Supergravity, Gaugings and Domain Walls

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ABSTRACT

The role of gauged supergravities within M-theory is discussed. In particular, we consider gauged maximal supergravities with so-called CSO gauge groups and their relation to the branes of string and M-theory. The gauge groups are characterised by n mass parameters, where n is the transverse dimension of the brane. We give the scalar potentials and construct the corresponding domain wall solutions. In addition, we show the higher-dimensional origin of the domain walls in terms of (distributions of) branes. Particular emphasis is put on the special features for $n = 2$ and $n = 3$.

1. Introduction

1.1. M-theory and Supergravity

It is known since the 1980's that string theory contains, among other forces, gravity and has finite scattering amplitudes; indeed, it is the most promising candidate for the unification of quantum field theory and general relativity. In addition it introduces a number of concepts which have not (yet) been observed in reality, such as supersymmetry, extra dimensions and extended objects. It was found that there are five different possibilities for strings, which all live in a ten-dimensional space-time and have either 16 or 32 supercharges Q:

- $Q = 16$: type I open strings with gauge group $SO(32)$ and heterotic closed strings with gauge groups $SO(32)$ or $E_8 \times E_8$,
- $Q = 32$: type IIA and IIB closed strings.

However, in the mid 1990's it was understood that these could be further unified to a single eleven-dimensional theory, M-theory. The different string theories are then understood as perturbative theories in different limits of the parameter space of M-theory. This appreciation goes under the name of U-duality [34] and has spectacularly changed our understanding of string theory and its non-perturbative regime.

Another important development is the AdS/CFT correspondence, which basically relates a string theory (in a particular background, i.e. IIB string theory on $AdS_5 \times S^5$) to a (particular and supersymmetric, i.e. $D = 4$ $N = 4$ SYM) quantum field theory. However, the link contains a non-perturbative duality: where the quantum field theory is well-behaved we have little control over the string theory and vice versa. Originally proposed by Maldacena [40] in 1997 with maximal supersymmetry, it has been extended to less supersymmetric cases.

Thus we have gained a better understanding of perturbative and non-perturbative string theory, its unification in M-theory and its relation to quantum field theory. However, there are still a lot of interesting open issues. Among the most important ranks cosmology: it remains to be seen how string theory gives rise to e.g. inflation, the expansion of the universe and, if recent astronomical observations are correct [45, 49], the acceleration of this expansion. The cosmological implications of string theory have gained an enormous interest recently.

An important tool to study the different phenomena in string theory is supergravity. This supersymmetric extension of general relativity appears as the low-energy effective description of string and M-theory. Many features of string and M-theory are also present in its supergravity limit, such as extended objects and U-duality, and one can learn a great deal about the underlying theory by studying its field theory limit. In particular, one can extract effective four-dimensional descriptions by considering string or M-theory on a compact "internal" manifold of very small size. This technique is known as dimensional reduction. Different reductions give rise to different four-dimensional supergravities.

Among these, gauged supergravities are very interesting from a phenomenological point of view. Supergravities have a global symmetry group which is called the U-duality group (and indeed is a consequence of the U-duality between string theories). In gauged supergravities a subgroup of this global group is made local, i.e. is gauged, by the introduction of mass parameters.

The combination of a gauge group and local supersymmetry implies the appearance of a scalar potential. This generically breaks the Minkowski vacuum to solutions like Anti-de Sitter space-time, domain walls (DWs) or time-dependent solutions. The first two are relevant due to i) AdS/CFT [40] and its generalisation, the DW/QFT correspondence [7, 13, 36], and ii) braneworlds scenarios [47, 48], in which our four dimensions are a hypersurface in space-time. The time-dependent solutions can play important roles in cosmological applications and can be uplifted to S-branes. Thus it is clearly very desirable to have a proper understanding of the different lower-dimensional gauged supergravities and their higher-dimensional origin. Before embarking on the construction of gauged supergravities, let us discuss application i) in more detail.

1.2. AdS/CFT and DW/QFT

Due to the AdS/CFT correspondence [40], it has been realised that there is an intimate relationship between certain branes of string and M-theory and lower-dimensional $SO(n)$ gauged supergravities [13]. The relation is established via a maximally supersymmetric vacuum configuration of string or M-theory, which is the direct product of an AdS space and a sphere. For a p-brane with n transverse directions we are dealing with an $AdS_{p+2} \times S^{n-1}$ vacuum configuration, where $p + n + 1 = 10$ or 11. On the one hand, this vacuum configuration arises as the near-horizon limit of the p-brane in question; on the other hand, the coset reduction over the sphere part leads to the related $SO(n)$ gauged supergravity in $p + 2$ dimensions which allows for a maximally supersymmetric AdS_{p+2} vacuum configuration. The gauge theory of the AdS/CFT correspondence can be taken at the boundary of this AdS_{p+2} space. All lower-dimensional dilatons are vanishing for this vacuum configuration. This is related to the conformal invariance of the gauge theory. For the branes occurring in the AdS/CFT correspondence, the situation is summarized in table 1).

Table 1: The branes of the AdS/CFT correspondence.

Brane	Vacuum configuration	Gauged SUGRA
M2	$AdS_4 \times S^7$	$D = 4\ SO(8)$
M5	$AdS_7 \times S^4$	$D = 7\ SO(5)$
D3	$AdS_5 \times S^5$	$D = 5\ SO(6)$

There are two ways to depart from the conformal invariance, which both involve exciting some of the dilatons in the vacuum configuration. For the cases given in the table, the scalar potential of the gauged supergravity contains $n-1$ dilatons. By exciting some of these dilatons one obtains a deformed AdS vacuum configuration, which can also be seen as a domain wall. In the AdS/CFT correspondence this corresponds to considering the (non-conformal) Coulomb branch of the gauge theory, see e.g. [17,28,38]. Alternatively, one can obtain a non-conformal theory by considering the other branes of string and M-theory, for which there is an extra dilaton present in the scalar potential of the gauged supergravities. In these cases the (maximally supersymmetric) AdS vacuum is replaced by a (non-conformal and half-supersymmetric) domain wall solution. This situation is encountered when one tries to generalise the AdS/CFT correspondence to a DW/QFT correspondence [7,13,36].

A natural generalisation is to excite some of the $n-1$ dilatons, leading to the Coulomb branch of the CFT, and the extra dilaton that leads to a non-conformal QFT *simultaneously*. This gives rise to domain wall solutions of $SO(n)$ gauged supergravities [18] that describe the Coulomb branch of the (non-conformal) QFT. The uplift of these domain walls leads to the near-horizon limit of brane distributions in string and M-theory.

1.3. Outline

This talk is outlined as follows. In section 2 we review some salient features of $CSO(p,q,r)$ gauged supergravities, their construction by dimensional reduction and their scalar potentials. The domain wall solutions of $CSO(p,q,r)$ gauged supergravities are presented in section 3. The higher-dimensional origin of these solutions as brane solutions, in terms of a harmonic function or, equivalently, a brane distribution, is discussed in section 4. We end with a discussion in section 5. The results presented here are mainly based on [12].

2. Gauged Supergravity

2.1. Gaugings and Higher-dimensional Origin

It has been known for long that certain gauged maximal supergravities with global symmetry groups $SL(n,\mathbb{R})$ allow for the gauging of the $SO(n)$ subgroup of the global symmetry. An example is the $SO(8)$ gauging in four dimensions [23]. Subsequently, it was realised that such gauged supergravities could be obtained by the reduction of a higher-dimensional supergravity over a sphere, with a flux of some field strength through the sphere. For example, the $SO(8)$ theory can be constructed by the reduction of 11D supergravity over S^7, with magnetic flux of the four-form field strength through the seven-sphere [24]. Other examples are given in table 2).

Table 2: This table indicates the D-dimensional gauged maximal supergravities and the corresponding $(D-2)$-branes, with n the number of mass parameters as well as the number of transverse directions. The third column indicates whether the scalar potential depends on the extra dilaton ϕ; the fourth column gives the higher-dimensional origin of the $SO(n)$ prime examples. The last row corresponds to a (conjectured) reduction of Euclidean IIB supergravity on a nine-sphere.

D	n	ϕ	Origin	Brane
10	1	$\surd$	Massive IIA [50]	D8
9	2	$\surd$	IIB with $SO(2)$ twist [41]	D7
8	3	$\surd$	IIA on S^2 [52]	D6
7	5	–	11D on S^4 [42,43]	M5
6	5	$\surd$	IIA on S^4 [20]	D4
5	6	–	IIB on S^5 [21,37]	D3
4	8	–	11D on S^7 [24]	M2
3	8	$\surd$	IIA on S^7 [27]	F1A
2	9	$\surd$	IIA on S^8 [44]	D0
1	10	$\surd$	$\mathrm{IIB_E}$ on S^9	D-instanton

In addition to $SO(n)$, the global symmetry group $SL(n,\mathbb{R})$ has more subgroups that can be gauged. It was found that many more gaugings could be obtained from the $SO(n)$ prime examples by analytic continuation or group contraction of the gauge group [30,31]. This leads one from $SO(n)$ to the group $CSO(p,q,r)$ with $p+q+r=n$. These gaugings are described in terms of a symmetric matrix Q, which can always be diagonalised as $Q=\mathrm{diag}(q_1,\ldots,q_n)$. Furthermore, the individual q_i's can always be chosen to be ±1 or

0:

$$Q = \begin{pmatrix} \mathbb{I}_p & 0 & 0 \\ 0 & -\mathbb{I}_q & 0 \\ 0 & 0 & 0_r \end{pmatrix} . \tag{1}$$

This generalises the $SO(n)$ gauging to $[n^2/4 + n]$ different possible gaugings.

The question of the higher-dimensional origin of the CSO gaugings was clarified in [35], where the same operations of analytic continuations and group contractions were applied to the internal manifold. The resulting manifolds are hypersurfaces in $\mathbb{R}^n$ (with Cartesian coordinates μ_i) defined by

$$\sum_{i=1}^{n} q_i {\mu_i}^2 = 1 \,, \tag{2}$$

where $q_i = 0, \pm 1$ are the diagonal entries of the matrix Q (1). The manifold corresponding to (2) is denoted by [35]

$$H^{p,q} \circ T^r \,, \tag{3}$$

where we use the symbol $\circ$ instead of $\times$ to indicate that the manifold is not a direct product. The corresponding reduction should first be performed over the toroidal part, followed by the hyperbolic manifold $H^{p,q}$. The latter can be endowed with a positive-definite metric, which generically is inhomogeneous [16]; the exceptions are the (maximally symmetric) coset spaces

$$S^{n-1} = H^{n,0} \simeq \frac{SO(n)}{SO(n-1)} \,, \qquad H^{n-1} = H^{1,n-1} \simeq \frac{SO(1,n-1)}{SO(n-1)} \,, \tag{4}$$

i.e. the sphere and the hyperboloid. Generically the spaces $H^{p,q}$ are non-compact; the only exception is the sphere with $q = 0$.

Thus non-compact gauge groups $CSO(p, q, r)$ with $q \neq 0$ are obtained from reduction over non-compact manifolds, as first suggested in [46]. It can be argued that the corresponding reduction, albeit a non-compactification, is a consistent truncation provided the compact case with $q = 0$ has been proven consistent [35].

2.2. Twists and Group Manifolds

We now discuss the special case of $p + q \leq 2$. The manifold (2) is then understood as a one-dimensional manifold S^1. It is instructive to consider a chain of contractions, starting from a coset reduction over an $(n-1)$-sphere yielding an $SO(n)$ gauge group:

$$S^{n-1} \to S^{n-2} \circ S^1 \to \cdots \to S^2 \circ T^{n-3} \to S^1 \circ T^{n-2} \to S^1 \circ T^{n-2} \,. \tag{5}$$

From (1) and (2), the resulting gauge groups should be

$$(n, 0, 0) \to (n-1, 0, 1) \to \cdots \to (3, 0, n-3) \to (2, 0, n-2) \to (1, 0, n-1) \,, \tag{6}$$

where we have used a short-hand notation $(p,0,r)$ for the group $CSO(p,0,r)$. The following puzzle arises for the last two links of this chain: the reduction over the torus $S^1 \circ T^{n-2}$ is supposed to lead to the gauge group $CSO(2,0,n-2)$ and its further contraction $CSO(1,0,n-1)$, while toroidal reduction naïvely leads to an ungauged theory with trivial gauge group $U(1)^{n-1}$.

This puzzle is yet more apparent when we consider the simplest case of $n=2$. In this case the reduction of IIB over S^1 is supposed to lead to the gauge group $SO(2)$ or its contraction $\mathbb{R}$. The resolution lies in a *twisted* reduction over S^1 [53], making use of the $SL(2,\mathbb{R})$ duality group of IIB supergravity (see [8, 10, 14, 32, 39, 41]). These twisted reductions give rise to CSO gauged supergravity in 9D with $n=2$. The three different possibilities correspond to twisting with the subgroup $SO(2)$ of $SL(2,\mathbb{R})$, the analytic continuation $SO(1,1)$ and the contraction $\mathbb{R}$, respectively[a].

The reduction Ansatz for a twisted reduction involves an $SL(2,\mathbb{R})$ transformation of the form [41]

$$\Omega = \exp(Cy)\,, \tag{7}$$

where y is the internal coordinate and C is a traceless matrix. Note that general $SL(n,\mathbb{R})$ twisted reductions [22, 53] give rise to a traceless matrix C. Only when twisting with an $SL(2,\mathbb{R})$ subgroup of $SL(n,\mathbb{R})$ can one relate the traceless matrix via

$$C_p{}^q = \epsilon_{pr} Q^{qr}\,, \qquad Q^{qr} = \mathrm{diag}(q_1, q_2)\,, \tag{8}$$

to a symmetric matrix Q. Due to (8), the traceless matrix C can be related to the parameters q_1 and q_2 of (2). The explicit relation between y and the Cartesian coordinates μ_i reads

$$\mu_1 = \sin(\sqrt{q_1 q_2}\,y)/\sqrt{q_1}\,, \qquad \mu_2 = \cos(\sqrt{q_1 q_2}\,y)/\sqrt{q_2}\,. \tag{9}$$

This explains the relation between twisted reduction and the case $p+q \leq 2$ of (2).

The cases $n>2$ can be treated in a similar way: at the two last links of the above chain of contractions, one must perform an $SL(2,\mathbb{R})$-twisted reduction. The required $SL(2,\mathbb{R})$-symmetry is always present for $n \geq 4$, because any reduction over a T^2 factor yields this symmetry. For instance, for $n=4$ we have:

$$S^3 \to S^2 \circ S^1 \to S^1 \circ T^2\,. \tag{10}$$

The reduction over the two-torus gives rise to the $SL(2,\mathbb{R})$-symmetry that is needed to perform the twisted reduction over the remaining S^1. The same happens for all cases $n \geq 4$. The difference between $(p,q,r) = (2,0,n-2)$, $(1,1,n-2)$ and $(1,0,n-1)$ is the flux of the scalars: the different values correspond to twisting with the subgroups $SO(2)$, $SO(1,1)$ and $\mathbb{R}$ of $SL(2,\mathbb{R})$, respectively.

[a] In the case of $SO(2)$ twisting, the reduction can be given an alternative interpretation as a Kaluza-Klein reduction with a consistent truncation to a higher mode [22].

The $n = 3$ case needs special attention. In this case we are dealing with two-dimensional spaces, e.g. S^2 and H^2, over which one can perform coset reductions. However, by contraction we get

$$S^2 \text{ or } H^2 \;\rightarrow\; S^1 \circ S^1 \,, \tag{11}$$

and we seemingly lack the $SL(2,\mathbb{R})$-symmetry due to the absence of a T^2 factor. However, the case $n = 2$ has a peculiar feature: the reduction over S^2 or H^2 (i.e. the first link of (11)) is only allowed for theories that have an origin in one dimension higher. From the higher-dimensional point of view, the reduction Ansatz over S^2 or H^2 corresponds to a group manifold reduction [13, 15]:

$$\mathcal{G}^{3\text{D}} = (S^2 \text{ or } H^2) \circ S^1 \rightarrow S^1 \circ T^2 \,. \tag{12}$$

Due to the hidden higher-dimensional origin, a two-torus appears on the right hand side, allowing for an $SL(2,\mathbb{R})$ twisted reduction over the remaining circle.

The higher-dimensional connection corresponds to the relation between M-theory and type IIA. One can either perform a two-dimensional coset reduction of IIA or a three-dimensional group manifold reduction of 11D to obtain the $SO(3)$ and $SO(2,1)$ gauged supergravities in 8D [1, 52]. The contracted versions are obtained by first reducing 11D over T^2 to 9D which produces an $SL(2,\mathbb{R})$ symmetry in 9D. Next, one applies a twisted reduction form 9D to 8D. Twisting with the subgroups $SO(2), SO(1,1)$ and $\mathbb{R}$ of $SL(2,\mathbb{R})$ leads to $D = 8$ gauged supergravities with gauge groups $ISO(2), ISO(1,1)$ and Heisenberg, respectively.

In the reduction (12) one uses group manifolds of class A of the Bianchi classification, whose structure constants can be written as

$$f_{mn}{}^p = \epsilon_{mnq} Q^{pq} \,, \qquad Q^{mn} = \text{diag}(q_1, q_2, q_3) \,. \tag{13}$$

Note that the structure constants can only be written in terms of a symmetric matrix Q for *three-dimensional* group manifolds. In the group manifold reduction, the internal metric is described in terms of the Maurer-Cartan 1-forms $\sigma^m = U^m{}_n \, dy^n$, which in turn combine into the structure constants (13) of the 8D gauged supergravity and therefore the mass parameters q_i. For more details, see [1].

Reducing from 11D to 10D, one finds a relation between the three-dimensional group manifold reductions (with coordinates y^1, y^2, y^3) and the reductions over the two-dimensional hypersurface (2), which boils down to the following expression for the Cartesian coordinates

$$\begin{aligned}
\mu_1 &= \sin(\sqrt{q_2 q_3}\, y^2)/\sqrt{q_1} \,, \\
\mu_2 &= \sin(\sqrt{q_1 q_3}\, y^1) \cos(\sqrt{q_2 q_3}\, y^2)/\sqrt{q_2} \,, \\
\mu_3 &= \cos(\sqrt{q_1 q_3}\, y^1) \cos(\sqrt{q_2 q_3}\, y^2)/\sqrt{q_3} \,,
\end{aligned} \tag{14}$$

where $y^{1,2}$ are the two coordinates of the 3D group manifold that remain after reduction over y^3 to 10D.

2.3. Scalar Potentials

We consider truncations of maximal gauged supergravities to the sector with only gravity and the dilatons. The consistency of such truncations have been discussed in e.g. [17,18]. The corresponding Lagrangian in D dimensions is given by the kinetic terms and a scalar potential (due to the gauging):

$$\mathcal{L} = \sqrt{-g}(R - \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{4}\text{Tr}[\partial M \partial M^{-1}] - V)\,, \qquad M = \text{diag}(e^{\vec{\alpha}_1\cdot\vec{\phi}}, \ldots, e^{\vec{\alpha}_n\cdot\vec{\phi}})\,. \tag{15}$$

The scalars in M are a truncated parametrisation of the scalar coset of the particular maximal supergravity we are considering. In all cases this scalar coset G/H will be of the form

$$M \in \frac{SL(n,\mathbb{R})}{SO(n)}\,, \tag{16}$$

and it is described by the n vectors $\vec{\alpha}_i = \{\alpha_{iI}\}$, which are weights of $SL(n,\mathbb{R})$ and satisfy the following relations

$$\sum_{i=1}^{n} \alpha_{iI} = 0\,, \quad \sum_{i=1}^{n} \alpha_{iI}\,\alpha_{iJ} = 2\,\delta_{IJ}\,, \quad \vec{\alpha}_i \cdot \vec{\alpha}_j = 2\,\delta_{ij} - \frac{2}{n}\,, \tag{17}$$

with indices $i,j = (1,\ldots,n)$ and $I,J = (1,\ldots,n-1)$. In addition we allow for an extra dilaton ϕ, which would correspond to an extra $\mathbb{R}^+$ factor on the scalar manifold. It is present in some maximal supergravities and absent in others. Explicit information on n, D and ϕ can be found in table 2).

Note that M and ϕ generically do not describe the full scalar coset of maximal supergravities, however, they do constitute the part that is relevant to the CSO gauging and scalar potential. Similarly, the full global symmetry will often be larger than $SL(n,\mathbb{R})$. Its $SL(n,\mathbb{R})$ subgroup will generically be the largest symmetry of the Lagrangian, however, and is the only part of the symmetry group that is relevant for the present discussion.

The corresponding scalar potential, coming from the gauging of the group $CSO(p,q,r)$, takes the following form

$$V = e^{a\phi}\Big(\text{Tr}[QMQM] - \tfrac{1}{2}(\text{Tr}[QM])^2\Big)\,, \qquad Q = \text{diag}(q_1,\ldots,q_n)\,, \tag{18}$$

in terms of n mass parameters q_i. The dilaton coupling a is given by

$$a^2 = \frac{8}{n} - 2\,\frac{D-3}{D-2}\,, \tag{19}$$

for the different cases. For later purposes, it is convenient to write the potential V in terms of the superpotential W

$$V = \tfrac{1}{2}(\vec{\partial}W)^2 + \tfrac{1}{2}(\partial_\phi W)^2 - \frac{D-1}{4(D-2)}\,W^2\,, \qquad W = e^{a\phi/2}\,\text{Tr}[QM]\,, \tag{20}$$

where $\vec{\partial} W = \partial W / \partial \vec{\phi}$ and $\partial_\phi W = \partial W / \partial \phi$. Note that this form is only possible for dilaton couplings that satisfy (19).

In accordance with table 2), a vanishes for $(D, n) = (7, 5)$, $(5, 6)$ and $(4, 8)$, for which the extra dilaton ϕ is absent. The $SL(2, \mathbb{R})$ twisted reduction of IIB [10, 41] and class A group manifold reduction of 11D [1] yield scalar potentials that coincide with (18) for $(D, n) = (9, 2)$ and $(8, 3)$, respectively. In addition, the scalar potential of massive IIA supergravity [50] is also of exactly this form with $(D, n) = (10, 1)$ and is therefore included in table 2).

In [12], the dimensional reduction and group contraction (which amounts to setting one q_i to zero) of the scalar potential are discussed. We show that the only effect of these operations is to decrease D or n by one, respectively. Thus, we expect the following relations between operations on the brane and the gauged supergravity, see also [12, 13]:

Brane		Gauged supergravity
direct dimensional reduction	⇔	group contraction
double dimensional reduction	⇔	circle reduction

3. Domain Walls

In this section, we give a unified description for a class of domain wall solutions for gauged supergravities in various dimensions. We consider the following Ansatz for the domain wall with $D-1$ world-volume coordinates $\vec{x}$ and one transverse coordinate y:

$$ds^2 = g(y)^2 d\vec{x}^2 + f(y)^2 dy^2 \,, \qquad M = M(y) \,, \qquad \phi = \phi(y) \,. \tag{21}$$

The idea is to substitute this Ansatz into the action and write it as a sum of squares, as was done for the conformal cases, i.e. all $q_i = 1$ and $a = 0$, in [4]. Using (20), the reduced one-dimensional action can be written as a sum of squares, up to a boundary term [12]. Minimalisation of this action gives rise to the first-order Bogomol'nyi equations

$$\frac{1}{f}\frac{d\vec{\phi}}{dy} = -\vec{\partial} W \,, \quad \frac{1}{f}\frac{d\phi}{dy} = -\partial_\phi W \,, \quad \frac{2(D-2)}{fg}\frac{dg}{dy} = W \,. \tag{22}$$

Note that one should not expect a Bogomol'nyi equation associated to f since it can be absorbed in a reparametrisation of the transverse coordinate y.

The Bogomol'nyi equations can be solved by the domain wall solution, generalising [17, 18]

$$\begin{aligned} ds^2 &= h^{1/(2D-4)} d\vec{x}^2 + h^{(3-D)/(2D-4)} dy^2 \,, \\ M &= h^{1/n} \mathrm{diag}(1/h_1, \ldots, 1/h_n) \,, \qquad e^\phi = h^{-a/4} \,, \end{aligned} \tag{23}$$

written in terms of the n harmonic functions $h_i = 2q_i y + l_i^2$ and their product $h = h_1 \cdots h_n$. The functions h_i are necessarily positive since the entries of M are postive. For $q_i > 0$, this implies that y can range from 0 to ∞; if $q_i < 0$, the range of y is bounded from above.

The solution is parametrised by n integration constants l_i. However, if a charge q_i happens to be vanishing, the corresponding l_i can always be set equal to one (by $SL(n, \mathbb{R})$ transformations that leave the scalar potential invariant). In addition, one can eliminate one of the remaining l_i's by a redefinition of the variable y. Therefore we effectively end up with $p + q - 1$ independent constants.

It should not be a surprise that all scalar potentials of table 2) satisfy the relation (19) since these are embedded in a supergravity theory, whose Lagrangian "is the sum of the supersymmetry transformations" and therefore always yields first-order differential equations. For this reason, domain wall solutions to the Bogomol'nyi equations (22) will always preserve half of supersymmetry. The corresponding Killing spinor is given by

$$\epsilon = h^{1/(8D-16)} \epsilon_0 \,, \qquad (1 + \Gamma_y)\epsilon_0 = 0 \,, \tag{24}$$

where the projection constraint eliminates half of the components of ϵ_0. An exception is $a = 0$, $q_i = 1$ and $l_i = 0$, in which case the domain wall solution (23) becomes a maximally (super-)symmetric Anti-De Sitter space-time in horospherical coordinates. In this case the singularity at $y = 0$ is a coordinate artifact.

4. Higher-dimensional Origin

4.1. Harmonic Functions

Upon uplifting the domain walls (23), one obtains higher-dimensional solutions, which are related to (near-horizon limits of) the 1/2 supersymmetric brane solutions of 11D, IIA and IIB supergravity, as indicated in table 2). Note that the number of mass parameters (and therefore the number of harmonic functions h_i of the transverse coordinate) always equals the transverse dimension of the brane. Thus, in D dimensions, the number of mass parameters is related to the co-dimension of the half-supersymmetric $(D-2)$-brane of IIA, IIB or M-theory.

The metric of the uplifted solution can in all cases be written in the form

$$ds^2 = H_n^{(2-n)/(D+n-3)}\, dx_{D-1}^2 + H_n^{(D-1)/(D-n-3)}\, ds_n^2 \,, \tag{25}$$

where H_n is a harmonic function on the transverse space, whose powers are appropriate for the corresponding D-brane solution in ten dimensions or M-brane solution in eleven dimensions. From the form of the metric, it is therefore seen that the solution corresponds to some kind of brane distribution. For all $q_i = 1$, these solutions were found in [4,5,17] for the D3-, M2- and M5-branes and in [6,18] for the other non-conformal branes. The transverse part of the metric and the harmonic function take the form [17]

$$ds_n^2 = H_n^{-1} h^{-1/2} dy^2 + \sum_{i=1}^{n} h_i\, d\mu_i^2 \,, \qquad H_n(y, \mu_i) = h^{-1/2} \Big(\sum_{i=1}^{n} \frac{q_i^2 \mu_i^2}{h_i} \Big)^{-1} \,, \tag{26}$$

with the μ_i fulfilling (2). With a change to coordinates z_i, it can be proven that the n-dimensional transverse space is flat [12,17,51] and that the function H_n is indeed harmonic on $\mathbb{R}^n$ for all values of q_i [5,12]. However, the explicit form of H_n in terms of z_i is difficult to obtain; see [12] for the case $n = 2$.

4.2. Brane Distributions for $SO(n)$ Harmonics

In this section we restrict ourselves to the $SO(n)$ cases and their group contractions.

Since the harmonic function H_n depends on the angular variables in addition to the radial, the uplifted solution will in general correspond to a distribution of branes rather than a single brane. For $D < 9$ and $q_i = 1$ (i.e. the $SO(n)$ cases[b] with $n \geq 3$) this means that the harmonic function can be written in terms of a charge distribution σ as follows [17,18]

$$H_n(\vec{z}) = \int d^n z' \frac{\sigma(\vec{z}\,')}{|\vec{z} - \vec{z}\,'|^{n-2}} \,, \qquad n \geq 3 \,, \tag{27}$$

and since H_n appears without an integration constant, the distributions will actually be a near-horizon limit of the brane distribution.

It turns out that the distributions are given in terms of higher dimensional ellipsoids [17,38]. The dimension of these ellipsoids are given in terms of the number m of non-vanishing constants l_i. It is convenient to define

$$x_m = 1 - \sum_{i=1}^{m} \frac{z_i^2}{l_i^2} \,, \qquad \vec{l} = (l_1, \ldots l_m, 0, \ldots, 0) \,, \tag{28}$$

where the last $n - m$ constants l_i are vanishing. Starting with the case $m = n - 1$, we have a negative charge[c] distributed inside the ellipsoid and a positive charge distributed on the boundary [17,28]:

$$\sigma_{n-1} \sim \frac{1}{l_1 \cdots l_{n-1}} \Big(- x_{n-1}^{-3/2} \Theta(x_{n-1}) + 2\, x_{n-1}^{-1/2} \delta(x_{n-1}) \Big) \, \delta^{(1)}(z_n) \,. \tag{29}$$

Upon sending l_{n-1} to zero, the charges in the interior of the ellipsoid cancel, leaving one with a positive charge on the boundary of a lower dimensional ellipsoid:

$$\sigma_{n-2} \sim \frac{1}{l_1 \cdots l_{n-2}} \, \delta(x_{n-2}) \, \delta^{(2)}(z_{n-1}, z_n) \,. \tag{30}$$

Next, the contraction of more constants will yield brane distributions over the inside of an ellipsoid. The distribution $\sigma(z_i)$ is then a product of a delta-function and a theta-function and the branes are localised along $n - m$ coordinates and distributed within an m-dimensional ellipsoid, defined by $x_m = 0$. For $1 \leq m \leq n - 3$ non-zero constants, one has

$$\sigma_m \sim \frac{1}{l_1 \cdots l_m} \, x_m^{(n-m-4)/2} \, \Theta(x_m) \, \delta^{(n-m)}(z_{m+1}, \ldots, z_n) \,. \tag{31}$$

[b]Note that the group contractions are included by taking some $q_i = 0$.

[c]However, these negative charges might be pathological, since the tension will also be negative [26,28].

Finally, one is left with all constant l_i vanishing, in which case the distribution has collapsed to a point and generically reads

$$\sigma_0 = \delta^{(n)}(z_1, \ldots, z_n)\,, \tag{32}$$

i.e. we are left with (the near-horizon limit of) a single brane. All these distributions satisfy

$$\sigma_{m-1} = \delta(z_m) \int \sigma_m\,, \tag{33}$$

consistent with the picture of distributions that collapse the z_m-coordinate upon sending l_m to zero. The case of D5-branes[d] is illustrated in figure 1.

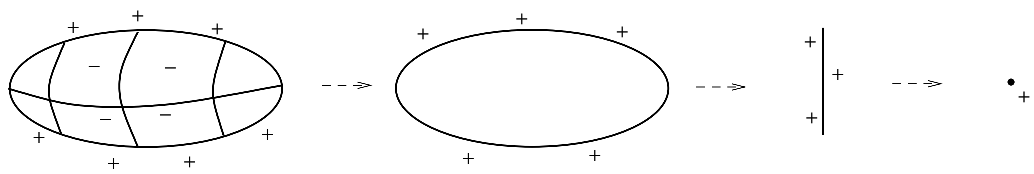

Figure 1: The distributions of D5-branes with three, two, one and zero non-vanishing l_i's, respectively.

The uplift and the corresponding distributions were found in [17] for the D3-, M2- and M5-branes. The extension to the other branes was treated in [18]. Special cases have also been studied in [26, 28]. For certain cases, i.e. when the l_i's are pairwise equal, the distributions above are related to the extremal limit of rotating branes [38, 54], and the l_i's then correspond to rotation parameters.

The special case of the D6-brane distributions (with all $q_i = 1$) was first discussed in [18]. This splits up in three separate possibilities, with $m = 2, 1$ or 0. The first distribution σ_2 consists of positive and negative densities and is given by the general formula (29). Upon sending l_2 to zero, this collapses to

$$\sigma_1 \sim \frac{1}{l_1}\,\delta\big(1 - \frac{{z_1}^2}{{l_1}^2}\big)\,\delta^{(2)}(z_2, z_3)\,. \tag{34}$$

This is a distribution at the boundary of a one-dimensional ellipse, i.e. it is localised at the points $z_1 = \pm l_1$. Upon sending l_1 to zero, the brane distribution σ_0 collapses to a point, as given in (32). The different distributions of D6-branes are shown in figure 2.

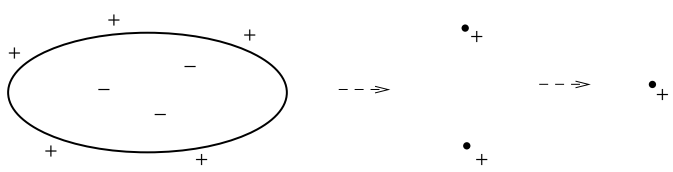

Figure 2: The distributions of D6-branes with two, one and zero non-vanishing l_i's, respectively.

[d]For remarks about the corresponding 7D gauged supergravity, see the discussion.

In the other special case, with $n = 2$ and $q_1 = q_2 = 1$, the IIB solution can be understood as a distribution of D7-branes. Without loss of generality we take $l_2 = 0$. Now (27) does not apply and the harmonic function will instead be given by

$$H_2(\vec{z}) = 1 + \int dz_1' dz_2' \, \sigma_1(z_1', z_2'; l_1) \log((z_1 - z_1')^2 + (z_2 - z_2')^2) \,, \tag{35}$$

with the D7-brane distribution

$$\sigma_1 = \frac{1}{2\pi l_1} \Big[-\big(1 - \frac{z_1'^{\,2}}{l_1^2}\big)^{-3/2} \Theta\big(1 - \frac{z_1'^{\,2}}{l_1^2}\big) + 2\big(1 - \frac{z_1'^{\,2}}{l_1^2}\big)^{-1/2} \delta\big(1 - \frac{z_1'^{\,2}}{l_1^2}\big) \Big] \,. \tag{36}$$

Note that this distribution consists of a line interval of negative D7-brane density with positive contributions at both ends of the interval. Both positive and negative contributions diverge but these cancel exactly:

$$\int dz_1' dz_2' \, \sigma_1(z_1', z_2') = 0 \,, \tag{37}$$

i.e. the total charge in the distribution (36) vanishes.

The parameter l_1 of the general $SO(2)$ solution can be set to zero. This corresponds to a collapse of the line interval to a point, as can be seen from (36). However, due to the fact that the total charge vanishes, this leaves us without any D7-brane density: $\sigma_0 = 0$. Albeit locally flat, the corresponding metric has a conical singularity [10]. Note that for this case the 9D scalar potential V vanishes, while the corresponding superpotential W is non-vanishing. The D7-brane distributions are shown in figure 3.

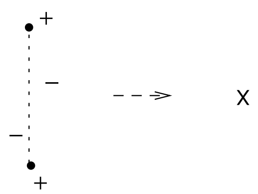

Figure 3: The distributions of D7-branes with one and zero non-vanishing l_i's, respectively. The cross indicates the conical singularity of the locally flat space-time.

Thus, the two-dimensional $SO(2)$ harmonic function (35) of the D7-brane differs in two important ways from the generic $SO(n)$ harmonic function with $n > 2$. Firstly, the total charge distribution of D7-branes vanishes, while it adds up to a finite and positive number in the other cases. Secondly, but not unrelated, one needs to include a constant in the harmonic function (35) in terms of the distribution. In the generic cases this constant was absent, corresponding to the near-horizon limit of these branes. For the D7-brane, the concept of a near-horizon limit is unclear, as we comment upon in the discussion.

5. Discussion

One of the themes of this talk has been that for every brane there is a corresponding gauged supergravity. Sofar, however, we have not treated all the branes of string and

M-theory. For one thing, we did not explicitly include the NS5A-brane in table 2). Since this brane is the direct dimensional reduction of the M5-brane, it leads to the $D = 7$ $ISO(4)$ gauged supergravity of [20]. Only the $SO(4)$ symmetries are linearly realized, the other four symmetries occur as Stueckelberg symmetries. A similar story holds for the D2-brane which is the direct dimensional reduction of the M2-brane. Furthermore, we did not consider the IIB doublets of NS5B/D5- and F1B/D1-branes. The associated theories are the reduction of IIB over S^3 or S^7 with an electric or magnetic flux of the NS-NS/R-R three form field-strength [15, 19]. For the $D = 3$ $SO(8)$ theories corresponding to the IIB strings (which are different than the F1A result), see [27]. The five-brane cases are supposed to lead to new $D = 7$ $SO(4)$ gauged supergravities, which might be related to the theories constructed in [2].

We would like to comment on the special features of the $SO(2)$ harmonic function corresponding to the D7-brane that we encountered in section 4. The near-horizon limit of D-branes does not yield a separated spherical part in Einstein frame; for this one needs to go to the so-called dual frame, in which the tension of the brane is independent of the dilaton

$$g_{\mu\nu}^{\text{dual}} = \exp\left(\frac{(3-p)}{2(p-7)}\,\phi\right) g_{\mu\nu}^{\text{Einstein}} \,. \tag{38}$$

In the dual frame, the near-horizon geometry of all D-branes with $p \leq 6$ reads[e] $AdS_{p+2} \times S^{n-1}$. Clearly, this formula does not hold for the D7-brane; a related complication is the fact that the dual object is the D-instanton, which lives on a Euclidean space.

Finally, we would like to mention that there are also gauged maximal supergravities which are not of the CSO class considered in this talk, see e.g. [3, 25, 33]. In addition, there are gauged maximal supergravities that do not have an action but only have field equations [9, 11]; such theories do not allow for domain wall solutions, however. It would be interesting to investigate the structure of such gaugings and to establish whether the absence of domain wall solutions is a general feature of these theories.

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