Regularization by Dimensional Reduction: New results on Factorization and Supersymmetry

Dominik Stöckinger

IPPP Durham

RADCOR 2005

Outline

1. Motivation
2. Factorization-Problem
3. Supersymmetry and M_h-calculations
4. Conclusions

Outline

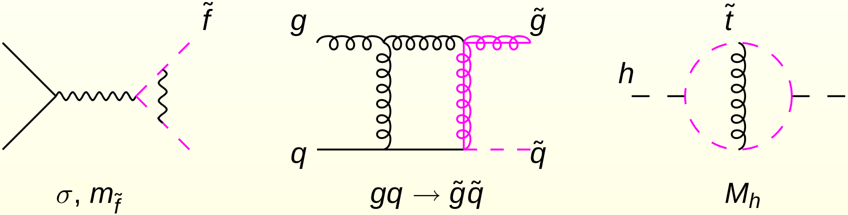
1 Motivation

2 Factorization-Problem

3 Supersymmetry and M_h-calculations

4 Conclusions

Precise measurement of SUSY observables justifies/necessitates SUSY loop calculations



Loop calculations require

- Regularization
- Renormalization

Common Regularization/Renormalization Schemes

- DREG
  breaks SUSY, complicated in practice
- DRED
  doesn't (?) break SUSY, usually applied

Dim. Regularization (DREG)

D dimensions
D Gluon-components
4 Gluino-components

Dim. Reduction (DRED)

D dimensions,
4 Gluon-components,
4 Gluino-components

Common Regularization/Renormalization Schemes

- DREG
  breaks SUSY, complicated in practice
- DRED
  doesn't (?) break SUSY, usually applied

Dim. Regularization (DREG)

D dimensions
D Gluon-components
4 Gluino-components

Dim. Reduction (DRED)

D dimensions,
4 Gluon-components,
4 Gluino-components

DRED useful for SUSY, also useful for QCD: Focus on DRED!

Problems of DRED

SUSY

- SUSY preserved?
- Math. inconsistency

[Siegel'80]

- Symmetry-restoring counterterms necessary in calculations?

QCD-Factorization

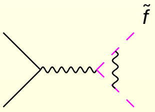
- Hadron processes:

$$\sigma_{\text{had}} = f_{\text{parton}} \otimes \sigma_{\text{parton}}$$

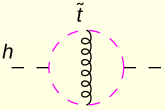
+non-factorizing terms?

[Beenakker,Kuijf,Neerven,Smith'88]

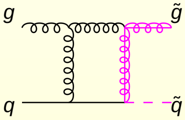
[Beenakker,Höpker,Spira,Zerwas'96]



σ, $m_{\tilde{f}}$



M_h



$gq \to \tilde{g}\tilde{q}$

Problems of DRED

SUSY

- SUSY preserved?
- Math. inconsistency

[Siegel'80]

- Symmetry-restoring counterterms necessary in calculations?

QCD-Factorization

- Hadron processes:

$$\sigma_{\rm had} = f_{\rm parton} \otimes \sigma_{\rm parton}$$

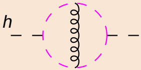
+non-factorizing terms?

[Beenakker,Kuijf,Neerven,Smith'88]

[Beenakker,Höpker,Spira,Zerwas'96]

Aims: [DS, '05]

- Method to verify SUSY
- Perform verification in important case



Aims: [Signer, DS, '05]

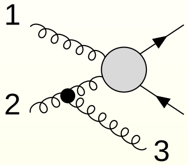
- "non-factorizing terms": Origin?
- Reconcile DRED with factorization!

Outline

1. Motivation
2. Factorization-Problem
3. Supersymmetry and M_h-calculations
4. Conclusions

$gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit 2||3:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

Result for DREG, $m = 0$

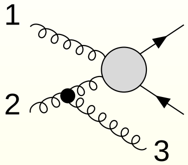
[Beenakker, Kuijf, van Neerven, Smith '88]

$$\sigma^{\mathrm{DREG}}(gg \to t\bar{t}g) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\, \sigma^{\mathrm{DREG}}(GG \to t\bar{t})$$

$$\sigma_{\mathrm{had}} \sim f^{\mathrm{DREG}} \otimes \sigma^{\mathrm{DREG}}_{\mathrm{parton}}$$

$gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

Result for DRED, $m = 0$

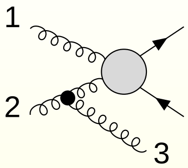
[Beenakker, Kuijf, van Neerven, Smith '88]

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\mathrm{DRED}}(GG \to t\bar{t})$$

$$\sigma_{\mathrm{had}} \sim f^{\mathrm{DREG}} \otimes \sigma^{\mathrm{DRED}}_{\mathrm{parton}}$$

$gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

Result for DREG, $m \neq 0$

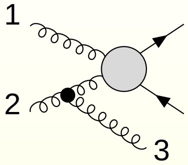
[Beenakker, Kuijf, van Neerven, Smith '88]

$$\sigma^{\text{DREG}}(gg \to t\bar{t}g) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\text{DREG}}(GG \to t\bar{t})$$

$$\sigma_{\text{had}} \sim f^{\text{DREG}} \otimes \sigma^{\text{DREG}}_{\text{parton}}$$

$gg \to t\bar{t}$ (massive quark) at NLO: $gg \to t\bar{t}g$

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

Result for DRED, $m \neq 0$

[Beenakker, Kuijf, van Neerven, Smith '88]

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\, \sigma^{\text{DRED}}(GG \to t\bar{t})$$

$$+\frac{1}{k_2 k_3} K_g\, \sigma^{\text{DRED}}(GG \to t\bar{t}, m\text{-terms only})$$

$$\sigma_{\text{had}} \sim f^{\text{DREG}} \otimes \sigma^{\text{DRED}}_{\text{parton}} + \text{non-factorizing terms?}$$

Problems, Questions, and Aims

- Problem discovered in [Beenakker, Kuijf, van Neerven, Smith '88] [van Neerven, Smith '04]
- Does this mean that DRED cannot be used for hadron processes?
- Resort to DREG $\Rightarrow$ SUSY-restoring cts necessary
  $\Rightarrow$ complication [Beenakker, Höpker, Zerwas '96] [Beenakker, Höpker, Spira, Zerwas '96]

- Aims: [Signer, DS, '05]
  - Identify origin of "non-factorizing terms"
  - Rewrite them in factorized form
  - Reconcile DRED with factorization

Analyze DRED

Consider 4-dim QCD, reduced to D space-time dimensions

Resulting Theory:

D space-time dimensions

G: 4-component gluon

$\equiv$ composition of

g: D-component "gluon/gauge field"

ϕ: $4 - D$ "ϵ-scalars"

$\Rightarrow$ D-dim QCD with extra scalars ϕ

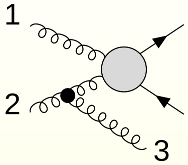
$$G = g \oplus \phi$$

$$\sigma(G \ldots) = \sigma(g \ldots) + \sigma(\phi \ldots)$$

Two partons g, ϕ $\longrightarrow$ consider $\sigma(g \ldots), \sigma(\phi \ldots)$ independently

Distinguish g and ϕ in DRED

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

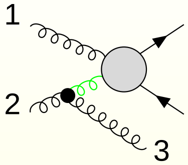
Result for DRED, $m \neq 0$

[Signer, DS '05]

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} \Big[P_{g \to gg}\, \sigma^{\text{DRED}}(GG \to t\bar{t}) + K_g\, \sigma^{\text{DRED}}(GG \to t\bar{t}, m\text{-terms only}) \Big]$$

Distinguish g and ϕ in DRED

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

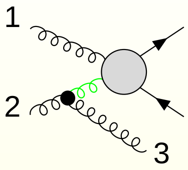
Result for DRED, $m \neq 0$

[Signer, DS '05]

$$\sigma^{\rm DRED}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3}\Big[P_{g\to gg}\, \sigma^{\rm DRED}(GG \to t\bar{t}) + K_g\, \sigma^{\rm DRED}(GG \to t\bar{t}, m\text{-terms only})\Big]$$

Distinguish g and ϕ in DRED

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

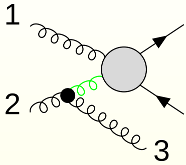
Result for DRED, $m \neq 0$

[Signer, DS '05]

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} \Big[P_{G\to gG}\, \sigma^{\mathrm{DRED}}(Gg \to t\bar{t}) + P_{G\to \phi G}\, \sigma^{\mathrm{DRED}}(G\phi \to t\bar{t}) \Big]$$

Distinguish g and ϕ in DRED

Collinear limit $2\|3$:



$\Rightarrow$ divergence $\sim \frac{1}{k_2 k_3}$

$\Rightarrow$ should factorize

Result for DRED, $m \neq 0$

[Signer, DS '05]

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3}\Big[P_{G\to gG}\,\sigma^{\text{DRED}}(Gg \to t\bar{t}) + P_{G\to\phi G}\,\sigma^{\text{DRED}}(G\phi \to t\bar{t})\Big]$$

Factorized form as expected

Understanding factorization in DRED

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3}\Big[P_{G\to gG}\,\sigma^{\mathrm{DRED}}(Gg \to t\bar{t}) + P_{G\to \phi G}\,\sigma^{\mathrm{DRED}}(G\phi \to t\bar{t})\Big]$$

Remarks:

- NLO calculation not modified
- Only collinear limit involves g, ϕ
- "non-factorizing terms" understood:
  $K_g\ \sigma(GG \to t\bar{t}, m\text{-terms only}) \to P_{\phi\to g\phi}\,[\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})]$

Understanding factorization in DRED

$$\sigma^{\text{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3}\Big[P_{G\to gG}\ \sigma^{\text{DRED}}(Gg \to t\bar{t}) + P_{G\to\phi G}\ \sigma^{\text{DRED}}(G\phi \to t\bar{t})\Big]$$

Remarks:

- NLO calculation not modified
- Only collinear limit involves g, ϕ
- “non-factorizing terms” understood:
  $K_g\ \sigma(GG \to t\bar{t}, m\text{-terms only}) \to P_{\phi\to g\phi}\,[\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})]$
- Possibility to evaluate hadron cross sections in DRED

Understanding factorization in DRED

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3}\Big[P_{G\to gG}\,\sigma^{\mathrm{DRED}}(Gg \to t\bar{t}) + P_{G\to \phi G}\,\sigma^{\mathrm{DRED}}(G\phi \to t\bar{t})\Big]$$

Hadron cross section

$$\sigma_{\mathrm{had}} = f_{\mathrm{parton}} \otimes \hat{\sigma},$$

$$\hat{\sigma} = \sigma_{\mathrm{NLO}} - \sigma_{\mathrm{coll.subtr.}}$$ in DRED or DREG

$$\hat{\sigma}^{\mathrm{DRED}} = \hat{\sigma}^{\mathrm{DRED}}$$

Consequences and Outlook

Outlook:

- Treat more general processes
- Include virtual NLO corrections
- Show that factorization holds in DRED in general

- What is more efficient to compute $\hat{\sigma}_{\mathrm{NLO}}$?
- DRED — distinguish between g, ϕ in subtraction terms?
- DREG — susy-restoring counterterms?

Outline

1. Motivation
2. Factorization-Problem
3. Supersymmetry and M_h-calculations
4. Conclusions

DRED and Supersymmetry

Mathematical Inconsistency

"DRED $\Rightarrow$ 1=0"!? [Siegel'80]

- No general proofs possible, in particular no proof of SUSY

Question:

To what extent is DRED SUSY-preserving?

DRED and Supersymmetry

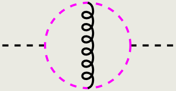
Mathematical Inconsistency

"DRED $\Rightarrow$ 1=0"!? [Siegel'80]

- No general proofs possible, in particular no proof of SUSY

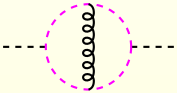
Question here:

Is DRED sufficiently SUSY-preserving

for two-loop calculations of M_h  ?

DRED and Supersymmetry

Is DRED SUSY-preserving at the level required for



?

If Yes:

- multiplicative renormalization o.k.
- has been assumed in all calculations

If No:

- additional, SUSY-restoring counterterms necessary
- tedious to determine and implement

DRED and SUSY —Relevant Case

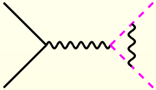
Status: many SUSY Identities checked:

1-Loop Ward identities [Capper,Jones,van Nieuvenhuizen'80]

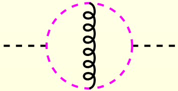
β-functions [Martin, Vaughn '93] [Jack, Jones, North '96]

1-Loop S-matrix relation [Beenakker,Höpker,Zerwas'96]

1-Loop Slavnov-Taylor identities [Hollik,Kraus,DS'99] [Hollik,DS'01] [Fischer,Hollik,Roth,DS'03]

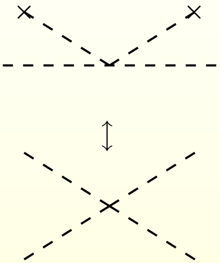


- sufficient for one-loop SUSY processes,
  $\Rightarrow$ multiplicative renormalization o.k.



- but not for two-loop Higgs mass calculations:
  $\Rightarrow$ SUSY-restoring counterterms required?
- Important case, result not obvious $\rightarrow$ Check that!

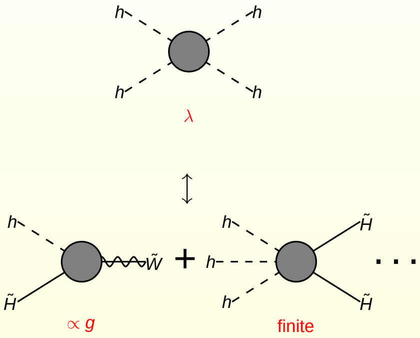
Higgs boson mass and quartic coupling



Higgs mass

- M_h governed by quartic Higgs self coupling λ
- $\lambda \propto g^2$ in SUSY

Quartic coupling and SUSY

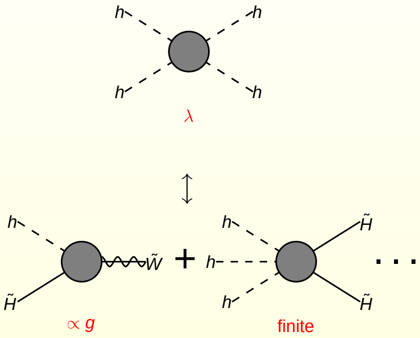


$$0 \stackrel{?}{=} \delta_{\mathrm{SUSY}} \langle hhh\tilde{H} \rangle$$

Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- can be evaluated for two-loop Green functions
- If it is satisfied by DRED $\Leftrightarrow$ multiplicative renormalization o.k.
- Needs to be verified

Quartic coupling and SUSY

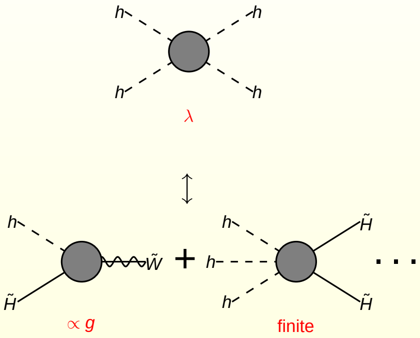


$$0 \stackrel{?}{=} \delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle$$

Obstacle:

- Two-loop evaluation:
  - up to 5-point functions
  - very diffi cult
- previously not feasible

Quartic coupling and SUSY



$$0 \stackrel{?}{=} \delta_{\rm SUSY}\langle hhh\tilde{H}\rangle \equiv \langle \Delta hhh\tilde{H}\rangle$$

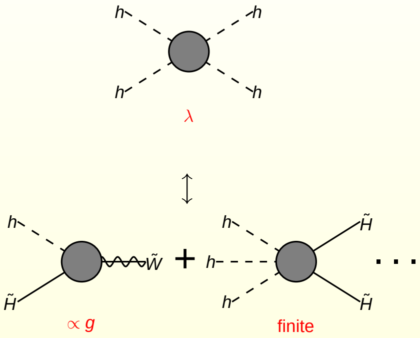
Obstacle:

- Two-loop evaluation:
  - up to 5-point functions
  - very diffi cult
- previously not feasible

Solution:

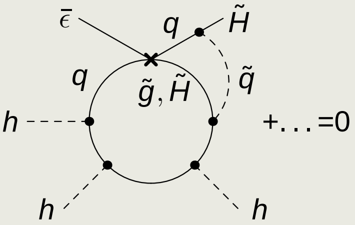
- Recent proof in DRED
- $\Delta = \delta_{\rm SUSY}\mathcal{L}_{\rm DRED}$ [DS '05]
- $\langle \Delta hhh\tilde{H}\rangle$ easier to evaluate

Quartic coupling and SUSY



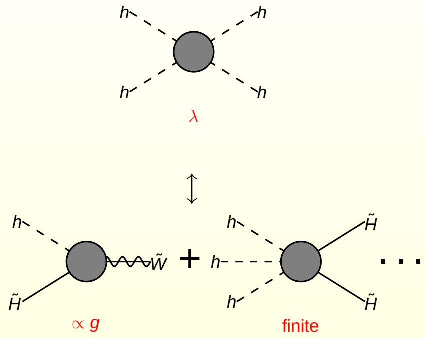
STI valid if

$\langle \Delta hhh\tilde{H} \rangle = 0 \Leftrightarrow$



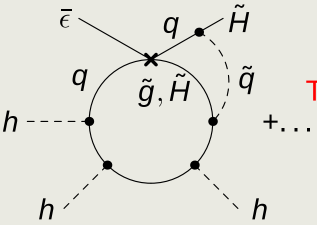
$$0 \stackrel{?}{=} \delta_{\text{SUSY}} \langle hhh\tilde{H} \rangle \equiv \langle \Delta hhh\tilde{H} \rangle$$

Quartic coupling and SUSY



STI valid if

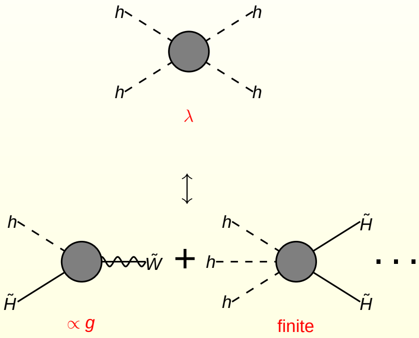
$\langle \Delta hhh\tilde{H} \rangle = 0 \Leftrightarrow$



True!
=0

Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Quartic coupling and SUSY



Results:

- Two-loop STI valid in DRED (in Yukawa-approximation, $\mathcal{O}(\alpha_{t,b}^2, \alpha_{t,b}\alpha_s)$)
- for M_h-calculation at this order, multiplicative renormalization correct
- Previous calculations sufficient

Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Outline

1 Motivation

2 Factorization-Problem

3 Supersymmetry and M_h-calculations

4 Conclusions

Summary and Results

SUSY

- Method to verify SUSY

$$\delta_{\mathrm{SUSY}}\langle hhh\tilde{H}\rangle \equiv \langle \Delta hhh\tilde{H}\rangle$$

- Performed verification for quartic Higgs coupling
- Relevant for M_h at $\mathcal{O}(\alpha_{t,b}^2, \alpha_{t,b}\alpha_s)$

[DS '05] [Hollik, DS '05]

QCD-Factorization

- Treat g, ϕ independently in subtraction terms

$$\sum_{m=g,\phi} P_{G\to mG}\, \sigma(Gm \to t\bar{t})$$

- Factorization holds in DRED!

[Signer, DS '05]

Consequences and Outlook

Hadron cross section

- subtracted hard scattering cross section in general:

$$\hat{\sigma}_{\text{NLO}} = \sigma_{\text{NLO}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P\right) \otimes \sigma_{\text{LO}}$$

Consequences and Outlook

Hadron cross section

- subtracted hard scattering cross section in general:

$$\hat{\sigma}_{\mathrm{NLO}} = \sigma_{\mathrm{NLO}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P\right) \otimes \sigma_{\mathrm{LO}}$$

- subtracted hard scattering cross section in DREG:

$$\hat{\sigma}_{\mathrm{NLO}} = \sigma_{\mathrm{NLO}}^{\mathrm{DREG}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P_{g\to gg}\right) \otimes \sigma_{\mathrm{LO}}^{\mathrm{DREG}}$$

Consequences and Outlook

Hadron cross section

- subtracted hard scattering cross section in general:

$$\hat{\sigma}_{\mathrm{NLO}} = \sigma_{\mathrm{NLO}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P\right) \otimes \sigma_{\mathrm{LO}}$$

- subtracted hard scattering cross section in DRED:

$$\hat{\sigma}_{\mathrm{NLO}} = \sigma^{\mathrm{DRED}}_{\mathrm{NLO}} - \sum_m \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P_{G\to mG}\right) \otimes \sigma^{\mathrm{DRED}}_{\mathrm{LO}}(m)$$

Consequences and Outlook

Hadron cross section

- subtracted hard scattering cross section in general:

$$\hat{\sigma}_{\text{NLO}} = \sigma_{\text{NLO}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P\right) \otimes \sigma_{\text{LO}}$$

- the same $\hat{\sigma}$ can be obtained in DREG and DRED

Consequences and Outlook

Hadron cross section

- subtracted hard scattering cross section in general:

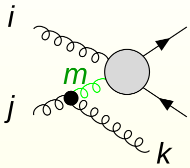
$$\hat{\sigma}_{\mathrm{NLO}} = \sigma_{\mathrm{NLO}} - \left(\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P\right) \otimes \sigma_{\mathrm{LO}}$$

- the same $\hat{\sigma}$ can be obtained in DREG and DRED

$$\sigma_{\mathrm{had,NLO}} = f_{\mathrm{parton}} \otimes \hat{\sigma}_{\mathrm{NLO}}$$

- with the same f_{parton} in DREG and DRED

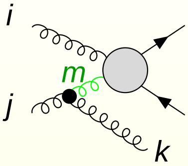
Understanding factorization in DRED



General result, $i, j, k, m = g, \phi$:

$$\sigma(ij \to t\bar{t}k) \xrightarrow{2\|3} \sim P_{j\to mk}\ \sigma(im \to t\bar{t})$$

Understanding factorization in DRED



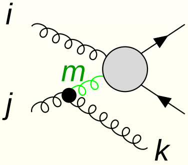
General result, $i, j, k, m = g, \phi$:

$$\sigma(ij \to t\bar{t}k) \xrightarrow{2\|3} \sim P_{j\to mk}\ \sigma(im \to t\bar{t})$$

Reconcile "non-factorizing terms" with factorization

$$K_g\, \sigma(GG \to t\bar{t}, m\text{-terms only}) \to P_{\phi\to g\phi}\, [\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})]$$

Understanding factorization in DRED



General result, $i, j, k, m = g, \phi$:

$$\sigma(ij \to t\bar{t}k) \xrightarrow{2\|3} \sim P_{j\to mk}\, \sigma(im \to t\bar{t})$$

Reconcile "non-factorizing terms" with factorization

$$K_g\, \sigma(GG \to t\bar{t}, m\text{-terms only}) \to P_{\phi\to g\phi}\, [\sigma(Gg \to t\bar{t}) - \sigma(G\phi \to t\bar{t})]$$

$$\sigma(Gg \to t\bar{t}) \neq \sigma(G\phi \to t\bar{t})?$$

- General criterium
- Our case: for $m = 0$ problem disappears