Electroweak observables beyond one-loop

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RADCOR, 4th Oct. '05

Outline:

- Motivation

- Effective weak mixing angle prediction
  - the fermionic part

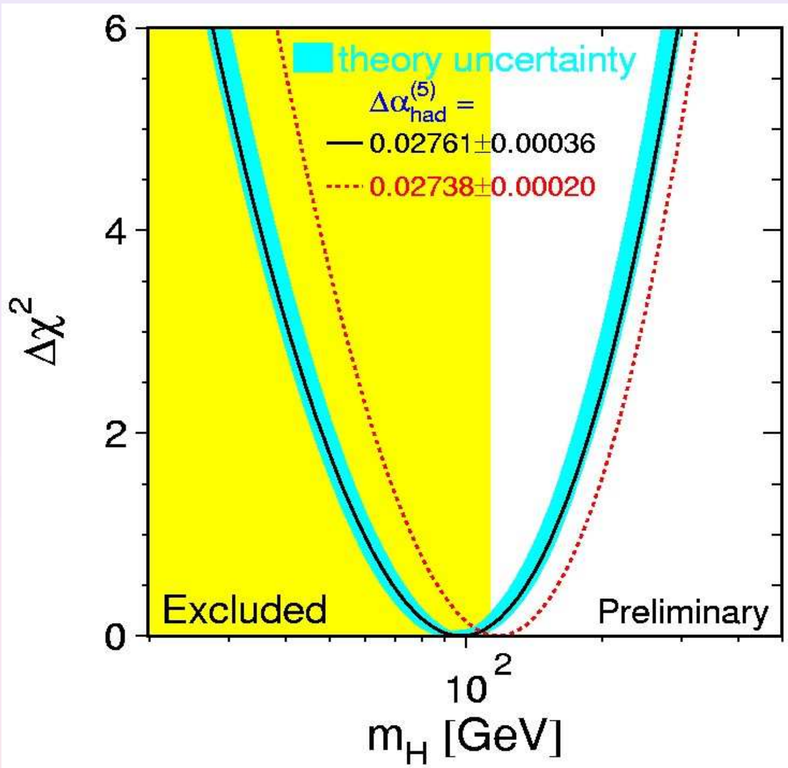
    in collaboration with : Małgorzata Awramik (DESY Zeuthen)
    Ayres Freitas (Fermilab)
    Georg Weiglein (Durham)

  - the bosonic part

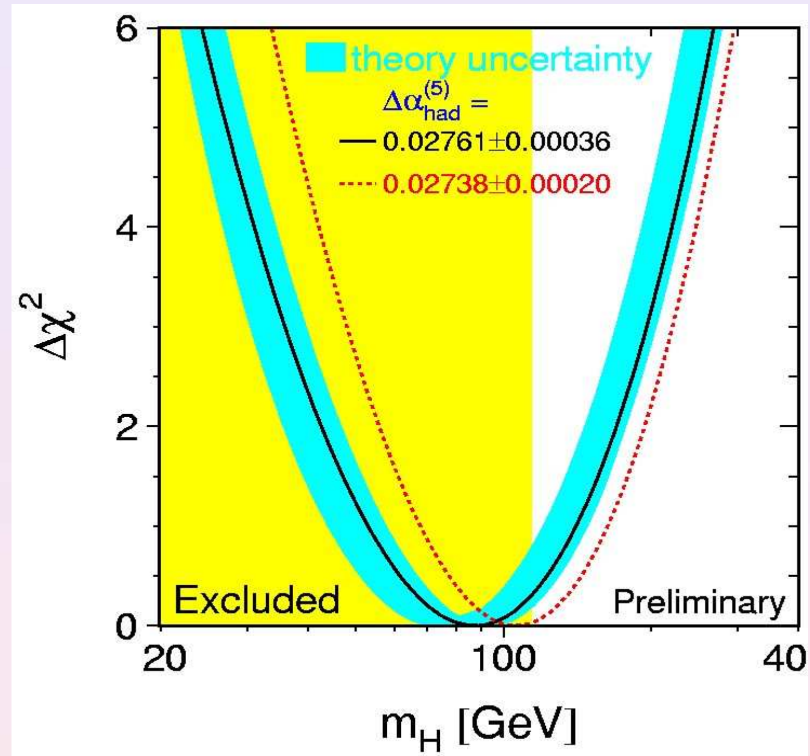
    in collaboration with: Małgorzata Awramik & Ayres Freitas

Motivation: Standard Model precision tests

Winter 2001



Summer 2001



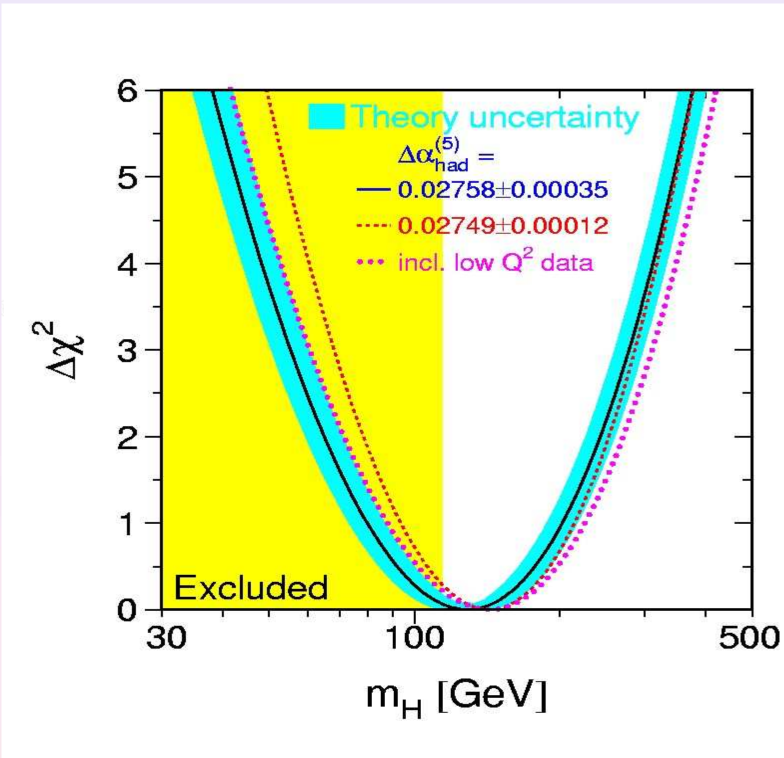
theory prediction of $\sin^2\theta^{lep}_{eff}$, M_W:

expansions up to m_t^2
(Degrassi et al)

exact calculation of M_W (Freitas et al)
also included in error estimation
of $\sin^2\theta^{lep}_{eff}$

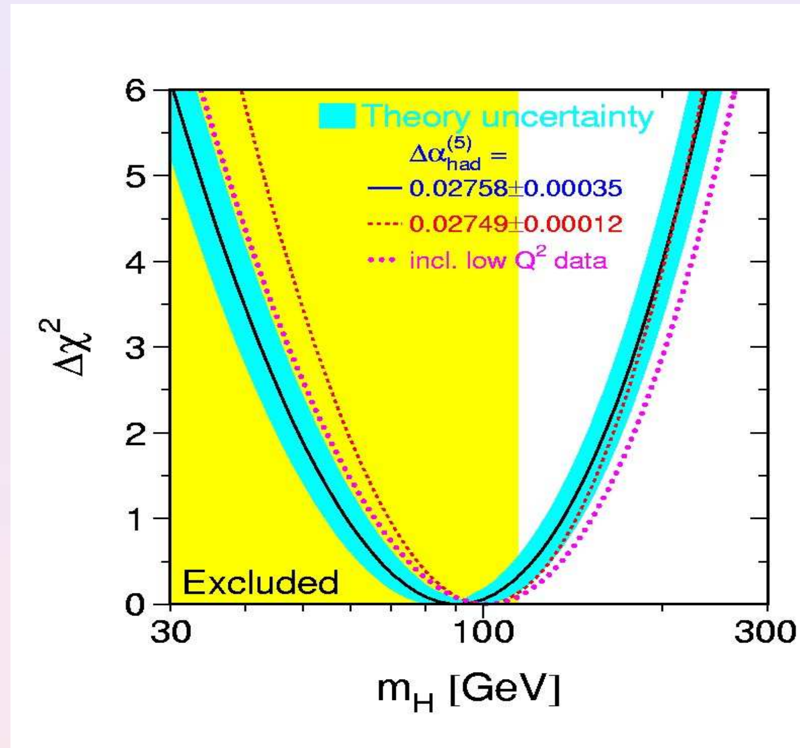
Motivation: Standard Model precision tests

Early Summer 2005



M_H=129 GeV,M_H<285 GeV

Summer 2005

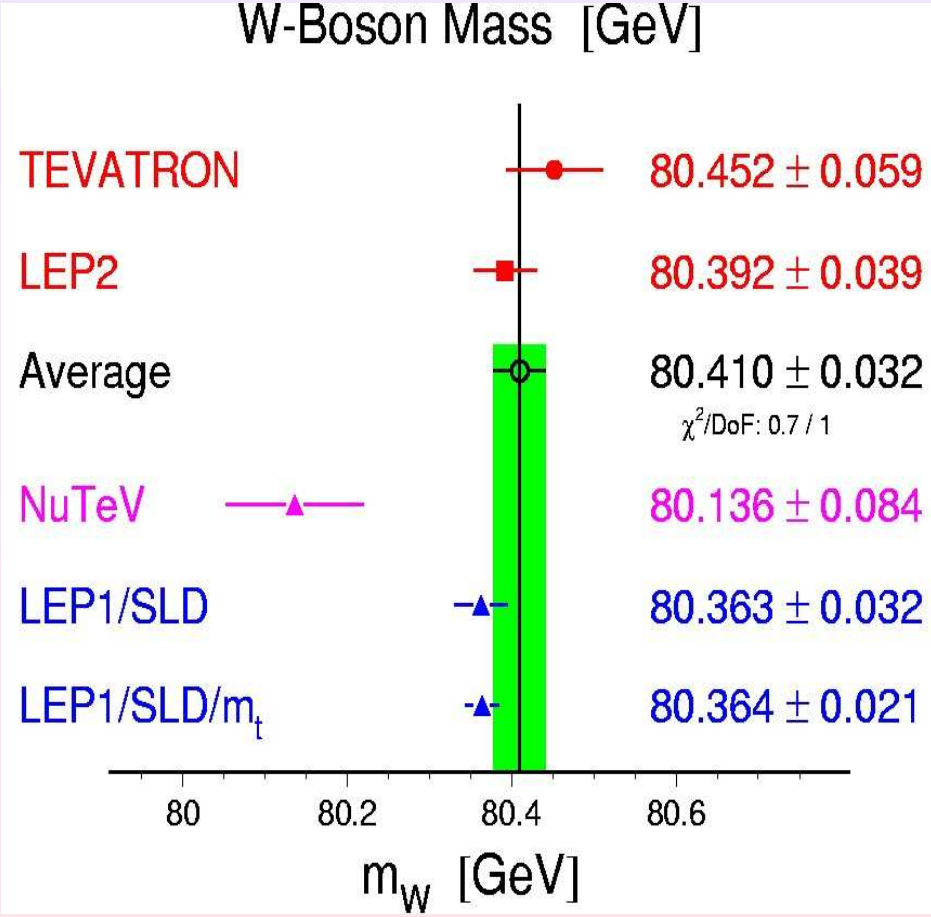


M_H=90.7 GeV, M_H<186 GeV

ZFITTER:

- written by: Dima Bardin, Penka Christova, Mark Jack, Lida Kalinovskaya, Sacha Olshevski, Sabine Riemann, Tord Riemann
- for e+e- annihilation into a fermion pair calculates several observables:
  - M_W
  - Γ_Z and Γ_W
  - cross sections (differential and total)
  - asymmetries
- since May 2005: http://www-zeuthen.desy.de/theory/research/zfitter
- supported by: Andrej Arbuzov, Malgorzata Awramik, Michal Czakon, Ayres Freitas, Martin Gruenewald, Klaus Moenig, Sabine Riemann, Tord Riemann
- recent changes:
  - Higher order QED corrections to fermion-pair production, of importance at energies off the Z boson peak
  - Electroweak corrections to the weak charge Q_W, describing the parity violation effects
  - in atoms, of importance for so-called global Standard Model fits
  - Electroweak corrections to νe νe production, of importance for a precise description of $\nu\nu\gamma$ production;
  - Electroweak two-loop corrections to M_W and the effective weak mixing angle $\sin^2\theta_{eff}$, of importance for global Standard Model fits and for precise predictions of the Higgs mass M_H

W boson mass



- experimental uncertainty

  32 MeV - present

  15 MeV - at LHC

  07 MeV - at LC

- theoretical uncertainty

  - parametric:

    m_t (30 MeV -> 10 MeV-> 1 MeV)

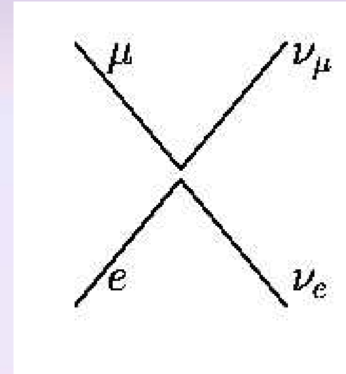
    $\Delta\alpha$ (6 MeV)

  - intrinsic

    4 MeV for M_H < 300 GeV

• Using Fermi Model (with effects of light states only)

$$\mathcal{L}_{\mathsf{Fermi}} = \mathcal{L}_{\mathsf{QED}} + \mathcal{L}^{(5)}_{\mathsf{QCD}} + \frac{G_\mu}{\sqrt{2}} \overline{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu \otimes \overline{e}\gamma_\alpha(1-\gamma_5)\nu_e$$



find muon decay lifetime (or Fermi constant):

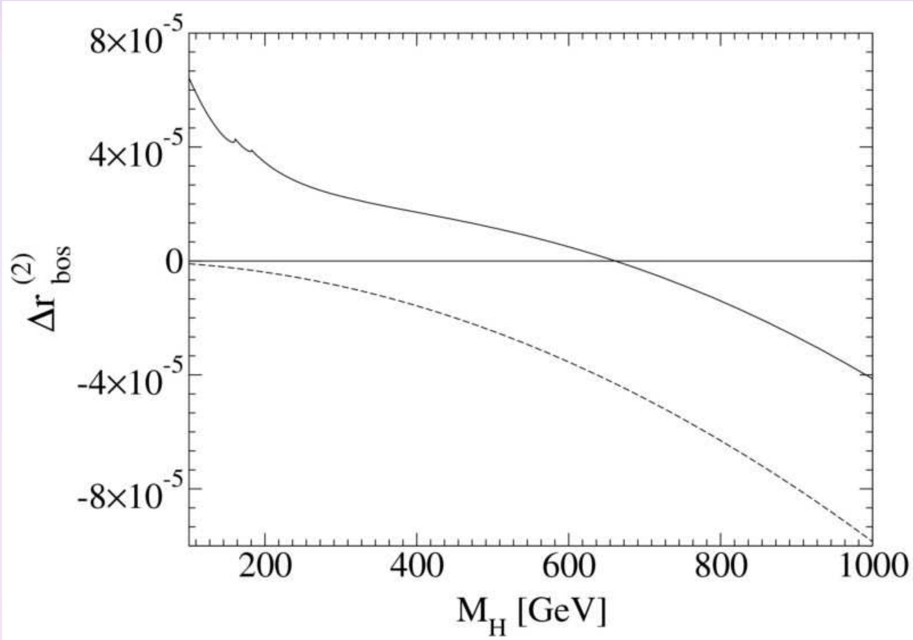
$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 + \frac{\alpha}{4\pi}1.810 + \frac{\alpha^2}{16\pi^2}(6.701 \pm 0.002)\right)$$

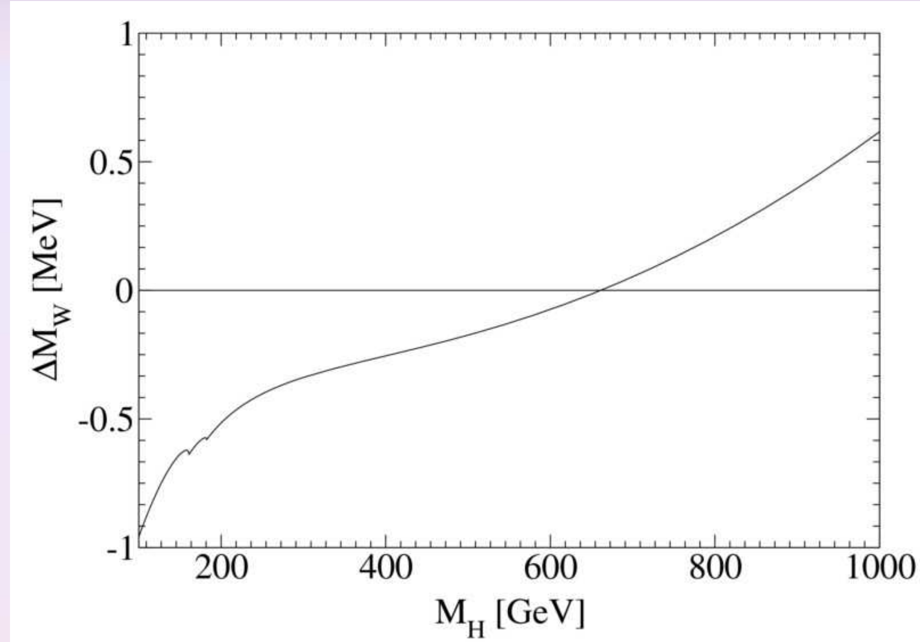
=> $G_\mu = 1.16637 \pm 0.00001\ \mathrm{GeV}^{-2}$

• Point interaction corrected in SM (effects of heavy states):

$$G_\mu = \frac{\alpha\pi}{2 s_W^2 M_W^2}\left(1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t)\right)$$

=> $M_W(G_\mu, m_t, \alpha, M_H, \ldots)$





M. Awramik, M.C., Phys. Rev. Lett. 89 (2002) 241801

- **Bosonic** contributions to **muon decay**
- completion of the 2-loop programme for M_W
- Found to be small, in particular $|\Delta M_W| < 1 MeV$

Theory versus experiment:

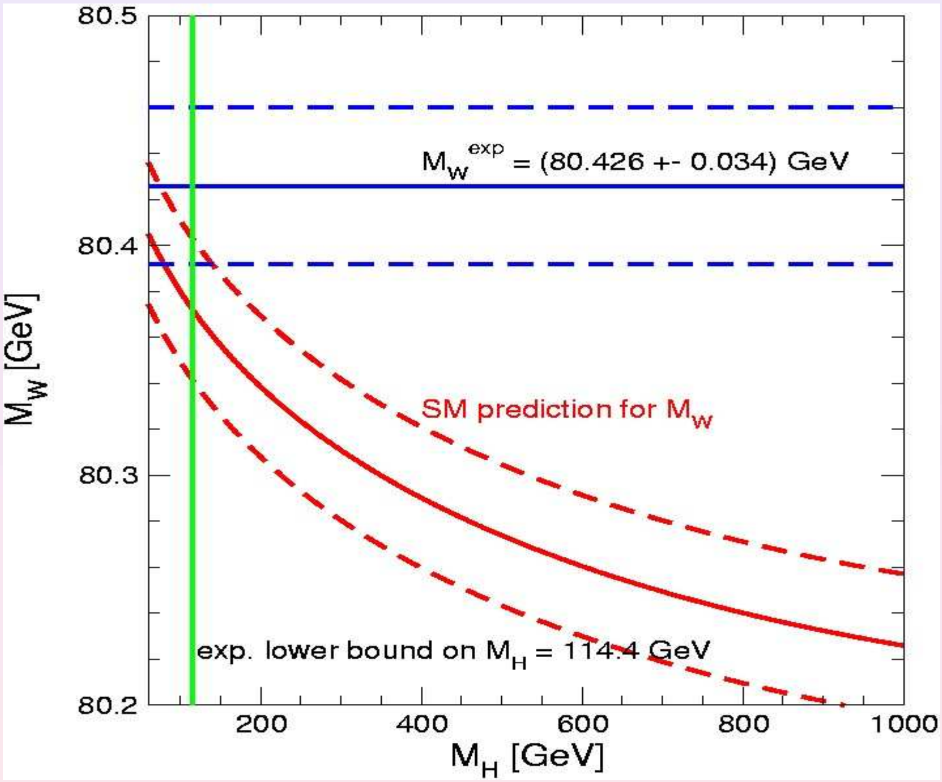
the newest theory prediction for M_W:

- M.Awramik,M.C.,A.Freitas,G.Weiglein, '04
  Phys.Rev.D69:053006,2004

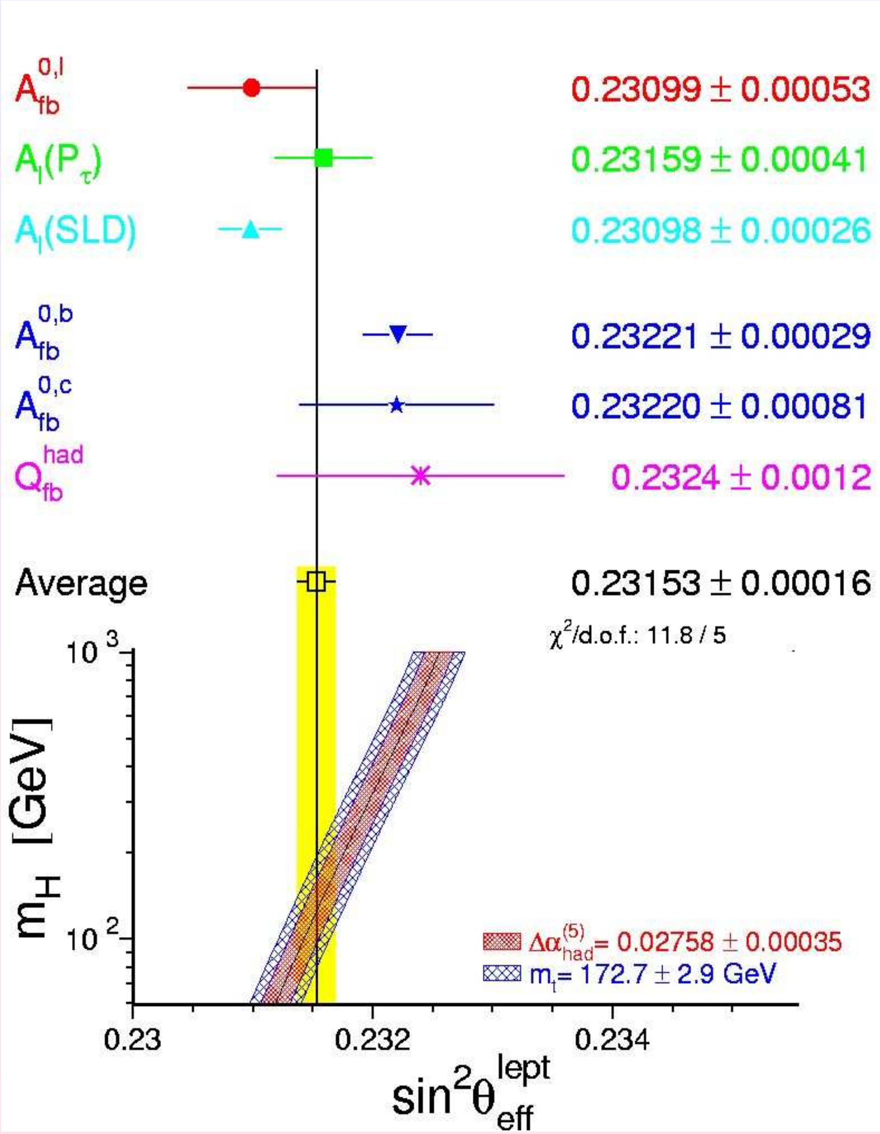
- contains all known corrections,
  in particular two loop part is completed

- error from unknown higher order terms:
  4 MeV (previous: 6 MeV)

- suggests very low Higgs boson mass
  ~ 43.3 GeV



Effective weak mixing angle:



- experimental uncertainty
  - 0.00016 - present
  - 0.00014 - at LHC
  - 0.000013 - at LC

- theoretical uncertainty
  - parametric
    - m_t: 0.00014 (3 months ago)
      - 0.00009 (present)
      - 0.00001 (at GigaZ)
    - $\Delta\alpha$: 0.00013
  - intrinsic
    - 0.000049

Known contributions to the effective weak mixing angle:

One-loop corrections $O(\alpha)$

W.J. Marciano, A. Sirlin, '80

G. Degrassi, A. Sirlin, '93, P. Gambino, A. Sirlin, '94

QCD corrections $O(\alpha\,\alpha_s)$ and $O(\alpha\,\alpha_s^2)$

A. Djouadi, '88, F. Halzen, B.A. Kniehl, '91

L. Avdeev et al., '94

K. Chetyrkin, J. Kuhn, M. Steinhauser, '95

Expansions in M_H, m_t of the $O(\alpha^2)$ contributions

J. v.d.Bij, M. Veltman, '84

J. v.d.Bij, F. Hoogeveen, '87

R. Barbieri et al., '93

J. Fleischer, O.V. Tarasov, F. Jegerlehner, '95

G. Degrassi, P. Gambino, A. Sirlin '97 $O(\alpha^2\, m_t^2/M_Z^2)$

Exact fermionic of $O(\alpha^2)$

M. Awramik, M. Czakon, A. Freitas, G. Weiglein, '04

W.Hollik, U. Meier, S. Uccirati, '05

Higgs mass dependence of the bosonic result

W. Hollik, U. Meier, S. Uccirati, '05

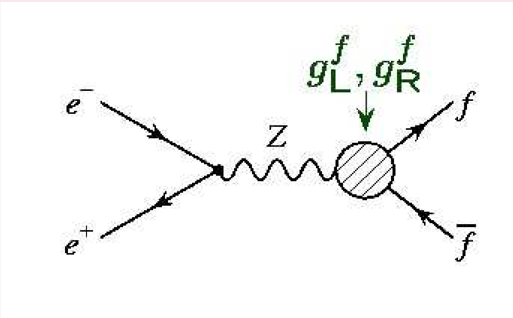
The amplitude for four fermion process at e+e- colliders:

$$\mathcal{A}(e^+e^- \to Z \to f\overline{f}) = \frac{4ie^2 I_e^{(3)} I_f^{(3)}}{s - M_Z^2 + iM_Z\Gamma_Z}\rho_{ef}$$
$$\times [\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5)$$
$$-4|Q_e|s_W^2\kappa_e\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu$$
$$+16|Q_eQ_f|s_W^4\kappa_{ef}\gamma_\mu\otimes\gamma^\mu]$$

defines effective weak mixing angle at Z pole:

$$\Rightarrow \qquad \sin^2\theta_{\text{eff}}^{\text{lept}} = \text{Re}[\kappa_l(s = M_Z^2)]s_W^2$$

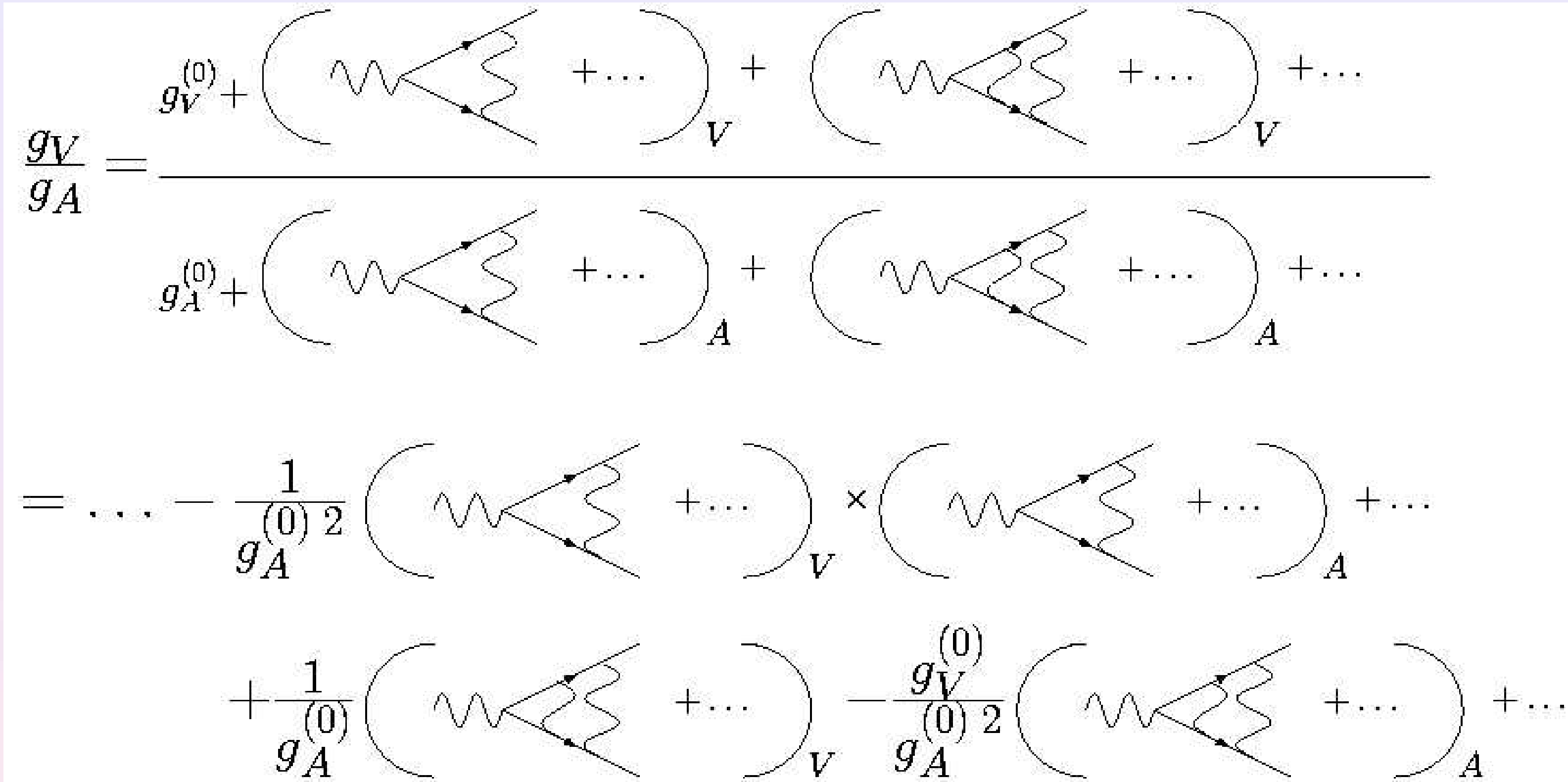
equivalently, the effective Z boson vertex



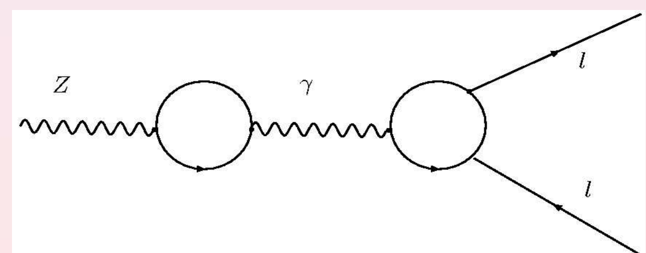
defines:

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = \frac{1}{4}\left(1 - \text{Re}\left(\frac{g_V}{g_A}\right)\right)$$

Contributions to the effective weak mixing angle:



plus some 1PR contributions, as:

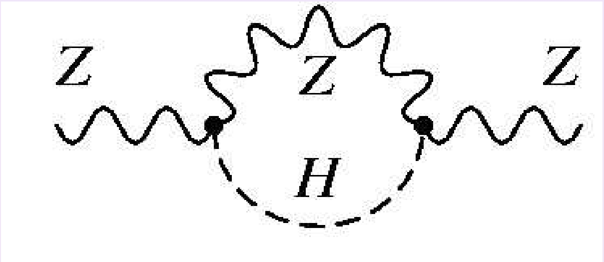


Higgs boson mass dependence:

- already at tree level, through W boson mass

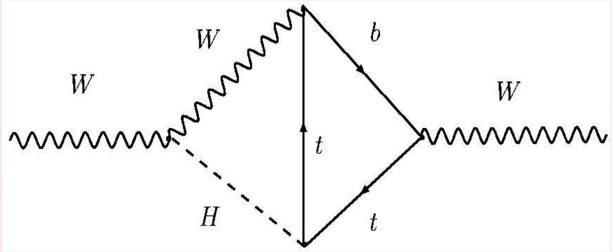
- at one loop level, through renormalization

  -> dominant $\log(M_H)$



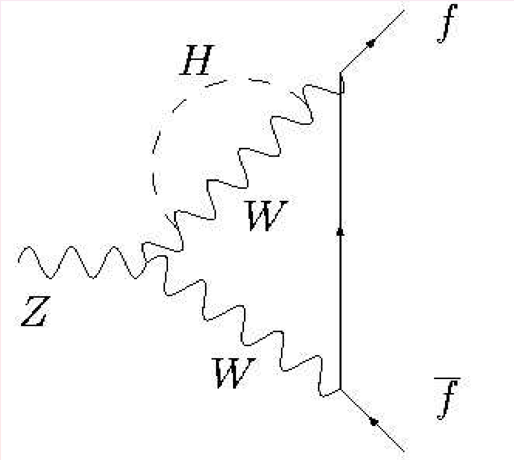
- at two loop level, for fermionic part:

  through renormalization



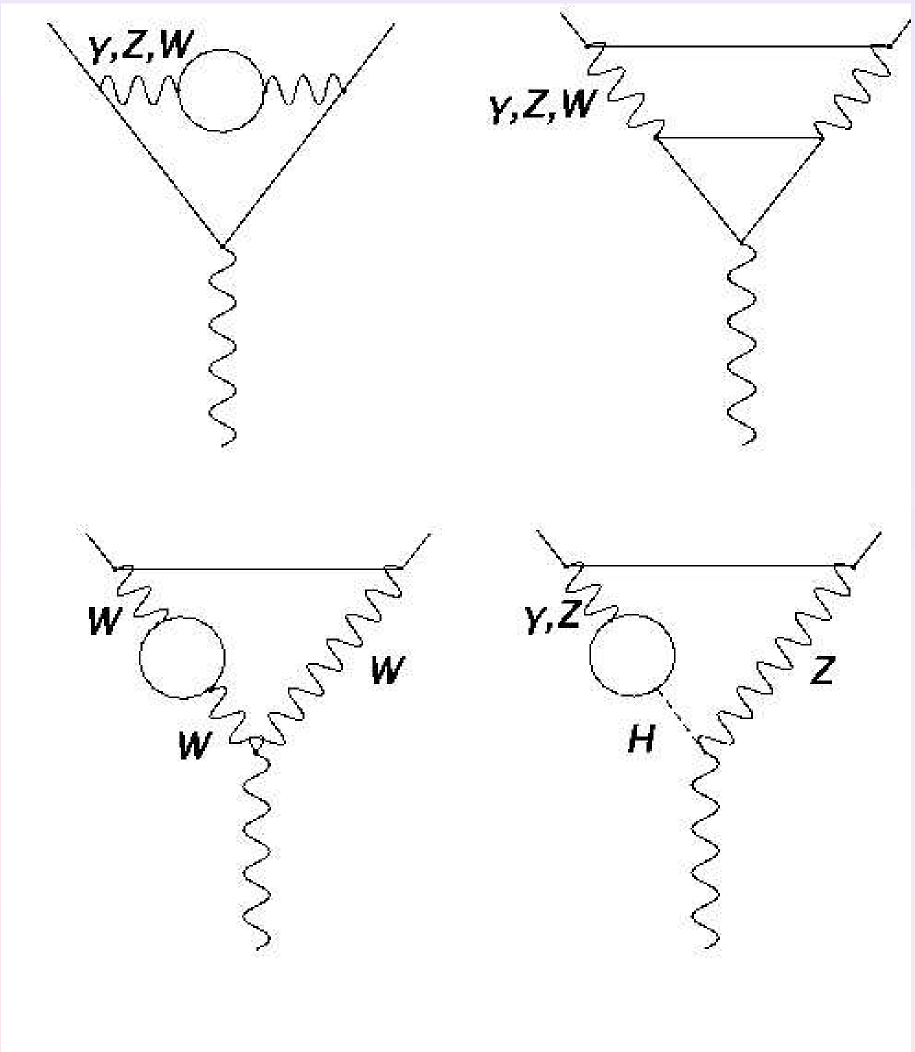
- at two loop level, for bosonic part:

  also through generic vertex diagrams



Two loop vertex contributions: the fermionic part

M.Awramik,M.C., A.Freitas, G.Weiglein Phys.Rev.Lett.93:201805,2004



- diagrams with Higgs cancel
- diagrams with top quark:

  semi-analytic result
- diagrams with light fermions:

  analytic result

Light fermion contributions:

- one variable only: M_W/M_Z
- reduction by *Integration By Parts Identities* (Chetyrkin, Tkachov '81)

  and *Lorentz Invariance Identities* *(Gehrman, Remiddi '00)*
- use IdSolver: *C++* library (M.C.)
- for new master integrals differential equations work
- exact result in terms of polylogarithms

Top Quark Contributions:

- depend on two variables:

$$m_t/M_Z,\ M_W/M_Z$$

- use *asymptotic expansions*

  in heavy top quark mass

 - justified by size of

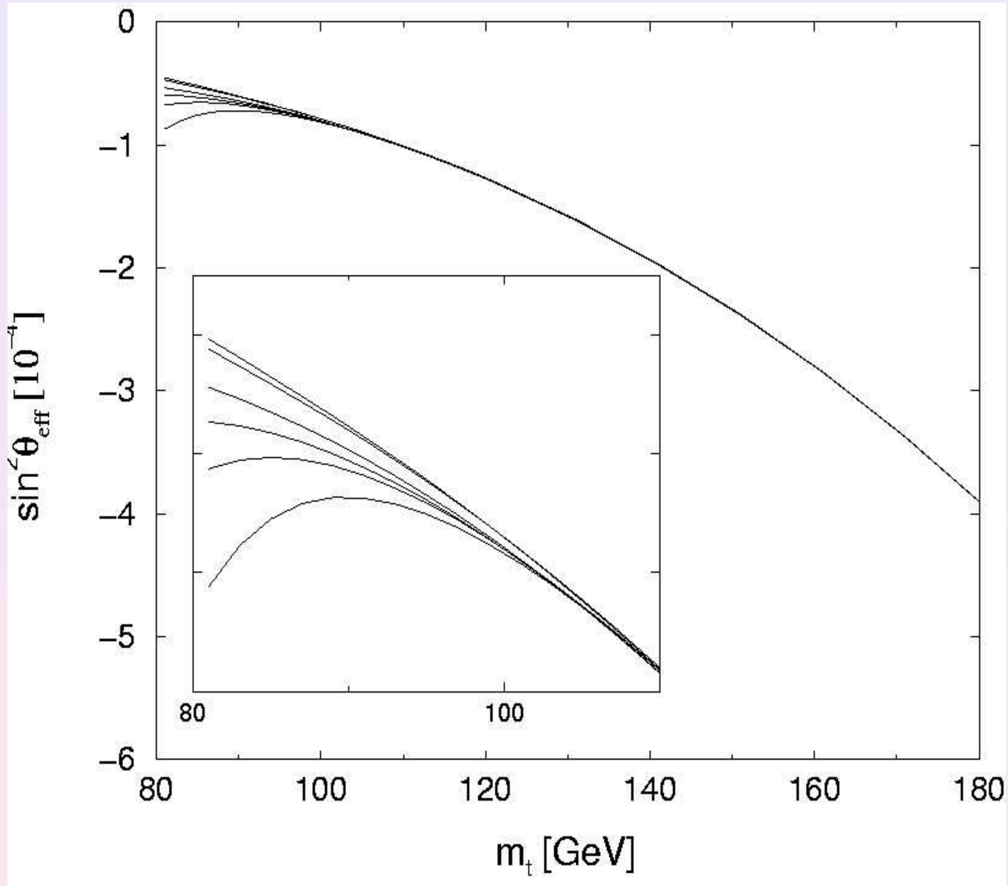
$$M_Z^2/m_t^2 \sim 1/4$$

 - up to two loop vacuum

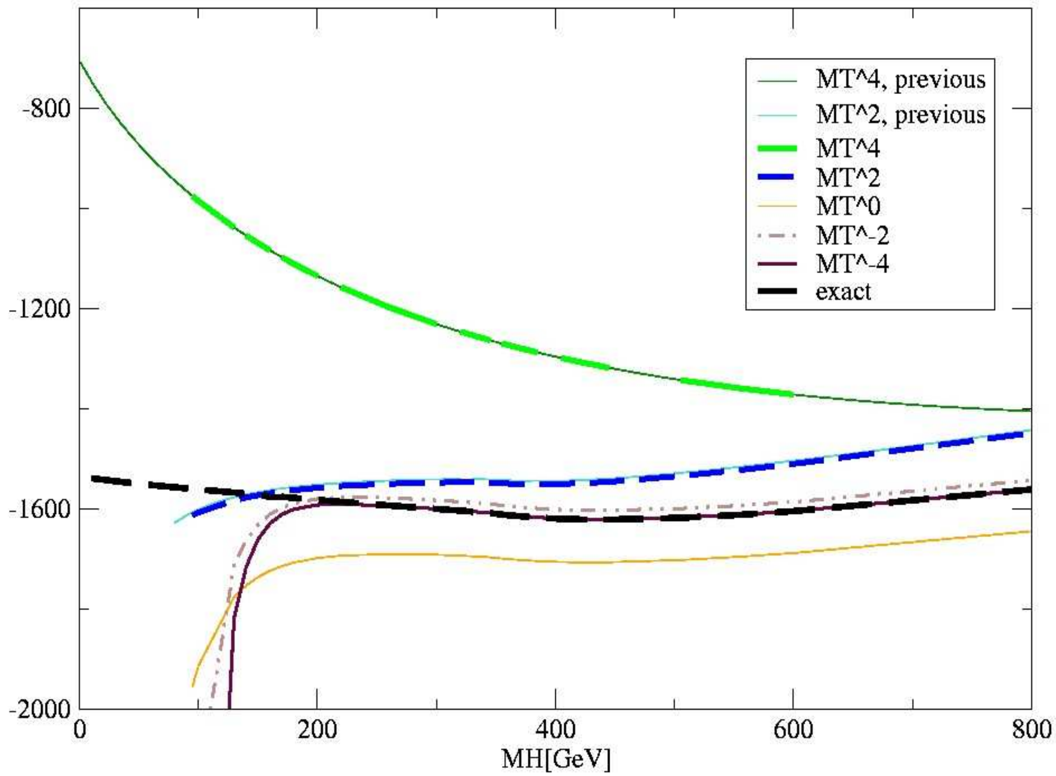
   diagrams required -> easy

- 10^{th} term gives relative

   error around $\sim 10^{-5}$



Tests:



1) expansion of the whole result
- agreement of leading and next to leading term with results of Gambino and Degrassi

2) some integrals tested with low momentum expansion

3) some integrals tested by means of Pade re-summed Mellin-Barnes representation

4) independent internal calculation

Size of different contributions to $\sin^2\theta^{lep}_{eff}$:

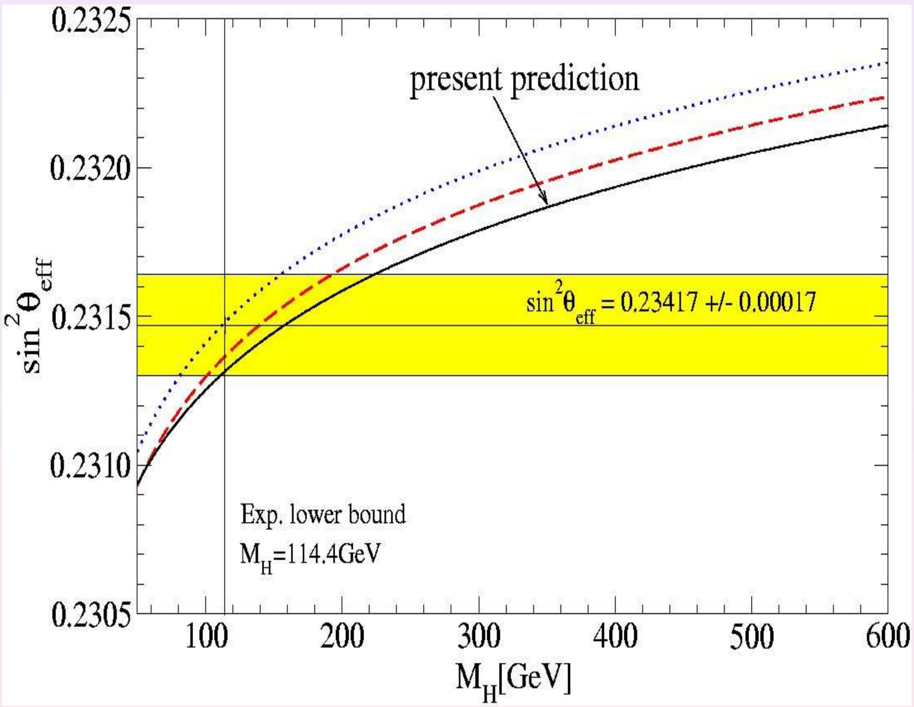
M_H [GeV]	$\mathcal{O}(\alpha)$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{ferm}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tb}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{lf}}$ $\times 10^{-4}$	$\mathcal{O}(\alpha^2)_{\text{tr}\gamma_5}$ $\times 10^{-4}$
100	97.47	-0.14	-3.77	-0.63	0.06
200	93.17	-0.48	-3.80	-0.68	0.06
600	84.28	-1.11	-3.75	-0.84	0.06
1000	79.63	-1.05	-3.31	-0.94	0.06

agreement in recent calculation of Hollik, Meier, Uccirati

M_H [GeV]	$\mathcal{O}(\alpha\alpha_s)$ $\times 10^{-4}$	$\mathcal{O}(\alpha\alpha_s^2)$ $\times 10^{-4}$	$c_W^2\Delta\rho^{\alpha^2\alpha_s}$ $\times 10^{-4}$	$c_W^2\Delta\rho^{\alpha^3}$ $\times 10^{-4}$	reducible $\times 10^{-4}$
100	-8.18	-1.60	0.28	0.04	0.20
200	-8.18	-1.60	0.46	0.02	0.21
600	-8.18	-1.60	0.90	0.02	0.21
1000	-8.18	-1.60	1.11	0.22	0.22

Faisst, Kühn, Seidensticker, Veretin, '03

Theory versus experiment:



The newest theory prediction for $\sin^2\theta^{lep}_{eff}$:

- M.Awramik,M.C.,A.Freitas,G.Weiglein

  Phys.Rev.Lett.93:201805,2004

- contains all known corrections, but two loop bosonic part is still missing

- error from unknown higher order terms: $4.9*10^{-5}$ (previous: $14*10^{-5}$)

M_H= 125 GeV (for m_t=172.7GeV)

shift due to new formula: +15GeV

shift due to measurements:

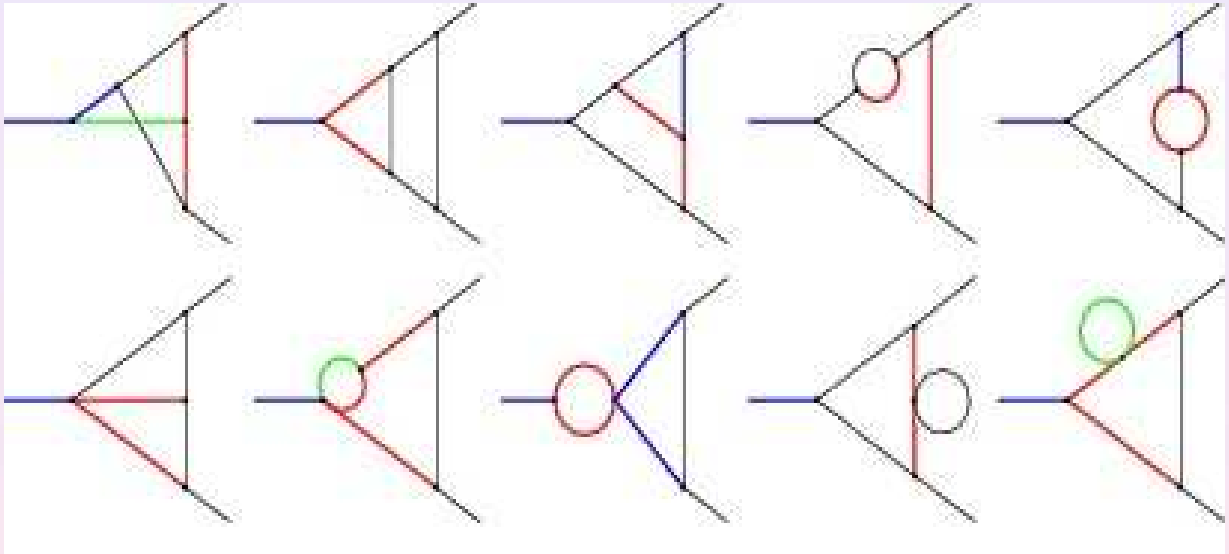
of m_t: ±30GeV

of $\sin^2\theta^{lep}_{eff}$: -10GeV

Two loop vertex contributions: the bosonic part

(in collaboration with M. Awramik and A. Freitas)

Possible topologies:



Possible problems:

- number of scales: M_Z (blue line), M_W (red line) and M_H (green line)
  - => around 100 of different mass patterns with up to 3 scales
- finiteness of the result and treatment of divergences

General approach:

- to reduce the number of scales perform a ***mass difference expansion*** in $\sin^2\theta_W$
- for the desired precision of 3-5 digits, 8 terms should be enough
- As a first step, expand in the difference between the Higgs and the Z boson masses
- Later, keep the Higgs boson mass arbitrary and integrate numerically

Realization:

- Threshold expansion
- Reduction of the Hard part with IdSolver
- Analytic evaluation of masters to get exact divergence cancellation

Masters of the hard part:

- Serveral taken from papers of

  Davydychev, Kalmykov, Veretin, Kotikov

- A few masters with a single massive line taken from

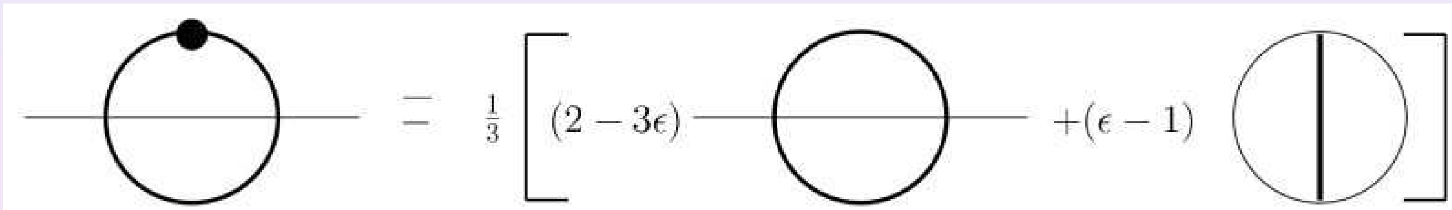
  Aglietti, Bonciani

- The rest evaluated with

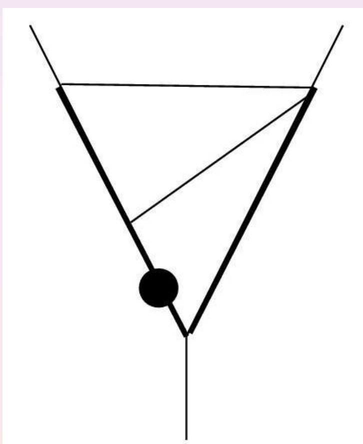
  - sector decomposition and numerical evaluation
  - taylor series either resummed empirically of with
    Pade with or without conformal mapping (Tarasov)
  - Mellin-Barnes representations
  - differential equations
  - PSLQ

Some aspects

- Funny relations



- Analytic values of finite parts of some graphs



$$\frac{2}{9}\zeta_3 - \frac{1}{54}\frac{\pi^3}{\sqrt{3}} + \frac{3}{2}\frac{\pi S2}{\sqrt{3}}$$

Result for the complete bosonic vertices: **PRELIMINARY**

- At $M_H = 100\,GeV$

$$\Delta\kappa = -4.4\times10^{-6}\frac{1}{s_w^4}\left(1-1.1\,s_w^2-3.9\,s_w^4-0.8\,s_w^6+0.2\,s_w^8+0.5\,s_w^{10}+1.1\times10^{-3}\log(s_w^2)\left(s_w^8+10.2\,s_w^{10}\right)+\ldots\right)$$

- The highest terms under brackets that we have is $0.6\,s_w^{14}$ and amounts to a relative error of 2×10^{-5}
- In combination with the remaining parts this gives

$\Delta\kappa = -0.47\times10^{-4}+0.03\times10^{-4} = -0.44\times10^{-4}$ or $\Delta\sin^2\theta_{eff} = -0.10\times10^{-4}$

- One still has to add the effect of the bosonic corrections on the W boson mass prediction

$\Delta\sin^2\theta = 0.18\times10^{-4}$

- **The total contribution at 100 GeV is** 8×10^{-6}

Variation with the Higgs boson mass:

$M_H\,[GeV]$	$\Delta\kappa^{(\alpha^2)}_{ferm} \times 10^{-4}$	$\Delta\kappa^{(\alpha^2)}_{ferm,sub} \times 10^{-4}$	$\Delta\kappa^{(\alpha^2)}_{bos,sub} \times 10^{-4}$
100	-0.637(1)	0	0
200	-2.165(1)	-1.528	0.265
600	-5.012(1)	-4.375	0.914
1000	-4.737(1)	-4.100	1.849

from Hollik, Meier, Uccirati, hep-ph/0509302

- From the expansion

$$-4.8\times10^{-5}\left(1+0.13\, s_h^2+0.04\, s_h^4+0.02\, s_h^6+0.01\, s_h^8\right)$$

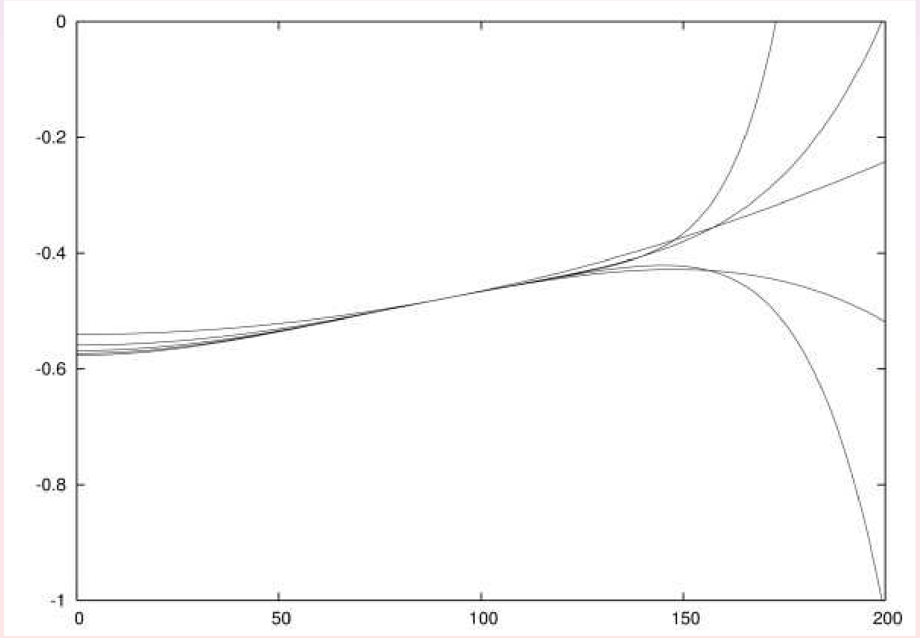
- From Pade resummation at 200 GeV

$$0.26\sim0.27\times10^{-4}$$

- Contribution to the mixing angle

at 200 GeV: 6×10^{-6}

at 600 GeV: 1.2×10^{-5}



Publicity:

Similar methods and software used for other projects

- *NNLO b->s* γ *M.C., U. Haisch, M. Misiak, in preparation*
- *4-loop QCD* β *-function M.C., Nucl. Phys.* **B710** *(2005) 485*

Summary:

- the theory error on Higgs boson mass prediction is dominated by unknown corrections to the effective weak mixing angle

- the completion of the fermionic corrections gave large numerical contributions, with a shift ~ 15GeV of the Higgs boson mass prediction

- The bosonic corrections are now available, which in principle closes the two-loop electroweak programme at the Z peak

- to match the precision at LC/GigaZ higher order terms: $O(\alpha^2\alpha_s)$ beyond leading term, $O(\alpha^3)$, $O(\alpha^2\alpha^2_s)$ will be required