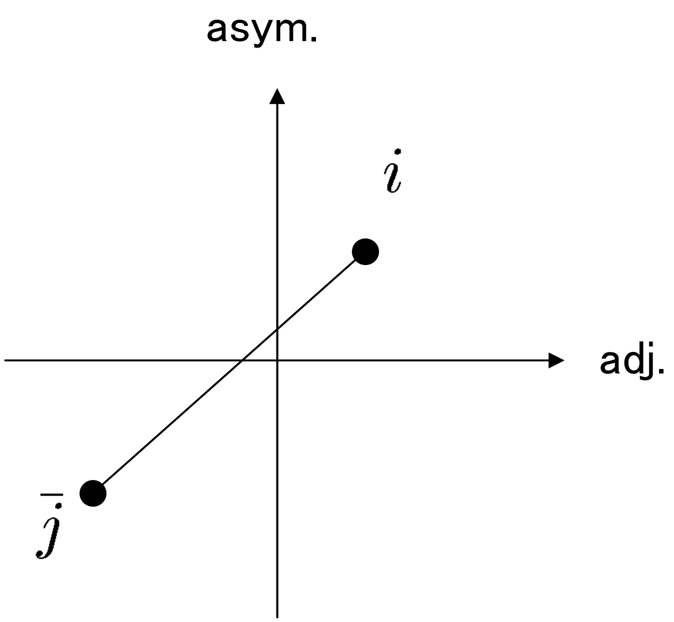
adjoint方向のみ

$T_{(ij)}$ に対して

$$x_\mu^{ij} = x_\mu^i - x_\mu^j \quad \rightarrow \quad \tilde{x}_\mu^{ij} = x_\mu^i + x_\mu^j$$
$$\xi_a^{ij} = \xi_a^i - \xi_a^j \quad \rightarrow \quad \tilde{\xi}_a^{ij} = \xi_a^i + \xi_a^j$$



$\Rightarrow$



$$\sim \xi^{ij} = \xi^i - \xi^j$$

$$\sim \tilde{\xi}^{ij} = \xi^i + \xi^j$$

USp

1-loop 有効作用

$$e^{-S_{\text{eff}}^{\text{1-loop}}[x]} = \int d\xi \prod \left[1 + \frac{\mathrm{tr}(T_{(ij)}^4)}{4} + \left(\frac{1}{2}\left(\frac{\mathrm{tr}(T_{(ij)}^4)}{4} \right)^2 + \frac{\mathrm{tr}(T_{(ij)}^8)}{8} \right) \right]$$

$$T = S - U$$

$$U = \begin{pmatrix} T_1^{\mu\nu} & T_0^{\mu b} \\ T_0^{a\nu} & T_1^{ab} \end{pmatrix}$$

$$S \leftrightarrow T$$

IIB ⟷ USp

・USp(2) case

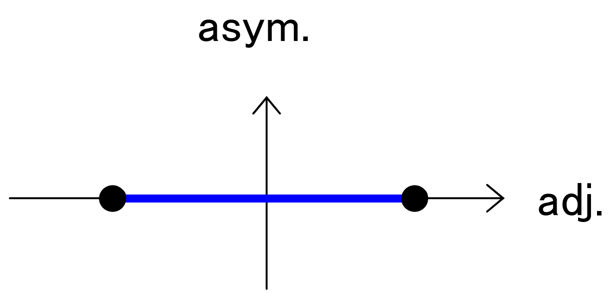
$$x_\mu = \begin{pmatrix} x_\mu & 0 \\ 0 & -x_\mu \end{pmatrix}, \qquad x_a = \begin{pmatrix} x_a & 0 \\ 0 & x_a \end{pmatrix},$$

$$\tilde{v}_\mu = \begin{pmatrix} m_\mu & n_\nu \\ n_\mu^* & -m_\mu \end{pmatrix}, \qquad \tilde{v}_a = \begin{pmatrix} a_a & b_a \\ b_a^* & a_a \end{pmatrix},$$

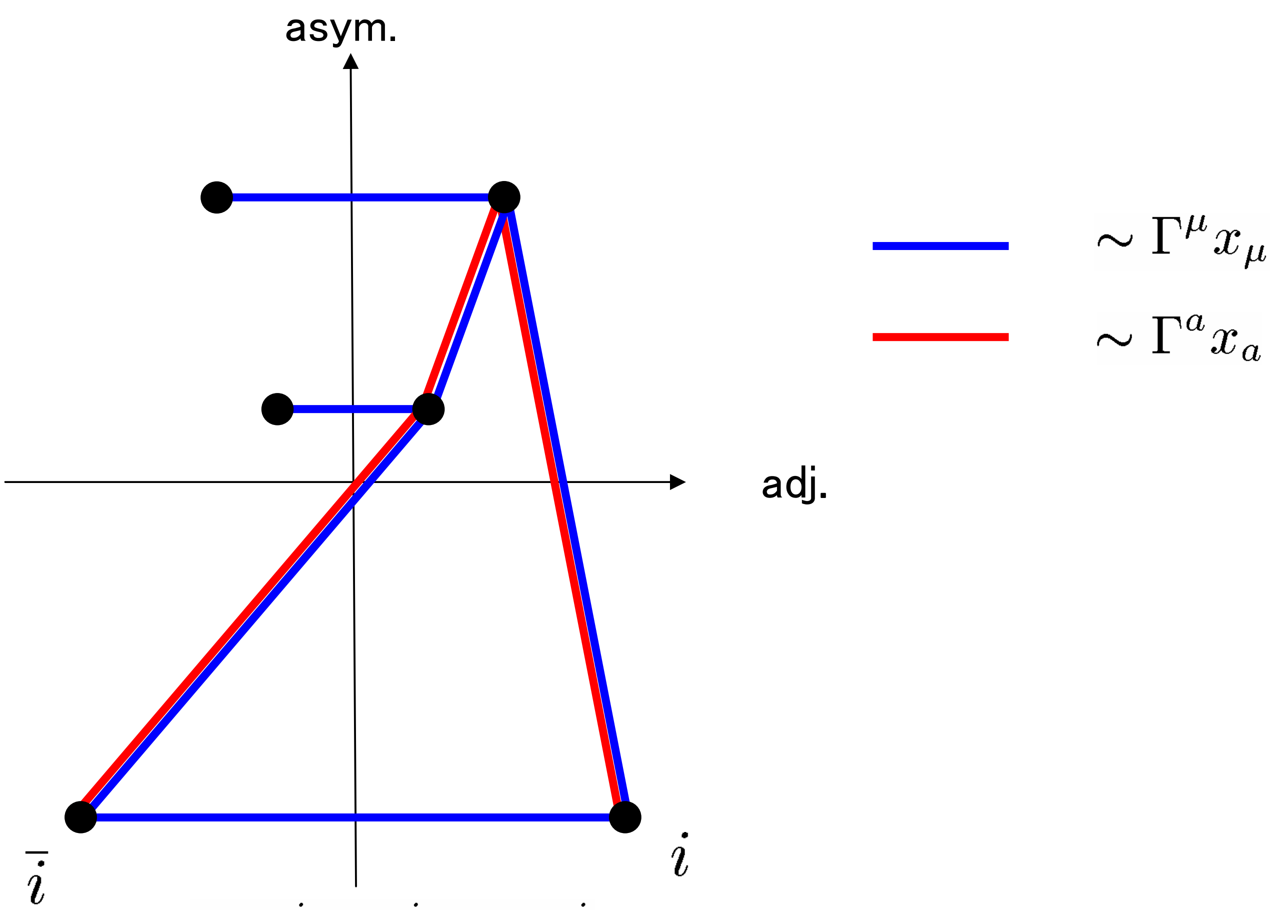
= SU(2)

$$x_a = a_a = 0$$

背景場はadjoint方向に制限されている



Aspect of interactions

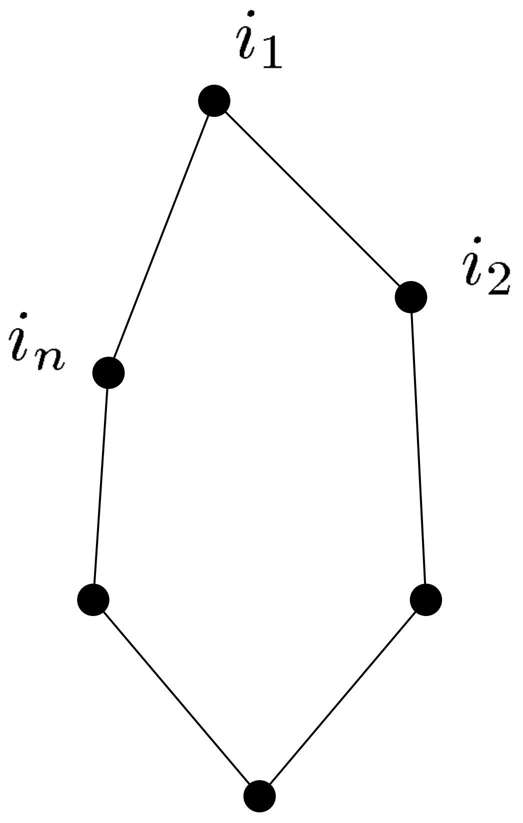


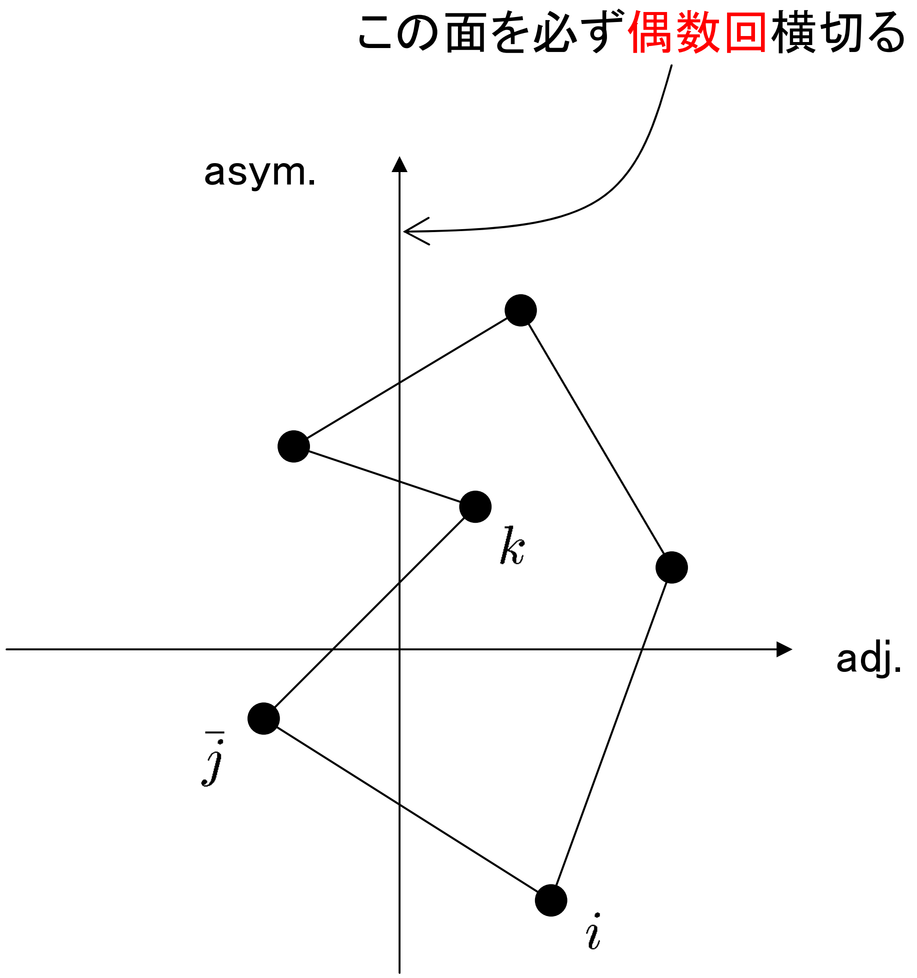
$$\sim (\xi^{i}_{a} + \xi^{i}_{a}) = 2\xi^{i}_{a}$$

$$\sim (\xi^{i}_{a'} - \xi^{i}_{a'}) = 0$$

$\Rightarrow$ ある点とその点自身の鏡像の間の相互作用は SU(2)行列模型と同じ

Maximal trees in USp





$$\sim (\xi^{i_1} - \xi^{i_2})(\xi^{i_2} - \xi^{i_3}) \cdots (\xi^{i_n} - \xi^{i_1})$$
$$= 0$$

$$\sim \cdots (\xi^{i} + \xi^{j})(\xi^{j} + \xi^{k}) \cdots$$
$$= 0$$

このようにloopは寄与せず、USp行列模型においてもmaximal treeのみが現れる。

Summery

・USp行列模型とIIB行列模型の有効相互作用の構造を比較した。

$$S \to T = S - U$$

・ξ 積分に関して2種類の相互作用が存在する。

$$\sim \Gamma^{\mu} x_{\mu} \ , \quad \sim \Gamma^{a} x_{a}$$

・ある点とその鏡像点の間の相互作用はSU(2)行列模型と同じ。

・Maximal treeのみが寄与する。