# Automated Calculation Scheme for $\alpha^n$ Contributions of QED to Lepton g - 2

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## Introduction

Primary concern of the talk is:

 $\triangleright A_1^{(10)}$  term of  $\alpha^5$  correction of electron anomalous magnetic moment.

Automated scheme for diagrams with no closed lepton loops.

- Anomalous magnetic moment is the best source of  $\alpha$ , and the most stringent test of QED as well.
- From recent improvement of measurement (Harvard Univ.), we find

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lpha^{-1}(a_e) = 137.035999708(12)(31)(68)
(lpha^4)(lpha^5)(expr)
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Preliminary. Do not quote until published.

- *cf.* Kinoshita's talk.
- Seliable estimates of  $\alpha^5$  term should be requested.

•  $A_1$  term is QED correction purely due to electron contributions. It is evaluated by perturbation theory in terms of  $\alpha$ :

$$A_1 = A_1^{(2)} \left(\frac{\alpha}{\pi}\right) + A_1^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + A_1^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + \cdots$$

 $\alpha^5$  contribution is denoted by  $A_1^{(10)}$ .

- The number of diagrams contributing to  $A_1^{(10)}$  is **12672**.
- They are classified into 32 gauge invariant groups within 6 distinct sets.

#### **Classification of 10th order diagrams**



### **Obstacles in set V diagrams**

Set V consists of 6354 Feynman diagrams that have no closed lepton loops.

9 vertex diagrams are related to 1 self-energy diagram by the Ward-Takahashi identity:

$$\Lambda^
u(p,q)\simeq -q_\mu\left[rac{\partial\Lambda^\mu(p,q)}{\partial q_
u}
ight]_{q
ightarrow 0}-rac{\partial\Sigma(p)}{\partial p_
u}$$

e.g. 4th order case:

By this, the number of diagrams reduces to 706.

Time reversal invariance reduces further to 389.

To (m) $\frac{1}{2}$ , m (The second m (ATA)  $\overline{(m)}$ (TAM)  $( \frown )$  $\overline{m}$ 6 the second  $\mathcal{X}$ and a 660  $6 \, \text{cm}$ ( ( ) )(m)(The m  $d \rightarrow$ the mini (The second (D) (h)TAD m 6 m and  $d \sigma d$  $(\Delta)$ (TA)  $(\mathcal{A})$ (A) and the tom  $\mathcal{A}$ (m) $(\mathcal{A})$  $\overline{\Delta}$ (The second  $(\overline{})$ 6 and a പ ത്ര  $\sim$ de la toto )  $(\Box )$  $(\overline{\mathcal{M}})$ (m)tran  $\mathcal{A}$ (a)tom  $(\pi)$  $(\Delta)$ 40 (TAM)  $\left( \alpha \right)$ d d a(AA)  $(\overline{m})$ tran  $d \overline{m}$ (A) (KA)  $\widehat{}$ (m)6.00  $\mathcal{M}$ (m) $( f_{\Delta} )$ <u></u> (m)tra  $\int$ (MA) A A (m) <u>`</u>@` ton (from  $(\overline{m})$  $(\mathcal{A})$ (m)tra han (a)((m)) $\mathcal{A}$  $\widehat{}$ ( )(The second tron  $(\overline{m})$ A D  $(\bigcirc$ (TAR) (d d) $\left( \right)$ dd m  $\sim \infty$  $\mathcal{L}$  $(\bigcirc$ ton  $\mathcal{H}_{m}$ 6 m them ( $(\overline{A})$ (A) CA  $\bigcirc$ (A) ton  $( \widehat{\ } )$ (Tran) ton to a  $\mathcal{H}_{\mathcal{O}}$  $\bigcirc$  $\bigcirc$ tra ff and  $\langle \rangle$ ( ) $(\mathcal{A})$  $(\bigcirc)$  $\mathcal{M}$ m la  $( \square )$ (The second  $(\bigcirc)$  $(\Delta)$  $\mathcal{A}$ A S TA A. Tom  $(\bigcirc)$ (A)  $(\overline{a})$ (ACA) (B)  $(\overline{\Delta})$  $(\overline{})$ (A) 600 600 (A) the  $\mathcal{H}$  $(\pi)$  $(\mathcal{A})$ (a) $(\overline{m})$ ton lloo (m)600  $\alpha$ (MA) (the de too and (A) (A) (m)that  $(\mathcal{A})$  $(\mathcal{A})$ (( ) )(m) $\mathcal{M}_{\mathcal{A}}$ ator (M) (Ma) (f) $(\mathcal{A})$ (m)6 ക്ര d a  $(\pi)$ A MM <u>a</u> <u></u> than lo  $(\overline{m})$  $(\Box)$ ((a))lad <u>A</u>  $\mathcal{A}$ ക്ര <u>((</u>) ( Do And  $(\bigcirc)$  $\overline{\mathcal{A}}$ (A) (m (A) (m) and to. atan Tota (A) 1 (ATA)  $(\bigcirc)$ A M (A) ( )(Trada) (A) and ( h) $\square$  $(\pi)$ to (A) (A) The and ( m  $\frac{1}{2}$ (m) (A)  $( \land )$ ( f)m alla) 600 an  $\mathcal{A}$ (A) (A) ( ) $\overline{}$ (m) $\pi$  $\widehat{}$  $\frac{1}{2}$ 6 (A) À 600 (A) and (R) (m)(m)

# Each diagram is known to have a large number of UV divergent parts, and is difficult to construct.

#### Maximally 47 UV subtraction terms are required.

e.g.

$$\begin{split} \Delta M_{\rm X001} &= M_{\rm X001} \\ &\quad - L_{2\rm v} M_{\rm m01}(\ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9) - L_{2\rm v} M_{\rm m01}(\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7) - L_{4\rm alv} M_{6\rm f}(\ell_5, \ell_6, \ell_7, \ell_8, \ell_9) - L_{4\rm alv} M_{6\rm f}(\ell_1, \ell_2, \ell_3, \ell_4, \ell_5) \\ &\quad - L_{6\rm flv} M_{4\rm a}(\ell_1, \ell_2, \ell_3) - L_{6\rm flv} M_{4\rm a}(\ell_7, \ell_8, \ell_9) - L_{\rm m011v} M_2(\ell_1) - L_{\rm m011v} M_2(\ell_9) \\ &\quad + L_{2\rm v} L_{2\rm v} M_{6\rm f}(\ell_5, \ell_6, \ell_7, \ell_8, \ell_9) + L_{2\rm v} L_{2\rm v} M_{6\rm f}(\ell_3, \ell_4, \ell_5, \ell_6, \ell_7) + L_{2\rm v} L_{2\rm v} M_{6\rm f}(\ell_1, \ell_2, \ell_3, \ell_4, \ell_5) \\ &\quad + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_7, \ell_8, \ell_9) + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_3, \ell_4, \ell_5) + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_1, \ell_2, \ell_3) \\ &\quad + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_7, \ell_8, \ell_9) + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_5, \ell_6, \ell_7) + L_{2\rm v} L_{4\rm alv} M_{4\rm a}(\ell_1, \ell_2, \ell_3) \\ &\quad + L_{2\rm v} L_{6\rm flv} M_2(\ell_3) + L_{2\rm v} L_{6\rm flv} M_2(\ell_9) + L_{2\rm v} L_{6\rm flv} M_2(\ell_9) + L_{2\rm v} L_{6\rm flv} M_2(\ell_1) + L_{2\rm v} L_{6\rm flv} M_2(\ell_1) + L_{2\rm v} L_{6\rm flv} M_2(\ell_7) \\ &\quad + L_{4\rm alv} L_{4\rm alv} M_2(\ell_1) + L_{4\rm alv} L_{4\rm alv} M_2(\ell_5) + L_{4\rm alv} L_{4\rm alv} M_2(\ell_9) \\ &\quad - L_{2\rm v} L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_5) - L_{2\rm v} L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) \\ &\quad - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) \\ &\quad - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) \\ &\quad - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_1) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_3) \\ &\quad - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_9) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_1) - L_{2\rm v} L_{2\rm v} L_{4\rm alv} M_2(\ell_5) \\ &\quad + L_{2\rm v} L_{2\rm v} L_{2\rm v} M_2(\ell_6) + L_{2\rm v} L_{2\rm v}$$

Some automated scheme is required to get rid of human errors.

Our subjects are 1PI self-energy diagrams without closed lepton loops. They have quite simple structure:

- (1) All lepton propagators form a single path.
- (2) All vertices lie on the lepton path.
- (3) Photon propagators contract pairs of vertices.



A diagram is represented by "pattern of contraction".



Everything about a diagram is contained in this simple expression.

#### Therefore,

- We can generate all diagrams by combinatorics of contractions.
- Independent set of closed paths on a diagram are easily identified. (they are used for constructing Feynman integrals.)
- Subdiagrams relevant for UV divergence are easily identified.

 $\implies$  Automated procedure will readily be implemented.

### **General formalism**

Evaluating a diagram:

Amplitude

Integration over loop momentum  $k_r$  is converted into Feynman parametric integrals over  $\{z_i\}$ .

$$egin{aligned} &rac{1}{i}\Sigma_{\mathrm{G}} = (ie)^{2n} \left[\prod_{r=1}^n \int &rac{d^4k_r}{(2\pi)^4}
ight] \gamma^{\mu_1} rac{i}{
ot\!\!\!/ p_1 - m} \cdots rac{i}{
ot\!\!\!/ p_{2n\!-\!1} - m} \gamma^{\mu_{2n}} \prod_{r=1}^n rac{-ig_{\mu_i\mu_j}}{k_r^2} \ &= \left(rac{lpha}{\pi}
ight)^n rac{1}{4^n} \Gamma(n\!-\!1) \int (dz)_G \ \mathbb{F} rac{1}{U^2 V^{n-1}} \end{aligned}$$

- Subtracting divergences
  - UV divergence
  - IR divergence

### Amplitude

Feynman parametric integral over 13 dimensional space:

$$rac{1}{i} \Sigma_{
m G} = \left(rac{lpha}{\pi}
ight)^n rac{1}{4^n} \Gamma(n\!-\!1) \int (dz)_G \left[rac{F_0(B_{ij},A_j)}{U^2 V^{n-1}} + rac{F_1(B_{ij},A_j)}{U^3 V^{n-2}} + \cdots
ight]$$

- Integrand is expressed by  $B_{ij}, A_i, U, V$
- Building blocks  $B_{ij}, A_i, U, V$  are homogeneous forms of Feynman parameters  $\{z_i\}$ .
  - $B_{ij}$ : Related to loop momenta. They are determined by the topology of diagram.
    - *A<sub>i</sub>*: Related to flow of external momenta. They are the currents satisfying "Kirchhoff law".

Expressions of integrand and building blocks are obtained analytically by Computer Algebra System, FORM, Maple, *etc*.

### Subtracting divergences

The original integral is divergent and must be renormalized.

- Requirements:
  - Numerical approach is taken.
    - $\longrightarrow$  must be a finite value.
  - Diagram-by-diagram evaluation.
- Our strategy:
  - Intermediate renormalization scheme in 3 steps:
    - (1) K-operation for UV divergence.
    - (2) I-operation for IR divergence.
    - (3) residual renormalization to realize on-shell renormalization.
  - Numerical point-wise subtraction.

Prepare subtraction term as an integral defined on the same parameter space as the original

#### UV subtraction

We employ Zimmermann's forest formula to subtract UV divergent parts  $X_f$  associated with forest f, and obtain finite part  $\Delta M_G$ .

$$egin{aligned} &\Delta M_G = M_G - \sum_{f\in\mathfrak{F}} X_f \ &\equiv \int (dz)_G \, \left[ J_G - \sum_{f\in\mathfrak{F}} \mathbb{K}_f J_G 
ight] \end{aligned}$$

- We prepare UV subtraction terms  $\mathbb{K}_f J_G$  in the same Feynman parameter space as the original integrand  $J_G$ , so that they cancel out singularities of  $J_G$  point-by-point.
- This setup is crucial for numerical integration.

#### K-operation and Forest formula

Subtraction term  $\mathbb{K}J_G$  associated with a divergent subdiagram S is obtained by *K*-operation acted on  $J_G$ , via simple power counting in the limit:

 $z_i \sim O(\epsilon) \qquad z_i \in S$ 

Thus UV-divergent part associated with S is extracted  $(M_G^{UV})$ .

By construction,  $M_G^{UV}$  analytically factorizes into lower order term  $M_{G/S}$  exactly and counter term  $\hat{L}_S$  by:

$$M_{G/S}~ imes~\hat{L}_S$$

A forest with multiple of UV divergent subdiagrams is handled by successive operation of *K*-operation.

## **Automated flow**



#### Current status

- We obtained a program to list up all the topologically distinct diagrams without lepton loops.
- Program to generate Feynman parametric integral for each diagram is obtained.
  - Integrand
  - Building blocks,  $B_{ij}, A_j$ .
- Program for vertex renormalization is obtained. Program including self-energy subdiagrams is almost done.
- IR subtraction and residual renormalization step are in progress.
- All the steps are applicable to arbitrary order.
- All diagrams which contain only vertex renormalization are being processed by numerical integration (2232 diagrams).

## Diagrams which contains only vertex renormalization are shown below. They correspond to 2232 diagrams of set V.

(COM) T (m)and (A) (a) and (MA) ANA tom  $\langle \hat{} \rangle$  $\mathcal{T}$ (The second (A) (A) too the (KAN)  $\pi$ the for  $(\mathcal{R})$ A A (AR  $( \mathcal{A} )$ (a)МΚ TA  $\mathcal{C}$ de la 1 m  $\left( \int_{\partial \Omega} \right)$  $(\bigcirc)$ to a łЖ  $\overline{}$ (A) (ATA)  $(\mathcal{M})$ m

# Crude estimates of those diagrams are presented below, just to confirm that renormalization is working.

X003         -0.11310         (0.01383)         X117         0.03013         X210         0.75169         (0.00852)         X343         3.87608         (0.00300)           X013         -1.35322         (0.02153)         X119         -0.12694         (0.01957)         X221         -0.27760         (0.00967)         X344         3.41470         (0.00376)           X016         -0.96093         (0.0195)         X120         1.74905         (0.02757)         X232         0.37427         (0.01842)         X346         -0.48438         (0.00376)           X019         1.17519         (0.02029)         X122         -0.74104         (0.00672)         X259         0.01791         (0.00683)         X348         -0.48587         (0.00376)           X032         -0.224265         (0.00148)         X125         0.73918         (0.03300)         X265         -0.67469         (0.00424)         X349         2.80873         (0.00330)           X033         -1.3714         (0.00143)         X128         0.57693         (0.2218)         X271         0.24188         (0.00752)         X335         0.145479         (0.00252)         X335         0.13896         (0.00252)         X335         0.145479         (0.00230)         X335 <t< th=""><th>X001</th><th>-0.34042(0.04943)</th><th>X116</th><th><math>1.79114 \ (0.00882)</math></th><th>X209</th><th>0.14436(0.00400)</th><th>X322</th><th><math>0.91017 \ (0.00572)</math></th></t<>	X001	-0.34042(0.04943)	X116	$1.79114 \ (0.00882)$	X209	0.14436(0.00400)	X322	$0.91017 \ (0.00572)$
X013         -1.35322         (0.0020)         X118         -3.18650         (0.01925)         X225         0.27766         (0.00967)         X344         -3.1470         (0.00367)           X014         0.75314         (0.0195)         X120         1.74905         (0.01757)         X232         0.37427         (0.01842)         X346         0.02843         (0.00367)           X015         2.10198         (0.00195)         X121         -0.86333         (0.00484)         X325         0.67593         (0.01842)         X346         0.48587         (0.00337)           X011         -0.29674         (0.00489)         X123         -3.32503         (0.01328)         X260         -0.40509         (0.00424)         X349         2.08073         (0.00330)           X032         -0.29674         (0.00489)         X123         -3.3503         (0.0320)         X265         -0.47649         (0.0052)         X351         0.24897         (0.00340)           X033         -1.35884         (0.0017)         X128         0.57693         (0.0221)         X271         0.21488         (0.0052)         X351         0.24490         (0.00340)           X033         -0.58384         (0.00142)         X128         0.57693         (0.0221)<	X003	-0.11310 (0.01383)	X117	$0.32171 \ (0.00533)$	X210	$0.75169 \ (0.00852)$	X343	$3.87608 \ (0.00390)$
X014         0.75314         (0.02153)         X119         -0.12694         (0.01957)         X231         -0.72760         (0.00967)         X345         -1.00102         (0.00288)           X015         2.10198         (0.00195)         X120         1.74095         (0.02757)         X232         0.37427         (0.01842)         X346         0.28443         (0.00385)           X019         1.17519         (0.02029)         X122         -0.74104         (0.00672)         X259         (0.01791         (0.00424)         X347         -0.48557         (0.00335)           X021         -0.29674         (0.00489)         X123         -3.32503         (0.01300)         X265         -0.67469         (0.00424)         X349         -0.48577         (0.00230)           X032         -0.24265         (0.00127)         X127         1.13048         (0.00232)         X272         -0.73345         (0.0149)         X351         0.24490         (0.00230)           X033         -1.37714         (0.00143)         X128         0.57633         (0.2222)         X272         -0.73345         (0.0149)         X353         0.1836         (0.00252)           X033         -0.58384         (0.00143)         X172         1.36015         (	X013	-1.35322 (0.00620)	X118	$-3.18650 \ (0.01925)$	X225	$0.27706 \ (0.01599)$	$\mathbf{X344}$	$3.41470 \ (0.00367)$
X0152.10198 (0.00195)X1201.74905 (0.02757)X2320.37427 (0.01842)X3460.28443 (0.00367)X016-0.96093 (0.0192)X121-0.86533 (0.00484)X2350.67593 (0.01684)X347-2.67776 (0.00335)X0191.17519 (0.02029)X122-0.74104 (0.00672)X2590.01791 (0.00639)X348-0.48587 (0.00376)X0312.29316 (0.00288)X1250.73918 (0.03300)X266-0.40509 (0.00424)X3492.08073 (0.00619)X033-1.37714 (0.00143)X1280.57693 (0.02218)X2660.11937 (0.00582)X3510.24490 (0.00340)X033-1.37714 (0.00143)X1280.57693 (0.02218)X2710.24188 (0.00872)X352-0.13189 (0.00252)X0341.25388 (0.00205)X1291.41734 (0.02232)X272-0.73345 (0.01469)X353-1.8836 (0.00252)X037-0.74165 (0.01199)X166-2.07775 (0.02188)X276-0.55445 (0.00283)X355-1.05668 (0.00551)X0390.31638 (0.00441)X1721.36015 (0.03942)X2772.77770 (0.00265)X3562.06867 (0.00617)X047-4.45507 (0.00326)X1780.7338 (0.00485)X278-1.09061 (0.00737)X3570.36337 (0.00367)X048-0.80512 (0.00160)X1780.02543 (0.00567)X280-1.09061 (0.00464)X359-0.15207 (0.00467)X049-0.02951 (0.00133)X1800.02543 (0.00567)X280-1.09061 (0.00464)X364-2.5207 (0.00467)X049-0.1733 (0.00202)<	X014	$0.75314 \ (0.02153)$	X119	$-0.12694 \ (0.01957)$	$\mathbf{X231}$	-0.72760(0.00967)	X345	-1.00102(0.00288)
X016         -0.96093         (0.00192)         X121         -0.86533         (0.00484)         X235         0.67593         (0.01684)         X347         -2.67776         (0.00335)           X019         1.17519         (0.00289)         X122         -0.74104         (0.00672)         X259         0.01791         (0.00639)         X348         -0.48587         (0.00376)           X021         -0.29674         (0.00489)         X125         0.73918         (0.0320)         X266         -0.64569         (0.00388)         X350         1.45479         (0.00320)           X032         -0.24265         (0.00142)         X127         1.13048         (0.00885)         X266         -0.67469         (0.00388)         X350         1.45479         (0.00230)           X033         -1.37714         (0.0143)         X128         0.57693         (0.2218)         X271         0.24188         (0.00445)         X354         -0.3177         (0.00439)           X034         -0.54344         (0.00445)         X354         -2.03177         (0.00439)         X355         -1.05668         (0.00561)           X037         -0.74165         (0.00412)         X167         -0.74381         (0.00473)         X357         0.36337	X015	$2.10198 \ (0.00195)$	X120	$1.74905 \ (0.02757)$	X232	$0.37427 \ (0.01842)$	$\mathbf{X346}$	$0.28443 \ (0.00367)$
X019         1.17519         (0.02029)         X122         -0.74104         (0.00672)         X259         0.01791         (0.00639)         X348         -0.48587         (0.00376)           X021         -0.29674         (0.00288)         X123         -3.32503         (0.01330)         X265         -0.67469         (0.0038)         X350         1.45479         (0.00230)           X032         -0.24265         (0.00127)         X127         1.13048         (0.00985)         X266         0.11937         (0.00592)         X351         0.24490         (0.00340)           X033         -1.37714         (0.00142)         X128         0.57693         (0.02218)         X271         0.21385         (0.00252)           X034         1.25388         (0.00142)         X165         -2.10910         (0.01990)         X275         -0.74340         (0.00445)         X354         -2.03177         (0.00439)           X037         -4.45507         (0.00326)         X178         -0.73818         (0.00441)         X278         -0.14964         (0.00737)         X357         -0.6668         (0.00677)           X047         -4.45507         (0.00326)         X178         -0.3128         (0.00477)         X236         -0.14964 <t< th=""><th>X016</th><th>-0.96093 (0.00192)</th><th>X121</th><th>-0.86533 (0.00484)</th><th>X235</th><th><math>0.67593 \ (0.01684)</math></th><th>X347</th><th>-2.67776(0.00335)</th></t<>	X016	-0.96093 (0.00192)	X121	-0.86533 (0.00484)	X235	$0.67593 \ (0.01684)$	X347	-2.67776(0.00335)
X021         -0.29674         (0.00489)         X123         -3.32503         (0.01328)         X260         -0.40509         (0.00424)         X349         2.08073         (0.00230)           X031         2.29316         (0.00288)         X125         0.73918         (0.03300)         X265         -0.67469         (0.00388)         X351         0.24409         (0.00340)           X033         -1.37714         (0.00143)         X128         0.57693         (0.02218)         X271         0.24188         (0.00872)         X351         0.24409         (0.00340)           X033         -1.57388         (0.00142)         X165         -2.10910         (0.02232)         X272         -0.73345         (0.01469)         X353         -0.13189         (0.00252)           X037         -0.74165         (0.0019)         X167         -2.27775         (0.7770         (0.00455)         X355         -1.05668         (0.00554)           X039         0.31638         (0.00441)         X172         1.36015         (0.03942)         X277         2.77770         (0.00265)         X356         2.03377         (0.00367)         X349         2.08677         (0.00377)         X357         0.36376         (0.000377)         X357         0.36376	X019	$1.17519 \ (0.02029)$	X122	$-0.74104 \ (0.00672)$	$\mathbf{X259}$	$0.01791 \ (0.00639)$	X348	-0.48587 (0.00376)
X031         2.29316         (0.00288)         X125         0.73918         (0.03300)         X266         -0.67469         (0.00388)         X350         1.45479         (0.00230)           X032         -0.24265         (0.00143)         X128         0.57693         (0.02218)         X271         0.24188         (0.00872)         X351         0.24490         (0.00340)           X033         -1.5714         (0.00142)         X165         -2.10910         (0.01232)         X272         -0.73345         (0.00445)         X354         -2.03177         (0.00439)           X037         -0.74165         (0.00199)         X166         -2.27775         (0.02188)         X276         -0.74340         (0.00445)         X355         -1.05668         (0.00657)           X039         0.31638         (0.00441)         X172         1.36015         (0.0384)         X277         2.77770         (0.00265)         X356         -2.08677         (0.0067)           X048         -0.80512         (0.00160)         X179         -0.43781         (0.00341)         X279         0.82134         (0.00439)         X358         0.03325         (0.00467)           X049         -0.02951         (0.0133)         X180         0.025431         (	X021	-0.29674 (0.00489)	X123	$-3.32503 \ (0.01328)$	$\mathbf{X260}$	-0.40509(0.00424)	X349	$2.08073 \ (0.00619)$
X032       -0.24265 (0.00127)       X127       1.13048 (0.00985)       X266       0.11937 (0.00592)       X351       0.24490 (0.00340)         X033       -1.37714 (0.00143)       X128       0.57693 (0.02218)       X271       0.24188 (0.00872)       X352       -0.13189 (0.00252)         X034       1.25388 (0.00142)       X165       -2.10910 (0.01990)       X272       -0.7345 (0.01469)       X353       0.18836 (0.00252)         X037       -0.74165 (0.00199)       X166       -2.27775 (0.02188)       X277       -0.7700 (0.00265)       X356       2.06867 (0.00617)         X047       -4.45507 (0.00326)       X178       0.70381 (0.00441)       X172       1.36015 (0.03942)       X277       2.77770 (0.00265)       X356       2.06867 (0.00617)         X048       -0.80512 (0.00160)       X179       -0.43781 (0.00341)       X279       -0.14964 (0.00737)       X357       -0.36332 (0.00465)         X049       -0.02951 (0.00133)       X180       0.02543 (0.00567)       X280       -1.00961 (0.00464)       X359       -0.51207 (0.00467)         X050       -1.22223 (0.00176)       X185       -0.13128 (0.00497)       X281       -1.37236 (0.00477)       X360       -0.47233 (0.00563)         X051       -0.17333 (0.00202)       X186       -0.13128 (0.00497) <th>X031</th> <th><math>2.29316 \ (0.00288)</math></th> <th>X125</th> <th><math>0.73918 \ (0.03300)</math></th> <th><math>\mathbf{X265}</math></th> <th>-0.67469(0.00388)</th> <th><math>\mathbf{X350}</math></th> <th><math>1.45479 \ (0.00230)</math></th>	X031	$2.29316 \ (0.00288)$	X125	$0.73918 \ (0.03300)$	$\mathbf{X265}$	-0.67469(0.00388)	$\mathbf{X350}$	$1.45479 \ (0.00230)$
X033-1.37714(0.00143)X1280.57693(0.0218)X2710.24188(0.00872)X352-0.13189(0.00252)X0341.25388(0.00143)X165-2.10910(0.02232)X272-0.73345(0.01469)X3530.18836(0.00252)X035-0.58384(0.00142)X165-2.10910(0.01990)X275-0.73450(0.00445)X354-2.03177(0.00439)X037-0.74165(0.00199)X166-2.27775(0.02188)X276-0.55445(0.00283)X355-1.05668(0.00617)X047-4.45507(0.00326)X1790.70338(0.00485)X278-0.14964(0.00737)X3570.36337(0.00617)X048-0.80512(0.00133)X1800.02543(0.00457)X280-1.00961(0.00464)X359-0.15207(0.00467)X050-1.22223(0.00176)X185-0.13128(0.00497)X281-1.37236(0.00407)X360-0.47233(0.00563)X051-0.17333(0.0022)X186-1.14242(0.00878)X282-0.48566(0.00385)X361-2.5071(0.01084)X053-0.36339(0.0142)X196-2.03753(0.00288)X284-0.27114(0.00320)X363-2.34078(0.00262)X076-5.19446(0.03379)X197-0.38704(0.00425)X2870.1690(0.00389)X3642.38344(0.0037)X0773.18404(0.069	X032	$-0.24265 \ (0.00127)$	X127	$1.13048 \ (0.00985)$	$\mathbf{X266}$	$0.11937 \ (0.00592)$	$\mathbf{X351}$	$0.24490 \ (0.00340)$
X034         1.25388 (0.00205)         X129         1.41734 (0.02322)         X272         -0.73345 (0.01469)         X353         0.18886 (0.00252)           X035         -0.58384 (0.00142)         X165         -2.10910 (0.01990)         X275         -0.73340 (0.00445)         X354         -2.03177 (0.00439)           X037         -0.74165 (0.00199)         X166         -2.27775 (0.02188)         X276         -0.75445 (0.00253)         X355         -1.05668 (0.00554)           X047         -4.45507 (0.00326)         X178         0.70338 (0.00485)         X278         -0.14964 (0.00737)         X357         0.36337 (0.00367)           X048         -0.80512 (0.00160)         X179         -0.43781 (0.00341)         X279         0.82134 (0.00439)         X358         0.03252 (0.00425)           X049         -0.02951 (0.00133)         X180         0.02543 (0.00567)         X281         -1.37236 (0.00407)         X360         -0.47233 (0.00465)           X050         -1.2223 (0.00176)         X185         -0.13128 (0.0047)         X281         -1.37236 (0.00407)         X360         -0.47233 (0.00563)           X051         -0.17333 (0.00202         X186         1.14242 (0.00878)         X282         0.48596 (0.00385)         X361         2.52071 (0.01048)           X053         0.36460	X033	$-1.37714 \ (0.00143)$	X128	$0.57693 \ (0.02218)$	$\mathbf{X271}$	$0.24188 \ (0.00872)$	X352	$-0.13189 \ (0.00252)$
X035-0.58384 (0.00142)X165-2.10910 (0.01990)X275-0.74340 (0.00445)X354-2.03177 (0.00439)X037-0.74165 (0.00199)X166-2.27775 (0.02188)X276-0.55445 (0.00283)X355-1.05668 (0.00554)X0390.31638 (0.00441)X1721.36015 (0.03942)X2772.77770 (0.00265)X3562.06867 (0.00617)X047-4.45507 (0.00326)X1780.70338 (0.00485)X278-0.14964 (0.00737)X3570.36337 (0.00367)X048-0.02951 (0.00133)X1800.02543 (0.00567)X280-1.00961 (0.00464)X359-0.15207 (0.00467)X050-1.22223 (0.00176)X185-0.13128 (0.00497)X281-1.37236 (0.00477)X360-0.47233 (0.00563)X051-0.17333 (0.00202)X1861.14242 (0.00878)X2820.48596 (0.00385)X3612.52071 (0.01484)X0530.36460 (0.00153)X195-1.06649 (0.00450)X283-0.05080 (0.00561)X362-0.5599 (0.00358)X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00422)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.7571	X034	$1.25388 \ (0.00205)$	X129	$1.41734 \ (0.02232)$	X272	$-0.73345 \ (0.01469)$	X353	$0.18836 \ (0.00252)$
X037-0.74165 (0.00199)X166-2.27775 (0.02188)X276-0.55445 (0.00283)X355-1.05668 (0.00554)X0390.31638 (0.00441)X1721.36015 (0.03942)X2772.77770 (0.00265)X3562.06867 (0.00617)X047-4.45507 (0.00326)X1780.70338 (0.00485)X278-0.14964 (0.00737)X3570.36337 (0.00367)X048-0.80512 (0.00160)X179-0.43781 (0.00341)X2790.82134 (0.00439)X3580.03225 (0.00425)X049-0.02951 (0.00133)X1800.02543 (0.00567)X280-1.00961 (0.00464)X359-0.15207 (0.00467)X050-1.22223 (0.00176)X185-0.13128 (0.00497)X281-1.37236 (0.00407)X360-0.47233 (0.00563)X0530.36460 (0.00153)X196-1.04649 (0.00450)X283-0.05080 (0.00385)X3612.52071 (0.01084)X0530.36460 (0.00153)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.0389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X093-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.85164 (0.0714)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X094-1.05792 (0	X035	$-0.58384 \ (0.00142)$	$\mathbf{X165}$	-2.10910(0.01990)	X275	-0.74340(0.00445)	$\mathbf{X354}$	$-2.03177 \ (0.00439)$
X0390.31638 (0.00441)X1721.36015 (0.03942)X2772.77770 (0.00265)X3562.06867 (0.00617)X047-4.45507 (0.00326)X1780.70338 (0.00485)X278-0.14964 (0.00737)X3570.36337 (0.00367)X048-0.80512 (0.00160)X179-0.43781 (0.00341)X2790.82134 (0.00439)X3580.0325 (0.00425)X049-0.02951 (0.00133)X1800.02543 (0.00567)X280-1.00961 (0.00464)X359-0.15207 (0.00467)X051-0.17333 (0.00202)X1861.14242 (0.00878)X2820.48596 (0.00385)X3612.52071 (0.01084)X0530.36460 (0.00153)X195-1.06649 (0.00450)X283-0.05080 (0.00561)X362-0.56599 (0.00358)X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.11610)X202-0.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.0	X037	$-0.74165 \ (0.00199)$	$\mathbf{X166}$	-2.27775(0.02188)	X276	$-0.55445 \ (0.00283)$	X355	$-1.05668 \ (0.00554)$
X047-4.45507(0.00326)X1780.70338(0.00485)X278-0.14964(0.00737)X3570.36337(0.00367)X048-0.80512(0.00160)X179-0.43781(0.00341)X2790.82134(0.00439)X3580.03325(0.00425)X049-0.02951(0.00133)X1800.02543(0.00567)X280-1.00961(0.00464)X359-0.15207(0.00467)X050-1.2223(0.00176)X185-0.13128(0.00497)X281-1.37236(0.00407)X360-0.47233(0.00563)X051-0.17333(0.00202)X1861.14242(0.00878)X2820.48596(0.00385)X3612.52071(0.01084)X0530.36460(0.00153)X195-1.06649(0.00450)X283-0.05080(0.00561)X362-0.56599(0.00262)X076-5.19446(0.03379)X197-0.38704(0.00222)X2850.01690(0.00389)X3642.38344(0.0037)X0773.18404(0.6924)X198-2.33747(0.00442)X2860.76614(0.00587)X367-0.14907(0.00453)X091-1.85164(0.07314)X2000.00793(0.00703)X2960.54479(0.00457)X371-0.00744(0.00453)X093-1.75719(0.00771)X201-0.48774(0.00369)X297-0.47919(0.00468)X372-1.28486(0.00428)X094-1.05792(0.01610	X039	$0.31638 \ (0.00441)$	X172	$1.36015 \ (0.03942)$	X277	$2.77770 \ (0.00265)$	X356	$2.06867 \ (0.00617)$
X048-0.80512(0.00160)X179-0.43781(0.00341)X2790.82134(0.00439)X3580.03325(0.00425)X049-0.02951(0.00133)X1800.02543(0.00577)X280-1.00961(0.00464)X359-0.15207(0.00467)X050-1.22223(0.00176)X185-0.13128(0.00497)X281-1.37236(0.00407)X360-0.47233(0.00467)X051-0.17333(0.00202)X1861.14242(0.00878)X2820.48596(0.00385)X3612.52071(0.01084)X0530.36460(0.00153)X195-1.06649(0.00450)X283-0.05080(0.00561)X362-0.56599(0.00388)X055-0.36339(0.01142)X196-2.03753(0.00282)X284-0.27114(0.00320)X363-2.34078(0.00262)X076-5.19446(0.03379)X197-0.38704(0.00222)X2850.01690(0.00389)X3642.38344(0.00337)X0773.18404(0.06924)X198-2.33747(0.00442)X2860.76614(0.00587)X367-0.71804(0.00490)X0780.82179(0.07104)X1991.04594(0.00455)X2870.17755(0.01168)X370-1.47907(0.00442)X093-1.75719(0.00711)X201-0.48774(0.00369)X297-0.47919(0.00468)X372-1.28486(0.00428)X094-1.05792(0.0	X047	-4.45507 (0.00326)	X178	$0.70338 \ (0.00485)$	$\mathbf{X278}$	$-0.14964 \ (0.00737)$	X357	$0.36337 \ (0.00367)$
X049-0.02951(0.00133)X1800.02543(0.00567)X280-1.00961(0.00464)X359-0.15207(0.00467)X050-1.22223(0.00176)X185-0.13128(0.00497)X281-1.37236(0.00407)X360-0.47233(0.00563)X051-0.17333(0.0020)X1861.14242(0.00878)X2820.48596(0.00385)X3612.52071(0.01084)X0530.36460(0.00153)X195-1.06649(0.00450)X283-0.05080(0.00561)X362-0.56599(0.00358)X055-0.36339(0.00142)X196-2.03753(0.00288)X284-0.27114(0.00320)X363-2.34078(0.00262)X076-5.19446(0.03379)X197-0.38704(0.00222)X2850.01690(0.00389)X3642.38344(0.00262)X0773.18404(0.06924)X198-2.33747(0.00442)X2860.76614(0.00587)X367-0.71804(0.00490)X0780.82179(0.07104)X1991.04594(0.00455)X2970.477919(0.00457)X371-0.00744(0.00453)X093-1.75719(0.00771)X201-0.48774(0.00369)X297-0.47919(0.00458)X372-1.28486(0.00428)X094-1.05792(0.01610)X2021.92431(0.00273)X3030.32133(0.00246)X3730.55778(0.00697)X0950.57719(0.007	X048	$-0.80512 \ (0.00160)$	X179	$-0.43781 \ (0.00341)$	$\mathbf{X279}$	$0.82134 \ (0.00439)$	X358	$0.03325 \ (0.00425)$
X050-1.22223 (0.00176)X185-0.13128 (0.00497)X281-1.37236 (0.00407)X360-0.47233 (0.00563)X051-0.17333 (0.00202)X1861.14242 (0.00878)X2820.48596 (0.00385)X3612.52071 (0.01084)X0530.36460 (0.00153)X195-1.06649 (0.00450)X283-0.05080 (0.00561)X362-0.56599 (0.00385)X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00703) (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00711)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3770.41220 (0.00524)X101-0.26275 (0.01629)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X102-1.43773 (0.05228)X2061.62847 (0.01119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540	X049	$-0.02951 \ (0.00133)$	X180	$0.02543 \ (0.00567)$	$\mathbf{X280}$	$-1.00961 \ (0.00464)$	$\mathbf{X359}$	-0.15207 (0.00467)
X051-0.17333 (0.00202)X1861.14242 (0.00878)X2820.48596 (0.00385)X3612.52071 (0.01084)X0530.36460 (0.00153)X195-1.06649 (0.00450)X283-0.05080 (0.00561)X362-0.56599 (0.00358)X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.0713)X3781.29109 (0.00583)X102-1.43773 (0.52	X050	-1.22223 (0.00176)	$\mathbf{X185}$	-0.13128(0.00497)	$\mathbf{X281}$	-1.37236(0.00407)	$\mathbf{X360}$	-0.47233 (0.00563)
X0530.36460 (0.00153)X195-1.06649 (0.00450)X283-0.05080 (0.00561)X362-0.56599 (0.00358)X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.0713)X3781.29109 (0.00583)X102-1.43773 (0.05228)X2061.62847 (0.01119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540 (0.03	X051	-0.17333(0.00202)	X186	$1.14242 \ (0.00878)$	$\mathbf{X282}$	$0.48596 \ (0.00385)$	$\mathbf{X361}$	$2.52071 \ (0.01084)$
X055-0.36339 (0.00142)X196-2.03753 (0.00288)X284-0.27114 (0.00320)X363-2.34078 (0.00262)X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.00713)X3781.29109 (0.00583)X102-1.43773 (0.05228)X2061.62847 (0.01119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540 (0.03423)X2070.28937 (0.00524)X321-0.92478 (0.01276)X3811.06166 (0.00659)	X053	$0.36460 \ (0.00153)$	$\mathbf{X195}$	$-1.06649 \ (0.00450)$	X283	$-0.05080 \ (0.00561)$	$\mathbf{X362}$	-0.56599 $(0.00358)$
X076-5.19446 (0.03379)X197-0.38704 (0.00222)X2850.01690 (0.00389)X3642.38344 (0.00337)X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.00713)X3781.29109 (0.00583)X102-1.43773 (0.05228)X2061.62847 (0.01119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540 (0.03423)X2070.28937 (0.00418)X3200.55630 (0.00518)X3811.06166 (0.00659)X115-0.59498 (0.01112)X2080.52057 (0.00524)X321-0.92478 (0.01276)	$\mathbf{X055}$	-0.36339(0.00142)	X196	-2.03753(0.00288)	$\mathbf{X284}$	-0.27114 (0.00320)	X363	-2.34078(0.00262)
X0773.18404 (0.06924)X198-2.33747 (0.00442)X2860.76614 (0.00587)X367-0.71804 (0.00490)X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.00713)X3781.29109 (0.00583)X102-1.43773 (0.05228)X2061.62847 (0.01119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540 (0.03423)X2070.28937 (0.00418)X3200.55630 (0.00518)X3811.06166 (0.00659)X115-0.59498 (0.01112)X2080.52057 (0.00524)X321-0.92478 (0.01276)X3811.06166 (0.00659)	X076	$-5.19446 \ (0.03379)$	X197	$-0.38704 \ (0.00222)$	$\mathbf{X285}$	$0.01690 \ (0.00389)$	$\mathbf{X364}$	$2.38344 \ (0.00337)$
X0780.82179 (0.07104)X1991.04594 (0.00455)X2870.17755 (0.01168)X370-1.47907 (0.00453)X091-1.85164 (0.07314)X2000.00793 (0.00703)X2960.54479 (0.00457)X371-0.00744 (0.00415)X093-1.75719 (0.00771)X201-0.48774 (0.00369)X297-0.47919 (0.00468)X372-1.28486 (0.00428)X094-1.05792 (0.01610)X2021.92431 (0.00297)X3030.32133 (0.00246)X3730.55778 (0.00697)X0950.57719 (0.00717)X2030.90371 (0.00233)X304-0.34223 (0.00489)X3761.03581 (0.00341)X0961.24779 (0.02784)X204-1.91907 (0.00671)X3050.46192 (0.00397)X3770.41220 (0.00524)X101-0.26275 (0.01629)X205-0.90380 (0.00489)X3130.94419 (0.00713)X3781.29109 (0.00583)X102-1.43773 (0.05228)X2061.62847 (0.1119)X3140.78814 (0.01293)X379-0.35067 (0.00901)X1030.76540 (0.03423)X2070.28937 (0.00418)X3200.55630 (0.00518)X3811.06166 (0.00659)X115-0.59498 (0.01112)X2080.52057 (0.00524)X321-0.92478 (0.01276)X3811.06166 (0.00659)	X077	$3.18404 \ (0.06924)$	X198	-2.33747(0.00442)	X286	$0.76614 \ (0.00587)$	$\mathbf{X367}$	-0.71804 (0.00490)
X091       -1.85164       (0.07314)       X200       0.00793       (0.00703)       X296       0.54479       (0.00457)       X371       -0.00744       (0.00415)         X093       -1.75719       (0.00771)       X201       -0.48774       (0.00369)       X297       -0.47919       (0.00457)       X371       -0.00744       (0.00415)         X094       -1.05792       (0.01610)       X202       1.92431       (0.00297)       X303       0.32133       (0.00246)       X373       0.55778       (0.00697)         X095       0.57719       (0.00717)       X203       0.90371       (0.00233)       X304       -0.34223       (0.00489)       X376       1.03581       (0.00341)         X096       1.24779       (0.02784)       X204       -1.91907       (0.00671)       X305       0.46192       (0.00397)       X377       0.41220       (0.00524)         X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)	X078	$0.82179 \ (0.07104)$	X199	$1.04594 \ (0.00455)$	X287	$0.17755 \ (0.01168)$	X370	-1.47907 (0.00453)
X093       -1.75719       (0.00771)       X201       -0.48774       (0.00369)       X297       -0.47919       (0.00468)       X372       -1.28486       (0.00428)         X094       -1.05792       (0.01610)       X202       1.92431       (0.00297)       X303       0.32133       (0.00246)       X373       0.55778       (0.00697)         X095       0.57719       (0.00717)       X203       0.90371       (0.00233)       X304       -0.34223       (0.00489)       X376       1.03581       (0.00341)         X096       1.24779       (0.02784)       X204       -1.91907       (0.00671)       X305       0.46192       (0.00397)       X377       0.41220       (0.00524)         X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00524)       X321       -0.92478       (0.01276)       X381       1.06166       (0.0659) </th <th>X091</th> <th><math>-1.85164 \ (0.07314)</math></th> <th>X200</th> <th><math>0.00793 \ (0.00703)</math></th> <th>X296</th> <th><math>0.54479 \ (0.00457)</math></th> <th>X371</th> <th><math>-0.00744 \ (0.00415)</math></th>	X091	$-1.85164 \ (0.07314)$	X200	$0.00793 \ (0.00703)$	X296	$0.54479 \ (0.00457)$	X371	$-0.00744 \ (0.00415)$
X094       -1.05792       (0.01610)       X202       1.92431       (0.00297)       X303       0.32133       (0.00246)       X373       0.55778       (0.00697)         X095       0.57719       (0.00717)       X203       0.90371       (0.00233)       X304       -0.34223       (0.00489)       X376       1.03581       (0.00341)         X096       1.24779       (0.02784)       X204       -1.91907       (0.00671)       X305       0.46192       (0.00397)       X377       0.41220       (0.00524)         X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00524)       X321       -0.92478       (0.01276)         X115       -0.59498       (0.01112)       X208       0.52057       (0.00524)       X321       -0.92478       (0.01276)	X093	-1.75719(0.00771)	X201	-0.48774(0.00369)	X297	-0.47919(0.00468)	X372	-1.28486 (0.00428)
X095       0.57719       (0.00717)       X203       0.90371       (0.00233)       X304       -0.34223       (0.00489)       X376       1.03581       (0.00341)         X096       1.24779       (0.02784)       X204       -1.91907       (0.00671)       X305       0.46192       (0.00397)       X377       0.41220       (0.00524)         X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00418)       X320       0.55630       (0.00518)       X381       1.06166       (0.00659)         X115       -0.59498       (0.01112)       X208       0.52057       (0.00524)       X321       -0.92478       (0.01276)	X094	$-1.05792 \ (0.01610)$	X202	$1.92431 \ (0.00297)$	X303	0.32133 (0.00246)	X373	$0.55778 \ (0.00697)$
X096       1.24779       (0.02784)       X204       -1.91907       (0.00671)       X305       0.46192       (0.00397)       X377       0.41220       (0.00524)         X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00418)       X320       0.55630       (0.00518)       X381       1.06166       (0.00659)         X115       -0.59498       (0.01112)       X208       0.52057       (0.00524)       X321       -0.92478       (0.01276)	X095	$0.57719 \ (0.00717)$	X203	$0.90371 \ (0.00233)$	X304	-0.34223(0.00489)	X376	$1.03581 \ (0.00341)$
X101       -0.26275       (0.01629)       X205       -0.90380       (0.00489)       X313       0.94419       (0.00713)       X378       1.29109       (0.00583)         X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00418)       X320       0.55630       (0.00518)       X381       1.06166       (0.00659)         X115       -0.59498       (0.01112)       X208       0.52057       (0.00524)       X321       -0.92478       (0.01276)	X096	1.24779(0.02784)	X204	-1.91907 (0.00671)	X305	0.46192 (0.00397)	X377	$0.41220 \ (0.00524)$
X102       -1.43773       (0.05228)       X206       1.62847       (0.01119)       X314       0.78814       (0.01293)       X379       -0.35067       (0.00901)         X103       0.76540       (0.03423)       X207       0.28937       (0.00418)       X320       0.55630       (0.00518)       X381       1.06166       (0.00659)         X115       -0.59498       (0.01112)       X208       0.52057       (0.00524)       X321       -0.92478       (0.01276)       X381       1.06166       (0.00659)	X101	-0.26275 (0.01629)	X205	-0.90380 (0.00489)	X313	0.94419 (0.00713)	X378	1.29109 (0.00583)
X103 $0.76540$ $(0.03423)$ X207 $0.28937$ $(0.00418)$ X320 $0.55630$ $(0.00518)$ X381 $1.06166$ $(0.00659)$ X115 $-0.59498$ $(0.01112)$ X208 $0.52057$ $(0.00524)$ X321 $-0.92478$ $(0.01276)$	X102	-1.43773 (0.05228)	X206	1.62847 (0.01119)	X314	0.78814 (0.01293)	X379	-0.35067 (0.00901)
X115 $-0.59498$ (0.01112)       X208 $0.52057$ (0.00524)       X321 $-0.92478$ (0.01276)	X103	$0.76540 \ (0.03423)$	X207	0.28937 (0.00418)	X320	0.55630 (0.00518)	X381	1.06166 (0.00659)
	X115	$-0.59498 \ (0.01112)$	X208	$0.52057 \ (0.00524)$	X321	-0.92478 (0.01276)		

### **Concluding remarks**

- Numerical estimates:
  - Typical integral is composed of 80,000 lines of FORTRAN code.
  - Rough estimates of time-scale for each diagram:
    - 10 20 min. for code generation
    - $10^6$  sampling points  $\times$  20 iterations take 5 7 hours on 32 CPU PC cluster
  - To evaluate within a few percent of accuracy, it will take:
    - a year for set V diagrams.
    - 2 3 years for full  $A_1^{(10)}$  contributions.
- Theoretical issues:
  - To complete the automated procedure to include IR subtraction, and residual renormalization.
  - To extend to general diagrams with lepton loops.