

Dressed Gluon Exponentiation for Inclusive B–decay Spectra

Einan Gardi (Cambridge)

Plan of the talk

- Introduction: inclusive B decay spectra — the challenge
- Sudakov resummation with NNLL accuracy, divergence of perturbation theory
- Dressed Gluon Exponentiation: renormalon resummation in the Sudakov exponent
- The quark distribution function in a meson and in an on-shell heavy quark
- cancellation of the leading renormalon ambiguity
- Numerical results for inclusive B-decay spectra by DGE; comparison to data

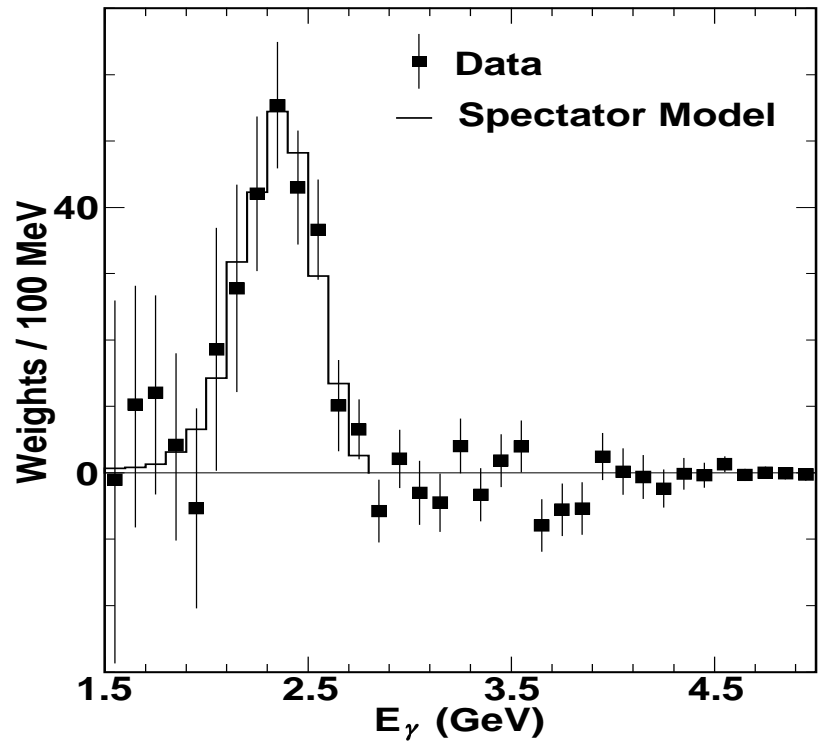
Dressed Gluon Exponentiation for Inclusive B–decay Spectra

References

- Inclusive spectra in charmless semileptonic B decays by DGE, J.R. Andersen, E. Gardi, [hep-ph/0509360].
- Taming the $\bar{B} \rightarrow X_s \gamma$ spectrum by Dressed Gluon Exponentiation, J.R. Andersen, E. Gardi, JHEP **0506** 030 (2005) [hep-ph/0502159].
- On the quark distribution in an on-shell heavy quark and its all-order relations with the perturbative fragmentation function, E. Gardi, JHEP **0502**, 053 (2005) [hep-ph/0501257].
- Radiative and semi-leptonic B-meson decay spectra: Sudakov resummation beyond logarithmic accuracy and the pole mass, E. Gardi, JHEP **0404**, 049 (2004) [hep-ph/0403249].

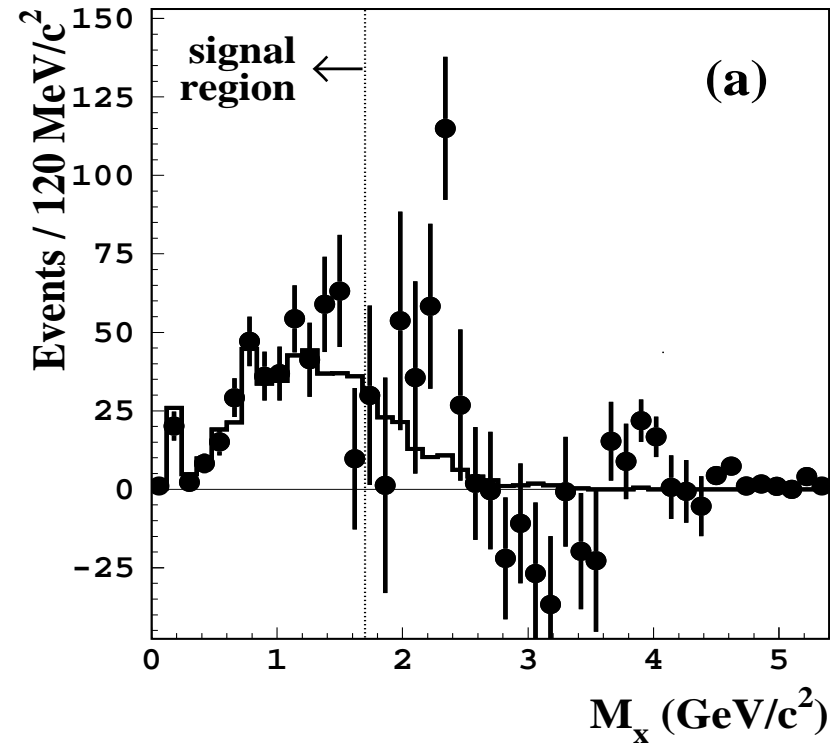
Inclusive B-decay Spectra

radiative decay: $\bar{B} \longrightarrow X_s \gamma$



CLEO

semi-leptonic decay: $\bar{B} \longrightarrow X_u l \bar{\nu}_l$



BELLE

The distribution **peaks** close to the **endpoint** ($E_\gamma \longrightarrow M_B/2$; small M_X)

Example: extracting $|V_{ub}|$ from the semi-leptonic decay

Precise measurements are restricted to the **small M_X region** (charm background)

Determination of $|V_{ub}|$ relies on calculation of the spectrum.

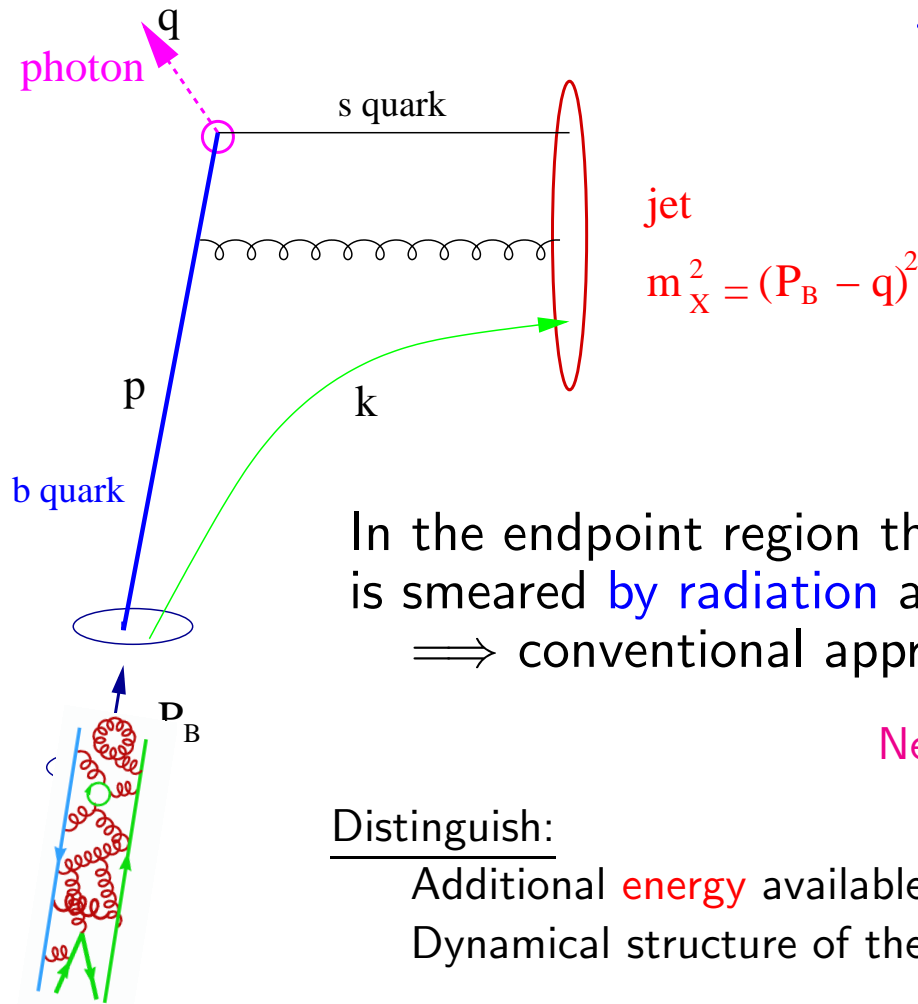
Kinematics in $\bar{B} \rightarrow X_s \gamma$

In the **B meson** the **b quark** is close to its mass shell.
 Therefore, perturbation theory (with an on-shell quark initial state) applies
 (up to power corrections...).

$$x \equiv \frac{2E_\gamma}{m_b}; \quad \left. \frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dx} \right|_{\text{LO}} = \delta(1-x)$$

Perturbative endpoint: $x = 1$

Physical endpoint: $x = M_B/m_b > 1$



In the endpoint region the distribution
 is smeared by **radiation** and by **the primordial motion of the quark**
 \implies conventional approach: **leading power NP “shape function”**.

Neubert; Bigi, Shifman, Uraltsev & Vainshtein (93)

Distinguish:

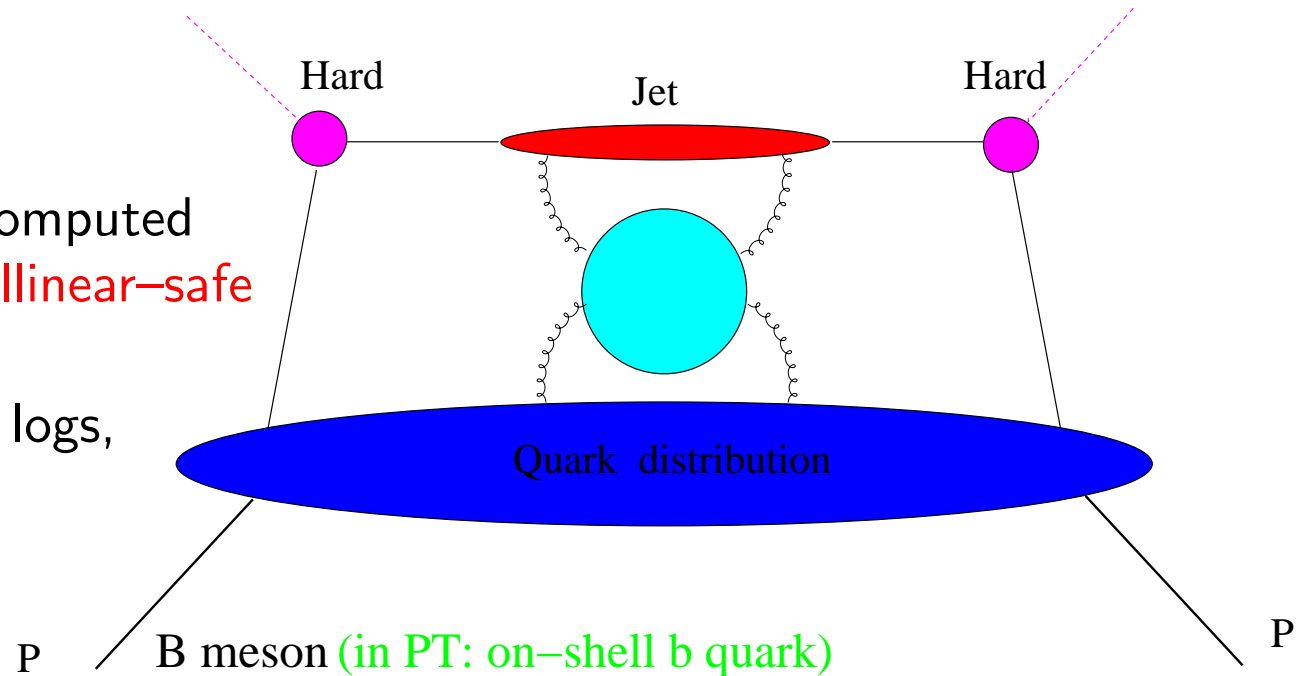
Additional **energy** available in the meson $\bar{\Lambda} = M_B - m_b$

Dynamical structure of the meson

Large- x factorization in inclusive B decays

The spectrum can be computed in PT: **infrared- and collinear-safe**

Dominated by Sudakov logs, $\ln(1-x)$



scales:

Hard: m

Jet: $m_X^2 = (P_b - q)^2 \simeq m^2(1-x) \implies m^2/N$

Soft: $m(1-x) \implies m/N$

Korchinsky & Sterman (94)

Spectral moments:

$$\begin{aligned} \Gamma_N^{\text{PT}} &\equiv \int_0^1 dx x^{N-1} \frac{1}{\Gamma_{\text{tot}}^{\text{PT}}} \frac{d\Gamma^{\text{PT}}}{dx} \\ &= H(m) J(m^2/N; \mu) S_{\text{PT}}(m/N; \mu) + \mathcal{O}(1/N) \\ &\equiv H(m) \text{Sud}(N, m) + \mathcal{O}(1/N) \end{aligned}$$

Coefficients in the Sudakov exponent

$$\text{Sud}(N, m) = \exp \left\{ - \sum_{n=1}^{\infty} \sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi} \right)^n \right\}$$

The coefficients $C_{n,k}$ are known **exactly** to **NNLL accuracy** [Gardi (2005)]

For $N_f = 4$ $C_{n,k}$ are:

	$k \longrightarrow$								
n	−1.564	0.667	0	0	0	0	0	0	0
↓	3.837	−0.078	1.389	0	0	0	0	0	0
	?	20.579	6.339	3.376	0	0	0	0	0
	?	?	116.464	33.024	9.042	0	0	0	0
	?	?	?	597.221	138.600	25.955	0	0	0
	?	?	?	?	2859.284	548.170	78.492	0	0
	?	?	?	?	?	13141.289	2129.058	247.233	0
	?	?	?	?	?	?	58941.217	8238.359	0
	?	?	?	?	?	?	?	260391.559	0

- At a given order in α_s the coefficients of **subleading logs** (lower k) get large...
- Is the **fixed-logarithmic-accuracy** approximation at **LL** / **NLL** / **NNLL** good?

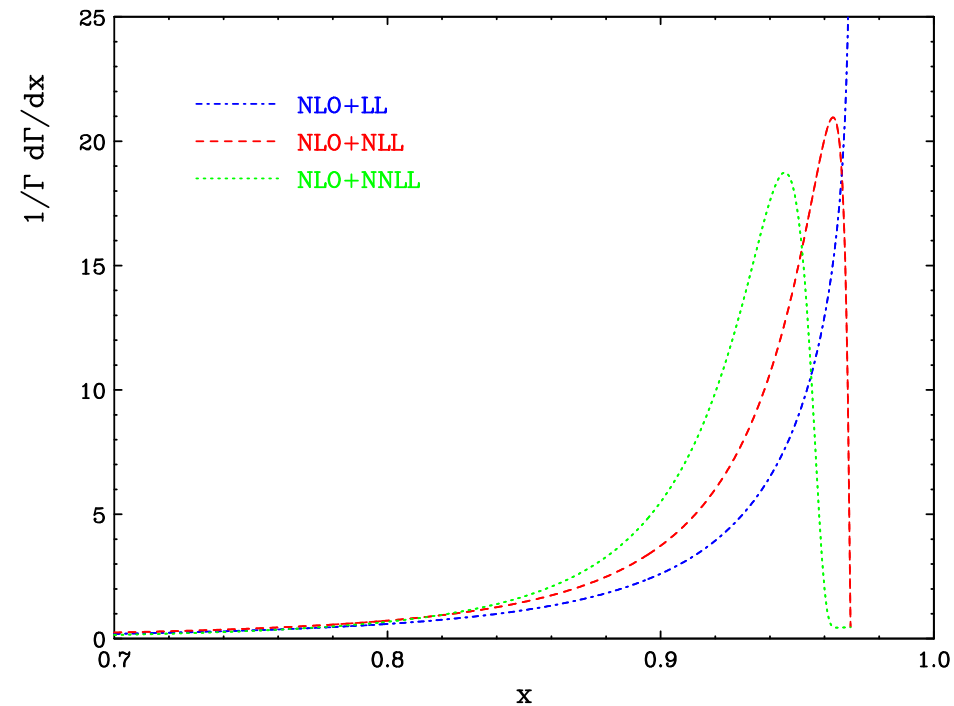
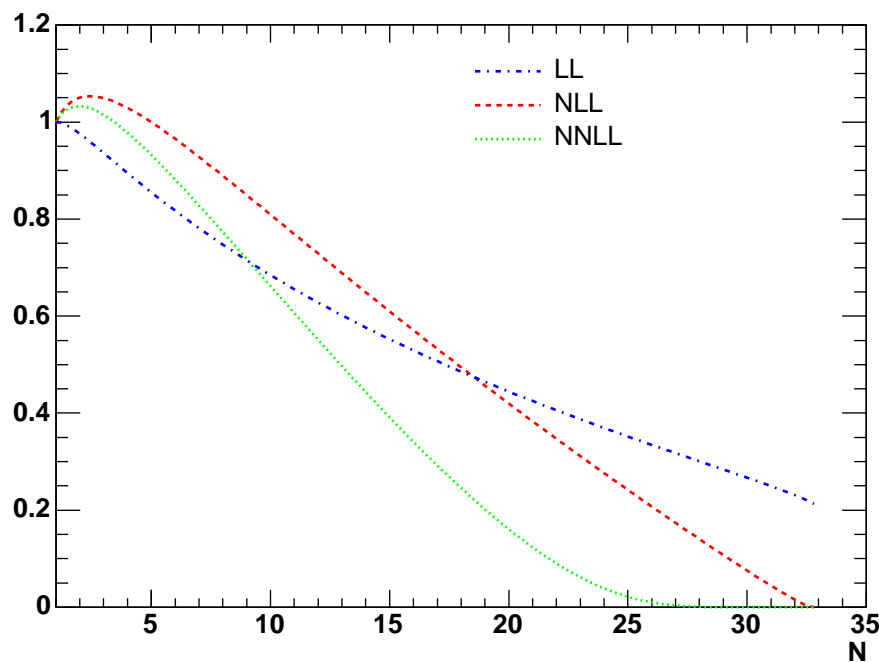
Conventional Sudakov resummation with NNLL accuracy

$$\text{Sud}(N, m) = \exp \left\{ \sum_{n=0}^{\infty} g_n(\lambda) \left(\frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi} \right)^{n-1} \right\}; \quad \lambda \equiv \frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi} \beta_0 \ln N$$

$$g_0(\lambda) = \frac{C_F}{\beta_0^2} \left[(1 - \lambda) \ln(1 - \lambda) - \frac{1}{2} (1 - 2\lambda) \ln(1 - 2\lambda) \right]$$

Sud(N, m)

Corresponding spectra



Coefficients in the Sudakov exponent in the large- β_0 limit

$$\text{Sud}(N, m) = \exp \left\{ - \sum_{n=1}^{\infty} \sum_{k=1}^{n+1} C_{n,k} \ln^k N \left(\frac{\alpha_s^{\overline{\text{MS}}}(m^2)}{\pi} \right)^n \right\}$$

The part in $C_{n,k}$ that is proportional to $(\beta_0)^{n-1}$ is known to all orders:

	$k \longrightarrow$						
n	-1.56	0.67	0	0	0	0	0
	1.24	0.90	1.39	0	0	0	0
↓	61.17	28.32	8.28	3.38	0	0	0
	1096.06	515.20	166.25	34.89	9.04	0	0
	20399.23	10078.43	3231.40	793.25	131.33	25.95	0
	444615.21	221481.03	73268.94	17791.58	3514.66	482.12	78.49
	11342675.74	5665794.49	1883129.50	468180.33	91361.30	15080.79	1768.50
	334032127.30	166960507.50	55609620.17	13867704.58	2760946.21	449959.01	63745.75

- $C_{n,k}$ increase for lower powers of $\ln N$, building up $\sum_{k=1}^{n+1} C_{n,k} \ln^k N \sim n! f_n(N)$
- Truncation at fixed logarithmic accuracy is **not** a good approximation.
- **Renormalon divergence** sets in already at low orders — requires a prescription!

Dressed Gluon Exponentiation — the jet function

Borel representation of the Sudakov exponent:

$$\begin{aligned} \ln J_N(Q; \mu_F) &= \int_0^1 dx \frac{x^{N-1} - 1}{1-x} \left[\int_{\mu_F^2}^{(1-x)Q^2} \frac{d\mu^2}{\mu^2} \mathcal{A}(\alpha_s(\mu^2)) + \mathcal{B}(\alpha_s((1-x)Q^2)) \right] \\ &= -\frac{C_F}{\beta_0} \int_0^\infty \frac{du}{u} \left(\frac{\Lambda^2}{Q^2} \right)^u \times \left[B_{\mathcal{J}}(u) \Gamma(-u) (N^u - 1) + \left(\frac{Q^2}{\mu_F^2} \right)^u B_{\mathcal{A}}(u) \ln N \right], \end{aligned}$$

we defined $B_{\mathcal{J}}(u) \equiv B_{\mathcal{A}}(u) - u B_{\mathcal{B}}(u)$ and used the Borel representation of the **anomalous dimensions**,

$$\mathcal{A}(\alpha_s(\mu^2)) = \frac{C_F}{\beta_0} \int_0^\infty du \left(\frac{\Lambda^2}{\mu^2} \right)^u B_{\mathcal{A}}(u); \quad \mathcal{B}(\alpha_s(\mu^2)) = \frac{C_F}{\beta_0} \int_0^\infty du \left(\frac{\Lambda^2}{\mu^2} \right)^u B_{\mathcal{B}}(u),$$

$$\int_0^1 dx x^{N-1} (1-x)^{-1-u} = \frac{\Gamma(-u) \Gamma(N)}{\Gamma(N-u)} \simeq \Gamma(-u) N^u \times (1 + \mathcal{O}(1/N)).$$

In the **large β_0 limit** $B_{\mathcal{J}}(u) = e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \times \frac{1}{2} \left(\frac{1}{1-u} + \frac{1}{1-u/2} \right) \times \left(1 + \mathcal{O}(u/\beta_0) \right).$

Infrared sensitivity appears as renormalon ambiguity in the Sudakov exponent
 \Rightarrow **parametrically-enhanced power corrections $\mathcal{O}(N\Lambda^2/Q^2)$ in the exponent**

Dressed Gluon Exponentiation — the soft function

Borel representation of the **soft** Sudakov exponent:

$$\begin{aligned} \ln S_N(Q; \mu_F) &= \int_0^1 dx \frac{x^{N-1} - 1}{1-x} \left[\int_{(1-x)^2 Q^2}^{\mu_F^2} \frac{d\mu^2}{\mu^2} \mathcal{A}(\alpha_s(\mu^2)) - \mathcal{D}(\alpha_s((1-x)^2 Q^2)) \right] \\ &= \frac{C_F}{\beta_0} \int_0^\infty \frac{du}{u} \left(\frac{\Lambda^2}{Q^2} \right)^u \left[B_{\mathcal{S}}(u) \Gamma(-2u) (N^{2u} - 1) + \left(\frac{Q^2}{\mu_F^2} \right)^u B_{\mathcal{A}}(u) \ln N \right], \end{aligned}$$

where we defined $B_{\mathcal{S}}(u) \equiv B_{\mathcal{A}}(u) - u B_{\mathcal{D}}(u)$.

What does one gain?

- **Resummation of running-coupling effects** beyond the available logarithmic accuracy
- Upon choosing a prescription (e.g. **PV**) for the Borel integral, *the divergent sum is defined*.
- **Cancellation** of **certain renormalon ambiguities** can then take place.
- Landau singularities are **absent**.
- The pattern of power corrections (observable dependent) can be studied:
singularities in $\Gamma(-2u) \implies$ power corrections $(N\Lambda/Q)^k$ **in the exponent**, except for $B_{\mathcal{S}}(u) = 0$.

However, QCD perturbation theory gives the power expansion: $B_{\mathcal{S}}(u) = 1 + s_1 u + \dots$

For DGE one needs to know $B_{\mathcal{S}}(u)$ also **away from the origin** — involves assumptions!

Soft anomalous dimensions in the large- β_0 limit

$$B_S(u) = e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} b_S(u) \times \left(1 + \mathcal{O}(u/\beta_0)\right)$$

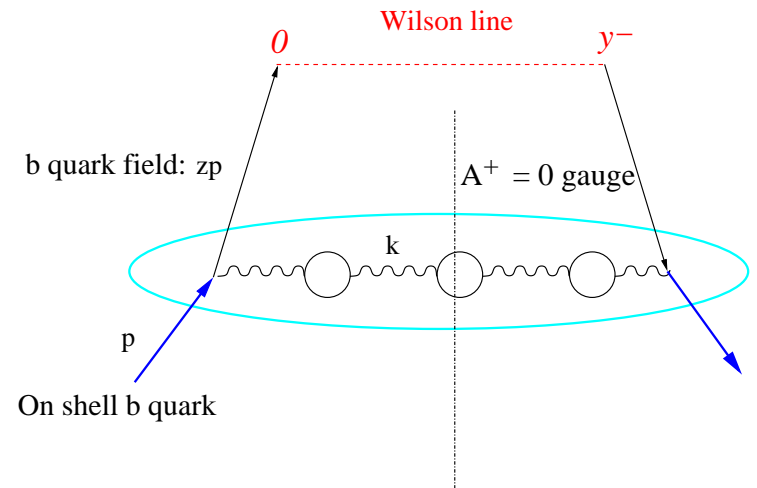
Observable	$b_S(u)$	$B_S(u) = 0$	power corrections
Drell-Yan (2)	$\frac{\Gamma^2(1-u)}{\Gamma(1-2u)}$	$u = \frac{1}{2}, \frac{3}{2}, \dots$	$\left(\frac{\Lambda N}{Q}\right)^k, k \text{ even}$
Heavy Jet Mass (1) / Thrust (2)	1		$\left(\frac{\Lambda N}{Q}\right)^k, k \text{ integer}$
c parameter (2)	$\frac{\Gamma^2(1+u)}{\Gamma(1+2u)}$		$\left(\frac{\Lambda N}{Q}\right)^k, k \text{ integer}$
Heavy Quark Fragmentation (1) Heavy Quark Distribution (1) ($Q^2 = m^2$)	$(1-u) \frac{\pi u}{\sin \pi u}$	$u = 1$	$\left(\frac{\Lambda N}{m}\right)^k, k \neq 2$

The quark distribution function

$$F_{\text{PT}}(N; \mu) \xrightarrow{\text{large } N} \left\langle b(p_b) \left| \left[\bar{\Psi}(y) \gamma^+ \Phi_y(0, y) \Psi(0) \right]_{\mu} \right| b(p_b) \right\rangle \Big|_{ip_b^+ y^- \rightarrow N} = H(m_b, \mu) \mathcal{S} \left(\frac{N\mu}{m_b} \right)$$

$$\mathcal{S} \left(\frac{N\mu}{m_b} \right) = \exp \left\{ \frac{C_F}{\beta_0} \int_0^\infty \frac{du}{u} \left(\frac{\Lambda^2}{\mu^2} \right)^u \left[B_S(u) \Gamma(-2u) \left(\left(\frac{N\mu}{m_b} \right)^{2u} - 1 \right) + B_A(u) \ln \left(\frac{N\mu}{m_b} \right) \right] \right\}$$

$$\begin{aligned} \text{with } B_S(u) &= e^{\frac{5}{3}u} (1-u) \times \left(1 + \mathcal{O}(u/\beta_0) \right) \\ &= 1 + s_1 u + s_2 u^2/2! + \dots \end{aligned}$$



Renormalon in the exponent and their interpretation:

- **Leading renormalon** $u = \frac{1}{2}$, $\mathcal{O}(\Lambda N/m_b)$, is related to the **mass** of $\langle b(p_b) \rangle$: $e^{-i \delta m y^-} = e^{-\delta m N/m_b}$
- Higher renormalons $u \geq \frac{3}{2}$, $(\Lambda N/m_b)^k$ with $k \geq 3$, correspond to the difference between the momentum distribution in the **on-shell quark** and the (unambiguous) distribution in the **meson**:

$$F(N; \mu) = \left\langle B(P_B) \left| \left[\bar{\Psi}(y) \gamma^+ \Phi_y(0, y) \Psi(0) \right]_{\mu} \right| B(P_B) \right\rangle \Big|_{iP_B^+ y^- \rightarrow N} + \mathcal{O}(1/N)$$

Cancellation of the leading renormalon ambiguity

Owing to **kinematic power corrections**, the resummed E_γ spectrum is not influenced by the $u = \frac{1}{2}$ $\mathcal{O}(N\Lambda/m_b)$ ambiguity of the perturbative Sudakov exponent:

$$\begin{aligned} \frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dE_\gamma} &= \frac{2}{m_b} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \left(\frac{2E_\gamma}{m_b} \right)^{-N} H(m_b) \underbrace{J(m_b^2/N; \mu) S_{\text{PT}}(m_b/N; \mu)}_{\text{Sud}(m_b, N) - \text{ambiguous}} \\ &\simeq \frac{2}{M_B} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} \left(\frac{2E_\gamma}{M_B} \right)^{-N} H(m) J(m_b^2/N; \mu) \underbrace{S_{\text{PT}}(m_b/N; \mu) e^{-(N-1)\bar{\Lambda}/m_b}}_{u=\frac{1}{2} \text{ prescription independent}} \end{aligned}$$

The cancellation **is exact in all the moments**, but it requires

- renormalon resummation in the Sudakov exponent
- renormalon resummation in $\bar{\Lambda} = M_B - m_b$ using the same prescription.

Sudakov resummation beyond logarithmic accuracy

$$\text{Sud}(m, N)|_{\text{PV}} = \exp \left\{ \frac{C_F}{\beta_0} \text{PV} \int_0^\infty du T(u) \left(\frac{\Lambda^2}{m^2} \right)^u \right. \\ \left. \times \frac{1}{u} \left[B_{\mathcal{S}}(u) \Gamma(-2u) (N^{2u} - 1) - B_{\mathcal{J}}(u) \Gamma(-u) (N^u - 1) \right] \right\}.$$

What do we know about $B_{\mathcal{S}}(u)$?

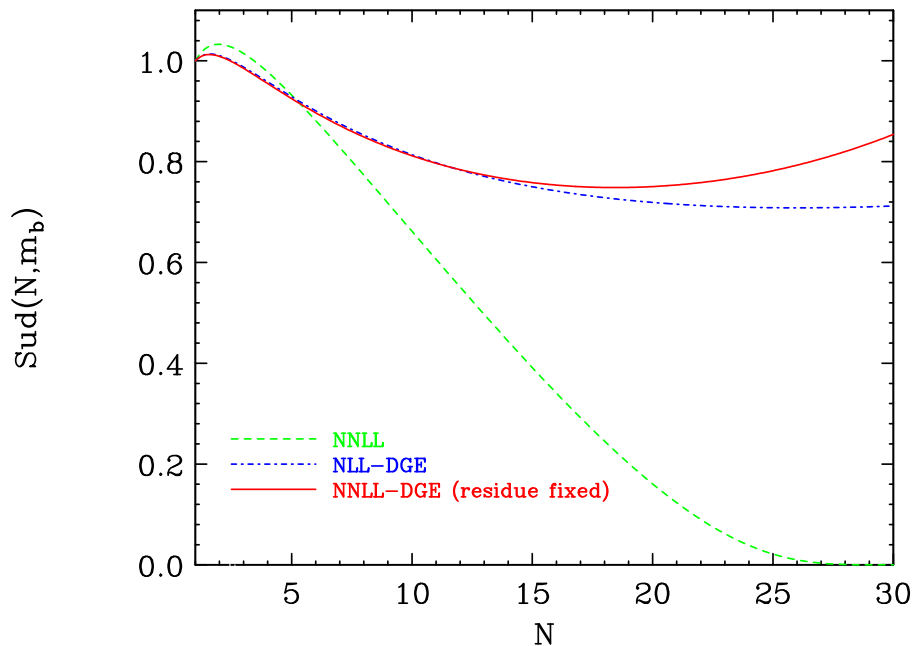
- NNLO in the full theory: $B_{\mathcal{S}}(u) = 1 + s_1 \frac{u}{1!} + s_2 \frac{u^2}{2!} + \dots$
- Renormalon cancellation in $\text{Sud}(m, N) e^{-(N-1)\bar{\Lambda}/M}$ implies:
 $B_{\mathcal{S}}(u = 1/2)$ is **equal in magnitude and opposite in sign** to the residue of the $u = 1/2$ **renormalon** in $m/m_{\overline{\text{MS}}}$, which can be determined from the known NNLO expansion in $\overline{\text{MS}}$ **within a few percent**.
- All orders in the large- β_0 limit: $B_{\mathcal{S}}(u) = e^{\frac{5}{3}u} (1 - u) + \mathcal{O}(1/\beta_0^2)$.
 The vanishing of $B_{\mathcal{S}}(u)$ at $u = 1$ is **assumed** to hold in general.

$\bar{B} \rightarrow X_s \gamma$ spectrum: from moment space to E_γ

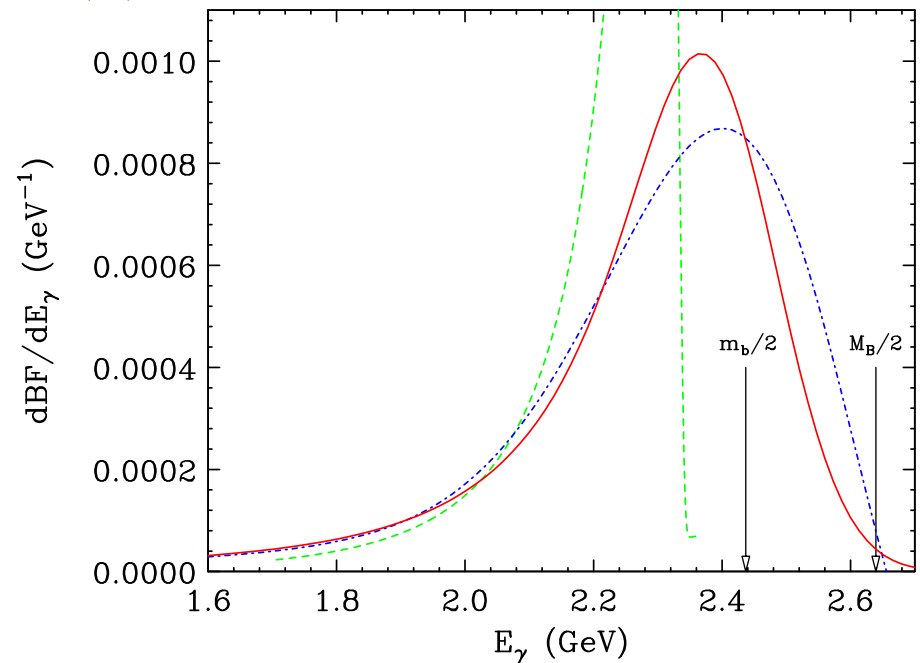
$$\text{Sud}(m, N)|_{\text{PV}} = \exp \left\{ \frac{C_F}{\beta_0} \text{PV} \int_0^\infty du T(u) \left(\frac{\Lambda^2}{m^2} \right)^u \right. \\ \left. \times \frac{1}{u} \left[B_S(u) \Gamma(-2u) (N^{2u} - 1) - B_J(u) \Gamma(-u) (N^u - 1) \right] \right\}. \\ \frac{d\Gamma(E_\gamma)}{dE_\gamma} = \frac{m_{\text{PV}}}{2} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} H(m) \text{Sud}(m, N)|_{\text{PV}} \left(\frac{2E_\gamma}{m_{\text{PV}}} \right)^{-N}$$

Modified support properties:

$\text{Sud}(N, m)|_{\text{PV}}$ with various approx. for $B_S(u)$



Corresponding spectra



$B_S(u)$ away from the origin

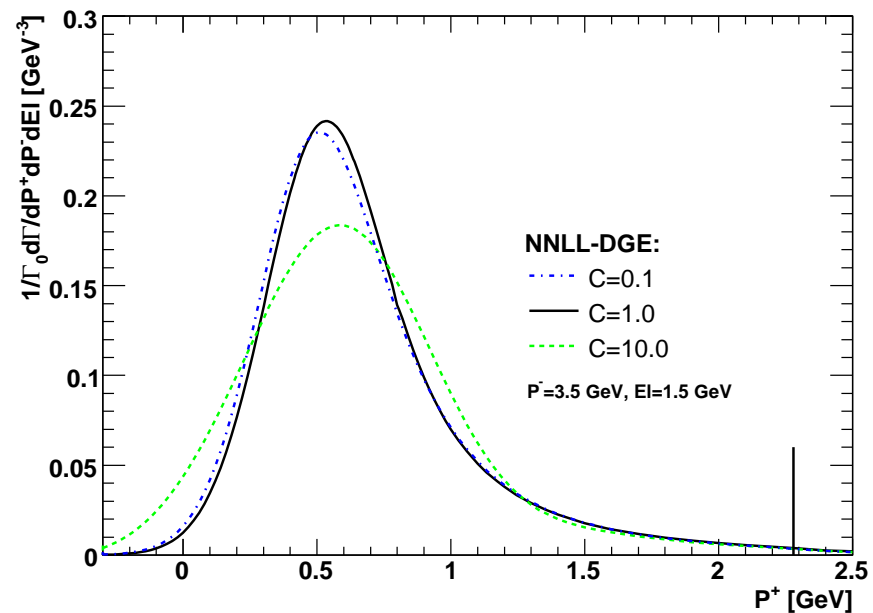
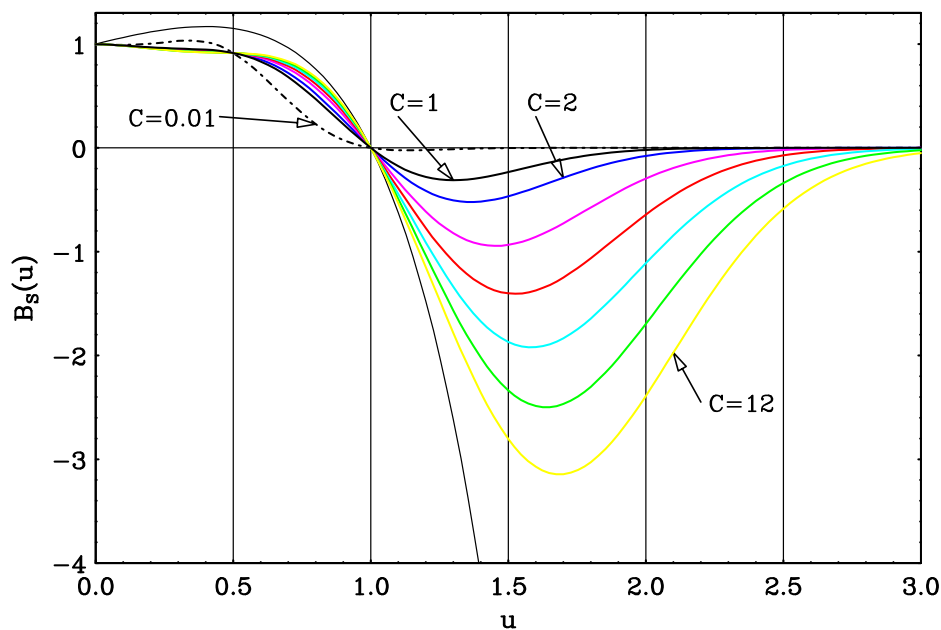
Ansatz for $B_S(u)$ that is consistent with the known $\mathcal{O}(u^2)$ result in QCD (and the large- β_0 limit):

$$B_S(u) = e^{\frac{5}{3}u}(1-u) \times \exp \left\{ c_2 u + \frac{1}{2} \left[c_3 - c_2^2 + \frac{C_A}{\beta_0} \left(\frac{5}{18} \pi^2 + \frac{7}{9} - \frac{9}{2} \zeta_3 \right) \right] u^2 \right\} \times W(u)$$

$$W(u) \equiv e^{t_1 u + \frac{1}{2} t_2 u^2} \left(1 - t_1 u + \frac{1}{2} (t_1^2 - t_2) u^2 \right) = 1 + \mathcal{O}(u^3).$$

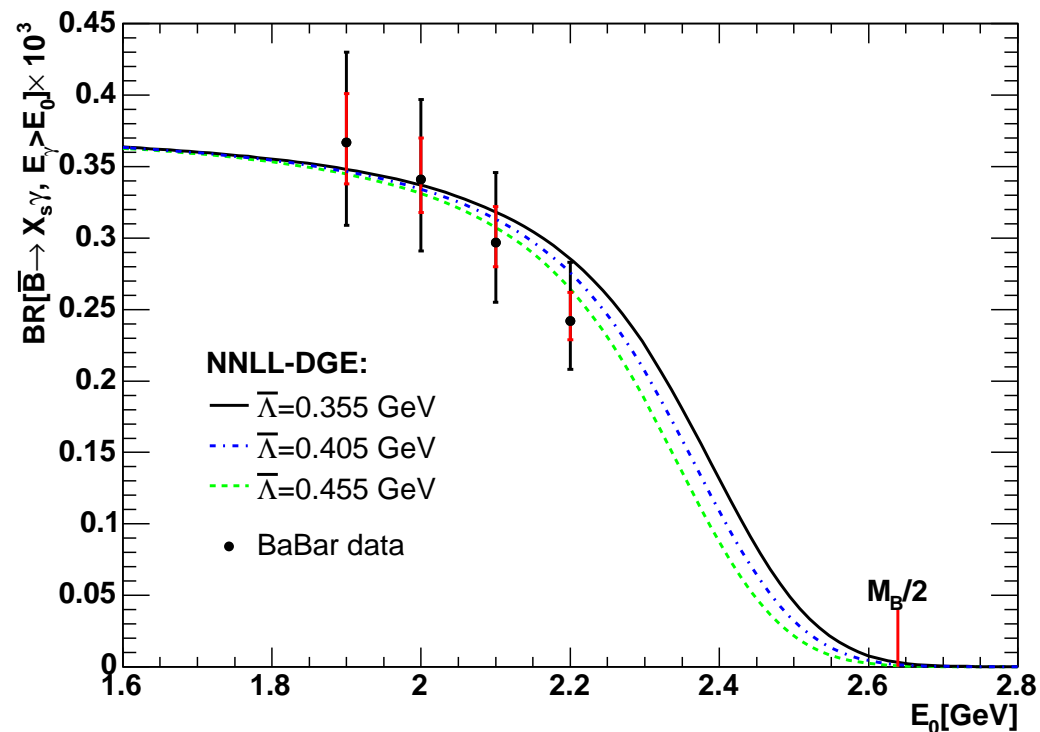
Here $t_{1,2}$ are fixed requiring:

$$B_S(u = 1/2) = 0.914 \pm 3\% \text{ (computed)}; \quad B_S(u = 3/2) = -0.23366 \times C,$$



comparison to data: $\bar{B} \rightarrow X_s \gamma$ branching fraction

- Theoretical uncertainty on the total BF $\sim 10\%$
- Experimental cuts on E_γ *do not* significantly increase the overall uncertainty.
- The measured BF is **consistent** with the Standard Model.

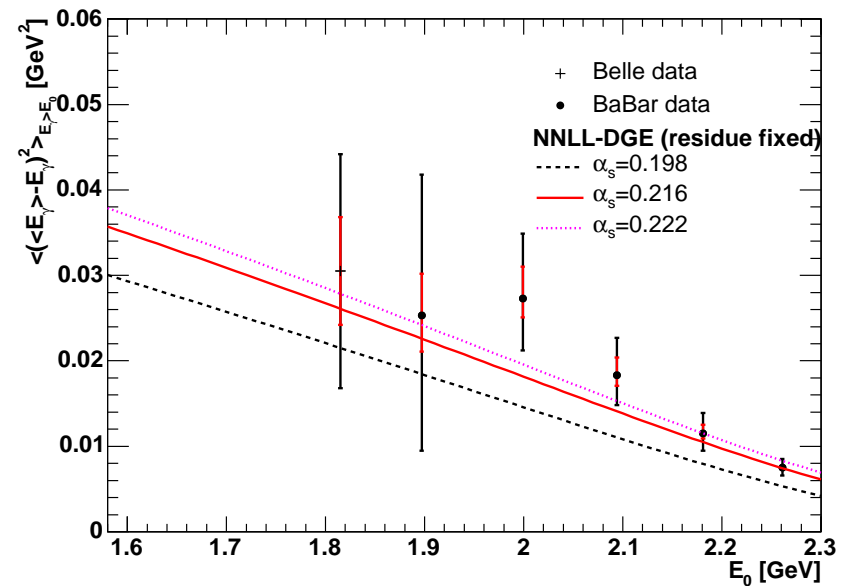
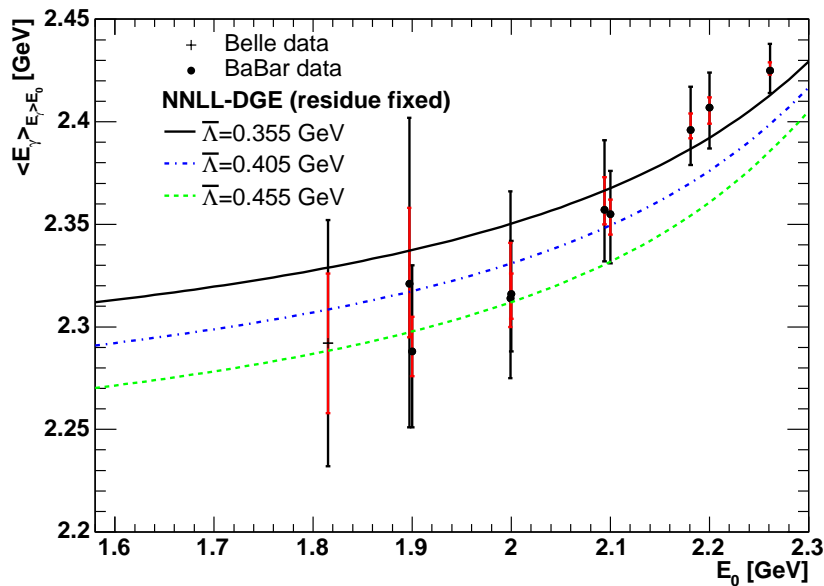


- Possible **determination of m_b** !

comparison to data: cut moments in $\bar{B} \rightarrow X_s \gamma$

$$\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} E_\gamma$$

$$\left\langle \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n \right\rangle_{E_\gamma > E_0} \equiv \frac{1}{\Gamma(E_\gamma > E_0)} \int_{E_0} dE_\gamma \frac{d\Gamma(E_\gamma)}{dE_\gamma} \left(\left\langle E_\gamma \right\rangle_{E_\gamma > E_0} - E_\gamma \right)^n.$$



- The comparison suggests that **power corrections** are indeed small.
- In future: possible measurement of **power corrections**.

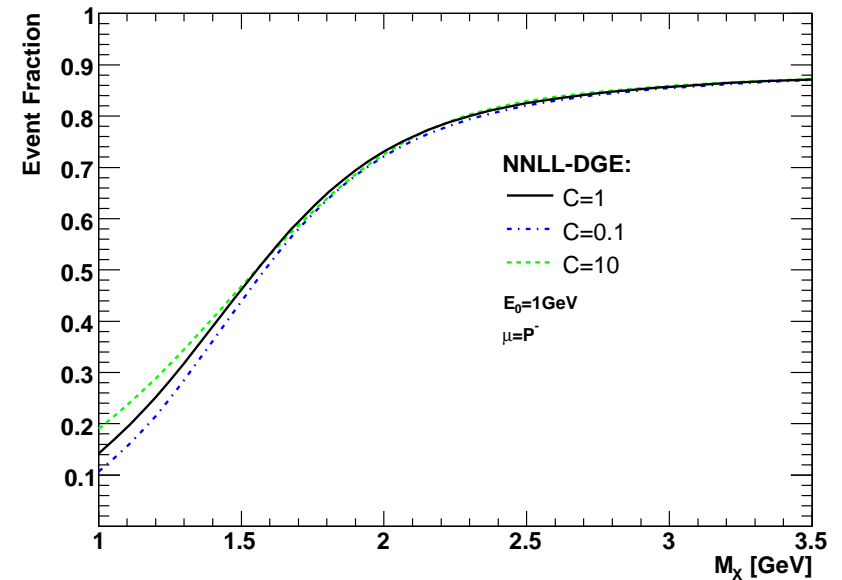
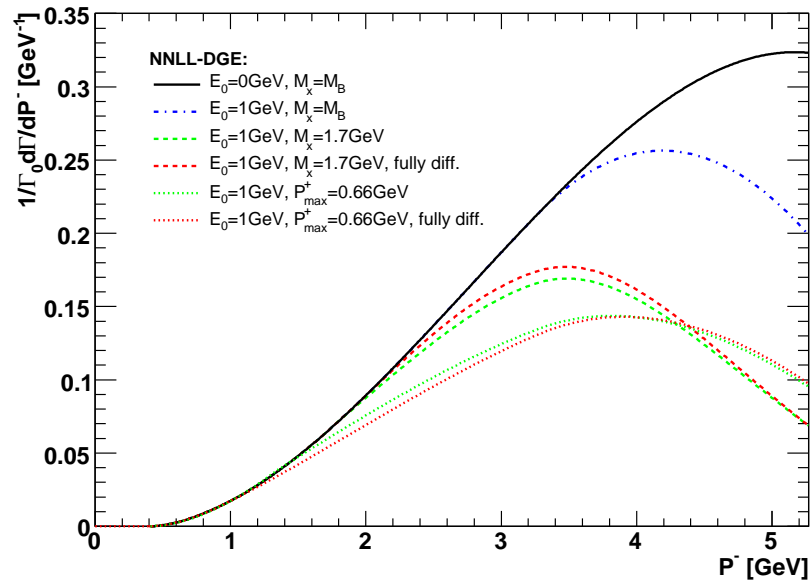
Integrated $\bar{B} \rightarrow X_u l \bar{\nu}$ spectrum

Integrating the spectrum with given experimental cuts:

- Hadronic Mass Cut: $P^+ P^- < (1.7 \text{ GeV})^2$, $E_l > 1 \text{ GeV}$
- Small Lightcone Component Cut: $P^+ < 0.66 \text{ GeV}$, $E_l > 1 \text{ GeV}$

The effect of cuts on the P^- spectrum

Sensitivity of the Event Fraction to C



Extracting for $|V_{ub}|$ from Belle data

$$\Delta\mathcal{B}(\bar{B} \longrightarrow X_u l \bar{\nu} \text{ restricted phase space}) = \tau_B \Gamma_{\text{tot}}(\bar{B} \longrightarrow X_u l \bar{\nu}) \times R_{\text{cut}}.$$

From Belle data

$$\Delta\mathcal{B}(P^+ P^- < (1.7 \text{ GeV})^2, E_l > 1 \text{ GeV}) = 1.24 \cdot 10^{-3} \quad (\pm 13.4\%)$$

$$\Delta\mathcal{B}(P^+ < 0.66 \text{ GeV}, E_l > 1 \text{ GeV}) = 1.10 \cdot 10^{-3} \quad (\pm 17.2\%)$$

and the computed event fraction

$$R_{\text{cut}}(P^+ P^- < (1.7 \text{ GeV})^2, E_l > 1 \text{ GeV}) = 0.615 \quad (\pm 9.6\%)$$

$$R_{\text{cut}}(P^+ < 0.66 \text{ GeV}, E_l > 1 \text{ GeV}) = 0.535 \quad (\pm 15.2\%),$$

we obtain

$$|V_{ub}| = \left(4.35 \pm 0.28_{[\text{exp}]} \pm 0.14_{[\text{th-total}(m_b^{\overline{\text{MS}}})]} \pm 0.22_{[\text{th-cuts}]} \right) \cdot 10^{-3}$$

$$|V_{ub}| = \left(4.39 \pm 0.36_{[\text{exp}]} \pm 0.14_{[\text{th-total}(m_b^{\overline{\text{MS}}})]} \pm 0.38_{[\text{th-cuts}]} \right) \cdot 10^{-3}$$

Conclusions

- Resummed perturbation theory can be directly used as **can approximation to inclusive B meson decay spectra**, **without a leading power non-perturbative function**.
- The leading renormalon **cancels out** when using the **same prescription** for renormalons in both the **Sudakov exponent** and the **pole mass** that appears upon conversion to physical variables.
- Beyond the **logarithmic** accuracy at hand (NNLL), the Borel sum of the exponent is constrained by information on renormalon residues. For the quark distribution in an on-shell heavy quark $B_S(u = 1/2)$ was **computed(!)** and $B_S(u = 1)$ **vanishes(?)**
- Contrary to Sudakov resummation with fixed logarithmic accuracy, the **DGE** prediction is **free of Landau singularities** and **stable**.
- The DGE spectrum smoothly extends **beyond the perturbative endpoint**
Its support is close to the physical one, provided that $B_S(u)$ is not too large at **intermediate u** (e.g. around $u \sim 3/2$).
- Application to **charmless semileptonic decay***:
the **event fraction** associated with an **invariant mass cut** $P^+P^- < (1.7 \text{ GeV})^2$
is obtained with **$\pm 10\%$ accuracy**
Consistent values for $|V_{ub}|$ are obtained from two different cuts.

*The program can be found at: www.hep.phy.cam.ac.uk/~andersen/BDK/B2U