THRESHOLD RESUMMATION IN $B \to X_u \ell \nu$

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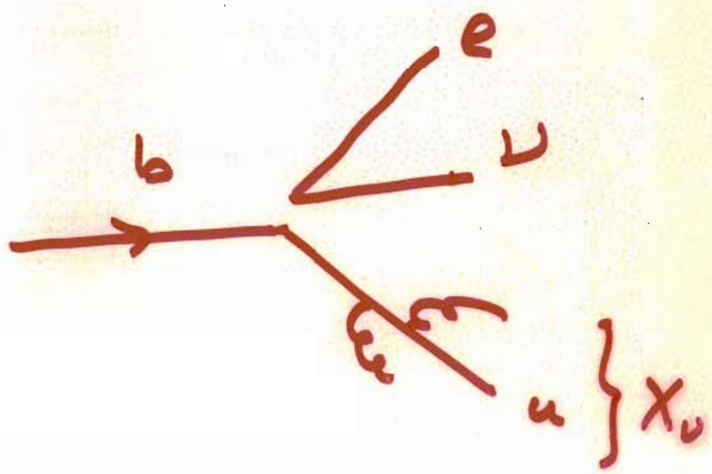
RADCOR05

TRIPLE-DIFFERENTIAL DISTRIBUTION

(MOST GENERAL)



$$\frac{1}{\Gamma}\frac{d^3\Gamma}{dx\,dw\,du} = C(x, w; \alpha)\,\sigma(u; \alpha(w\, m_b)) + d(x, w, u; \alpha)$$

(U. A. 2001,
G. FERRERA,
G. RICCIARDI
& U. A. 2005)

WHERE :

$$x \equiv \frac{2E_e}{m_b} \qquad (0 \le x \le 1)$$

$$w \equiv \frac{2E_X}{m_b} \qquad (0 \le w \le 2)$$

$$u \equiv \frac{E_X - \sqrt{E_X^2 - m_X^2}}{E_X + \sqrt{E_X^2 - m_X^2}} \simeq \left(\frac{m_X}{2E_X}\right)^2$$

FOR $m_X \ll E_X$ $\qquad (0 \le u \le 1)$

$$C(x, w; \alpha) = C^{(0)}(x, w) + \alpha C^{(1)}(x, w) + \dots$$

SHORT-DISTANCE (PROCESS-DEPENDENT) COEFFICIENT FUNCTION, DEPENDING ON 2 INDEPENDENT ENERGIES

$$\left(x + w + x_\nu = 2, \quad x_\nu \equiv \frac{2 E_\nu}{m_b} \right)$$

- $$G(u; \alpha) = \delta(u) - \frac{\alpha_s C_F}{\pi} \left(\frac{\ln u}{u} \right)_+ + - \frac{7}{4} \frac{\alpha_s C_F}{\pi} \left(\frac{1}{u} \right)_+ + O(\alpha_s^2)$$

LONG-DISTANCE DOMINATED, (UNIVERSAL) QCD FORM FACTOR, TO BE RESUMMED TO ALL ORDERS IN α_s

- $$d(x, w, u; \alpha_s) = \alpha_s \, d^{(1)}(x, w, u) + \alpha_s^2 \, d^{(2)}(x, w, u) + \dots$$

SHORT-DISTANCE (PROCESS-DEPENDENT) REMAINDER FUNCTION, TO HAVE GOOD APPROXIMATION ALSO IN THE REGION $w_x \lesssim E_x$, DEPENDENT ON ALL KINEMATICAL VARIABLES.

$$d^{(1)}(x, w, u) \underset{u \to 0}{\sim} \ln u$$

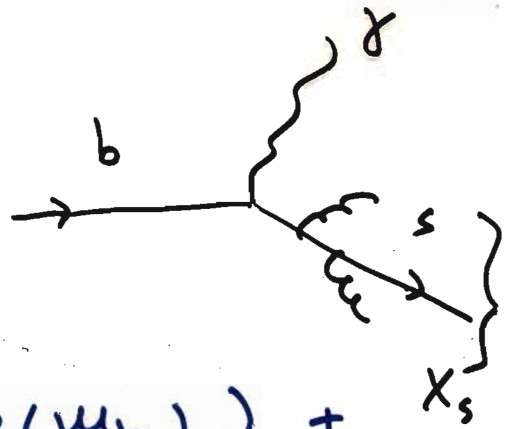
OR

$$\int_0^u du' \, d^{(1)}(x, w, u') \underset{u \to 0}{\longrightarrow} 0$$

REMARKS:

- "COMPLICATION" OF RESUMMATION FORMULA FOR

$$B \to X_s \gamma$$



$$\frac{1}{\Gamma_r}\frac{d\Gamma_r}{dt_r} = C_r(\alpha_s)\,\sigma(t_r;\alpha(m_b)) + d_r(t_r;\alpha(m_b))$$

(*)

$$t_r \equiv \frac{m_{X_s}^2}{m_b^2} \qquad (0 \le t_r \le 1)$$

(*) SAME QCD FORM FACTOR AS BEFORE, EVALUATED NOW IN

$$\alpha = \alpha(m_b) \simeq 0.22$$

$\longrightarrow$ INDEPENDENT ON KINEMATICS

- HARD SCALE Q IN SEMILEPTONIC CASE FIXED BY TOTAL FINAL HADRON ENERGY :

$$\alpha_s = \alpha_s(2E_X)$$

$$L = \ln \frac{m_X^2}{(2E_X)^2}$$

NOT m_b !

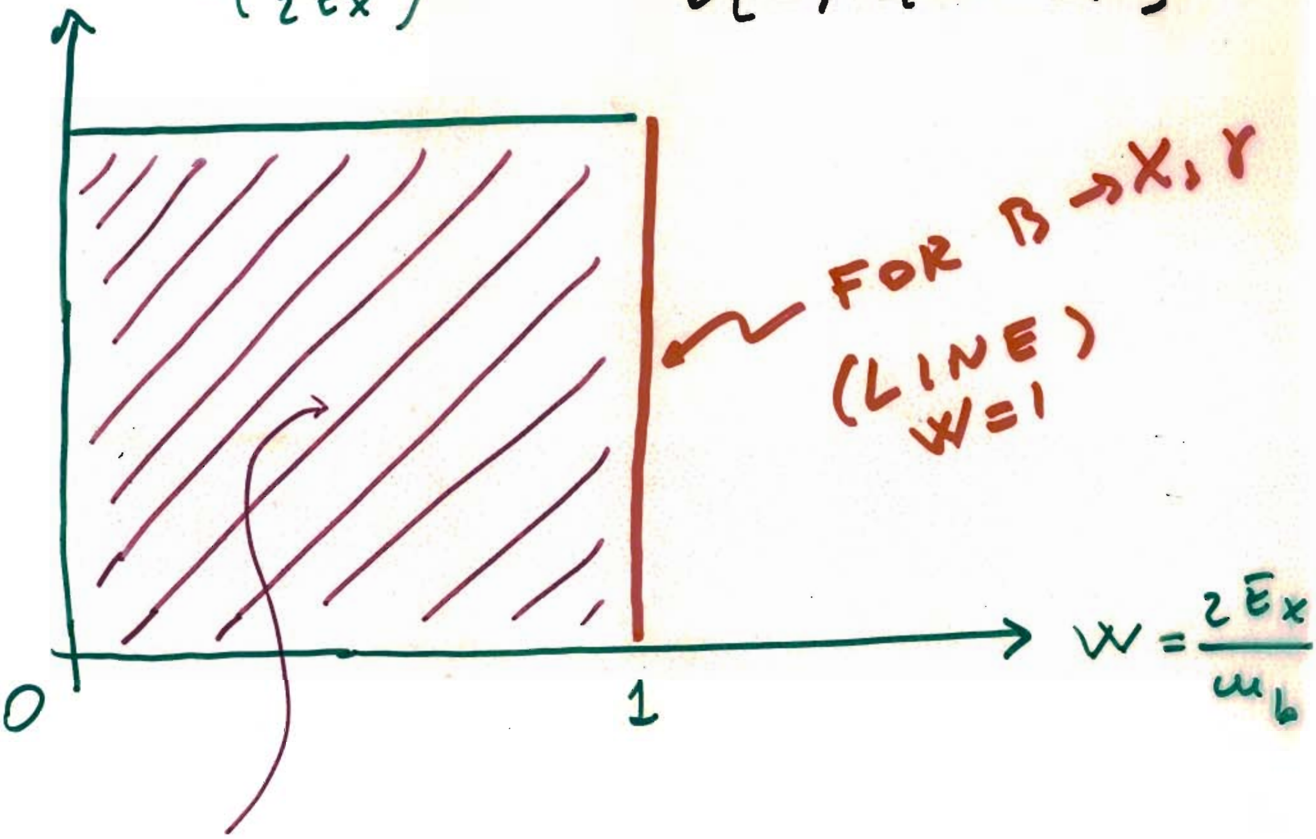
LARGE LOG IN QCD FORM FACTOR

$(0 \le E_X \le m_b)$

(6b)

QCD FORM-FACTOR NEEDED:

$u \approx \left(\frac{m_X}{2E_X}\right)^2$ $\qquad G[u; d/(w\,m_b)]$



FOR $B \to X_u \ell\nu$,

(SQUARE $0 \le u \le 1$, $0 \le w \le 1$):

FUNCTION OF $\underline{\underline{2}}$ VARIABLES

SEMILEPTONIC SPECTRA

i) NOT INTEGRATED OVER E_X,
i.e. OVER HARD SCALE, SIMPLER

$$\frac{1}{\Gamma}\frac{d\Gamma}{dE_X}$$ → SUDAKOV SHOULDER

(CATANI AND WEBBER, NEUBERT AND DI FAZIO)

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dE_X\, dE_e}$$ → COMPLICATION OF SUDAKOV SHOULDER

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dE_X\, d(m_X^2/4E_X^2)}$$

↓ IN THE HADRONIC VARIABLES

ii) SPECTRA INTEGRATED OVER HARD SCALE $2E_X$ => MIXING FROM MANY DIFFERENT HADRONIC SUBPROCESSES – HARDER

$$\frac{1}{\Gamma}\frac{d\Gamma}{dm_X}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{dE_e}$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d(m_X/E_X)}$$

. . . .

$$\approx \int_0^{m_Z} d\sqrt{s}\;\phi(\sqrt{s})\;\frac{1}{\sigma}\frac{d\sigma}{dm_H^2}$$

ANALOGY WITH $e^+e^- \to$ hadrons

WEIGHT FUNCTION

HEAVY JET MASS

HADRON ENERGY SPECTRUM

$$\frac{1}{\Gamma}\frac{d\Gamma}{dw} = \begin{cases} L_0(w) + \alpha_s L_1(w) + \ldots \quad w \le 1 \\ \text{NO LARGE LOGS, FIXED ORDER EXPANSION OK} \\ C_{w1}(\alpha_s)\left\{1 - C_{w2}(\alpha_s)\Sigma(w-1;\alpha_s) + d(w;\alpha_s)\right\} \quad w > 1 \end{cases}$$

$\alpha_s(m_b)$

$$\Sigma(w-1;\alpha_s) = 1 - \frac{\alpha_s C_F}{2\pi}\ln^2(w-1) - \frac{7\alpha_s C_F}{4\pi}\ln(w-1) + O(\alpha_s^2)$$

PARTIALLY-INTEGRATED QCD FORM FACTOR

$$\Sigma = \int 6$$

- $C_{W1}(\alpha_s) = 1 + \alpha_s C_{W1}^{(1)} + \alpha_s^2 C_{W1}^{(2)} + \ldots\ldots$

$C_{W2}(\alpha_s) = 1 + \alpha_s C_{W2}^{(1)} + \alpha_s^2 C_{W2}^{(2)} + \ldots\ldots$

$\uparrow$

SHORT - DISTANCE COEFFICIENT FUNCTIONS

- $d(w; \alpha_s) = \alpha_s d^{(1)}(w) + \alpha_s^2 d^{(2)}(w) + \ldots$

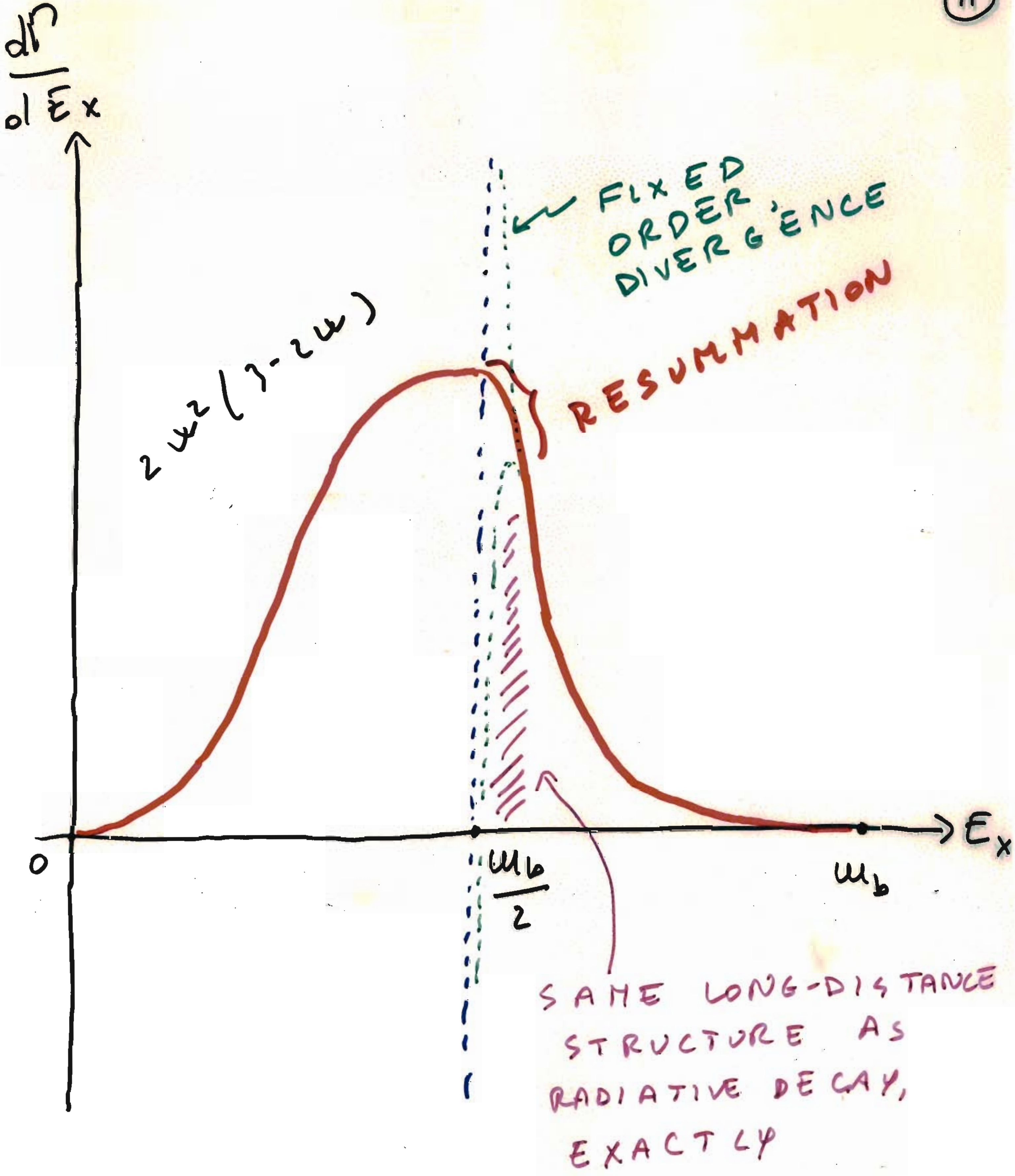
$$\Big\downarrow \; w \to 1^+$$

0

SHORT-DISTANCE REMAINDER FUNCTION

C_{W1} : FIXED IMPOSING CONTINUITY IN $w = 1$

C_{W2} : FIXED IMPOSING VANISHING OF REMAINDER FUNCTION FOR $w \to 1^+$ AND MATCHING WITH FIXED ORDER



$\frac{d\Gamma}{dE_x}$
E_x
0
$\frac{m_b}{2}$
m_b
$2w^2(3-2w)$
FIXED ORDER DIVERGENCE
RESUMMATION
SAME LONG-DISTANCE STRUCTURE AS RADIATIVE DECAY, EXACTLY

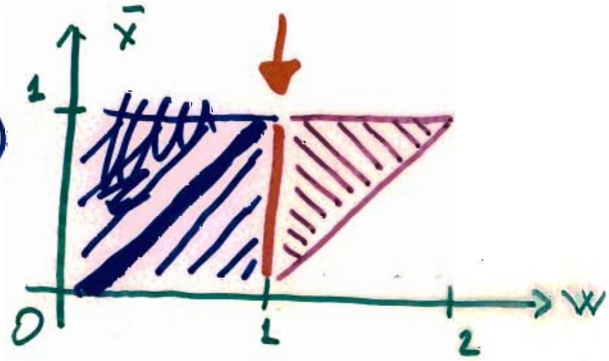
DOUBLE ENERGY DISTRIBUTION

$$
\frac{1}{\Gamma}\frac{d^2\Gamma}{d\bar{x}\,dw} = \begin{cases} C_<(w;\alpha_s)\,\Sigma\left(\frac{\bar{x}}{w};\alpha_s(w\,m_b)\right) + d_<(\bar{x},w;\alpha_s) & w \le 1 \\ C_{xw1}(\bar{x};\alpha)\left\{1 - C_{xw2}(\bar{x};\alpha)\,\Sigma(w-1;\alpha)\right. \\ \left. + d_>(\bar{x},w;\alpha_s)\right\} & w > 1 \end{cases}
$$

$\bar{x} = 1 - x$

CONTINUITY NOW IN THE LINE

$(w = 1, \bar{x})$



REQUIRE:

$d_<(\bar{x}, w; \alpha_s) \to 0$ FOR $\bar{x} \to 0$

$d_>(\bar{x}, w; \alpha_s) \to 0$ FOR $w \to 1^+$

CONCLUSIONS

- THRESHOLD RESUMMATION OF SEMILEPTONIC DECAY SPECTRA WELL UNDERSTOOD;

- "PURE" SHORT-DISTANCE RELATION BETWEEN HADRON ENERGY SPECTRUM IN SEMILEPTONIC AND PHOTON SPECTRUM IN RADIATIVE;

- FOR SPECTRA INTEGRATED OVER HADRONIC ENERGY, NOT SIMPLE RELATION WITH RADIATIVE DECAY:

$$\int_0^1 dw\, \underset{(\propto w^2)}{C_H(w;\alpha_S)}\; \Sigma\left(\frac{t}{w^2};\, \alpha_S(w\, m_b)\right)$$

$$t \equiv m_X^2/m_b^2$$

STATES WITH SMALL HARD SCALE

$$w \ll 1$$

HOWEVER SUPPRESSED BY SPECIFIC FORM OF COEFFICIENT FUNCTION

$$C_H(w) \approx w^2$$

$\Rightarrow$ NON TRIVIAL PICTURE OF RELATION BETWEEN RADIATIVE AND SEMILEPTONIC SPECTRA.