Automated resummation and hadron collider event shapes

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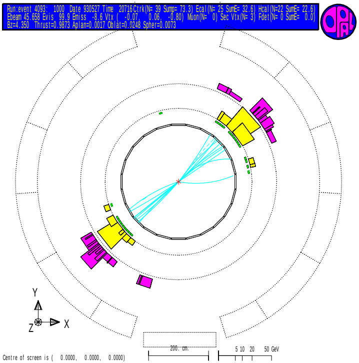
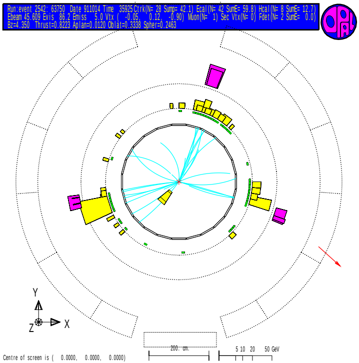
RADCOR
Shonan Village, Japan, October 2005

Final states

A wealth of information about QCD lies in its final states. Problem is how to extract it.





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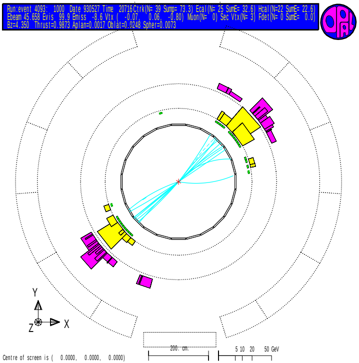

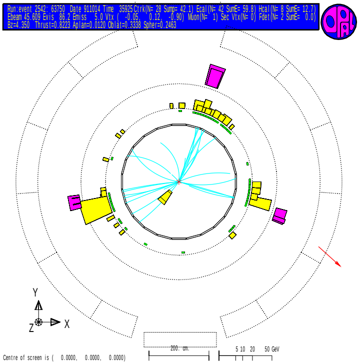

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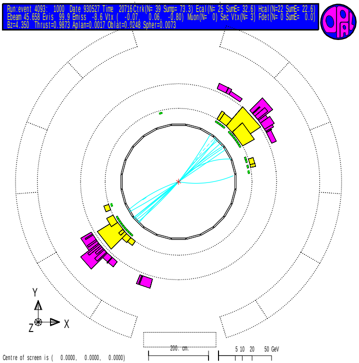

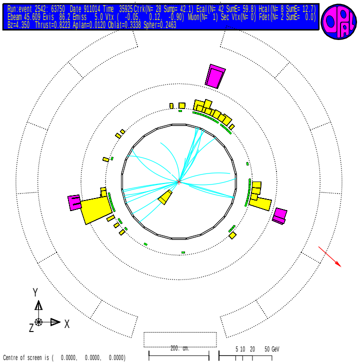

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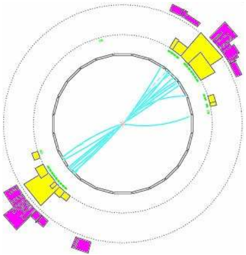
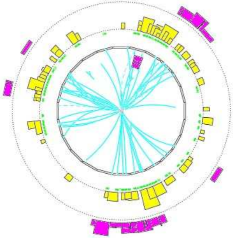
Event Shapes

First discussion goes back to 1964. Serious work got going in late '70s. Various proposals to measure *shape* of events. Most famous example is Thrust:

$$T = \max_{\vec{n}_T} \frac{\sum_i |\vec{p}_i.\vec{n}_T|}{\sum_i |\vec{p}_i|},$$



2-jet event: $T \simeq 1$



3-jet event: $T \simeq 2/3$

There exist many other measures of aspects of the shape: Thrust-Major, C-parameter, broadening, heavy-jet mass, jet-resolution parameters,. . .

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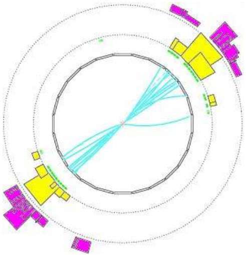

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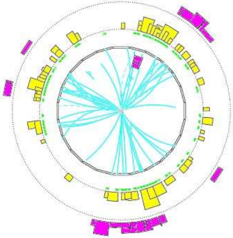

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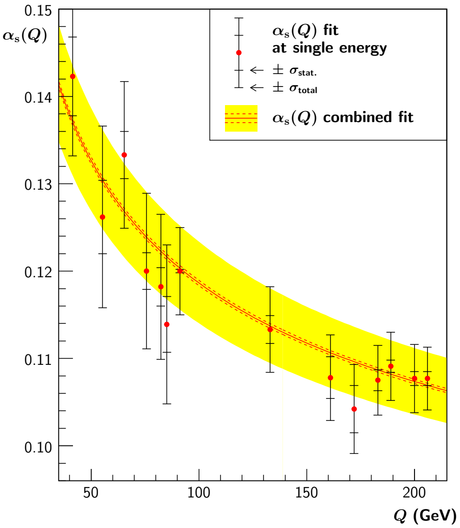
Event shapes: high information content



Much learnt from event-shapes in e^+e^- and DIS:

- α_s fits
- Tuning of Monte Carlos
- Colour factor fits $(C_A, C_F, \ldots)$
- Studies of analytical hadronisation models ($1/Q$, shape functions, ...)

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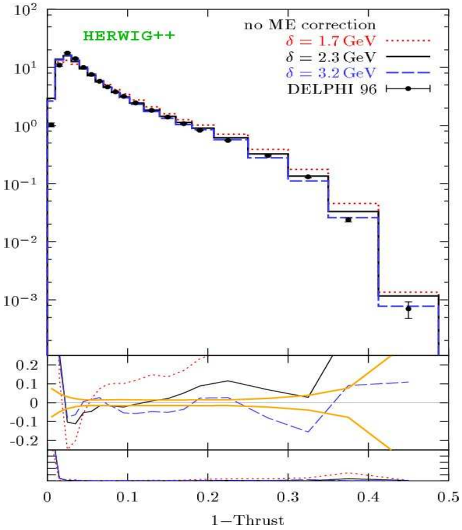

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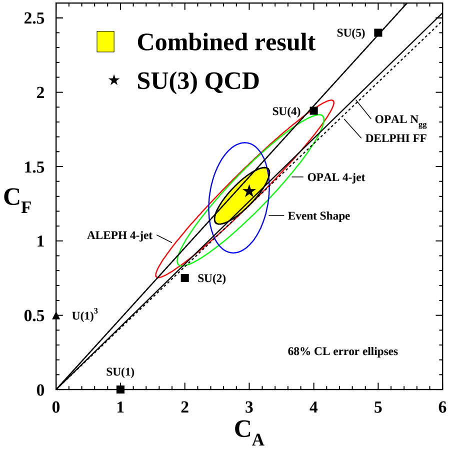

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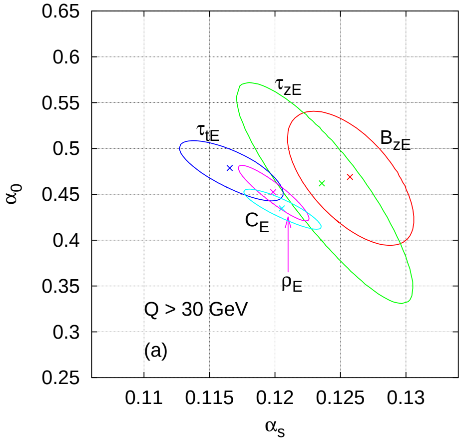

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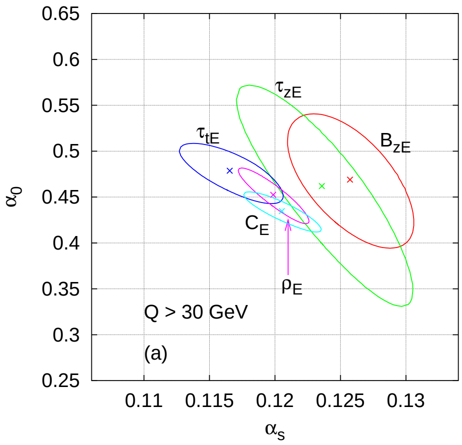

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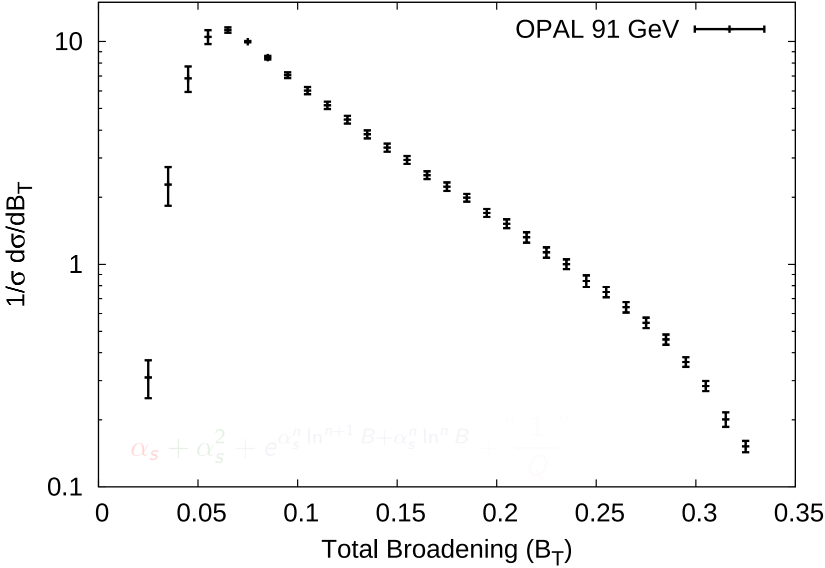
Neglected at hadron colliders despite (measurements: CDF Broad, D0 Thr)

- Rich structure of multi-jet events [e.g. Stony Brook soft colour logs]
- big source of gluon jets [*e.g.* for hadronisation studies]
- potential for studying underlying event

Inputs to event-shape distribution?



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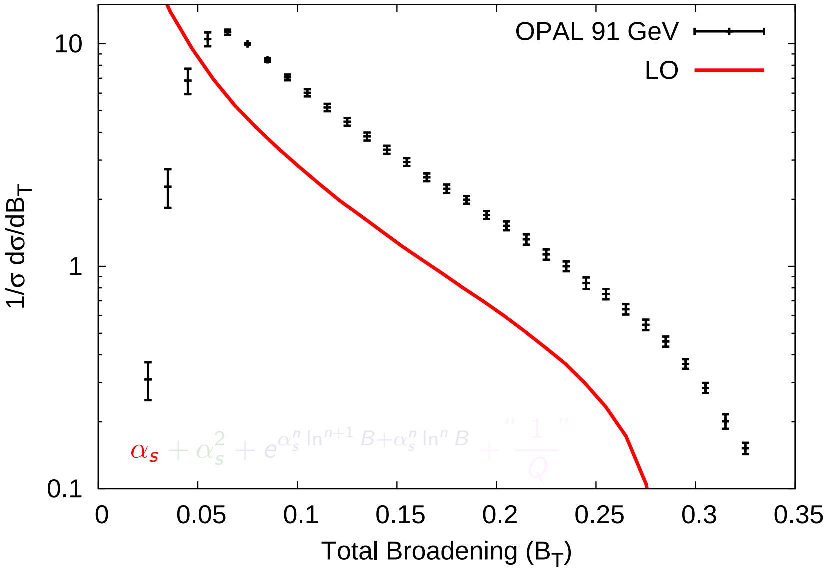

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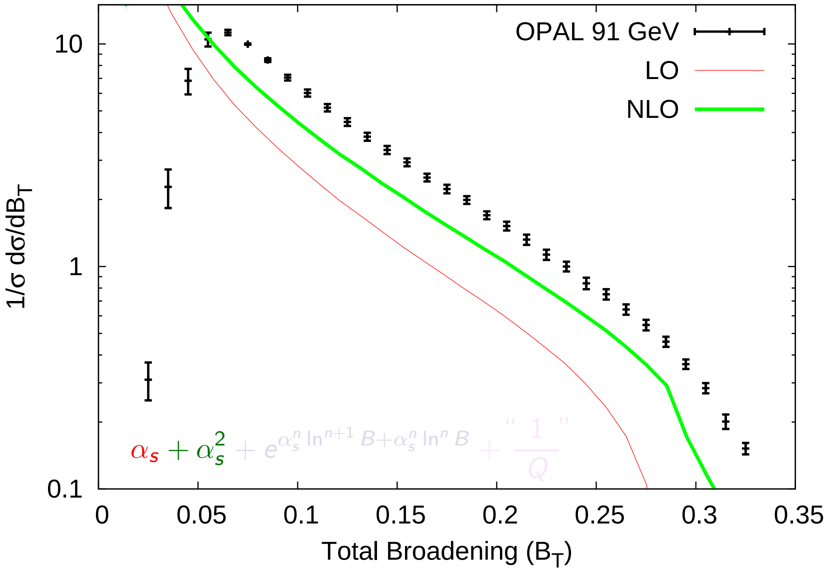

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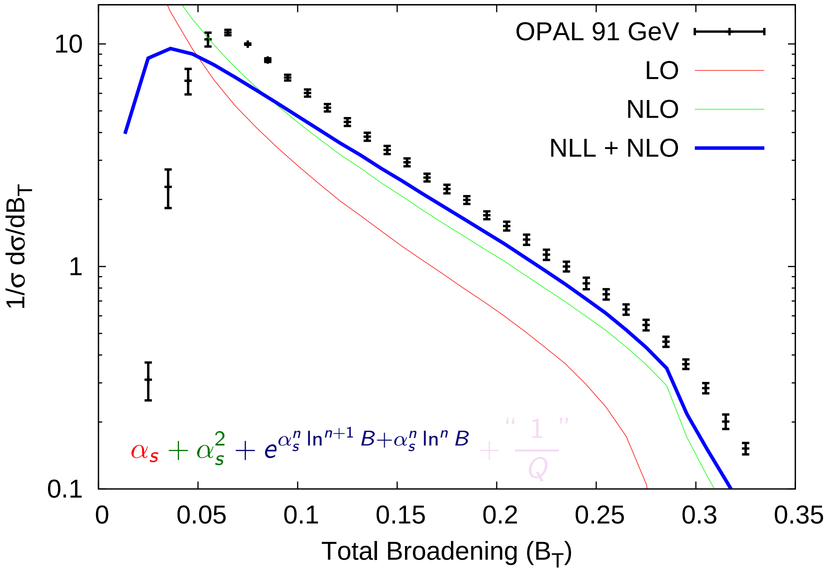

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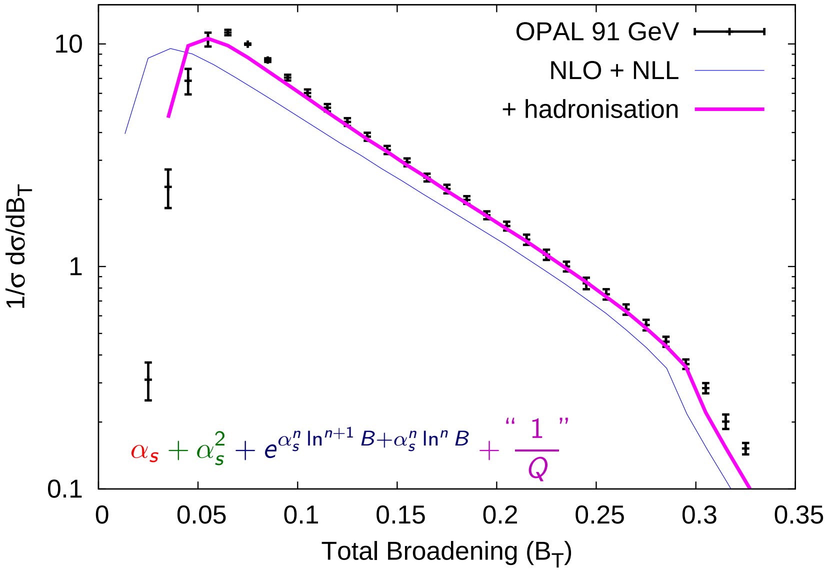

Perturbative tools

Fixed order

- Event shapes trivial for Born events (*e.g.* $p\bar{p} \to 2$ jets, thrust=1)
- First non-trivial order (LO) is Born + 1 parton, *i.e.* $p\bar{p} \to 3$ jets

$$\frac{1}{\sigma}\frac{d\sigma}{dV} \equiv \Sigma'(V) = \alpha_s f_1(V) + \alpha_s^2 f_2(V) + \dots$$

Given computer subroutine for $V(p_1, \dots, p_n)$ program gives you f_1(V), $f_2(V)$

NLOJET++, Nagy, '01–'03; also Kilgore-Giele code

Resummation

- For $V \ll 1$ (most data), soft-collinear logs dominate, $L = \ln 1/v$:

$$\Sigma(V) \simeq \sum_m \sum_{n=0}^{2m} \alpha_s^m L^n H_{mn} = \underbrace{h_1(\alpha_s L^2)}_{LL} + \underbrace{\sqrt{\alpha_s}\, h_2(\alpha_s L^2)}_{NLL} + \dots$$

- *Sometimes* series 'exponentiates', *i.e.* $\ln \Sigma$ is simpler:

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Exponentiating final-state resummations

$e^+e^- \to 2$ jets

S. Catani *et al.*, *Thrust distribution in e^+e^- annihilation,* Phys. Lett. B **263** (1991) 491.
S. Catani, G. Turnock and B. R. Webber, *Heavy jet mass distribution [...],* Phys. Lett. B **272** (1991) 368.
S. Catani *et al.*, *New clustering algorithm for multi-jet cross-sections in e^+e^- annihilation,* Phys. Lett. B **269** (1991) 432.
S. Catani, *et al. Resummation of large logarithms in e^+e^- event shape distributions,* Nucl. Phys. B **407** (1993) 3.
S. Catani, G. Turnock and B. R. Webber, *Jet broadening measures in e^+e^- annihilation,* Phys. Lett. B **295** (1992) 269.
G. Dissertori and M. Schmelling, *[...] two jet rate in e^+e^- annihilation,* Phys. Lett. B **361** (1995) 167.
Y. L. Dokshitzer *et al. On the QCD analysis of jet broadening,* JHEP **9801** (1998) 011
S. Catani and B. R. Webber, *Resummed C-parameter distribution in e^+e^- annihilation,* Phys. Lett. B **427** (1998) 377
S. J. Burby and E. W. Glover, *[...] light hemisphere mass and narrow jet broadening [...]* JHEP **0104** (2001) 029
M. Dasgupta and GPS, *Resummation of non-global QCD observables,* Phys. Lett. B **512** (2001) 323
E. Gardi and J. Rathsman, *Renormalon resummation [...] in the thrust distribution,* Nucl. Phys. B **609** (2001) 123
E. Gardi and J. Rathsman, *The thrust and heavy-jet mass distributions [...],* Nucl. Phys. B **638** (2002) 243
C. F. Berger, T. Kucs and G. Sterman, *Event shape / energy flow correlations,* Phys. Rev. D **68** (2003) 014012
F. Krauss and G. Rodrigo, *Resummed jet rates for e^+e^- annihilation into massive quarks,* Phys. Lett. B **576** (2003) 135
E. Gardi and L. Magnea, *The C parameter distribution in e+ e- annihilation,* JHEP **0308** (2003) 030
C. F. Berger and L. Magnea, *[...] angularities from dressed gluon exponentiation,* Phys. Rev. D **70**, 094010 (2004)

DIS 1+1 jet

V. Antonelli, M. Dasgupta and GPS, *Resummation of thrust distributions in DIS,* JHEP **0002** (2000) 001
M. Dasgupta and GPS, *Resummation of the jet broadening in DIS,* Eur. Phys. J. C **24** (2002) 213
M. Dasgupta and GPS, *Resummed event-shape variables in DIS,* JHEP **0208** (2002) 032

e^+e^-, DY, DIS 3 jets

A. Banfi, G. Marchesini, Y. L. Dokshitzer and G. Zanderighi, *QCD analysis of near-to-planar 3-jet events,* JHEP **0007** (2000) 002
A. Banfi, Y. L. Dokshitzer, G. Marchesini and G. Zanderighi, *Near-to-planar 3-jet events in and beyond QCD perturbation theory,* Phys. Lett. B **508** (2001) 269
A. Banfi, Y. L. Dokshitzer, G. Marchesini and G. Zanderighi, *QCD analysis of D-parameter in near-to-planar three-jet events,* JHEP **0105** (2001) 040
A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, *Out-of-plane QCD radiation in hadronic Z0 production,* JHEP **0108** (2001) 047
A. Banfi, G. Marchesini, G. Smye and G. Zanderighi, *Out-of-plane QCD radiation in DIS with high p(t) jets,* JHEP **0111** (2001) 066
A. Banfi, G. Marchesini and G. Smye, *Azimuthal correlation in DIS,* JHEP **0204** (2002) 024
A. Banfi and M. Dasgupta, *Dijet rates with symmetric E(t) cuts,* JHEP **0401**, 027 (2004)

Average: 1 observable per paper

Automate resummation?

Monte Carlo resummation:

Event generators (Herwig, Pythia, ...) = powerful automated resummation programs! *But:*

- Accuracy often unclear (depends on observable, no NLL for multi-jet processes)
- Difficult to estimate uncertainties of calculation
- Matching with fixed order is tricky
- No analytical information

What we would like:

Something as good as manual analytical resummation

- Guaranteed accuracy, exponentiation
- Separate LL, NLL functions, $g_1(\alpha_s L)$, $g_2(\alpha_s L)$
- Expansions of g_1 and g_2 to fixed order in α_s

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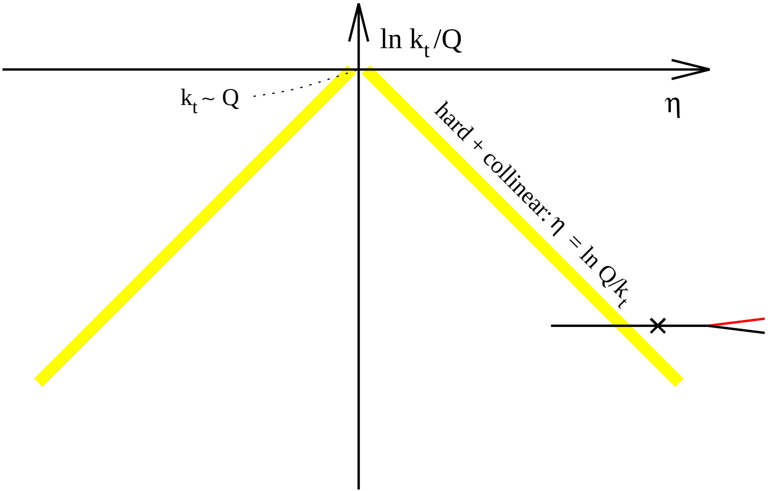
Phase space ($e^+e^- \to 2$ jets)

Use 'Lund' representation of kinematic plane: $\ln k_t$ and $\eta = -\ln\tan\theta/2$



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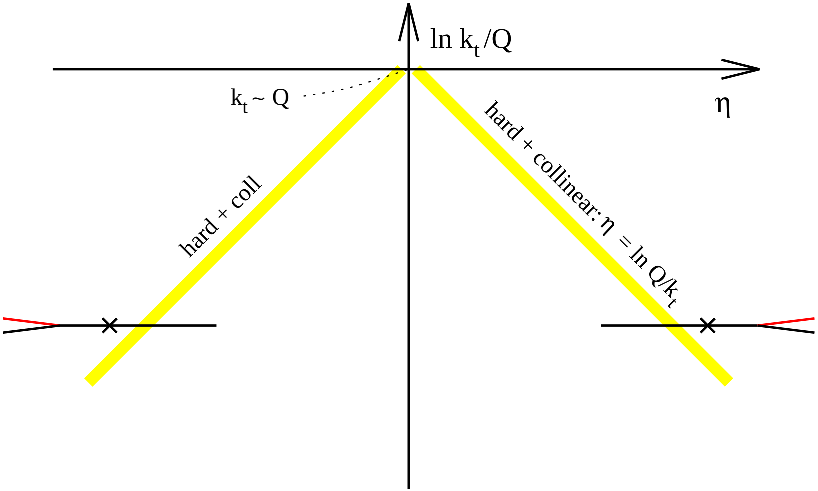

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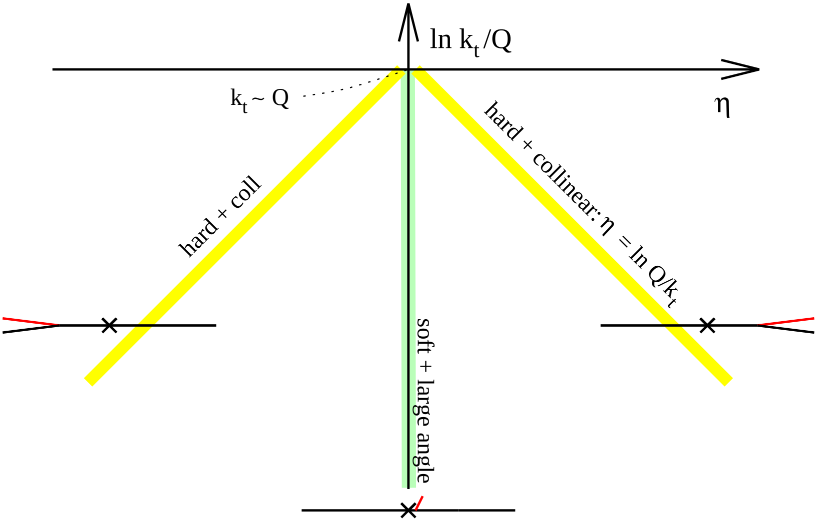

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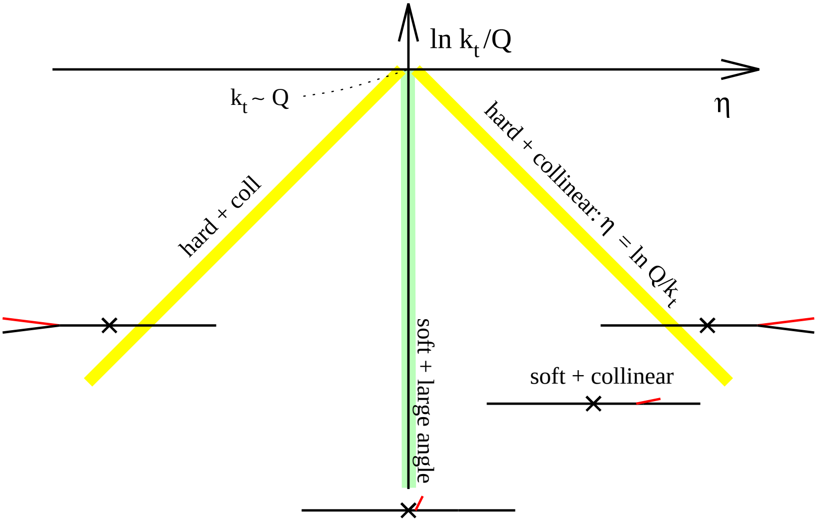
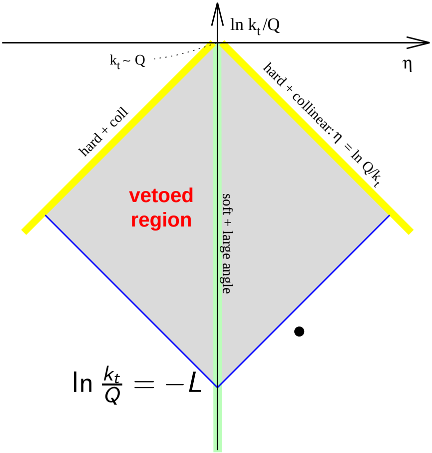


Introduce observable (& one emission)



Take observable, *e.g.* 1-Thrust (τ).

Dependence on *single soft collinear emission:*

$$\ln \tau = \ln \frac{k_t}{Q} - |\eta|$$

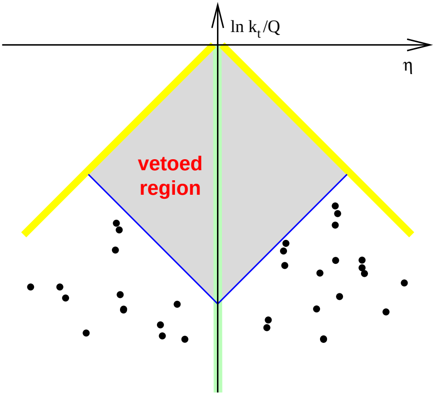
In general: linear comb. of $\ln \frac{k_t}{Q}$, $|\eta|$

Limit on τ, $\tau < \tau_{\max}$ defines *vetoed region* in $k_t - \eta$ plane.

Virtual-real cancellation occurs *everywhere except vetoed region* — left-over virtuals give ($\sim -\alpha_s \, d\eta \, d\ln k_t$):

$$\Sigma(\tau < \tau_{\max}) = 1 + \underbrace{G_{12}\alpha_s L^2}_{\text{Vetoed area}} + \underbrace{G_{11}\alpha_s L}_{\text{edges}}$$

What happens at all orders. . .



Virtual 'area' exponentiates:

$$\alpha_s L^2 \longrightarrow e^{\alpha_s^n L^{n+1}}$$

(Sudakov)

NLL edges stay NLL (and multiply LL exponential)

$$\alpha_s L \longrightarrow e^{\alpha_s^n L^n}$$

What of real emissions? Only cancel against virtuals if do not affect observable.

- *Require* non-canc. to be $\alpha_s^n L^n$, *i.e.* only emissions in band matter
- The rest cancel with virtual

- Require insensitivity to *secondary collinear splitting*
- '*cluster*' emissions

Like infrared-collinear (IRC) safety. But stronger: *recursive* IRC safety.

Low emission density → approximate M.E. by indep. emission (coherence)

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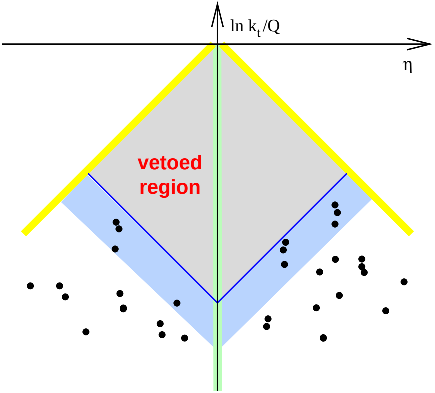

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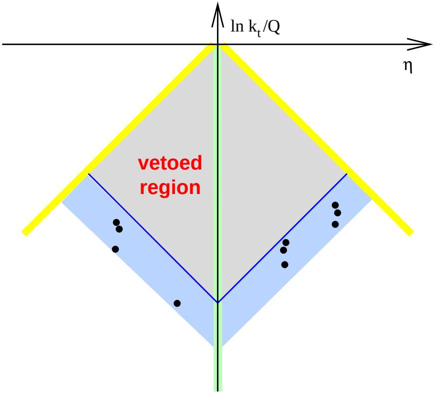

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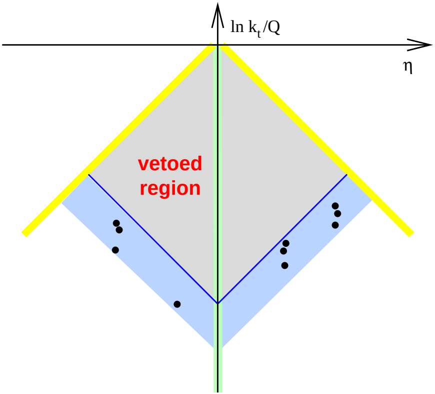

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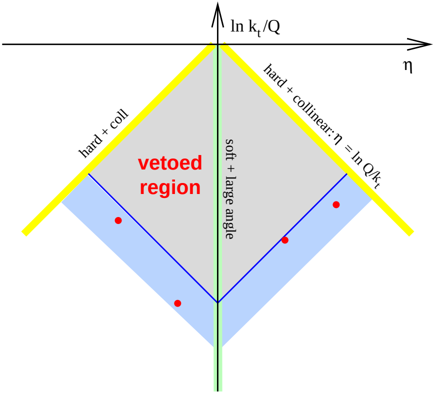

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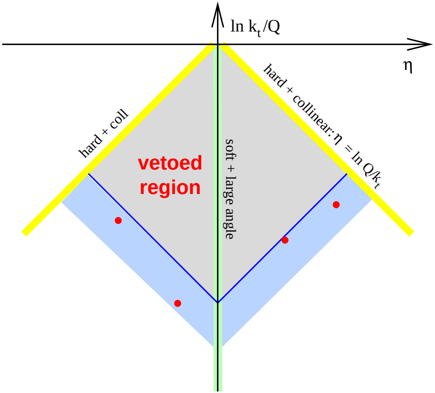

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- Recall $\ln \Sigma = \alpha_s^{-1} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots$.
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NB: observable must *scale properly* under remapping ($\to$ part of rIRC safety)

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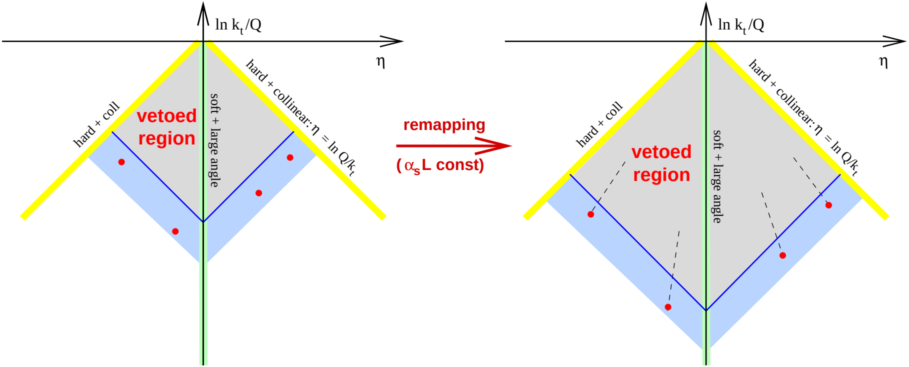

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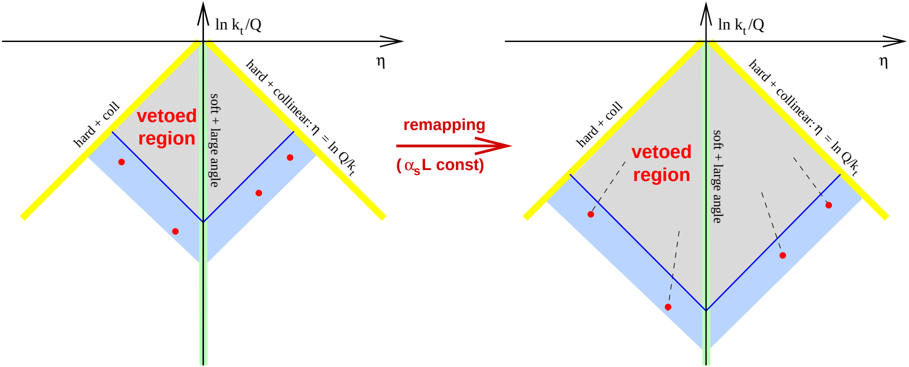

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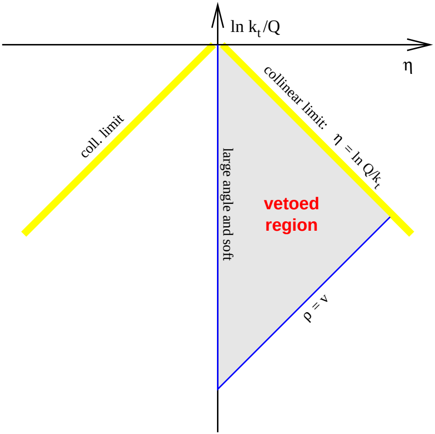

Some observables measure just part of phase space, *e.g.* single jet

non-global

Resummation is different:

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  Appleby & Seymour '02

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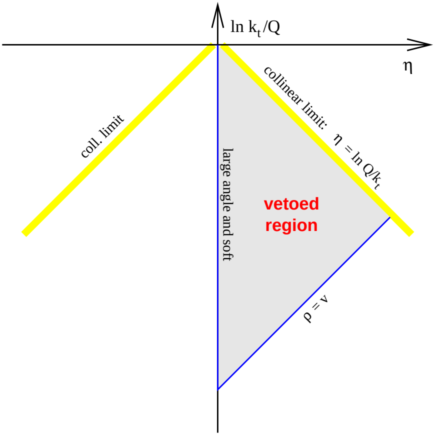

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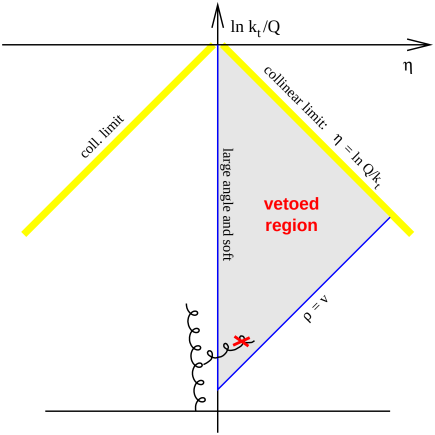

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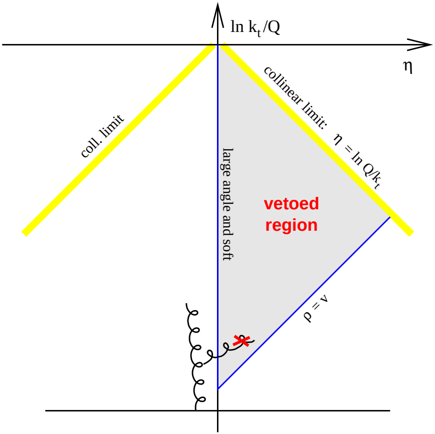

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Analytical work (done once and for all)

A1. formulate exact applicability conditions for the approach (its scope)

A2. derive a master formula for a generic observable in terms of simple properties of the observable

Numerical work (to be repeated for each observable)

N1. let an "expert system" investigate the applicability conditions

N2. it also determines the inputs for the master formula

N3. straightforward evaluation of the master formula, including phase space integration etc.

Note: N1 and N2 are core of automation

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In practice (1)?

Single emission properties

- Observable must have standard functional form for soft & collinear gluon emission

$$V(\{p\}, k) = d_\ell \left(\frac{k_t}{Q}\right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)\,.$$

Born momenta soft collinear emission

- *Determine coefficients* a_ℓ, b_ℓ, d_ℓ and $g_\ell(\phi)$ for emissions close to each hard Born parton (leg) ℓ.
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- Parametrize emission momenta by effect on observable:

$\kappa(\bar{v})$ is any momentum such that $V(\{p\}, \kappa(\bar{v})) = \bar{v}$

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The formulae

Given info from previous pages, *final answer is analytical:*

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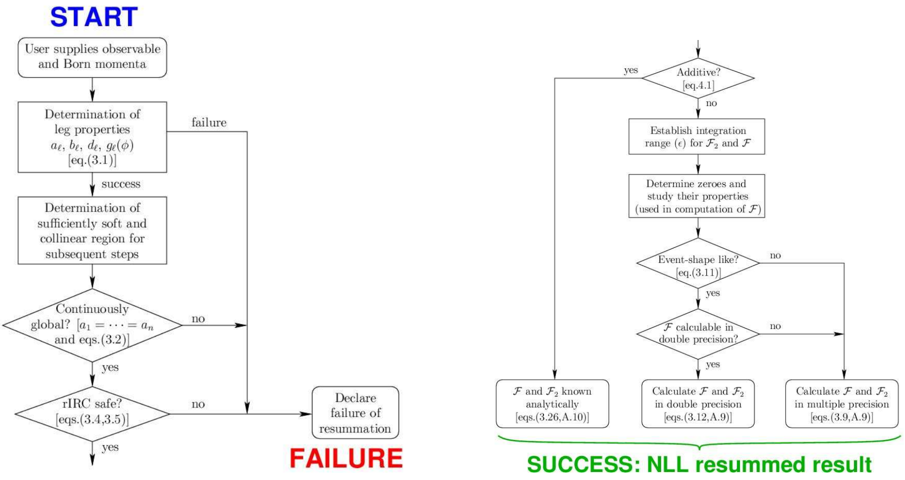
Except $\mathcal{F}$, which is calculated via MC integration

$$\mathcal{F} = \lim_{\epsilon \to 0} \frac{\epsilon^{R'}}{R'} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\prod_{i=1}^{m+1} \sum_{\ell_i=1}^{n} C_\ell r'_{\ell_i} \int_\epsilon^1 \frac{d\zeta_i}{\zeta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \right) \delta(\ln \zeta_1) \times \exp\left(-R' \ln \lim_{\bar{v} \to 0} \frac{V(\{\tilde{p}\}, \kappa_1(\zeta_1 \bar{v}), \ldots, \kappa_{m+1}(\zeta_{m+1} \bar{v}))}{\bar{v}} \right).$$

CAESAR flow chart

Computer Automated Expert Semi-Analytical Resummer



What it doesn't do

- Observables that vanish other than through suppression of radiation (*e.g.* Vector Boson p_t spectrum) have divergence in $g_2(\alpha_s L)$ beyond fixed value of $\alpha_s L$. Rakow & Webber '81; Dasgupta & GPS '02
  - for very-inclusive 2-jet cases analytical resummations are in any case more accurate (NNLL) Higgs p_t: Bozzi et al '03–05
    Back-to-back EEC: de Florian & Grazzini '04
  - For less-inclusive cases, this problem is sometimes 'academic' (in region of vanishing X-section).
- Non-global observables are beyond its scope (but perhaps could be included in future).
  - Individual jet properties, or subsets of jets
  - Gap resummations [various authors]
- Threshold resummations not yet thought about in this framework.

Hadron collider event shapes

Contradiction?

Theoretical calculations are for global observables.
But experiments only have detectors in limited rapidity range.
(Strictly: series of sub-detectors, of worsening quality as rapidity increases)

Model by cut around beam $|\eta| < \eta_{max}$
➥ Problems with globalness

Take cut as being edge of most forward detector with momentum or energy resolution:

	Tevatron	LHC
η_{max}	3.5	5.0

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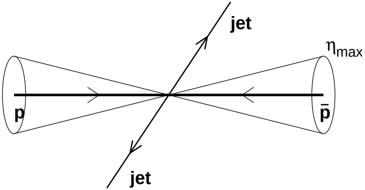

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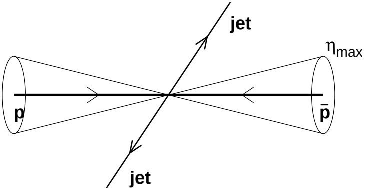

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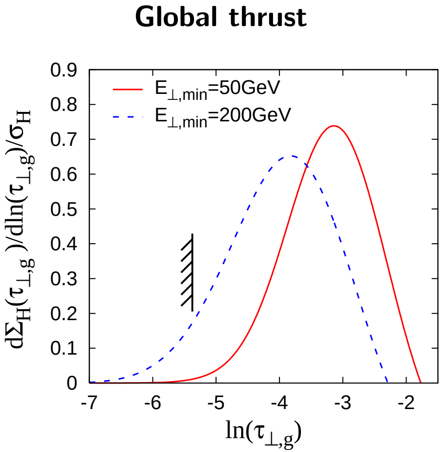
Practical hh event shapes

Particles from beyond max rapidity contribute significantly only for small $V \lesssim e^{-(a+b_\ell)\eta_{\max}}$.

Most of cross section may be *above that limit* — rapidity cut irrelevant.

Banfi et al. '01



Alternative

Measure just centrally & add recoil term (indirect sensitivity to rest of event):

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i\in\mathcal{C}} \vec{q}_{\perp i} \right| ,$$

Here $g_2(\alpha_s L)$ diverges for $L \sim 1/\alpha_s$ (due to cancellations in vector sum) – study distribution only before divergence.

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Most of cross section may be *above that limit* — rapidity cut irrelevant.

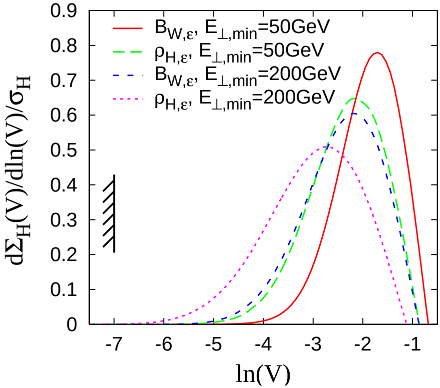
Banfi et al. '01

Alternative

Measure just centrally & add recoil term (indirect sensitivity to rest of event):

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i\in\mathcal{C}} \vec{q}_{\perp i} \right| ,$$

Jet-broadening, jet-mass
$(+k_t/Qe^{-|\eta|})$



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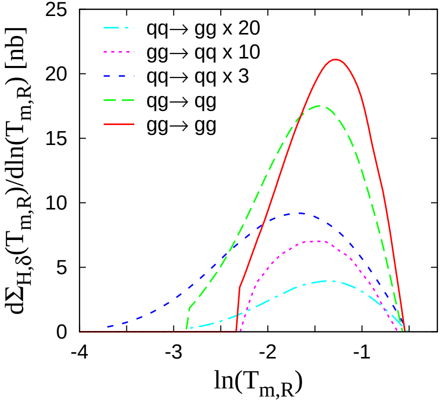
Recoil thrust minor



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Summary of observables

Event-shape	Impact of $\eta_{\max}$	Resummation breakdown	Underlying Event	Jet hadronisation
$\tau_{\perp,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/Q$
$T_{m,g}$	tolerable	none	$\sim \eta_{\max}/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
y_{23}	tolerable	none	$\sim \sqrt{y_{23}}/Q$	$\sim \sqrt{y_{23}}/Q$
$\tau_{\perp,\mathcal{E}}$, $\rho_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/Q$
$B_{X,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$T_{m,\mathcal{E}}$	negligible	serious	$\sim 1/Q$	$\sim 1/(\sqrt{\alpha_s}Q)$
$y_{23,\mathcal{E}}$	negligible	none	$\sim 1/Q$	$\sim \sqrt{y_{23}}/Q$
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NB: there may be surprises after more detailed study, *e.g.* matching to NLO...

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Note complementarity between observables

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Conclusions/Outlook

Status

- Powerful new tool
- Insight into structure of exponentiating resummations (rIRC safety)
- Many observables have been studied, and for first time, hadron-collider dijet event shapes http://qcd-caesar.org/

Short-term Outlook

- Matching with fixed order (DIS $2+1$ jets, e^+e^- 3 jets, then hadron-hadron)
- Making program public

NB: for accurate hadron-hadron matching, *crucial information is missing from fixed-order codes:*

To authors of fixed-order codes:
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EXTRA SLIDES

Interest of hadronic colliders?

Various processes:

- $pp \to$ W/Z/H boson + jet — Banfi Marchesini Smye Zanderighi '01
- $pp \to$ 2 jets — Main subject of this talk

Standard applications (*e.g.*)

- Measure α_s
- As for 3-jet/2-jet ratio in $p\bar{p}$, reduce dependence on PDFs
- But for event-shapes → *distribution*
- Far more information than 3-jet/2-jet ratio

New territory

- 4-jet (2 + 2) topology → novel perturbative structures
  soft colour evln matrices
  Botts, Kidonakis, Oderda, Sterman '89–99
- 3 & 4-jet topologies (& g-jets) → rich environment for analytical non-pert. studies
- Underlying event — test models (analytical & MC).

Variety of event-shape observables → complementary information → disentangle the different physics issues.

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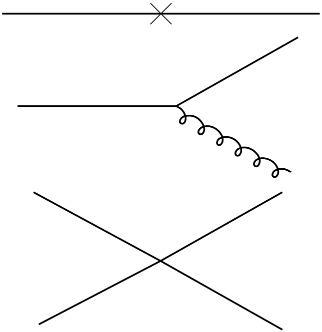
Soft colour evolution

Multi-jet final states: relative colour of pairs of hard partons determines soft large-angle radiation.



2 jets: always in a *colour singlet*

3 jets: colour state of any pair *fixed by third parton* (colour conservation).

4 jets: a given pair can be in various colour states. Soft virtual corrections mix colour states.

Resummation leads to *matrix evolution equation for colour state of amplitudes* ('soft anomalous dimenions')

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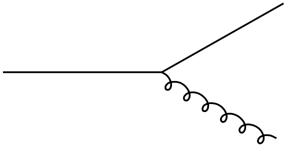

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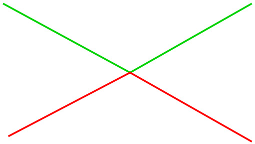

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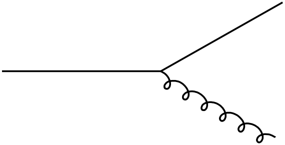

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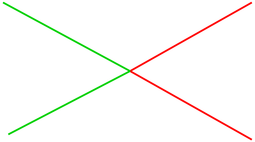

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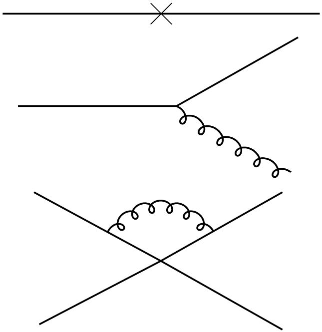

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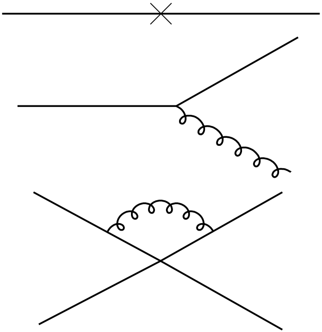

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rIRC v. IRC safety

IRC safety is subtle in two-scale problems. Say we have two scales: Q and $k_{t1} \ll Q$.

IRC safety says that if we add an extra emission k_{t2}, then

$$\lim_{k_{t2}\to 0} V(k_1, k_2) = V(k_1)$$

An example function that satisfies this is

$$V(k_1) = \frac{k_{t1}}{Q} \qquad V(k_1, k_2) = \frac{k_{t1}}{Q}\left(1 + \Theta(k_{t2} - k_{t1}^2/Q)\right)$$

But it is *not rIRC safe.* Take $k_{t1} = \bar{v}Q$ and $k_{t2} = \zeta_2 k_{t1}$

$$V(k_1, k_2) = \bar{v}(1 + \Theta(\zeta_2 - \bar{v}))$$

So

$$\lim_{\bar{v}\to 0}\lim_{\zeta_2\to 0} \frac{1}{\bar{v}} V(k_1, k_2) = 1\,, \qquad \text{while} \qquad \lim_{\zeta_2\to 0}\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(k_1, k_2) = 2\,.$$

Experimental considerations

Contradiction?

Theoretical calculations are for global observables.
But experiments only have detectors in limited rapidity range.
(Strictly: series of sub-detectors, of worsening quality as rapidity increases)

Model by cut around beam $|\eta| < \eta_{\mathrm{max}}$
➥ Problems with globalness

Take cut as being edge of most forward detector with momentum or energy resolution:

	Tevatron	LHC
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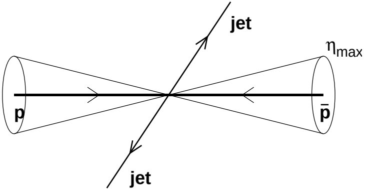

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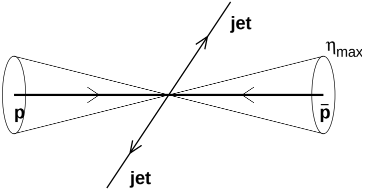

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Sidestepping non-globalness

Select events with central, hard jets (x_1, x_2 not too small), with transverse momentum $P_\perp$.

From kinematics, emissions (k) near forward detector edges typically have small transverse momentum:

$$k_\perp \sim P_\perp e^{-\eta_0} \ll P_\perp$$

If event-shape value is always sufficiently large that such an emission contributes negligibly, then:
we can ignore rapidity cut & pretend measurement is global

Proceed as follows:

- Calculate distribution without any rapidity cutoff
- Determine smallest 'typical' value of observable
- Check self-consistency: *i.e.* that in comparison, emissions beyond cutoff contribute negligbly.

Banfi, Marchesini, Smye & Zanderighi '01

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Event selection cuts

Results that follow based on this (illustrative) event selection:

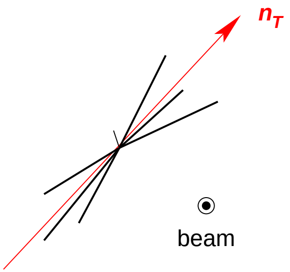
- Run longitudinally invariant inclusive k_t jet algorithm (could also use midpoint cone)
- Require hardest jet to have $P_{\perp,1} > P_{\perp,\mathrm{min}} = 50$ GeV
- Require two hardest jets to be central $|\eta_1|, |\eta_2| < \eta_c = 0.7$

Pure resummed results
no matching to NLO (or even LO)
Shown for Tevatron run II

Transverse thrust & minor

Some observables are naturally defined in terms of all particles in the event, *e.g.* *Global Transverse Thrust*

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}\,, \qquad \tau_{\perp,g} = 1 - T_{\perp,g}\,,$$



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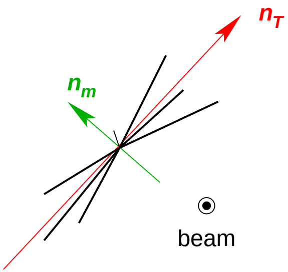

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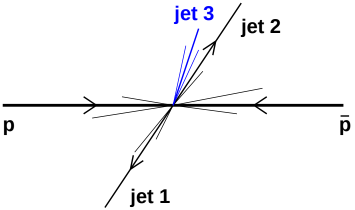
3-jet resolution threshold

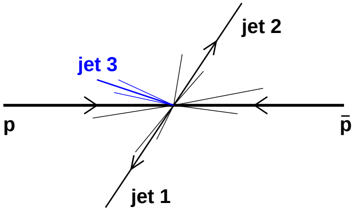
Use *exclusive* long. inv. k_t algorithm: successive recombination of pair with smallest closeness measure d_{kl}, d_{kB}:

$$d_{kB} = q_{\perp k}^2 \,, \qquad d_{kl} = \min\{q_{\perp k}^2, q_{\perp l}^2\} \left((\eta_k - \eta_l)^2 + (\phi_k - \phi_l)^2\right) \,.$$

Define $d^{(n)}$ as smallest d_{kl}, d_{kB} when only n pseudo-jets left. Examine (normalised) 3-jet resolution threshold

$$y_{23} = \frac{1}{(E_{\perp,1} + E_{\perp,2})^2} d^{(3)}$$





Generalisation of 3-jet cross section

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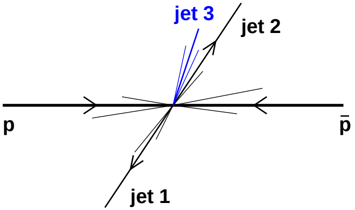

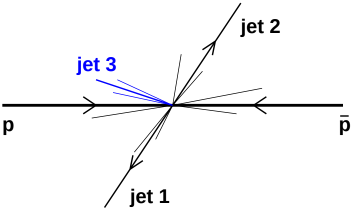

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$$P(v) = \exp\left[-G_{12}\frac{\alpha_s L^2}{2\pi} + \cdots\right], \quad L = \ln\frac{1}{v}$$

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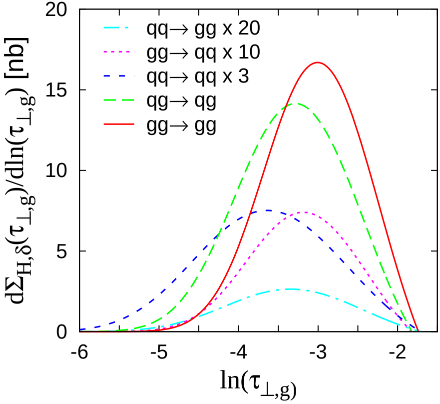

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$$P(v) = \exp\left[-G_{12}\frac{\alpha_s L^2}{2\pi} + \cdots\right], \quad L = \ln\frac{1}{v}$$

Ev.Shp.	G_{12}
$\tau_{\perp,g}$	$2C_B + C_J$
$T_{m,g}$	$2C_B + 2C_J$
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C_B = total colour of Beam partons
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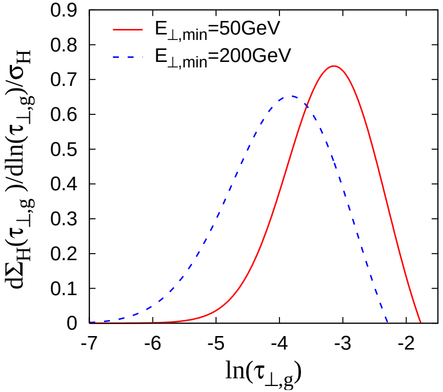

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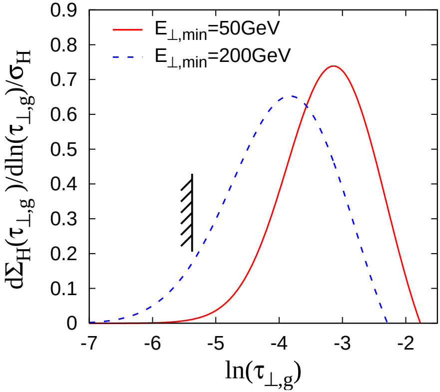

Beam cut: $\tau_{\perp,g} \gtrsim 0.15e^{-\eta_{\max}}$

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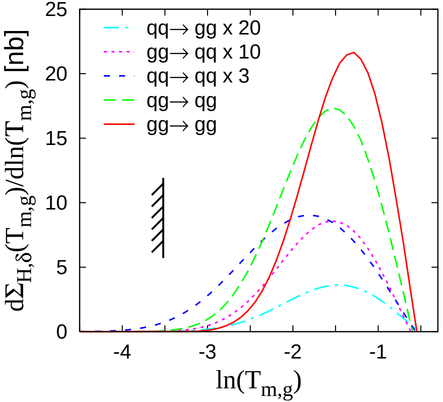

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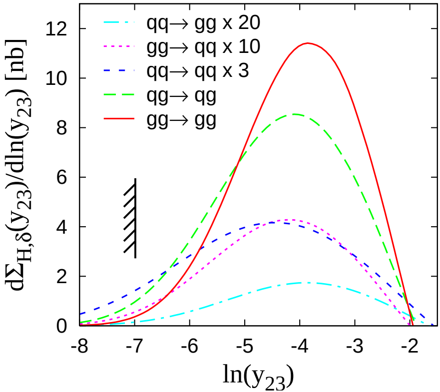


Beam cut: $y_{23} \gtrsim e^{-2\eta_{\max}}$ [because $y_{23} \sim k_t^2$]

Forward-suppressed observables

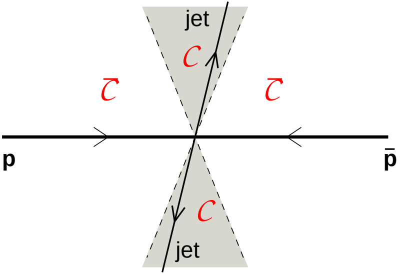
Divide event into central region ($\mathcal{C}$, say $|\eta| < 1.1$) and rest of event ($\bar{\mathcal{C}}$).
[NB: ∃ considerable freedom in definition of $\mathcal{C}$: *e.g.* can also be two hardest jets]

Define central ⊥ mom., and rapidity:

$$Q_{\perp,\mathcal{C}} = \sum_{i\in\mathcal{C}} q_{\perp i}\,, \quad \eta_{\mathcal{C}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\in\mathcal{C}} \eta_i\, q_{\perp i}$$

and an *exponentially suppressed forward term,*

$$\mathcal{E}_{\bar{\mathcal{C}}} = \frac{1}{Q_{\perp,\mathcal{C}}} \sum_{i\notin\mathcal{C}} q_{\perp i}\, e^{-|\eta_i - \eta_{\mathcal{C}}|}\,.$$



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Result is a global event shape, with suppressed sensitivity to forward region.

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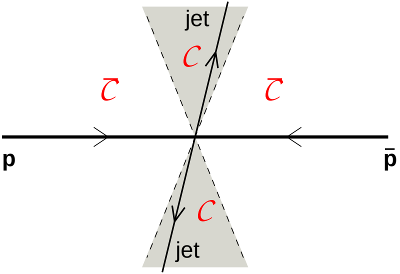

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Examples

- Split $\mathcal{C}$ into two pieces: *Up, Down*
- Define *jet masses* for each

$$\rho_{X,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}^2}\Big(\sum_{i\in\mathcal{C}_X} q_i\Big)^2\,, \qquad X = U, D\,,$$

Define sum and heavy-jet masses

$$\rho_{S,\mathcal{C}} \equiv \rho_{U,\mathcal{C}} + \rho_{D,\mathcal{C}}\,, \qquad \rho_{H,\mathcal{C}} \equiv \max\{\rho_{U,\mathcal{C}}, \rho_{D,\mathcal{C}}\}\,,$$

Define global extension, with extra forward-suppressed term

$$\rho_{S,\mathcal{E}} \equiv \rho_{S,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}\,, \qquad \rho_{H,\mathcal{E}} \equiv \rho_{H,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}\,.$$

- Similarly: *total and wide jet-broadenings*

$$B_{T,\mathcal{E}} \equiv B_{T,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}\,, \qquad B_{W,\mathcal{E}} \equiv B_{W,\mathcal{C}} + \mathcal{E}_{\bar{\mathcal{C}}}\,.$$

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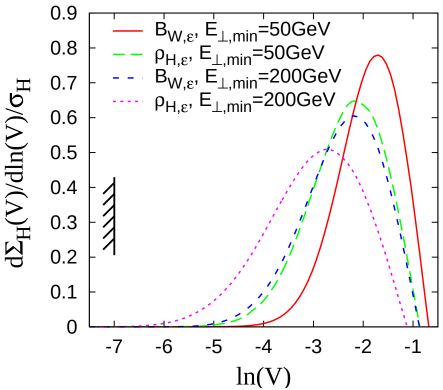

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Beam cuts: $B_{X,\mathcal{E}}, \rho_{X,\mathcal{E}} \gtrsim e^{-2\eta_{\max}}$ [because $\mathcal{E}_{\bar{\mathcal{C}}} \sim k_t e^{-|\eta|}$]

Recoil observables

By momentum conservation

$$\sum_{i\in\mathcal{C}} \vec{q}_{\perp i} = -\sum_{i\notin\mathcal{C}} \vec{q}_{\perp i}$$

Use central particles to define *recoil term*, which is *indirectly sensitive* to non-central emissions

$$\mathcal{R}_{\perp,\mathcal{C}} \equiv \frac{1}{Q_{\perp,\mathcal{C}}} \left| \sum_{i\in\mathcal{C}} \vec{q}_{\perp i} \right| ,$$

Define event shapes exclusively in terms of *central particles:*

$$\rho_{X,\mathcal{R}} \equiv \rho_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}} \,, \qquad B_{X,\mathcal{R}} \equiv B_{X,\mathcal{C}} + \mathcal{R}_{\perp,\mathcal{C}} \,, \ldots$$

These observables are *indirectly global*

First studied at HERA (B_{zE} broadening)

Results

CAESAR resummation works for observables having *direct exponentiation*:

$$P(v) = e^{Lg_1(\alpha_s L) + g_2(\alpha_s L) + \dots}$$

For recoil observables, exponentiation holds fully only after Fourier & other integral transforms (generalised *b*-space resummation).

Manifestation: NLLs ($g_2(\alpha_s L)$) diverge at some $\alpha_s L \sim 1$.

Consequently, cannot extend distribution to $v = 0$ — must cut before divergence.

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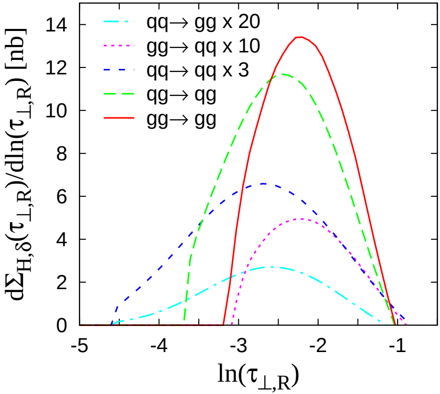
recoil transverse thrust



Quite large effect: $\sim$ 15% of X-sct is beyond cutoff

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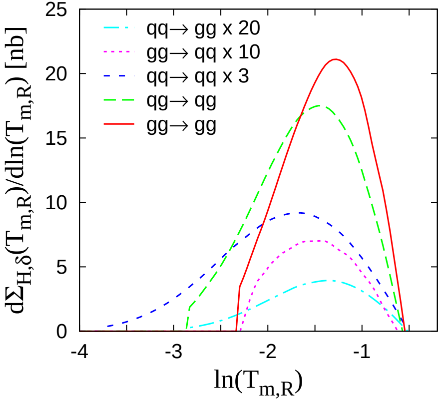
recoil thrust minor



Moderate effect: few % of X-sct is beyond cutoff