NLO QCD Corrections to Drell-Yan in TeV-scale Gravity Models

V. Ravindran

Harish-Chandra Research Institute, Allahabad

- Graviton mediated Drell-Yan
- QCD Factorisation scale ambiguity
- NLO corrections to new physics
- Conclusions

In collaboration with

Willy van Neerven, Prakash Mathews and K. Sridhar

Snap shot of my talk

Snap shot of my talk

- Perturbative QCD provides a frame work to compute infrared safe observables at high energies.

Snap shot of my talk

- Perturbative QCD provides a frame work to compute infrared safe observables at high energies.
- They are "often" sensitive to

  1) Renormalisation scale

  2) Factorisation scale

  3) Missing higher order contributions(stability of perturbation)

  4) Non-perturbative quantities that enter, such as parton density functions

Snap shot of my talk

- Perturbative QCD provides a frame work to compute infrared safe observables at high energies.
- They are "often" sensitive to

  1) Renormalisation scale

  2) Factorisation scale

  3) Missing higher order contributions(stability of perturbation)

  4) Non-perturbative quantities that enter, such as parton density functions

- Solution to (1) to (3) to compute higher order QCD corrections.

Snap shot of my talk

- Perturbative QCD provides a frame work to compute infrared safe observables at high energies.
- They are "often" sensitive to

  1) Renormalisation scale

  2) Factorisation scale

  3) Missing higher order contributions(stability of perturbation)

  4) Non-perturbative quantities that enter, such as parton density functions
- Solution to (1) to (3) to compute higher order QCD corrections.
- $P_1 + P_2 \to l^+ l^-$ is of course one of the most important processes to discover "new physics" at high energy colliders such as TeV scale gravity models (Large Extra-Dimensional theories)

Snap shot of my talk

- Perturbative QCD provides a frame work to compute infrared safe observables at high energies.
- They are "often" sensitive to

  1) Renormalisation scale

  2) Factorisation scale

  3) Missing higher order contributions(stability of perturbation)

  4) Non-perturbative quantities that enter, such as parton density functions
- Solution to (1) to (3) to compute higher order QCD corrections.
- $P_1 + P_2 \longrightarrow l^+ l^-$ is of course one of the most important processes to discover "new physics" at high energy colliders such as TeV scale gravity models (Large Extra-Dimensional theories)
- Higher order QCD corrections increases the reliablity of the predictions of the theory

Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM). They provide an alternate view of the "hierarchy" between the EW ($\sim \mathbf{1}$ TeV) and the Planck scale ($\mathbf{10^{16}}$ TeV)

Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM). They provide an alternate view of the "hierarchy" between the EW (~ 1 TeV) and the Planck scale (10^{16} TeV)

- Basically the "Geometry" of extra spacial dimensions may be responsible for the "Hierarchy"

Large vol of extra dim	Strong Curvature of extra dim
ADD	RS
ADD PLB 429(1998)	RS PRL 83 (1999) 3370

Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM). They provide an alternate view of the "hierarchy" between the EW ($\sim \mathbf{1}$ TeV) and the Planck scale ($\mathbf{10^{16}}$ TeV)

- Basically the "Geometry" of extra spacial dimensions may be responsible for the "Hierarchy"

Large vol of extra dim ADD ADD PLB 429(1998)	Strong Curvature of extra dim RS RS PRL 83 (1999) 3370

- These theories should have a "viable mechanism" to "hide" the "extra dim" such that "space time" is effectively "four" consistent with "known physics".

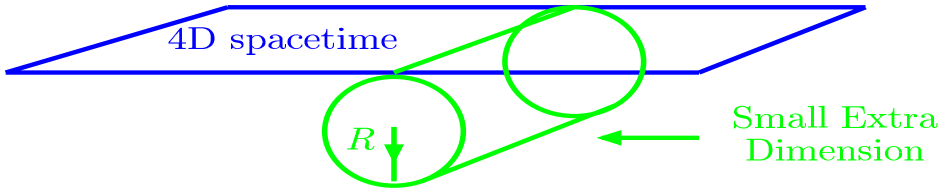
Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM). They provide an alternate view of the "hierarchy" between the EW ($\sim \mathbf{1}$ TeV) and the Planck scale ($\mathbf{10^{16}}$ TeV)

- Basically the "Geometry" of extra spacial dimensions may be responsible for the "Hierarchy"

Large vol of extra dim	Strong Curvature of extra dim
ADD	RS
ADD PLB 429(1998)	RS PRL 83 (1999) 3370

- These theories should have a "viable mechanism" to "hide" the "extra dim" such that "space time" is effectively "four" consistent with "known physics".
- No deviation at present accelerators leads to limits on extra dim $< \mathbf{10^{-16}}$ cm



Kaluza-Klein Picture

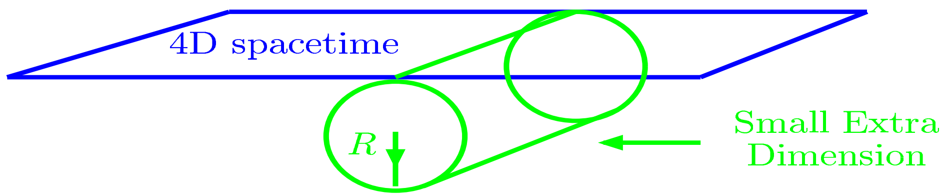
Large Extra Dimensions

Models of "Extra Dimensions" are now studied as serious contenders for "Physics Beyond SM"(BSM). They provide an alternate view of the "hierarchy" between the EW ($\sim \mathbf{1}$ TeV) and the Planck scale ($\mathbf{10^{16}}$ TeV)

- Basically the "Geometry" of extra spacial dimensions may be responsible for the "Hierarchy"

Large vol of extra dim	Strong Curvature of extra dim
ADD	RS
ADD PLB 429(1998)	RS PRL 83 (1999) 3370

- These theories should have a "viable mechanism" to "hide" the "extra dim" such that "space time" is effectively "four" consistent with "known physics".
- No deviation at present accelerators leads to limits on extra dim $< \mathbf{10^{-16}}$ cm



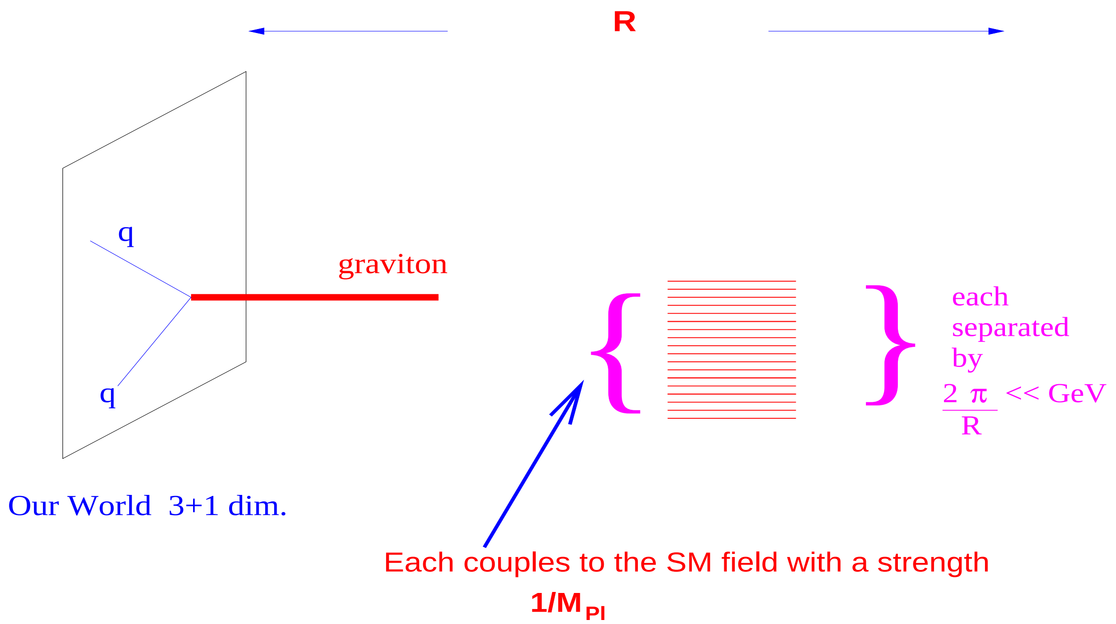
Kaluza-Klein Picture

Kaluza-Klein Modes

- Extra dimensions being compact, gravitational field will be periodic function in the extra dimension.
- In 4-dim it would correspond to nearly mass degenerate tower of KK modes $m_{\vec{n}}^2 \sim \vec{n}^2/R^2$

Kaluza-Klein Modes

- Extra dimensions being compact, gravitational field will be periodic function in the extra dimension.
- In 4-dim it would correspond to nearly mass degenerate tower of KK modes $m_{\vec{n}}^2 \sim \vec{n}^2/R^2$



Massless graviton and KK modes couple with SM fields with coupling $M_P^{-1} \sim R^{\frac{d}{2}}$

Gravity-QCD Coupling

Gravitational interaction with SM fields:

$$S = S_{SM} - \frac{\kappa}{2} \int d^n x \, T_{\mu\nu}(x) \, G^{\mu\nu}(x)$$

strength of interaction $\kappa \sim \sqrt{G_N} \sim M_P{}^{-1}$

Energy momentum tensor:

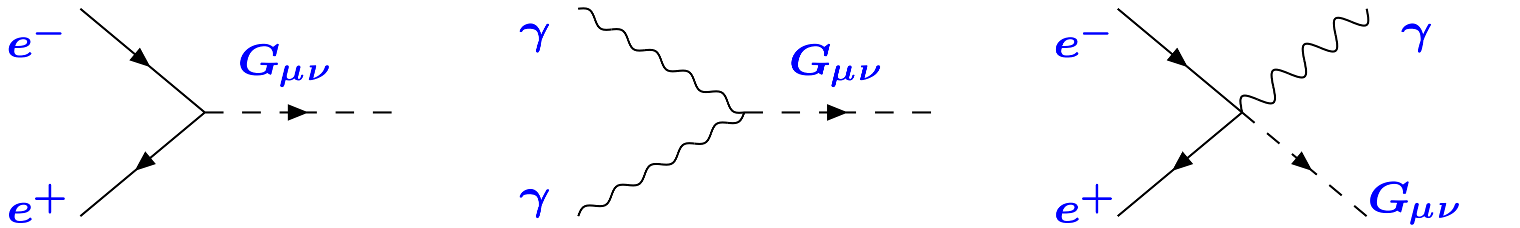
$$\begin{aligned} T^{QCD}_{\mu\nu} &= -g_{\mu\nu} \, \mathcal{L}_{QCD} - F^a_{\mu\rho} \, F^{a\rho}_{\nu} - g_{\mu\nu} \, \frac{1}{\xi} \, \partial^\rho \left(A_\rho \partial^\sigma A_\sigma \right) \\ &+ \left[\left(\frac{i}{4} \overline{\psi} \left[\gamma_\mu \left(\overrightarrow{\partial}_\nu - igT^a A^a_\nu \right) - \gamma_\mu \left(\overleftarrow{\partial}_\nu + igT^a A^a_\nu \right) \right] \psi \right. \right. \\ &+ \left. \left. \frac{1}{\xi} A^a_\nu \partial_\mu (\partial^\sigma A^a_\sigma) + \partial_\mu \overline{\omega}^a \left(\partial_\nu \omega^a - g f^{abc} A^c_\nu \omega^b \right) \right) + (\mu \leftrightarrow \nu) \right] \end{aligned}$$

A^a_μ	Gauge fields	ω^a	Ghost fields
ψ	Fermionic fields	$G_{\mu\nu}$	Graviton Fields

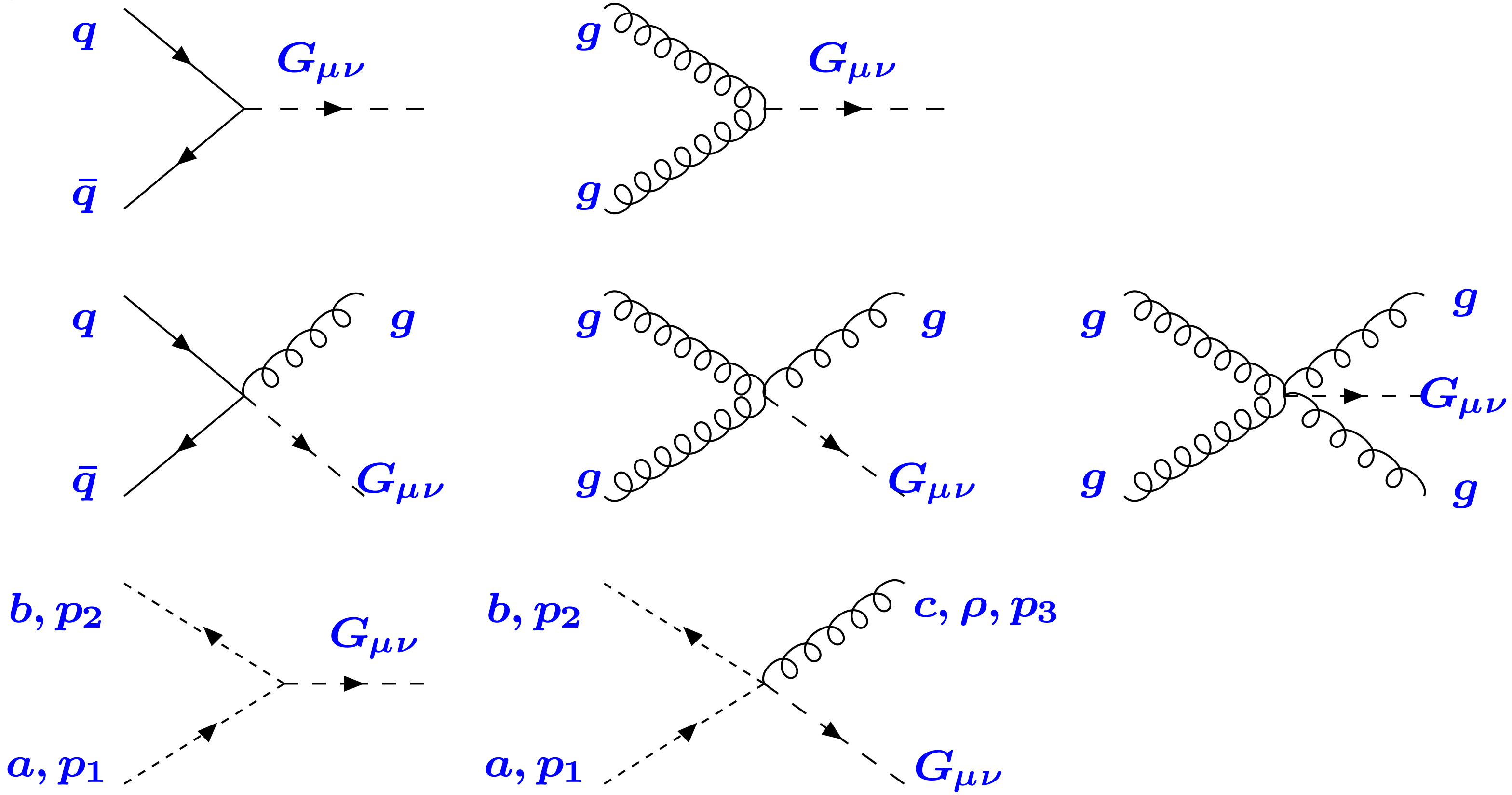
Gravitons couple to anything and everything

Feynman Rules

- QED

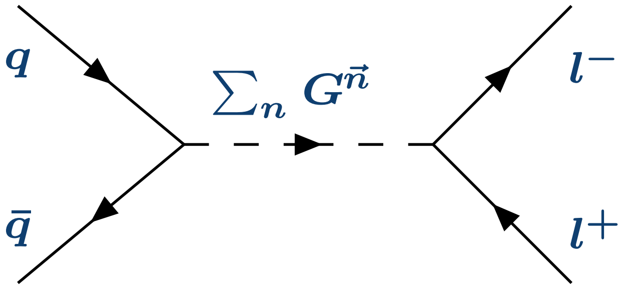


- QCD



Giudice, Rattazzi, Well hep-ph/9811291; Han, Lykken, Zhang hep-ph/9811350; PM, Ravindran, Sridhar hep-ph/0405292

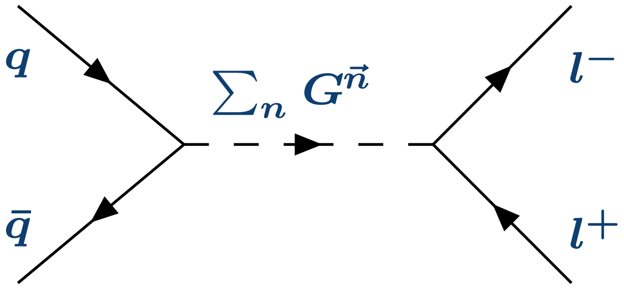
Kaluza-Klein suppression in ADD



$$\sim \frac{\kappa}{2} \quad \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \quad \frac{\kappa}{2}$$

$$\kappa \approx 10^{-16}$$

Kaluza-Klein suppression in ADD



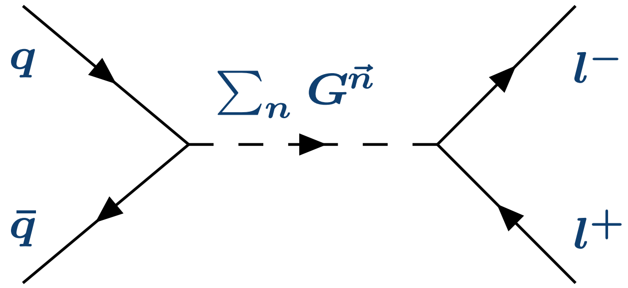
$$\sim \frac{\kappa}{2} \quad \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \quad \frac{\kappa}{2}$$

$$\kappa \approx 10^{-16}$$

- Summation of KK modes:

$$\sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} = \frac{16\pi}{\kappa^2} \left(\frac{Q^2}{M_S^2} \right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I\left(\frac{M_S}{Q} \right)$$

Kaluza-Klein suppression in ADD



$$\sim \frac{\kappa}{2} \quad \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \quad \frac{\kappa}{2}$$

$$\kappa \approx 10^{-16}$$

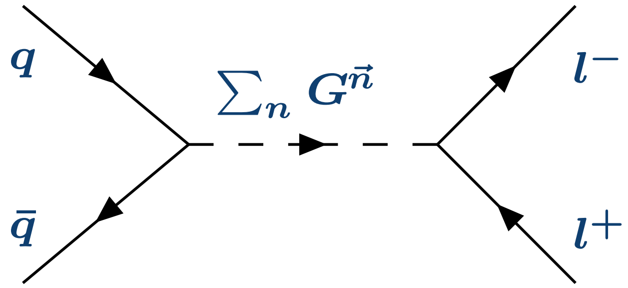
- Summation of KK modes:

$$\sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} = \frac{16\pi}{\kappa^2} \left(\frac{Q^2}{M_S^2} \right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I \left(\frac{M_S}{Q} \right)$$

- $(\textbf{Coupling})^2$ (Sum over the KK mode) leads to

$$(\kappa)^2 \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \Longrightarrow 16\pi \left(\frac{Q^2}{M_S^2} \right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I \left(\frac{M_S}{Q} \right)$$

Kaluza-Klein suppression in ADD



$$\sim \frac{\kappa}{2} \quad \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \quad \frac{\kappa}{2}$$

$$\kappa \approx 10^{-16}$$

- Summation of KK modes:

$$\sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} = \frac{16\pi}{\kappa^2} \left(\frac{Q^2}{M_S^2}\right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I\left(\frac{M_S}{Q}\right)$$

- $(\text{Coupling})^2$ (Sum over the KK mode) leads to

$$(\kappa)^2 \sum_n \frac{1}{Q^2 - m_{\vec{n}}^2 + i\epsilon} \Longrightarrow 16\pi \left(\frac{Q^2}{M_S^2}\right)^{\frac{d-2}{2}} \frac{1}{M_S^4} I\left(\frac{M_S}{Q}\right)$$

$$\frac{1}{M_S^{2+d}} \sim \frac{1}{(TeV)^{2+d}}$$

Planck suppression is compensated by High multiplicity of KK modes

Phenomenology with Extra-Dimension

In the Standard Model, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\hat{s} \frac{d}{dQ^2} \hat{\sigma}_{ab}^{SM} \left(\hat{s}, Q^2\right) \sim \frac{1}{Q^2}$$

Phenomenology with Extra-Dimension

In the Standard Model, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{SM}\left(\hat{s},Q^2\right) \sim \frac{1}{Q^2}$$

In the Gravity mediated processes, the partonic cross sections increase monotonically with the energy scale involved:

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{Gravity}\left(\hat{s},Q^2,M_S\right) \quad \sim \quad \frac{Q^6}{M_S^8}\left(\frac{Q^2}{M_S^2}\right)^{d-2} \qquad\qquad Q < M_S$$

Phenomenology with Extra-Dimension

In the Standard Model, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{SM}\left(\hat{s},Q^2\right)\sim\frac{1}{Q^2}$$

In the Gravity mediated processes, the partonic cross sections increase monotonically with the energy scale involved:

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{Gravity}\left(\hat{s},Q^2,M_S\right)\quad\sim\quad\frac{Q^6}{M_S^8}\left(\frac{Q^2}{M_S^2}\right)^{d-2}\qquad\qquad Q<M_S$$

- Gravity mediated cross sections can show up at high Q.

Phenomenology with Extra-Dimension

In the Standard Model, the partonic cross sections decreases with the energy scale (Q or p_T involved):

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{SM}\left(\hat{s},Q^2\right) \sim \frac{1}{Q^2}$$

In the Gravity mediated processes, the partonic cross sections increase monotonically with the energy scale involved:

$$\hat{s}\frac{d}{dQ^2}\hat{\sigma}_{ab}^{Gravity}\left(\hat{s},Q^2,M_S\right) \quad \sim \quad \frac{Q^6}{M_S^8}\left(\frac{Q^2}{M_S^2}\right)^{d-2} \qquad\qquad Q < M_S$$

- Gravity mediated cross sections can show up at high Q.
- The processes where the virtual/real KK gravitons contribute significantly:

  (1) Di-lepton or Drell-Yan production at large invariant mass Q

  (2) Di-photon or Di-boson production at large Q, P_T

  (3)Observables with missing energy
  (...) $\cdots$

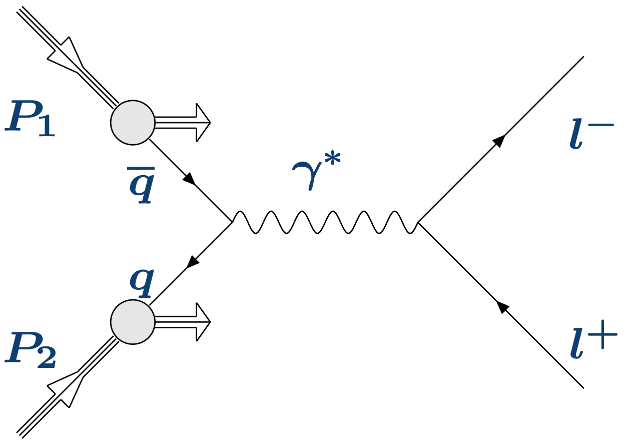
Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$

$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$

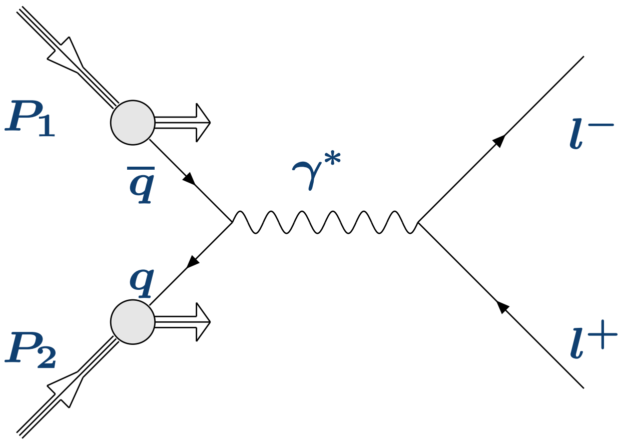
Drell-Yan Process

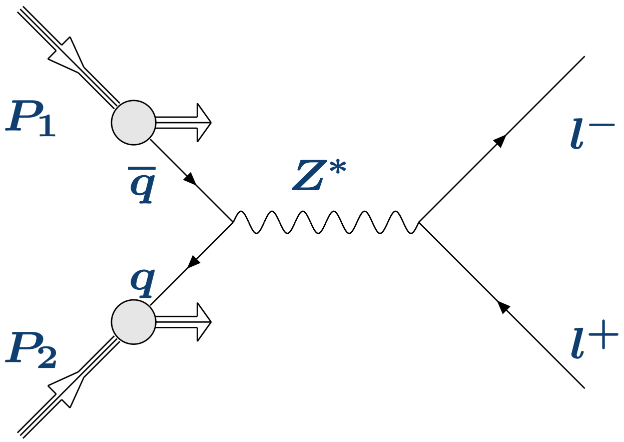
$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$



Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$

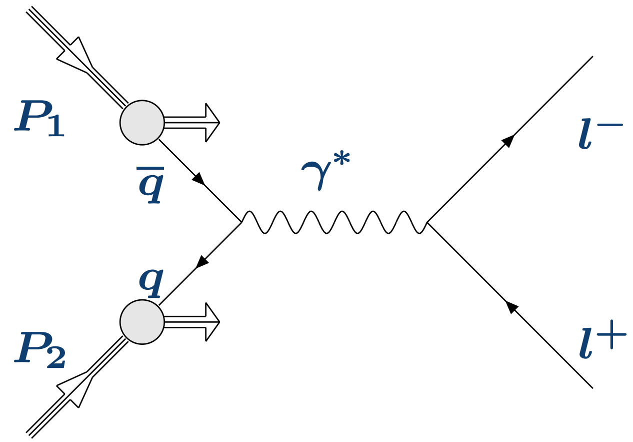


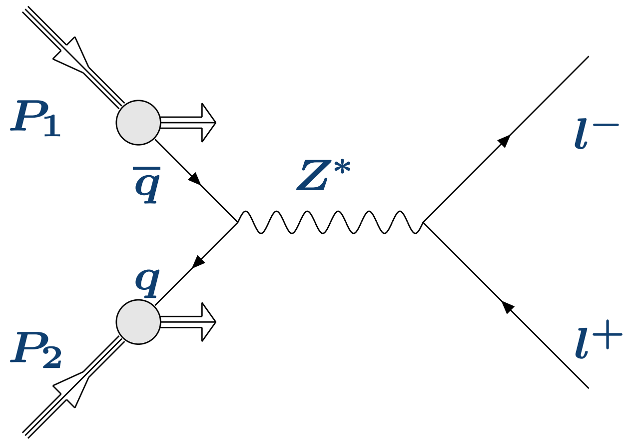


SM

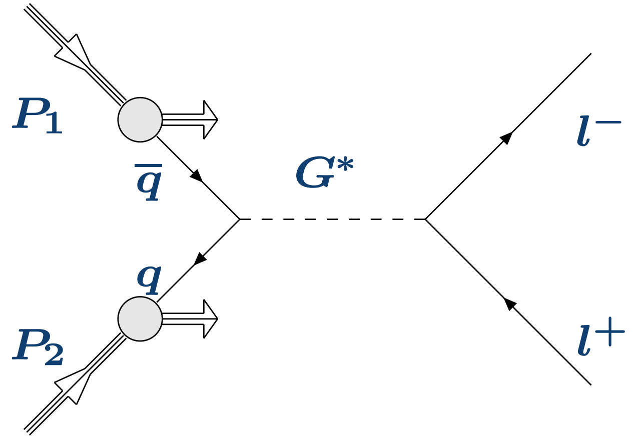
Drell-Yan Process

$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$





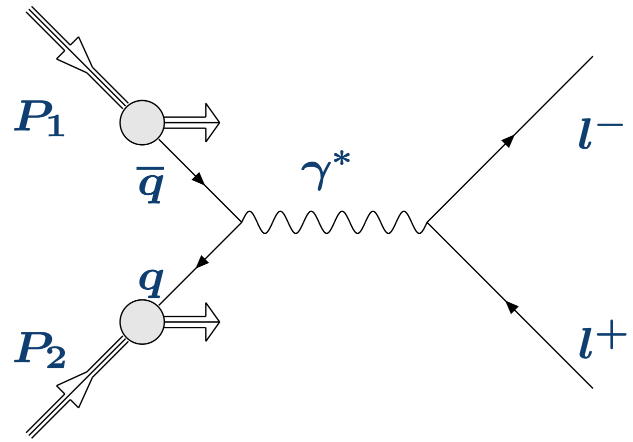
SM

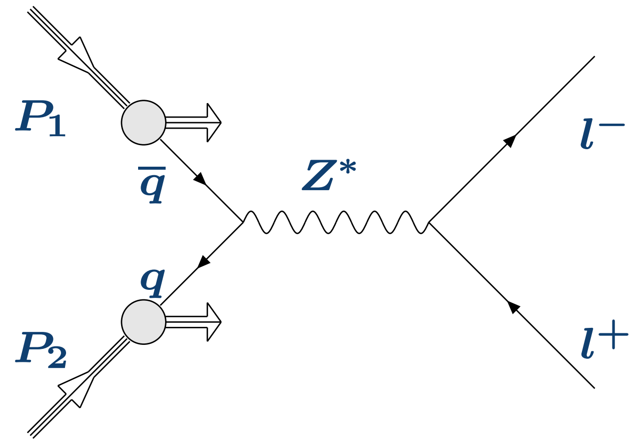


Drell-Yan Process

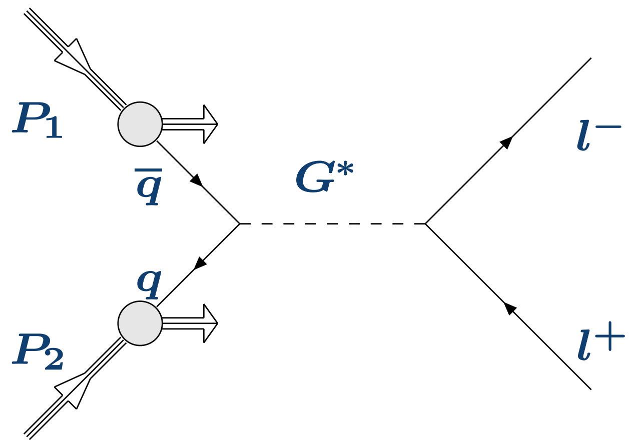
$$P_1(p_1) + P_2(p_2) \longrightarrow [\gamma, Z, G] + \text{hadronic states}(X)$$

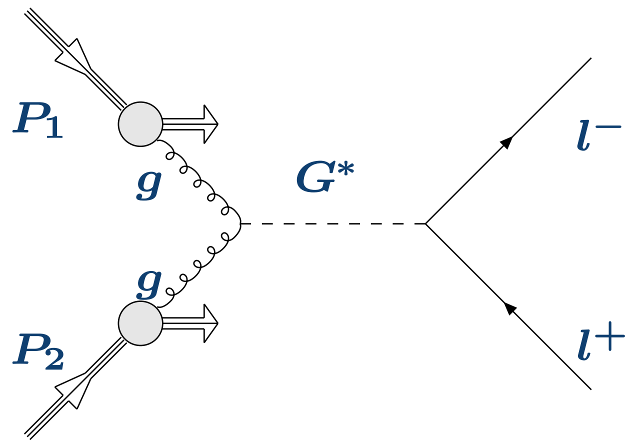
$$\hookrightarrow l^+(k_1) + l^-(k_2) \qquad (k_1 + k_2)^2 = Q^2$$





SM



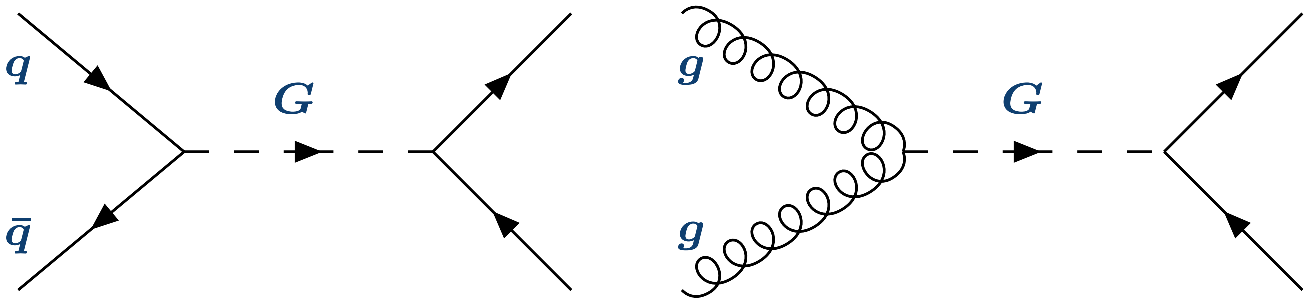


Gravity

Contributing Subprocess

Leading Order:

Standard Model	Gravity
$q + \bar{q} \rightarrow \gamma/Z$	$q + \bar{q} \rightarrow G$ $g + g \rightarrow G$



Born contributions

QCD improved Parton Model

$$P_1 + P_2 \rightarrow l^+ l^- + X \qquad m_h^2 = (l^+ + l^-)^2$$

QCD improved Parton Model

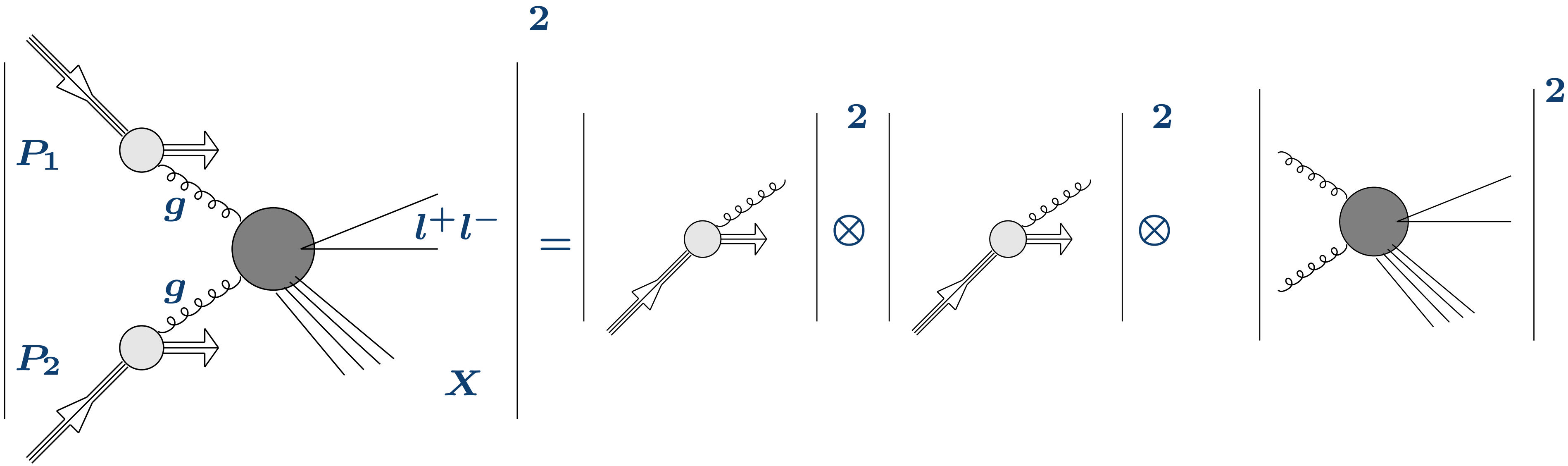
$$P_1 + P_2 \to l^+ l^- + X \qquad m_h^2 = (l^+ + l^-)^2$$

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \quad f_a\left(\tau\right) \quad \otimes \quad f_b\left(\tau\right) \quad \otimes \quad 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2\right), \qquad \tau = \frac{m_h^2}{S}$$

QCD improved Parton Model

$$P_1 + P_2 \to l^+ l^- + X \qquad m_h^2 = (l^+ + l^-)^2$$

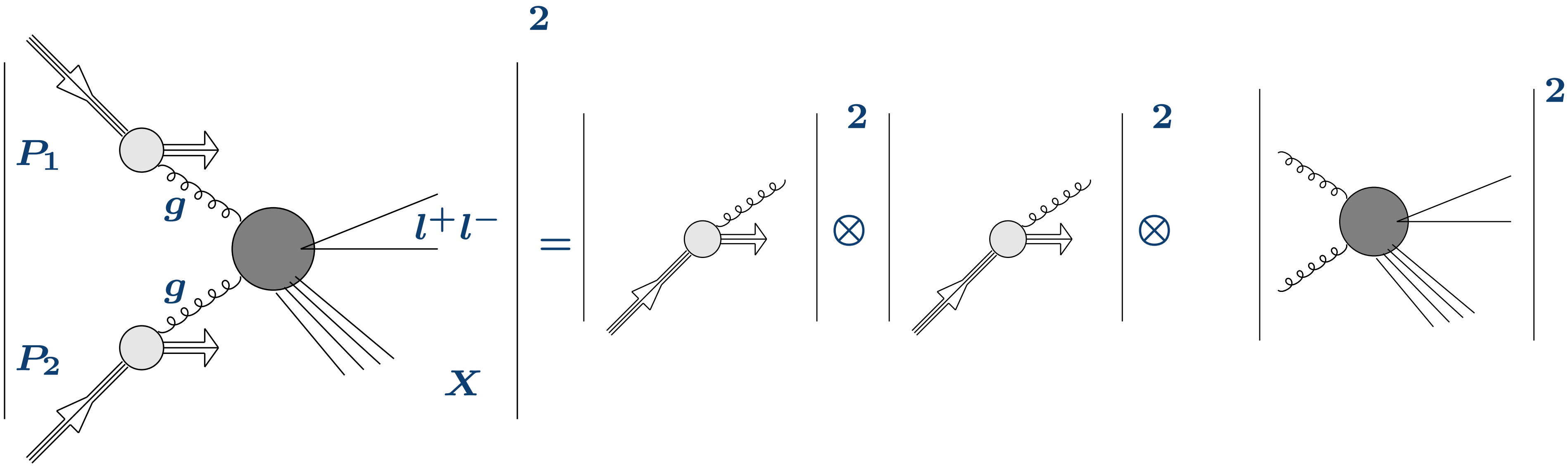
$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \quad f_a\left(\tau\right) \quad \otimes \quad f_b\left(\tau\right) \quad \otimes \quad 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2\right), \qquad \tau = \frac{m_h^2}{S}$$



QCD improved Parton Model

$$P_1 + P_2 \to l^+ l^- + X \qquad m_h^2 = (l^+ + l^-)^2$$

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \quad f_a\left(\tau\right) \quad \otimes \quad f_b\left(\tau\right) \quad \otimes \quad 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2\right), \qquad \tau = \frac{m_h^2}{S}$$

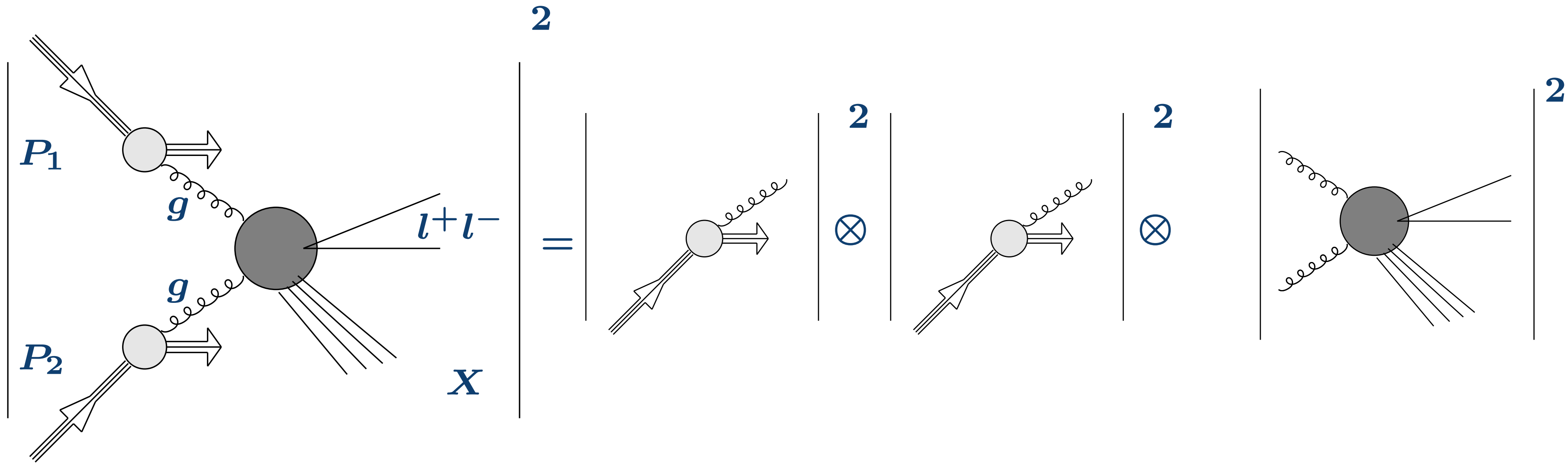


- $f_a(x)$ are parton distribution functions inside the hadron P.
- Non-perturbative in nature and process independent.

QCD improved Parton Model

$$P_1 + P_2 \rightarrow l^+ l^- + X \qquad m_h^2 = (l^+ + l^-)^2$$

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \quad f_a\left(\tau\right) \quad \otimes \quad f_b\left(\tau\right) \quad \otimes \quad 2\hat{s}\, d\hat{\sigma}^{ab}\left(\tau, m_h^2\right), \qquad \tau = \frac{m_h^2}{S}$$



- $f_a(x)$ are parton distribution functions inside the hadron P.
- Non-perturbative in nature and process independent.

- $\hat{\sigma}_{ab}$ are the partonic cross sections.
- Perturbatively calculable.

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

- The non-perturbative flux:

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

- The non-perturbative flux:

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

- $f_a^{P_1}(x, \mu_F)$ are Parton distribution functions with momentum fraction x.

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

- The non-perturbative flux:

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

- $f_a^{P_1}(x, \mu_F)$ are Parton distribution functions with momentum fraction x.
- μ_R is the Renormalisation scale and μ_F, Factorisation scale

Factorisation Theorem (Parton Model)

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

- The non-perturbative flux:

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

- $f_a^{P_1}(x, \mu_F)$ are Parton distribution functions with momentum fraction x.
- μ_R is the Renormalisation scale and μ_F, Factorisation scale
- The Renormalisation group invariance:

$$\frac{d}{d\mu}\sigma^{P_1 P_2}(\tau, m_h^2) = 0, \qquad \mu = \mu_F, \mu_R$$

Altarelli-Parisi/Renormalisation Group Equations

Altarelli-Parisi/Renormalisation Group Equations

Renormalised parton density:

$$f_a(z, \mu_F) = \Gamma_{ab}(z, \mu_F, \frac{1}{\varepsilon_{\mathrm{IR}}}) \otimes f_a^B(z)$$

Altarelli-Parisi/Renormalisation Group Equations

Renormalised parton density:

$$f_a(z, \mu_F) = \Gamma_{ab}(z, \mu_F, \frac{1}{\varepsilon_{\mathrm{IR}}}) \otimes \ f_a^B(z)$$

Altarelli-Parisi Evolution equation:

$$\mu_F \frac{d}{d\mu_F} f_a(x, \mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z, \mu_F) f_b\left(\frac{x}{z}, \mu_F\right), \qquad P \equiv \Gamma^{-1}\left(\mu_F \frac{d}{d\mu_F}\right)\Gamma$$

Altarelli-Parisi/Renormalisation Group Equations

Renormalised parton density:

$$f_a(z,\mu_F) = \Gamma_{ab}(z,\mu_F,\frac{1}{\varepsilon_{\mathrm{IR}}}) \otimes \ f_a^B(z)$$

Altarelli-Parisi Evolution equation:

$$\mu_F \frac{d}{d\mu_F} f_a(x,\mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z,\mu_F) f_b\left(\frac{x}{z},\mu_F\right), \qquad P \equiv \Gamma^{-1}\left(\mu_F \frac{d}{d\mu_F}\right)\Gamma$$

Perturbatively Calculable:

$$\begin{aligned} P_{ab}(z,\mu_F) \quad = \quad & \left(\frac{\alpha_s(\mu_F)}{4\pi}\right) P_{ab}^{(0)}(z) && \text{one loop} \quad (LO) \\ & +\left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^2 P_{ab}^{(1)}(z) && \text{two loop} \quad (NLO) \\ & +\left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^3 P_{ab}^{(2)}(z) && \text{three loop} \quad (NNLO) \end{aligned}$$

LO was computed by "Gross,Wilczek and Politzer"(Nobel prize paper also see "Altarelli and Parisi") and NNLO is computed recently (summer $\mathbf{2004}$) by "Moch,Vermaseren and Vogt"

UV Scale dependence of partonic cross section

UV Scale dependence of partonic cross section

- Collinear finite partonic cross sections are calculable in perturbative QCD in powers of α_s^B, bare strong coupling constant.

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty}\left(\frac{\alpha_s^B}{4\pi}\right)^i d\hat{\sigma}_{ab}^{B,(i)}\left(z, m_h^2, \mu_F, \frac{1}{\varepsilon_{UV}}\right)$$

UV singularities are regularised in dimensional regularisation $n = 4 + \varepsilon_{UV}$

UV Scale dependence of partonic cross section

- Collinear finite partonic cross sections are calculable in perturbative QCD in powers of α_s^B, bare strong coupling constant.

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s^B}{4\pi}\right)^i d\hat{\sigma}_{ab}^{B,(i)}\left(z, m_h^2, \mu_F, \frac{1}{\varepsilon_{UV}}\right)$$

UV singularities are regularised in dimensional regularisation $n = 4 + \varepsilon_{UV}$

- Ultraviolet divergences are removed by renormalisation in $\overline{MS}$, at the Renormalisation scale μ_R

UV Scale dependence of partonic cross section

- Collinear finite partonic cross sections are calculable in perturbative QCD in powers of α_s^B, bare strong coupling constant.

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty}\left(\frac{\alpha_s^B}{4\pi}\right)^i d\hat{\sigma}_{ab}^{B,(i)}\left(z, m_h^2, \mu_F, \frac{1}{\varepsilon_{UV}}\right)$$

  UV singularities are regularised in dimensional regularisation $n = 4 + \varepsilon_{UV}$

- Ultraviolet divergences are removed by renormalisation in $\overline{MS}$, at the Renormalisation scale μ_R

- UV Renormalised partonic cross section:

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty}\left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}_{ab}^{(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

UV Scale dependence of partonic cross section

- Collinear finite partonic cross sections are calculable in perturbative QCD in powers of α_s^B, bare strong coupling constant.

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s^B}{4\pi}\right)^i d\hat{\sigma}_{ab}^{B,(i)}\left(z, m_h^2, \mu_F, \frac{1}{\varepsilon_{UV}}\right)$$

UV singularities are regularised in dimensional regularisation $n = 4 + \varepsilon_{UV}$

- Ultraviolet divergences are removed by renormalisation in $\overline{MS}$, at the Renormalisation scale μ_R

- UV Renormalised partonic cross section:

$$d\hat{\sigma}_{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}_{ab}^{(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

- $Soft$ divergences disappear thanks to KLM theorem

Summary of Theoretical Uncertainties

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

- Renormalisation scale due to UV divergences

$$\alpha_s \longrightarrow \alpha_s(\mu_R)$$

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

- Renormalisation scale due to UV divergences

$$\alpha_s \longrightarrow \alpha_s(\mu_R)$$

- Parton Distribution Functions extracted from experiments

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

- Renormalisation scale due to UV divergences

$$\alpha_s \longrightarrow \alpha_s(\mu_R)$$

- Parton Distribution Functions extracted from experiments
- Stability of perturbative result and missing higher order contributions.

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

- Renormalisation scale due to UV divergences

$$\alpha_s \longrightarrow \alpha_s(\mu_R)$$

- Parton Distribution Functions extracted from experiments
- Stability of perturbative result and missing higher order contributions.
- Any "Fixed order" perturbative result is bound to depend on μ_R and μ_F

Summary of Theoretical Uncertainties

- Factorisation scale due to light quarks and massless gluon

$$f_a(x) \longrightarrow f_a(x, \mu_F) \qquad a = q, \bar{q}, g$$

- Renormalisation scale due to UV divergences

$$\alpha_s \longrightarrow \alpha_s(\mu_R)$$

- Parton Distribution Functions extracted from experiments
- Stability of perturbative result and missing higher order contributions.
- Any "Fixed order" perturbative result is bound to depend on μ_R and μ_F
- Observables are "free" of μ_R and μ_F.

$$\mu \frac{d}{d\mu} \sigma^{P_1 P_2} = 0, \qquad \mu = \mu_F, \mu_R$$

Scale Variation of Flux at LHC

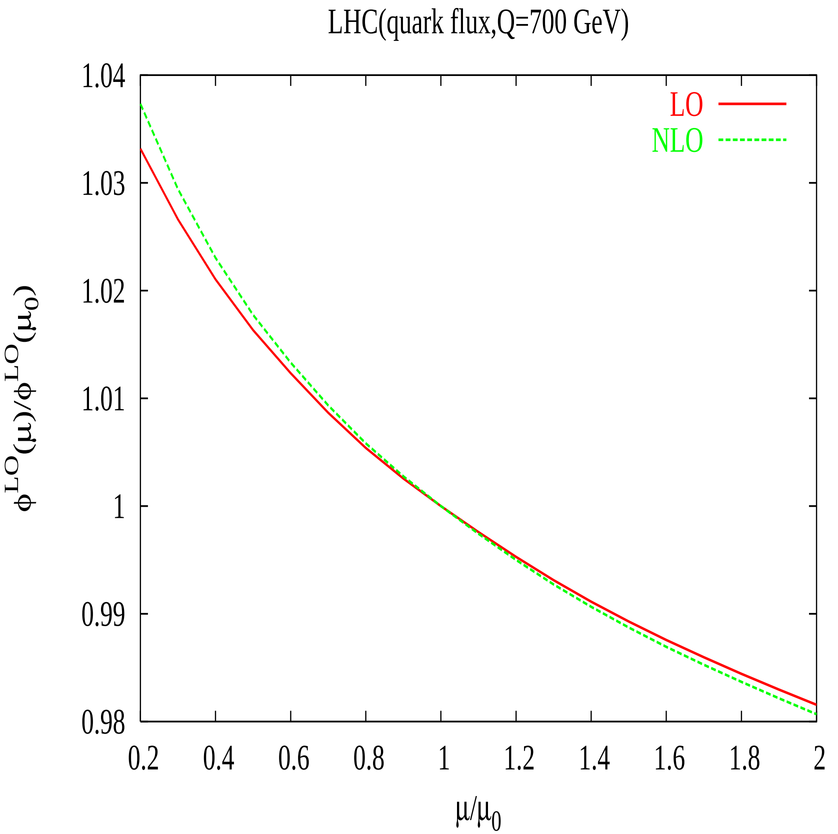
$$\Phi^{I}_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} \; f^I_a(z, \mu_F) \; f^I_b\left(\frac{x}{z}, \mu_F\right) \qquad I = LO, NLO$$

$$\mu_0 = 700 \text{ GeV}, \; x = \frac{Q}{\sqrt{S}}, \; Q = 700\text{GeV} \; \sqrt{S} = 14\, TeV$$

Scale Variation of Flux at LHC

$$\Phi^I_{ab}(x,\mu_F) \quad = \quad \int_x^1 \quad \frac{dz}{z} \quad f^I_a\,(z,\mu_F) \quad f^I_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad I = LO, NLO$$

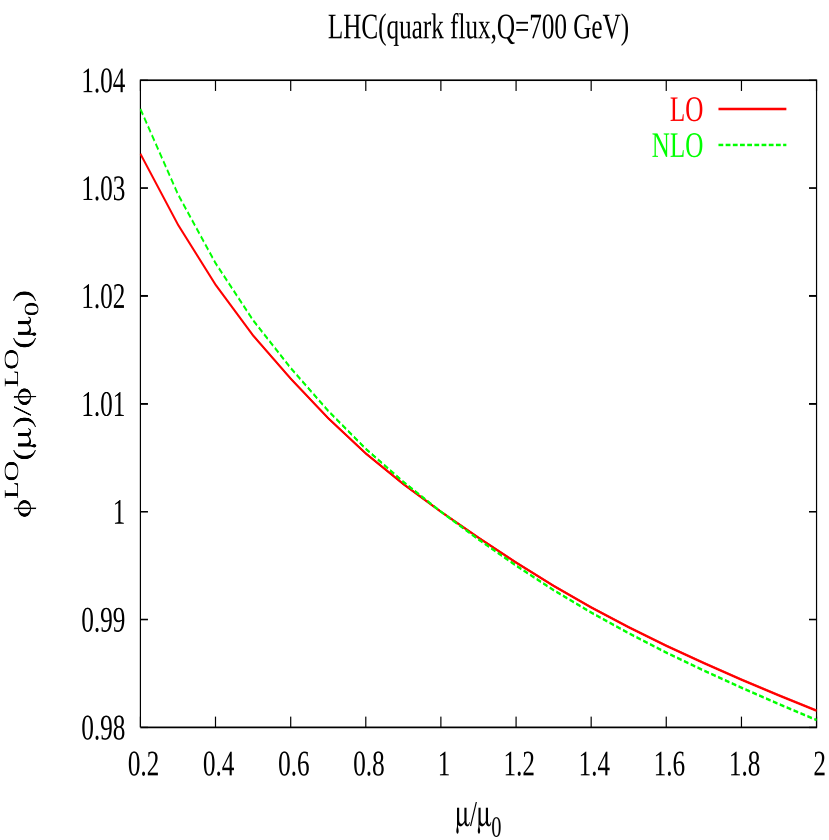
$$\mu_0 = 700\ \mathrm{GeV},\ \ x = \frac{Q}{\sqrt{S}},\ \ Q = 700\mathrm{GeV}\ \ \sqrt{S} = 14\,TeV$$

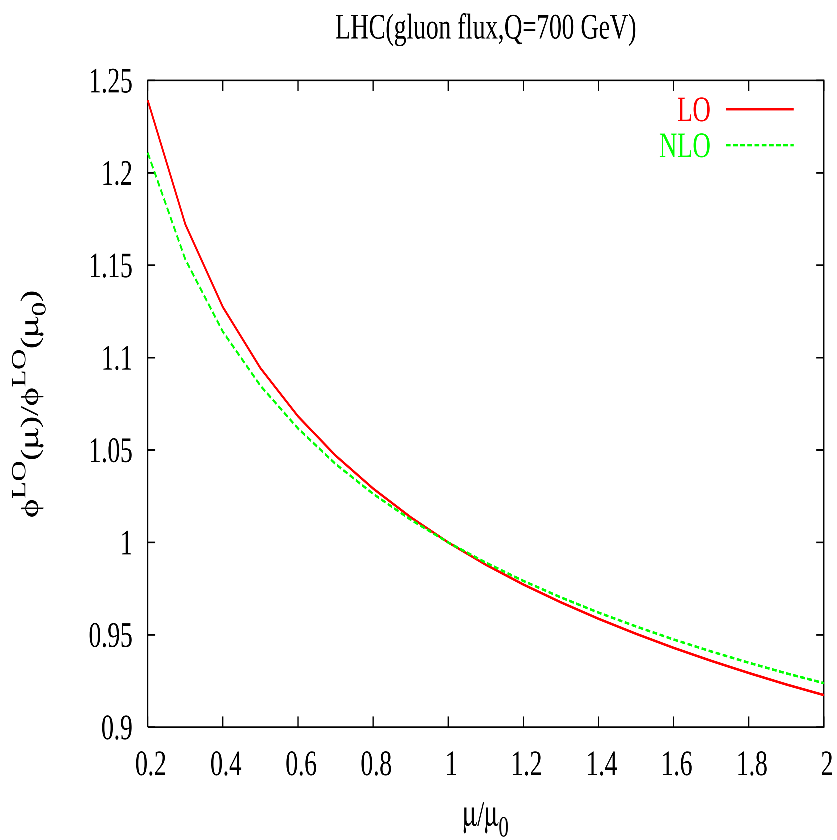


Scale Variation of Flux at LHC

$$\Phi^I_{ab}(x,\mu_F) = \int_x^1 \frac{dz}{z}\, f^I_a(z,\mu_F)\; f^I_b\left(\frac{x}{z},\mu_F\right) \qquad I = LO, NLO$$

$$\mu_0 = 700\ \mathrm{GeV},\ \ x = \frac{Q}{\sqrt{S}},\ \ Q = 700\mathrm{GeV}\ \ \sqrt{S} = 14\,TeV$$





Scale Variation of Flux at Tevatron

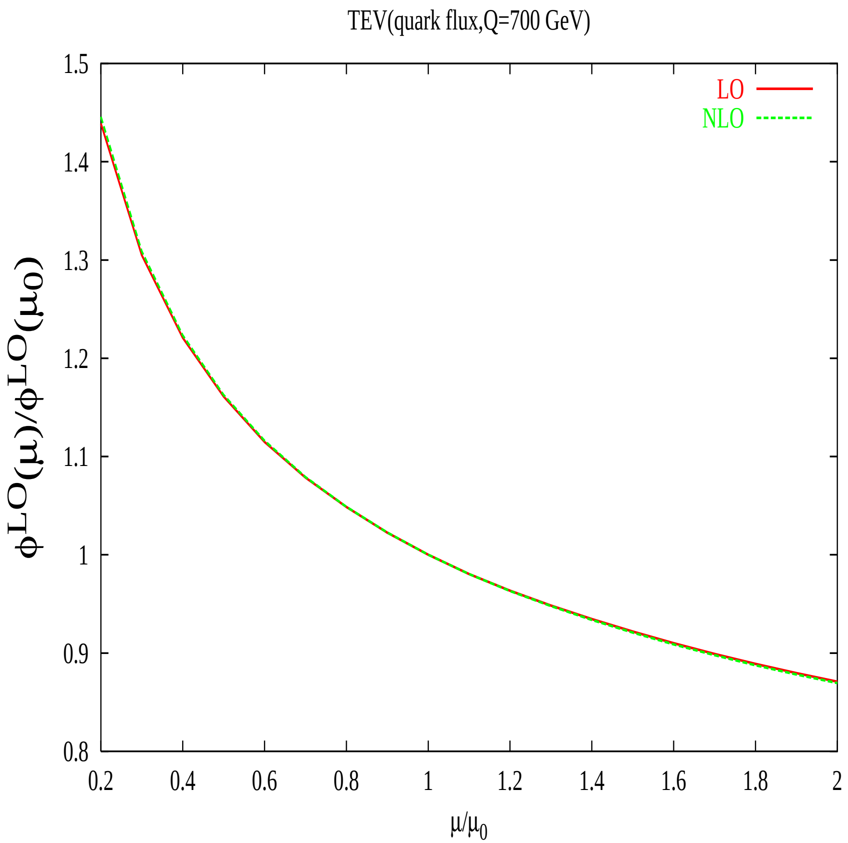
$$\Phi^I_{ab}(x, \mu_F) \quad = \quad \int_x^1 \quad \frac{dz}{z} \quad f^I_a(z, \mu_F) \quad f^I_b\left(\frac{x}{z}, \mu_F\right) \qquad\qquad I = LO, NLO$$

$$\mu_0 = 700 \text{ GeV}, \;\; x = \frac{Q_0}{\sqrt{S}}, \;\; Q = 700\text{GeV} \;\; (\sqrt{S} = 1.96 \; TeV$$

Scale Variation of Flux at Tevatron

$$\Phi^{I}_{ab}(x,\mu_F) \quad = \quad \int_x^1 \quad \frac{dz}{z} \quad f^{I}_{a}(z,\mu_F) \quad f^{I}_{b}\left(\frac{x}{z},\mu_F\right) \qquad I = LO, NLO$$

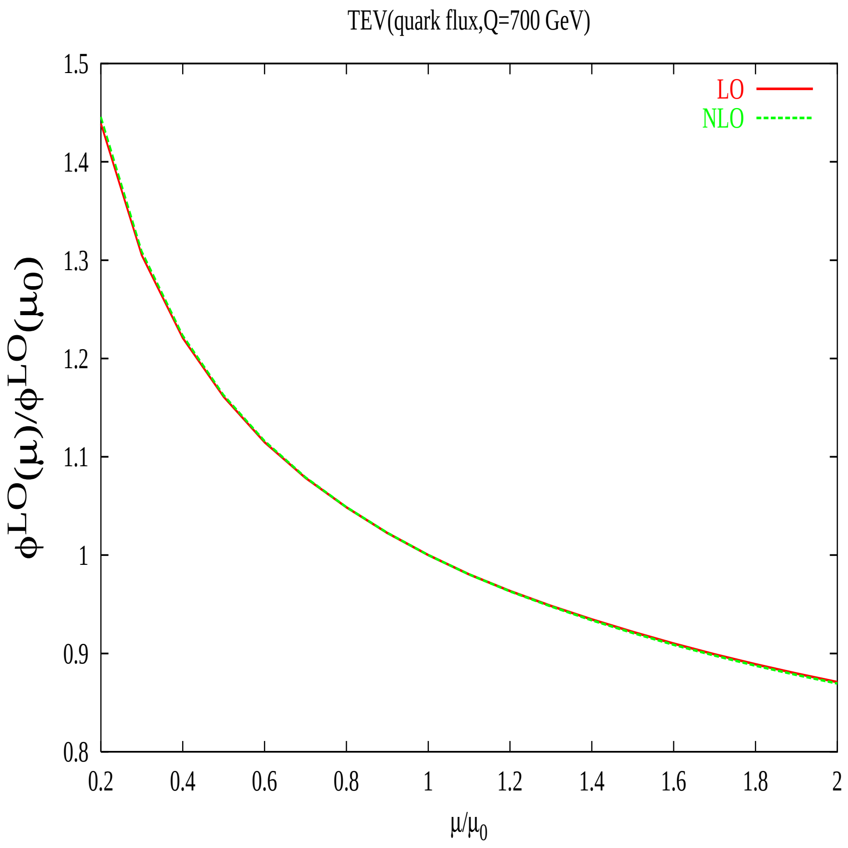
$$\mu_0 = 700 \text{ GeV}, \;\; x = \frac{Q_0}{\sqrt{S}}, \;\; Q = 700\text{GeV} \;\; (\sqrt{S} = 1.96\; TeV$$

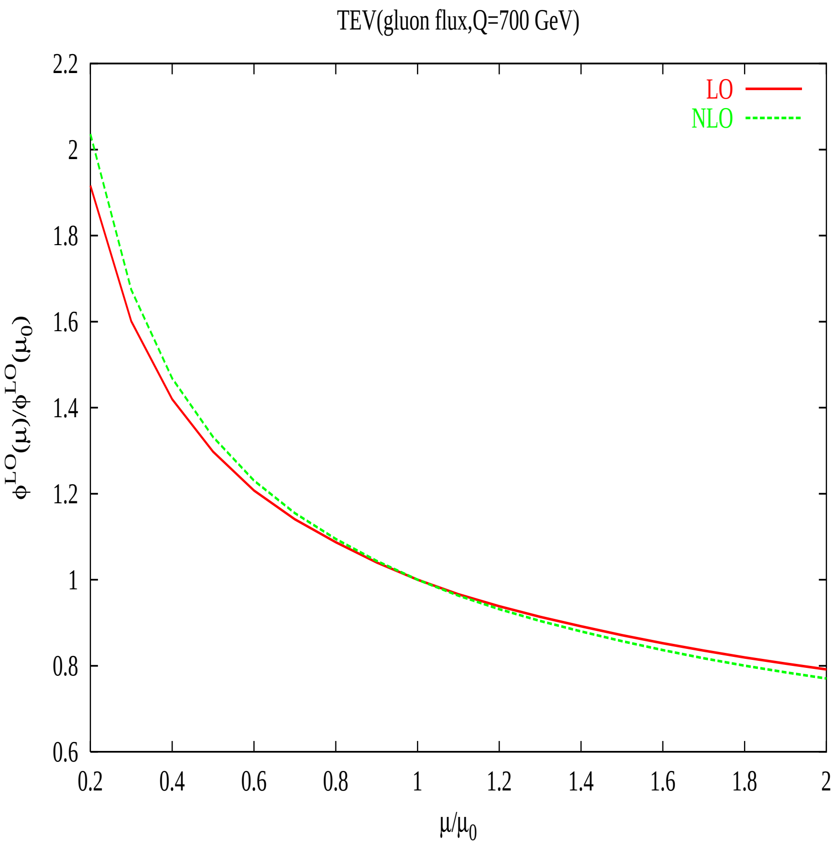


Scale Variation of Flux at Tevatron

$$\Phi^{I}_{ab}(x,\mu_F) = \int_x^1 \frac{dz}{z}\; f^I_a(z,\mu_F)\; f^I_b\left(\frac{x}{z},\mu_F\right) \qquad I = LO, NLO$$

$$\mu_0 = 700\ \mathrm{GeV},\ \ x = \frac{Q_0}{\sqrt{S}},\ \ Q = 700\mathrm{GeV}\ \ (\sqrt{S} = 1.96\ TeV$$





We are in Business!

We are in Business!

- Compute Next to leading order NLO QCD corrections to LO prcesses

We are in Business!

- Compute Next to leading order NLO QCD corrections to LO prcesses

$$d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{s}, Q^2, \mu_F^2)\left[1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\Delta_{ab}^{(1)}(\hat{s}, Q^2, \mu_F^2, \mu_R^2)\right]$$

QCD corrections are larger than other EW and gravity corrections.

Standard Model	Gravity
$q + \bar{q} \rightarrow \gamma/Z$ real emission one-loop	$q + \bar{q} \rightarrow G$ $g + g \rightarrow G$ real emission one-loop

We are in Business!

- Compute Next to leading order NLO QCD corrections to LO prcesses

$$d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{s}, Q^2, \mu_F^2)\left[1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\Delta_{ab}^{(1)}(\hat{s}, Q^2, \mu_F^2, \mu_R^2)\right]$$

QCD corrections are larger than other EW and gravity corrections.

Standard Model	Gravity
$q + \bar{q} \rightarrow \gamma/Z$ real emission one-loop	$q + \bar{q} \rightarrow G$ $g + g \rightarrow G$ real emission one-loop

- Energy momentum tensor is renormalised.

We are in Business!

- Compute Next to leading order NLO QCD corrections to LO prcesses

$$d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{s}, Q^2, \mu_F^2)\left[1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\Delta_{ab}^{(1)}(\hat{s}, Q^2, \mu_F^2, \mu_R^2)\right]$$

QCD corrections are larger than other EW and gravity corrections.

Standard Model	Gravity
$q + \bar{q} \to \gamma/Z$ real emission one-loop	$q + \bar{q} \to G$ $g + g \to G$ real emission one-loop

- Energy momentum tensor is renormalised.

- All the soft and collinear divergences are regulated in dimensional regularisation $n = 4 + \varepsilon$.

We are in Business!

- Compute Next to leading order NLO QCD corrections to LO prcesses

$$d\hat{\sigma}_{ab}(\hat{s}, Q^2, \mu_F^2) = d\hat{\sigma}_{ab}^{(0)}(\hat{s}, Q^2, \mu_F^2)\left[1 + \frac{\alpha_s(\mu_R^2)}{4\pi}\Delta_{ab}^{(1)}(\hat{s}, Q^2, \mu_F^2, \mu_R^2)\right]$$

QCD corrections are larger than other EW and gravity corrections.

Standard Model	Gravity
$q + \bar{q} \rightarrow \gamma/Z$ real emission one-loop	$q + \bar{q} \rightarrow G$ $g + g \rightarrow G$ real emission one-loop

- Energy momentum tensor is renormalised.

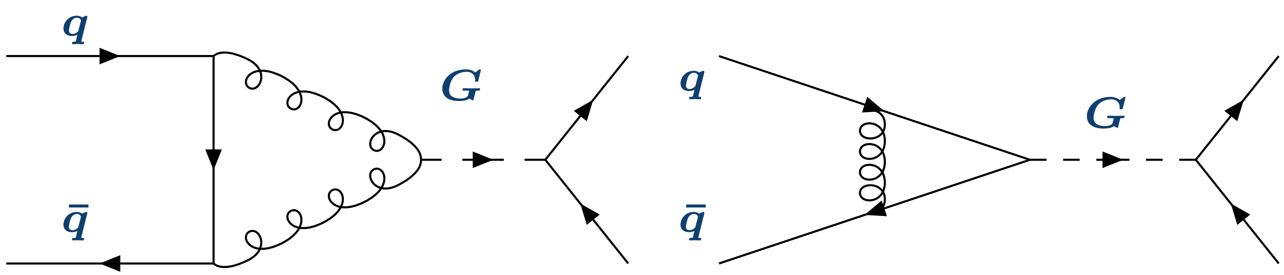
- All the soft and collinear divergences are regulated in dimensional regularisation $n = 4 + \varepsilon$.

- Collinear mass factorisation is done in $\overline{MS}$ scheme.

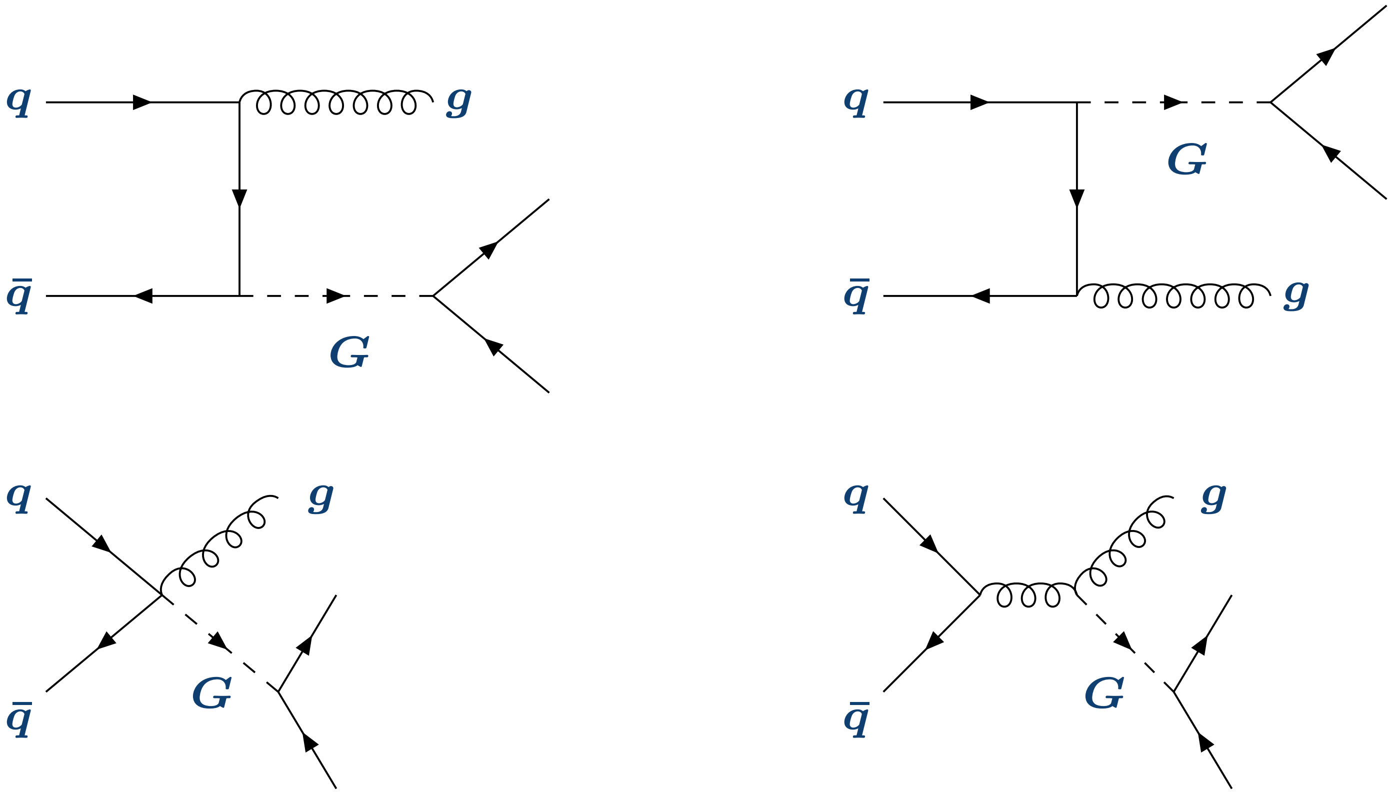
Virtual Corrections, $q\,\bar{q} \longrightarrow\ G$

$$\bar{\Delta}^{G}_{q\bar{q}} = \Delta^{(0)G}_{q\bar{q}} + a_s \frac{2}{\varepsilon} \Gamma^{(1)}_{qq} \otimes \Delta^{(0)G}_{q\bar{q}} + a_s \Delta^{(1)G}_{q\bar{q}}$$

$q + \bar{q} \longrightarrow G$ (1 loop):



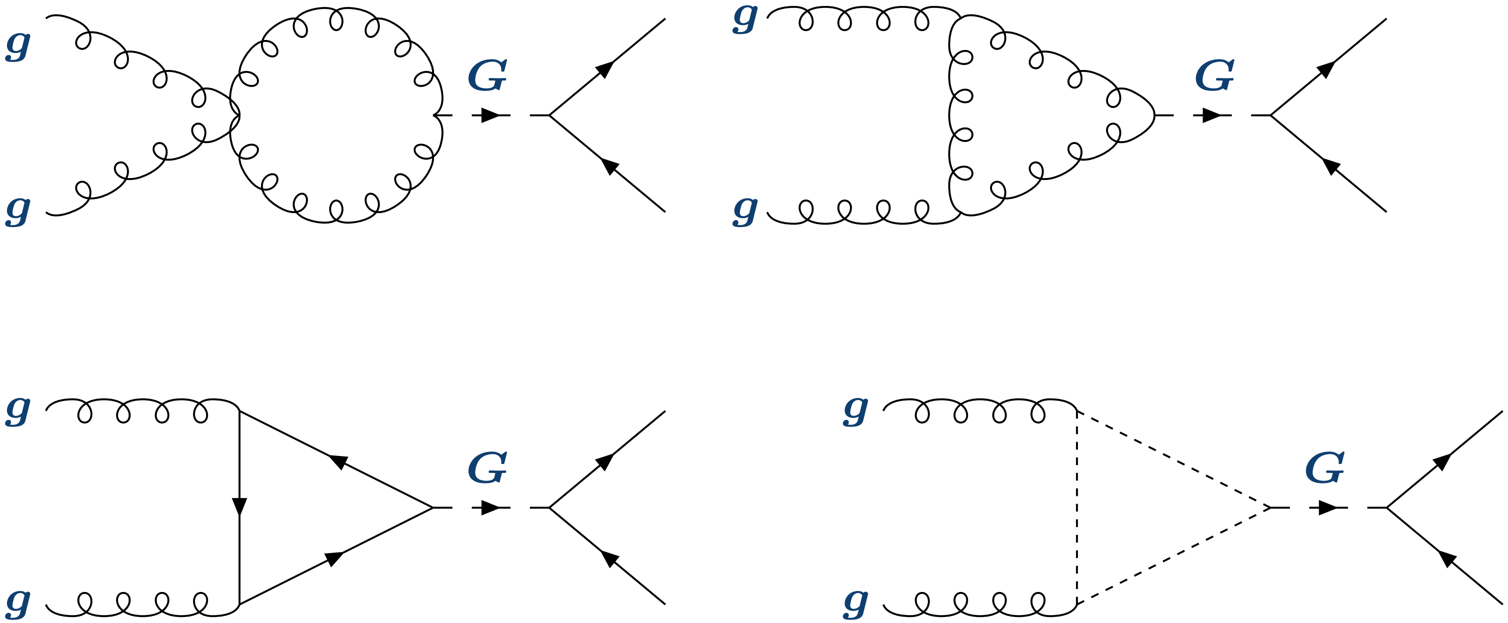
Real emission, $q\,\bar{q} \rightarrow g\,G$



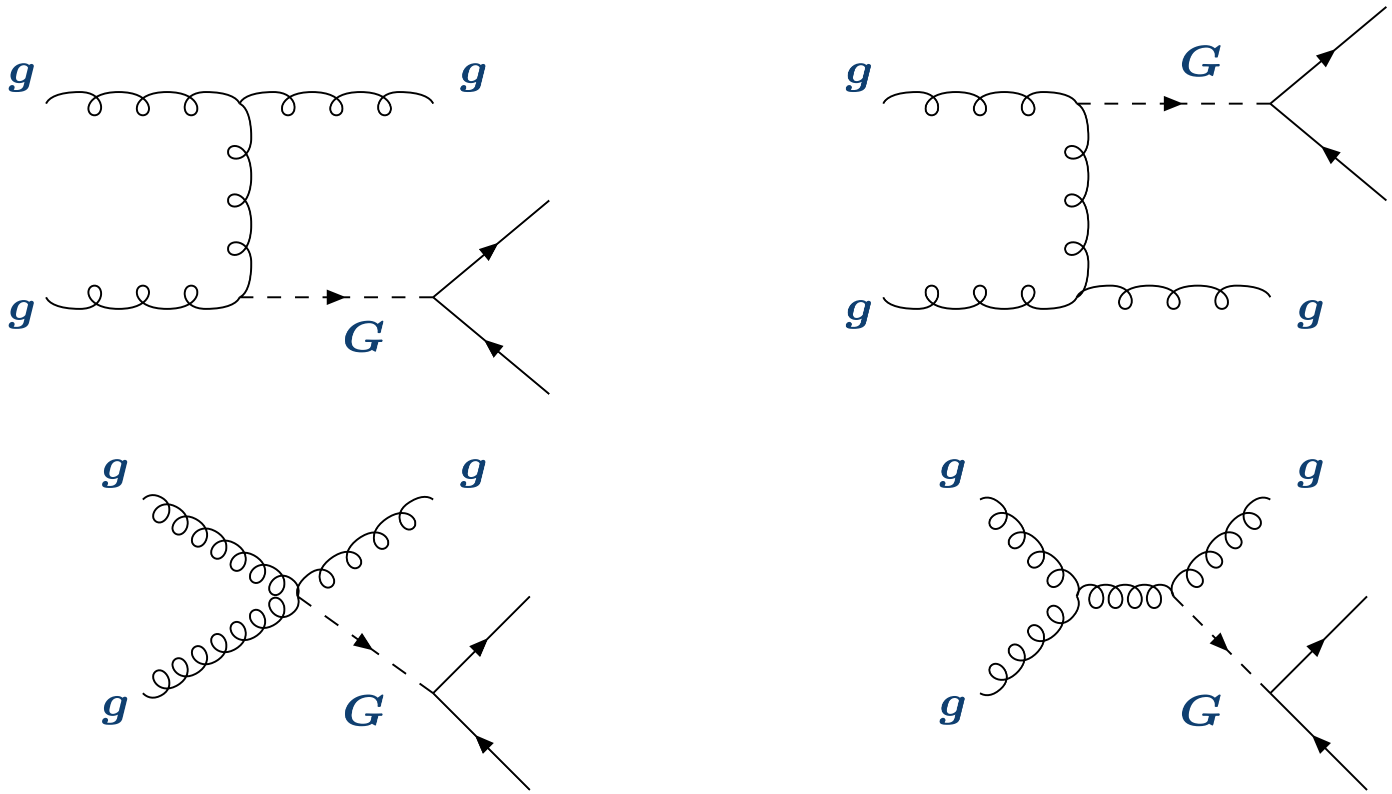
Virtual Corrections, $g\,\bar{g} \longrightarrow\ G$

$$\bar{\Delta}^{G}_{gg} = \Delta^{(0)G}_{gg} + a_s \frac{2}{\varepsilon} \Gamma^{(1)}_{gg} \otimes \Delta^{(0)G}_{gg} + a_s \Delta^{(1)G}_{gg}$$

$g + g \longrightarrow G$ (1 loop):



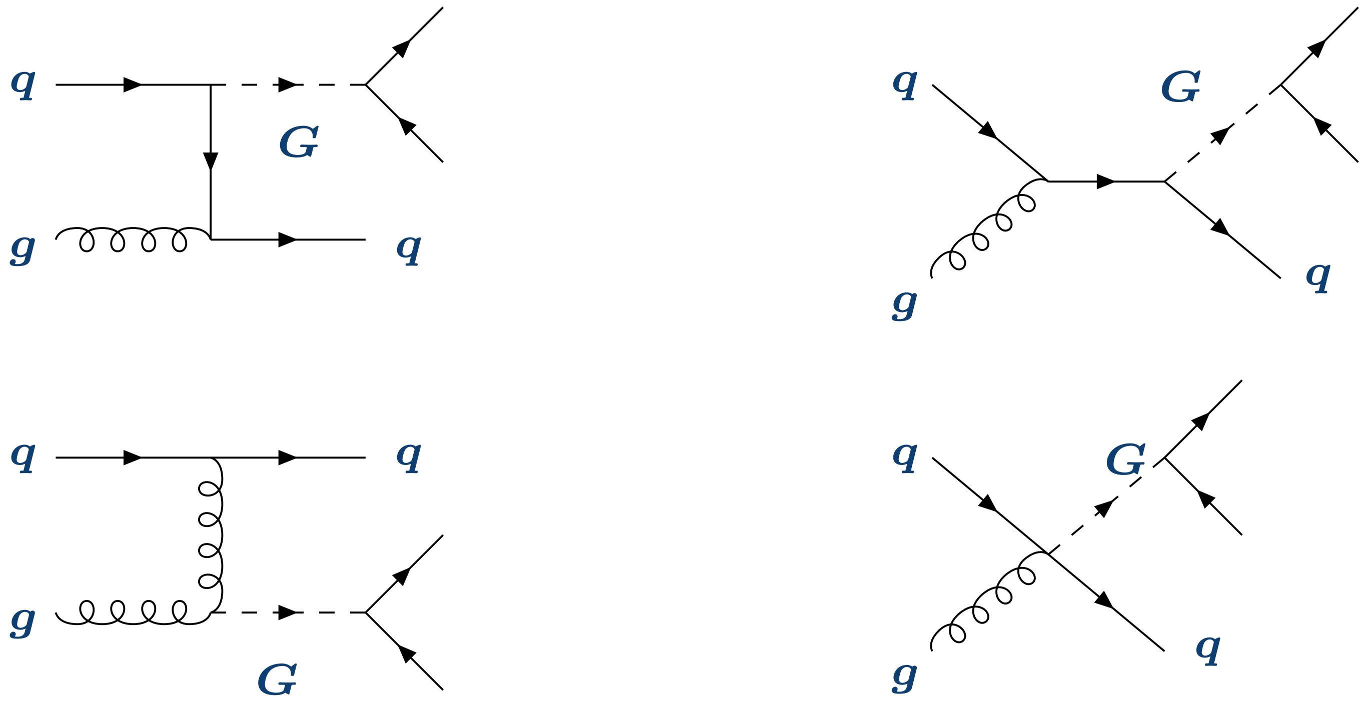
Real emission, $g\,g \rightarrow g\,G$



Real emissions, $q\,\bar{g} \longrightarrow q\,G$

$$\bar{\Delta}^{G}_{qg} = a_s \frac{1}{\varepsilon}\left(\Gamma^{(1)}_{qg} \otimes \Delta^{(0)G}_{q\bar{q}} + \Gamma^{(1)}_{gq} \otimes \Delta^{(0)G}_{gg}\right) + a_s \Delta^{(1)G}_{qg}$$

Real emission, $q\,g \longrightarrow q\,G$

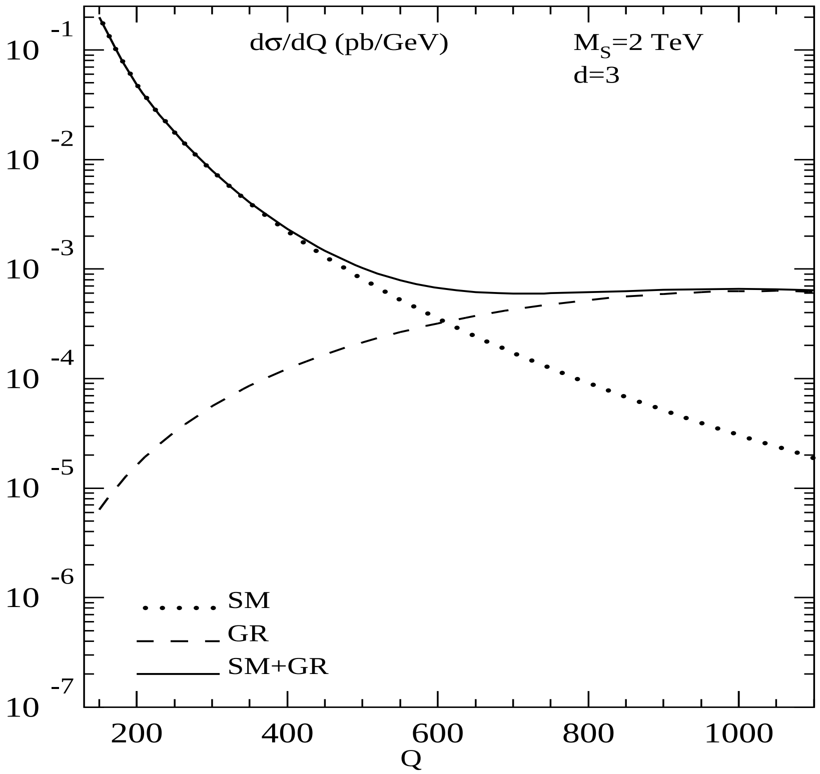


Invariant lepton pair mass Q distributions:

$$\frac{d\sigma^I(Q)}{dQ}$$

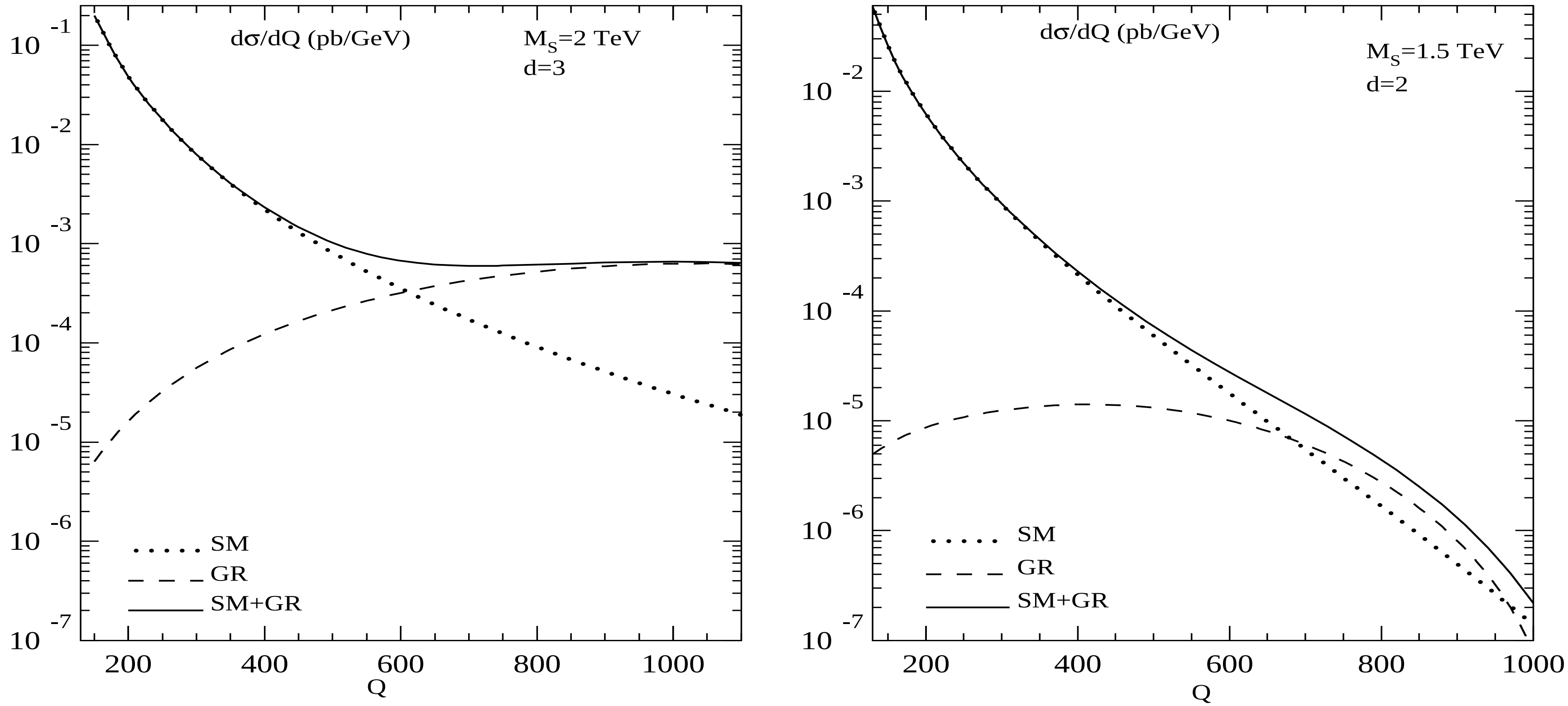
Invariant lepton pair mass Q distributions:

$$\frac{d\sigma^I(Q)}{dQ}$$



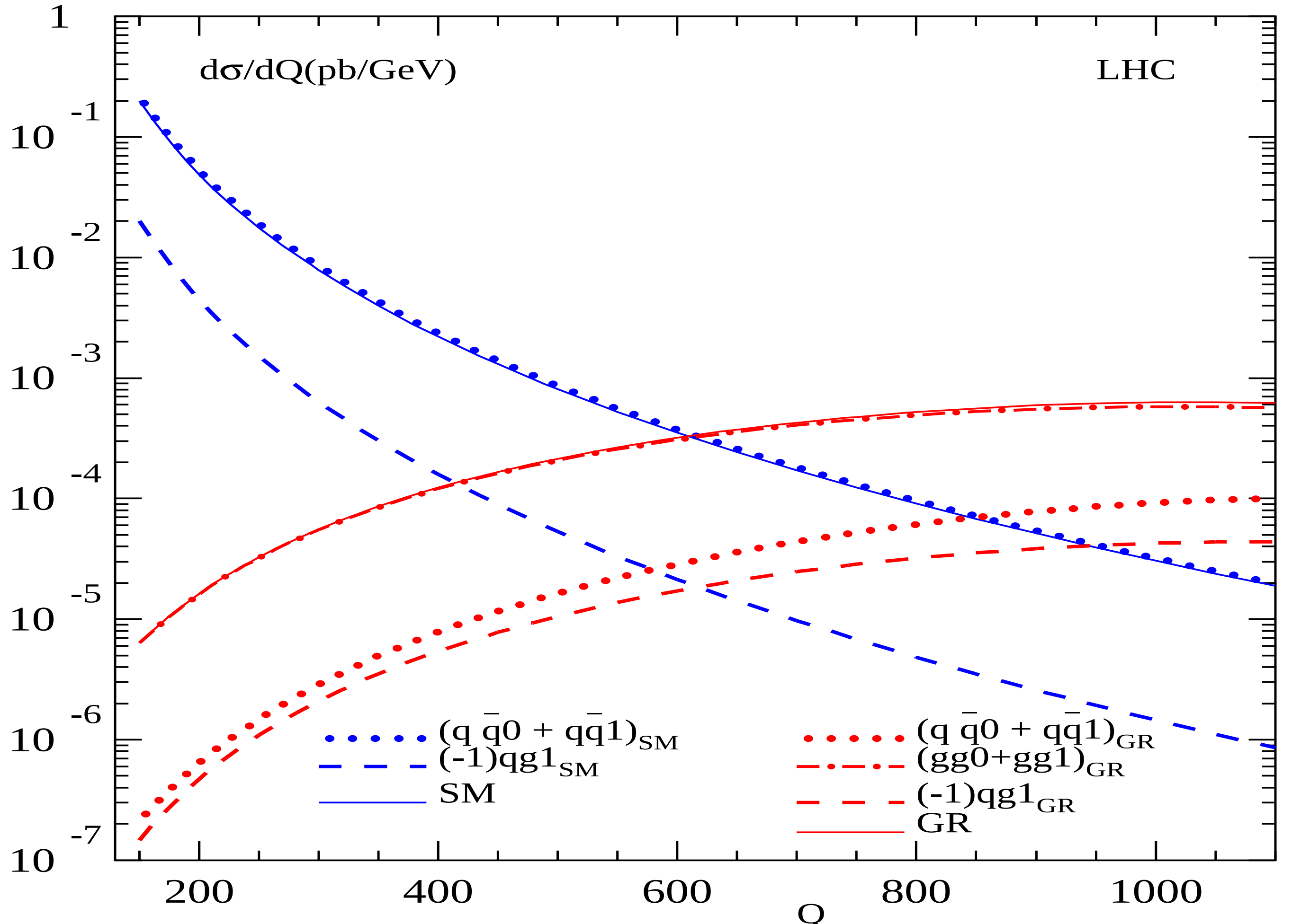
Invariant lepton pair mass Q distributions:

$$\frac{d\sigma^I(Q)}{dQ}$$



- LHC: SM dominates for $Q < 600$ GeV but for $Q > 600$ GeV the gravity mediated processes dominates
- TEV: for $Q > 700$ GeV the gravity mediated process becomes larger

Contributions at LHC

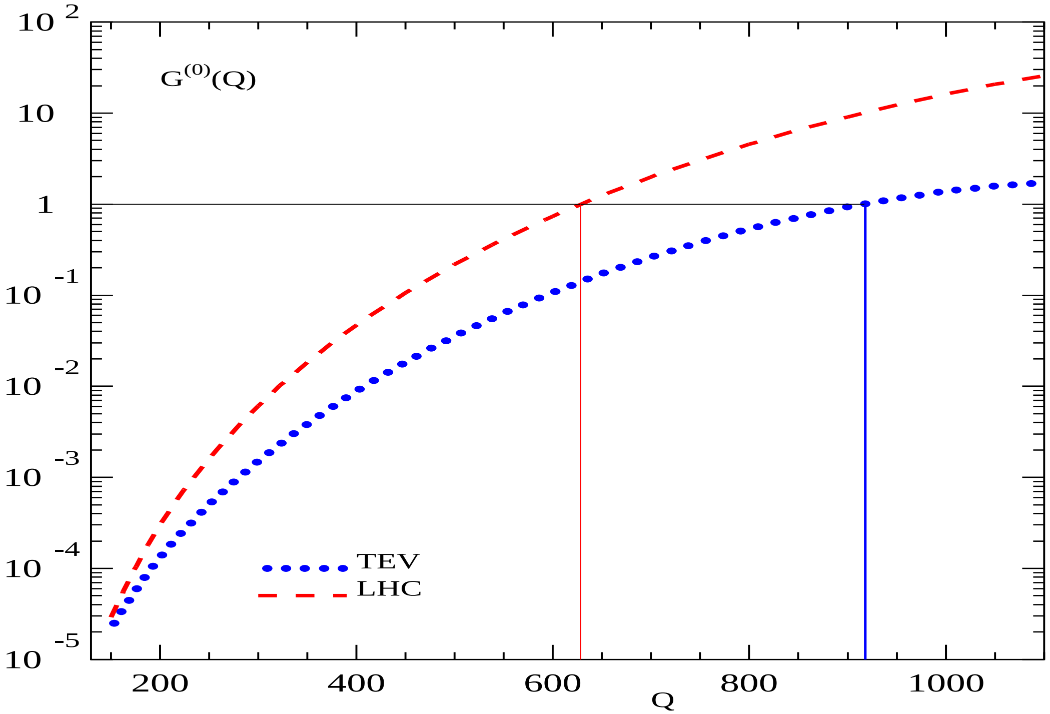


- SM the $q\bar{q}$ subprocess dominates (no gluon initiated process)
- Gravity mediated process gg sub process initiated process dominates and substantially contributes to the cross section at large Q^2

K-Factor

$$K^{(SM+GR)}(Q) = \frac{K^{SM} + K^{GR}\, G^{(0)}}{1 + G^{(0)}}$$

$$G^{(0)}(Q) = \left[\frac{d\sigma^{SM}_{LO}(Q)}{dQ}\right]^{-1} \left[\frac{d\sigma^{GR}_{LO}(Q)}{dQ}\right]$$



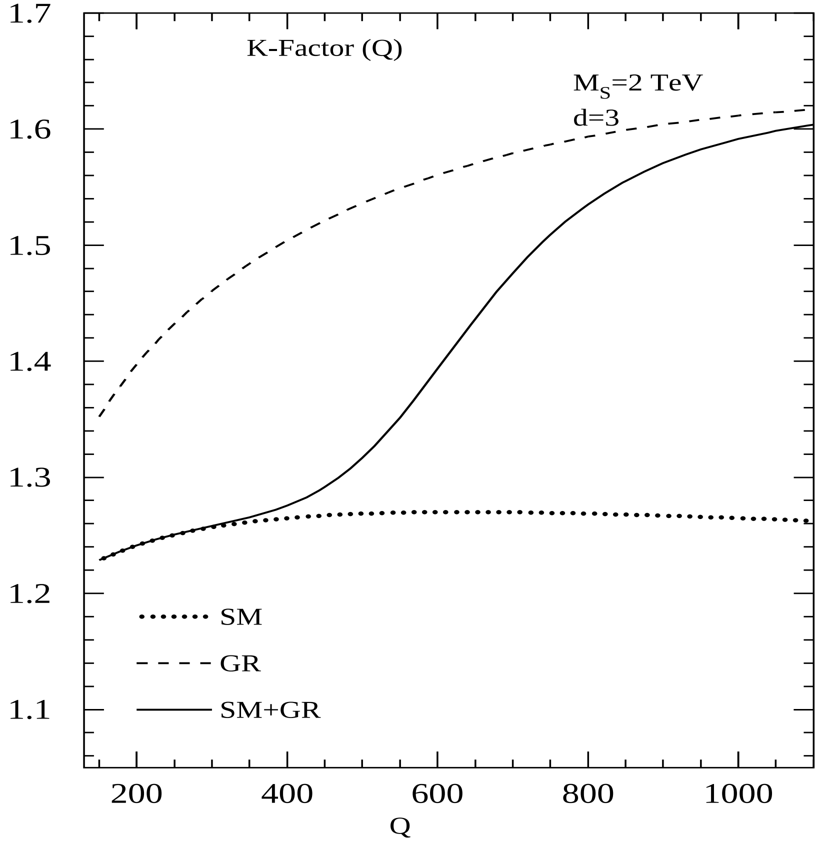
- $G^{(0)}(Q)$ behavior is governed by a competing 'couplings' and PDF flux at LHC and TEV
- At high Q when Gravity contribution becomes comparable to SM, the PDF flux dictates the proceedings

K-Factor:

$$K^{I} = \left[\frac{d\sigma^{I}_{LO}(Q)}{dQ}\right]^{-1}\left[\frac{d\sigma^{I}_{NLO}(Q)}{dQ}\right]$$

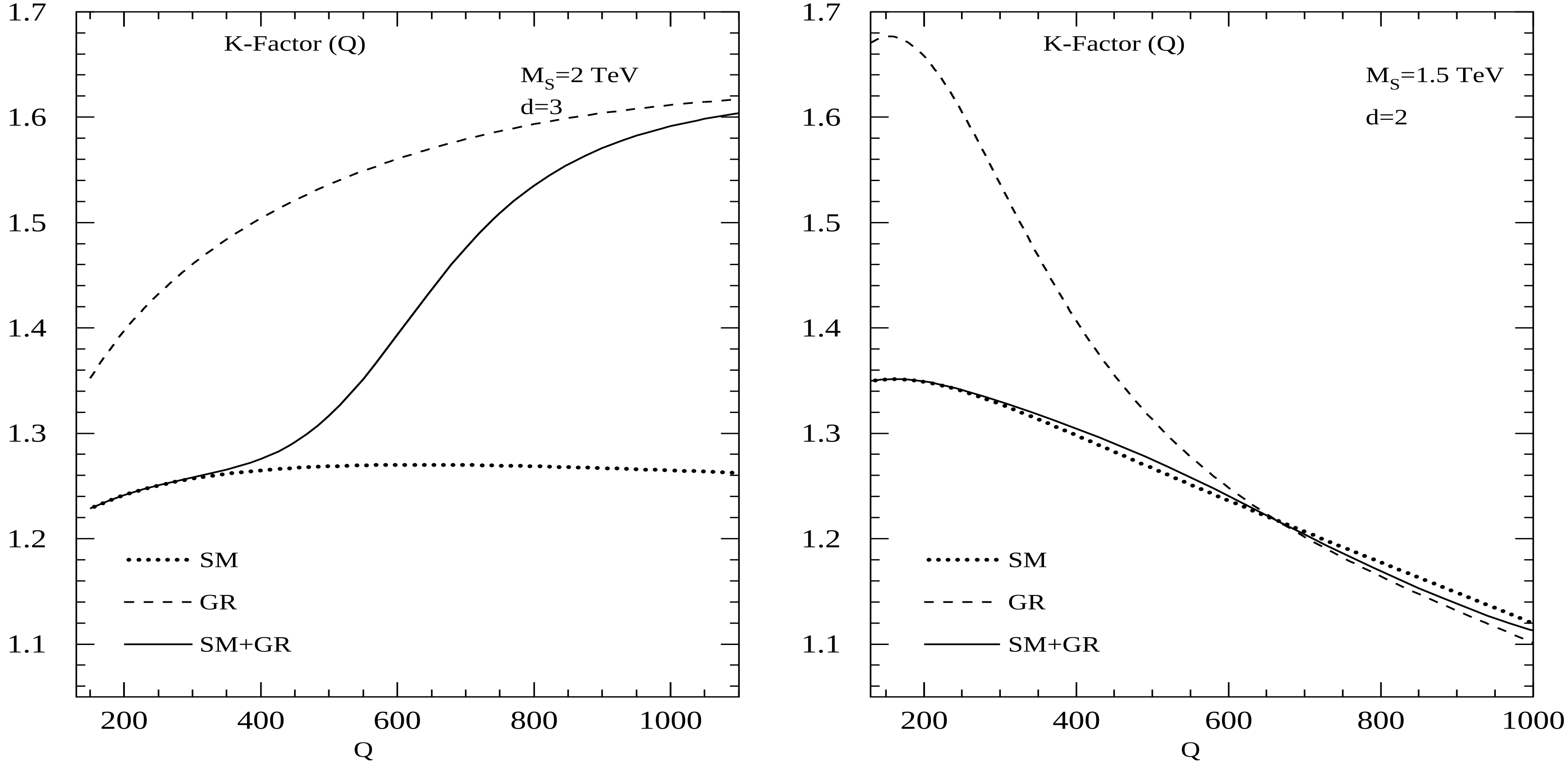
K-Factor:

$$K^I = \left[\frac{d\sigma^I_{LO}(Q)}{dQ}\right]^{-1}\left[\frac{d\sigma^I_{NLO}(Q)}{dQ}\right]$$



K-Factor:

$$K^I = \left[\frac{d\sigma^I_{LO}(Q)}{dQ}\right]^{-1}\left[\frac{d\sigma^I_{NLO}(Q)}{dQ}\right]$$



- LHC: K^{SM} is moderate for all values of Q while K^{GR} is much larger then K^{SM} at large Q. $Q > 700$ GeV, K^{GR} dominates the K^{SM+GR}. gg sub process contribute at LO itself via Gravity. NLO large effects due to small x terms in $\Delta^{(1)G}_{gg}$
- TEV: K^{SM} and K^{SM+GR} are not very different

R-Factor:

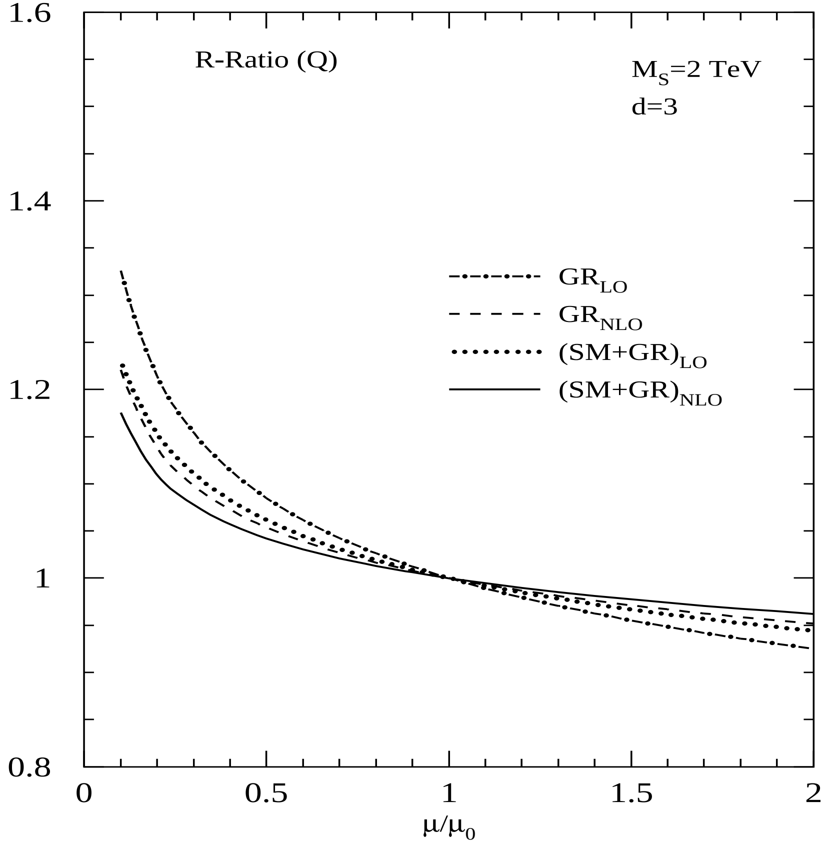
$$R^{I}_{LO,NLO} \quad = \quad \left[\frac{d\sigma^{I}_{LO,NLO}(Q, \mu = \mu_0)}{dQ}\right]^{-1} \left[\frac{d\sigma^{I}_{LO,NLO}(Q, \mu)}{dQ}\right]\Bigg|_{Q=Q_0}$$

$$\mu_0 = Q \qquad Q_0 = 700 \text{ GeV (LHC)} \qquad Q_0 = 400 \text{ GeV (TEV)}$$

R-Factor:

$$R^I_{LO,NLO} = \left[\frac{d\sigma^I_{LO,NLO}(Q,\mu=\mu_0)}{dQ}\right]^{-1}\left[\frac{d\sigma^I_{LO,NLO}(Q,\mu)}{dQ}\right]\Bigg|_{Q=Q_0}$$

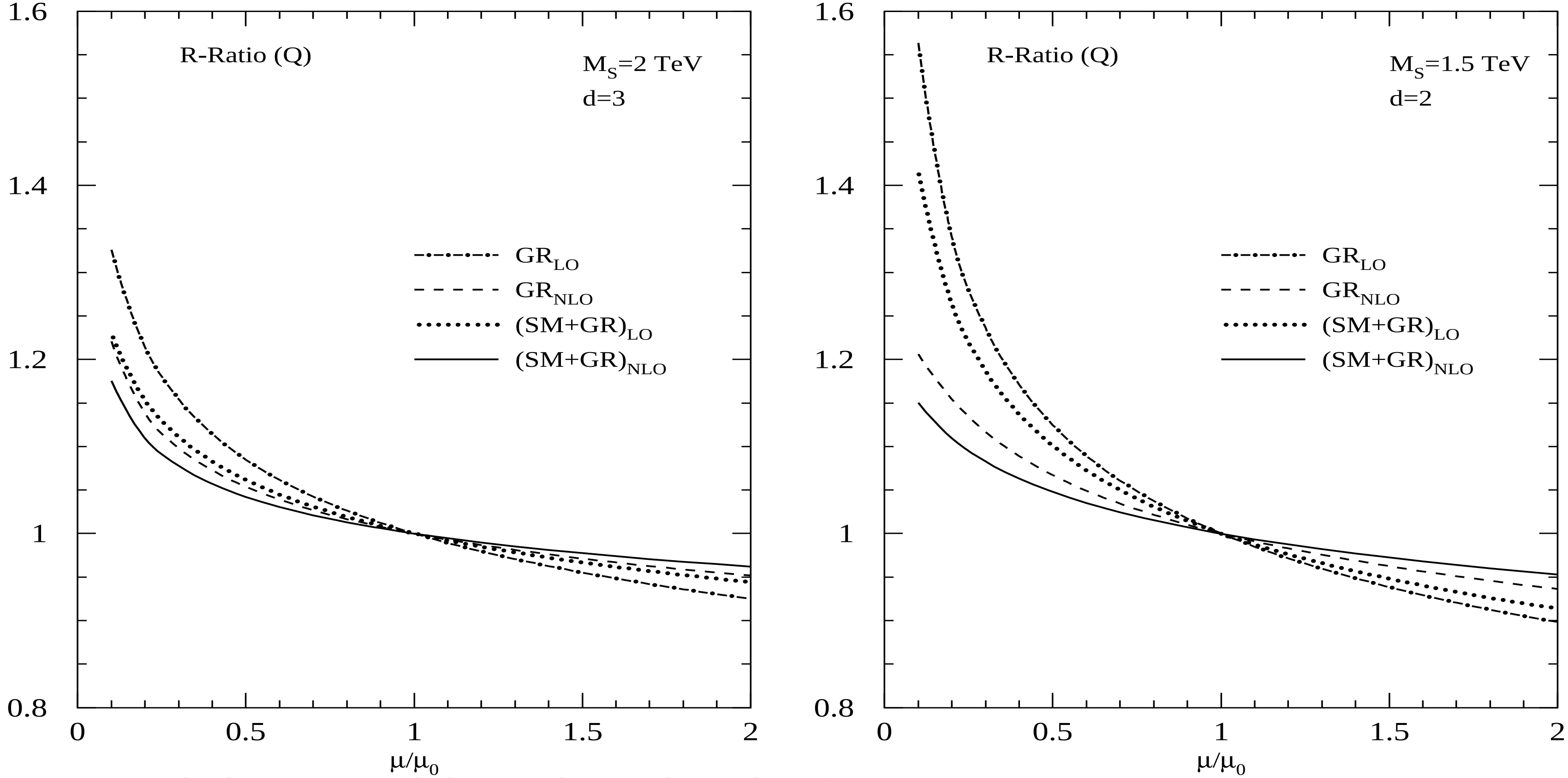
$$\mu_0 = Q \qquad Q_0 = 700 \text{ GeV (LHC)} \qquad Q_0 = 400 \text{ GeV (TEV)}$$



R-Factor:

$$R^I_{LO,NLO} = \left[\frac{d\sigma^I_{LO,NLO}(Q,\mu=\mu_0)}{dQ}\right]^{-1}\left[\frac{d\sigma^I_{LO,NLO}(Q,\mu)}{dQ}\right]\Bigg|_{Q=Q_0}$$

$$\mu_0 = Q \qquad Q_0 = 700\ \mathrm{GeV\ (LHC)} \qquad Q_0 = 400\ \mathrm{GeV\ (TEV)}$$



- Scale variation appreciably reduces in going from LO to NLO
- Inclusion of SM to GR also reduces scale variation

RS Scenario Results

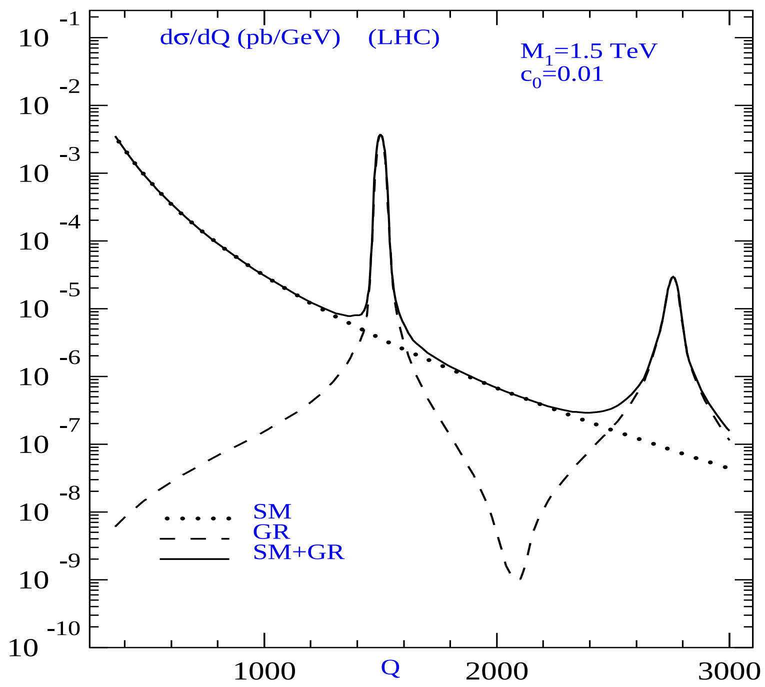
$$\mathcal{D}(Q^2) = \sum_{n=1}^{\infty} \frac{1}{Q^2 - M_n^2 + iM_n\Gamma_n} \equiv \frac{\lambda}{m_0^2}$$

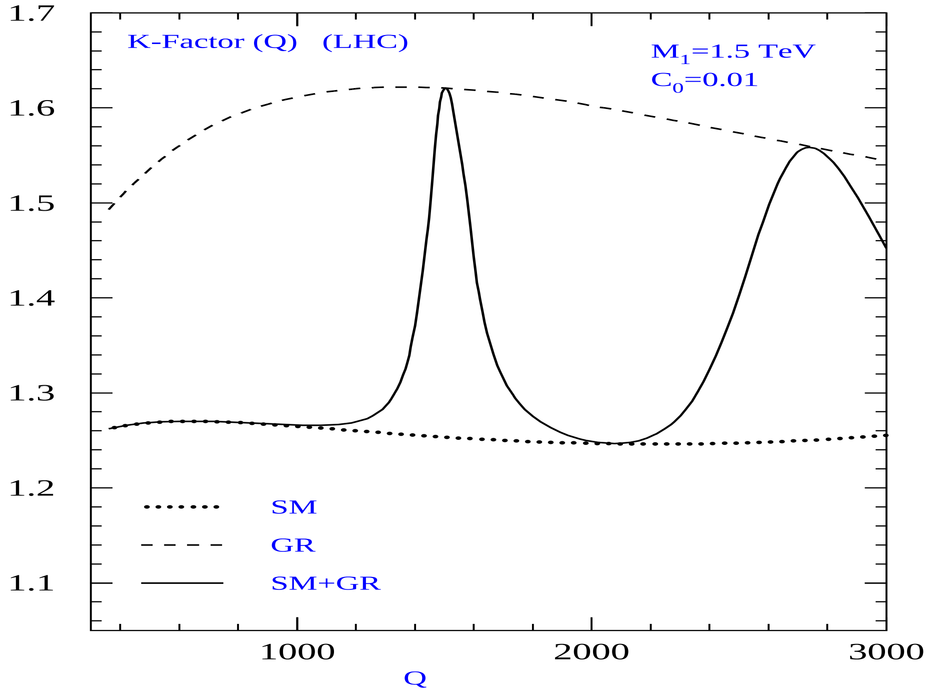
$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) = \frac{c_0^2}{m_0^4}\,\lambda$$

RS Scenario Results

$$\mathcal{D}(Q^2) = \sum_{n=1}^{\infty} \frac{1}{Q^2 - M_n^2 + iM_n\Gamma_n} \equiv \frac{\lambda}{m_0^2}$$

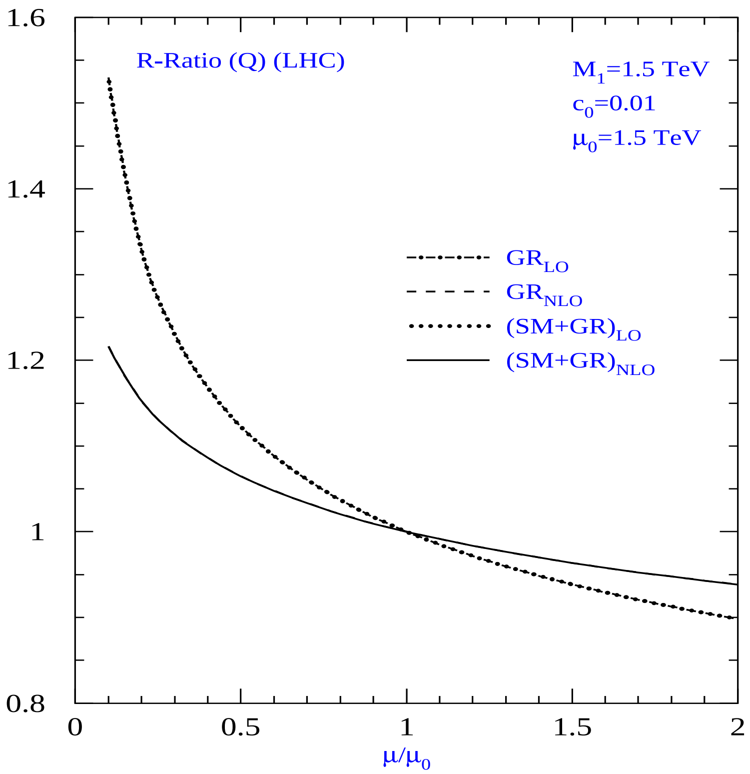
$$\frac{c_0^2}{m_0^2}\mathcal{D}(Q^2) = \frac{c_0^2}{m_0^4}\lambda$$

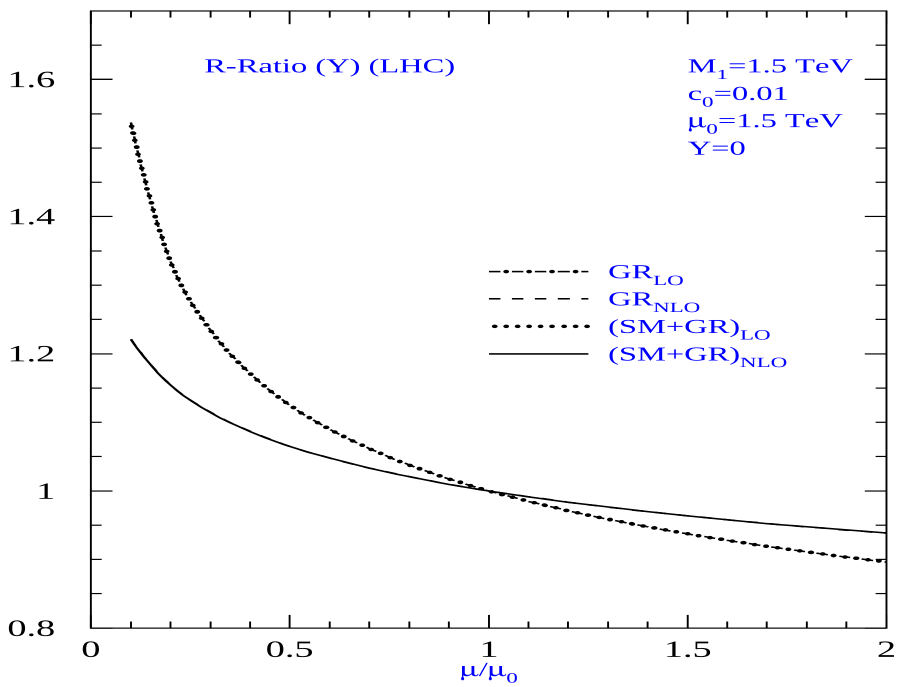




- Away from the resonance region gravity contribution is negligible
- K-Factor behavior can be understood from the $K^{(0)}$ behavior for the RS model.

R-Factor:





- Scale variation reduced considerably in going from LO $\longrightarrow$ NLO
- Inclusion of SM to GR also reduces scale variation

Summary

- Next to Leading Order coefficient functions for DY process in models of TeV-scale gravity are available now.
- Various distributions *viz.* $\boldsymbol{Q}$, $\boldsymbol{x_F}$, $\boldsymbol{Y}$ distributions and $\boldsymbol{A_{FB}}$ asymmetry at NLO are studied for ADD & RS models.
- Theoretical uncertainties get significantly reduced at NLO level
- Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders investigated