Explicit results for the anomalous three point functions and non-renormalization theorems

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Outline of Talk:

① **Motivation: Electroweak 2–loop contribution to muon magnetic moment**

② **Method: Gröbner basis technique**

③ **Results of two-loop calculations**

⑤ **Summary**

① Electroweak 2–loop contribution to muon magnetic moment

Electroweak two–loop calculations:

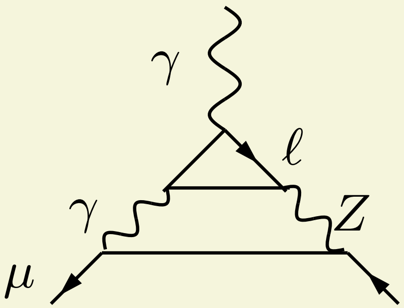
Leptons: yield large terms proportional to

$$\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$$

enhanced by a large logarithm.

(Kukhto, Kuraev, Schiller, Silagadze, 1992)

The most important diagrams VVA triangle diagrams ($VVV = 0, VVA \neq 0$)



$$a_\mu^{(4)\ \mathrm{EW}}(\ell) \simeq -\frac{\sqrt{2} G_\mu m_\mu^2}{16\pi^2} \frac{\alpha}{\pi} \left[3 \ln \frac{M_Z^2}{m_\ell^2} + C_\ell \right]$$

Anomaly cancellation by lepton quark duality:

$$\sum_f N_{cf} Q_f^2 T_{3f} = 0$$

Need consider complete family: $\Rightarrow$ hadronic effects in quark triangle graphs?

Quark parton model:

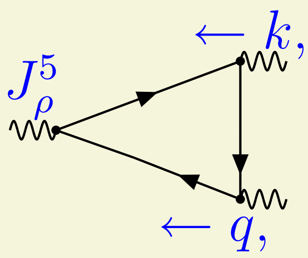
(Czarnecki, Krause, Marciano, 1995)

First family in QPM would yields

$$a_\mu^{(4)\,\mathrm{EW}}([e,u,d])_{\mathrm{QPM}} \simeq -\frac{\sqrt{2}G_\mu\, m_\mu^2}{16\pi^2}\,\frac{\alpha}{\pi}\,\left[\ln\frac{m_u^8}{m_\mu^6 m_d^2}+\frac{17}{2}\right]$$

need to know ill defined constituent quark masses. Note: large logs $\sim \ln M_Z$ have dropped!

Structure of $Z^* \gamma \gamma^*$ vertex:



$$J_\mu = \overline{q}^i_f V_{fh} \gamma_\mu q^i_h$$

$$J^5_\rho = \overline{q}^i_f A_{fh} \gamma_\rho \gamma_5 q^i_h$$

where V and A are diagonal matrices and $TrA = 0$ and considered the correlator

$$T_{\mu\nu\rho} = -\int d^4x d^4y e^{ikx - iqy} < 0|T \left\{ J_\mu(x) J_\nu(y) J^5_\rho(0) \right\} |0 >$$

Adler-Bardeen theorem proves nonrenormalization for the longitudinal part of triangles associated with the divergence of the axial current.

There was no general statement about the transversal part of triangle. This part, even it's existence, depends on the choice of external momenta.

At small k, multiplying correlator by polarization vector of the soft photon $e^{\mu}(k)$

$$T_{\mu\nu\rho}e^{\mu}(k) = -\frac{i}{4\pi^2}\left[w_T(q^2)(-q^2\widetilde{f}_{\nu\rho} + q_\nu q^\sigma \tilde{f}_{\sigma\rho} - q_\rho q^\sigma \widetilde{f}_{\sigma\nu}) + w_L(q^2)q_\rho q^\sigma \widetilde{f}_{\sigma\nu}\right]$$

$$\widetilde{f}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}f^{\rho\sigma}, \qquad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu$$

Contribution to the $(g-2)_\mu$

$$\Delta a_\mu^{EW} = \frac{\alpha}{\pi}\frac{G_\mu m_\mu^2}{8\pi^2\sqrt{2}}\int dQ^2\left(w_L(Q^2) + \frac{m_Z^2}{m_Z^2+Q^2}w_T(Q^2)\right)$$

Analysis in different integration regions is needed. At $Q^2 >> m^2$ Vainshtein proved theorem (A.Vainshtein Phys.Lett. B569(2003) 187):

$$w_L[m=0] = 2w_T[m=0].$$

Since w_L is of one-loop order Vainshtein's relation proves absence of perturbative corrections to the transversal part of fermion triangles.

Generalization of Vainshtein's theorem - Knecht et.al. (JHEP03 (2004) 035):

$$V_\mu^a = \overline{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi \quad , \quad A_\mu^a = \overline{\psi}\gamma_\mu\gamma_5 \frac{\lambda^a}{2}\psi \quad , \quad \psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} .$$

with $a, b, c = 3, 8$ they considered the QCD three point functions

$$\mathcal{W}^{abc}_{\mu\nu\rho}(q_1, q_2) = i \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \times \langle\, 0 \,|\, \mathsf{T}\{V_\mu^a(x_1) V_\nu^b(x_2) A_\rho^c(0)\} \,|\, 0 \,\rangle$$

$$\equiv \quad \frac{1}{2}\, d^{abc}\, \mathcal{W}_{\mu\nu\rho}(q_1, q_2)\, ,$$

The general decomposition of this function into invariant functions due to Ward identities $q_{1\mu}\mathcal{W}_{\mu\nu\rho}(q_1, q_2) = 0$, $q_{2\nu}\mathcal{W}_{\mu\nu\rho}(q_1, q_2) = 0$, has four terms

$$\begin{aligned} \mathcal{W}_{\mu\nu\rho}(q_1, q_2) \quad = \quad -\frac{1}{8\pi^2} \Big\{ & - w_L\left(q_1^2, q_2^2, q_3^2\right)(q_1 + q_2)_\rho\, \epsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \\ & + w_T^{(+)}\left(q_1^2, q_2^2, q_3^2\right) t^{(+)}_{\mu\nu\rho}(q_1, q_2) \\ & + w_T^{(-)}\left(q_1^2, q_2^2, q_3^2\right) t^{(-)}_{\mu\nu\rho}(q_1, q_2) + \widetilde{w}_T^{(-)}\left(q_1^2, q_2^2, q_3^2\right) \widetilde{t}^{(-)}_{\mu\nu\rho}(q_1, q_2) \Big\}\, , \end{aligned}$$

with the transverse tensors

$$t^{(+)}_{\mu\nu\rho}(q_1,q_2) = q_{1\nu}\,\epsilon_{\mu\rho\alpha\beta}\,q_1^\alpha q_2^\beta \;-\; q_{2\mu}\,\epsilon_{\nu\rho\alpha\beta}\,q_1^\alpha q_2^\beta$$
$$-\,(q_1\cdot q_2)\,\epsilon_{\mu\nu\rho\alpha}\,(q_1-q_2)^\alpha + \frac{q_1^2+q_2^2-q_3^2}{q_3^2}\,\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta (q_1+q_2)_\rho\,,$$

$$t^{(-)}_{\mu\nu\rho}(q_1,q_2) = \left[(q_1-q_2)_\rho - \frac{q_1^2-q_2^2}{(q_1+q_2)^2}\,(q_1+q_2)_\rho\right]\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta$$
$$\widetilde{t}^{(-)}_{\mu\nu\rho}(q_1,q_2) = q_{1\nu}\,\epsilon_{\mu\rho\alpha\beta}\,q_1^\alpha q_2^\beta + q_{2\mu}\,\epsilon_{\nu\rho\alpha\beta}\,q_1^\alpha q_2^\beta$$
$$-\,(q_1\cdot q_2)\,\epsilon_{\mu\nu\rho\alpha}(q_1+q_2)^\alpha\,.$$

Bose symmetry entails

$$w_T^{(+)}\left(q_2^2,q_1^2,q_3^2\right) = +w_T^{(+)}\left(q_1^2,q_2^2,q_3^2\right)$$
$$w_T^{(-)}\left(q_2^2,q_1^2,q_3^2\right) = -w_T^{(-)}\left(q_1^2,q_2^2,q_3^2\right), \qquad \widetilde{w}_T^{(-)}\left(q_2^2,q_1^2,q_3^2\right) = -\widetilde{w}_T^{(-)}\left(q_1^2,q_2^2,q_3^2\right)\,.$$

In addition, the longitudinal part is entirely fixed by the anomaly,

$$w_L\left(q_1^2,q_2^2,q_3^2\right) = -\frac{2N_C}{q_3^2}\,.$$

The following three non renormalization theorems were derived,

$$\left[w_T^{(+)} + w_T^{(-)}\right](q_1^2, q_2^2, q_3^2) - \left[w_T^{(+)} + w_T^{(-)}\right](q_3^2, q_2^2, q_1^2) = 0$$

$$\left[\widetilde{w}_T^{(-)} + w_T^{(-)}\right](q_1^2, q_2^2, q_3^2) + \left[\widetilde{w}_T^{(-)} + w_T^{(-)}\right](q_3^2, q_2^2, q_1^2) = 0$$

$$\left[w_T^{(+)} + \widetilde{w}_T^{(-)}\right](q_1^2, q_2^2, q_3^2) + \left[w_T^{(+)} + \widetilde{w}_T^{(-)}\right](q_3^2, q_2^2, q_1^2)\, w_L\,(q_3^2, q_2^2, q_1^2)$$
$$= -\frac{2\,(q_2^2 + q_1 \cdot q_2)}{q_1^2}\, w_T^{(+)}\,(q_3^2, q_2^2, q_1^2) + 2\,\frac{q_1 \cdot q_2}{q_1^2}\, w_T^{(-)}\,(q_3^2, q_2^2, q_1^2)\ ,$$

which hold for all values of the momentum transfers q_1^2, q_2^2 and q_3^2.

Vainshtein's non renormalization theorem appears as a particular case. Upon taking $q_1 = k \pm q$, $q_2 = -k$, and keeping only the terms linear in the momentum k, gives

$$
\begin{aligned}
t^{(+)}_{\mu\nu\rho}(k \pm q, -k) &= q^2 \epsilon_{\mu\nu\rho\sigma} k^\sigma - q_\mu \epsilon_{\nu\rho\alpha\beta} q^\alpha k^\beta - q_\rho \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta + \mathcal{O}(k^2) \\
t^{(-)}_{\mu\nu\rho}(k \pm q, -k) &= \mathcal{O}(k^2) \\
\widetilde{t}^{(-)}_{\mu\nu\rho}(k \pm q, -k) &= q^2 \epsilon_{\mu\nu\rho\sigma} k^\sigma - q_\mu \epsilon_{\nu\rho\alpha\beta} q^\alpha k^\beta - q_\rho \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta + \mathcal{O}(k^2) \,.
\end{aligned}
$$

Within this same kinematic configuration, the three non renormalization theorems then reduce to one single equality,

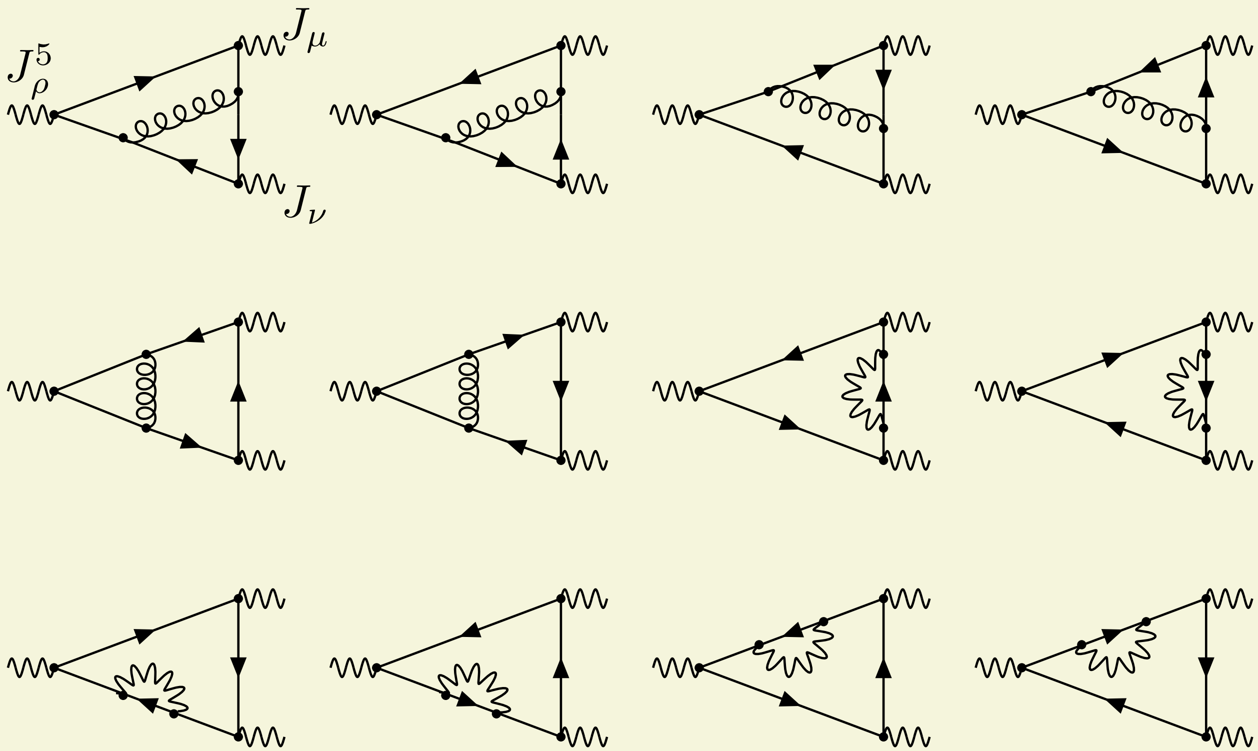
$$
w_L(Q^2) = 2\, w_T(Q^2) \,,
$$

where $Q^2 \equiv -q^2$ and

$$
\begin{aligned}
w_L(Q^2) &= w_L(-Q^2, 0, -Q^2) \\
w_T(Q^2) &= w_T^{(+)}(-Q^2, 0, -Q^2) + \widetilde{w}_T^{(-)}(-Q^2, 0, -Q^2) \,.
\end{aligned}
$$

Nonrenormalization theorems were not checked in explicit calculations. We decided to check them in massless QCD at the two-loop level for arbitrary external kinematics

Two-loop QCD diagrams contributing to $< AVV >$ correlator



Gröbner basis technique for Feynman integrals:

1. **O.V. Tarasov** *"Reduction of Feynman graph amplitudes to a minimal set of basic integrals"*
, Acta Physica Polonica, v B29 (1998) 2655

2. **O. V. Tarasov,** *"Computation of Groebner bases for two-loop propagator type integrals,"*
Talk at ACAT-2003
Nucl. Instrum. Meth. A 534 (2004) 293 [arXiv:hep-ph/0403253].

Gröbner Basis is a nice set of recurrence relations or differential relations for Feynman integrals allowing to reduce large (in principle infinite) number of integrals in terms of finite number of integrals

Main steps of the algorithm:

- Tensor integrals express in terms of scalar ones with shifted space-time dimension

$$I_{\mu\nu\ldots} = T_{\mu\nu\ldots}(\partial, \mathbf{d}^+) I$$

- Scalar integrals with dots on lines represent as derivatives w.r.t. masses

$$\int \frac{d^d k_1 \ldots d^d k_L}{\ldots (k_1^2 - m_1^2)^{\nu_1} ((k_1 - p_1)^2 - m_2^2)^{\nu_2} \ldots}$$
$$= \frac{1}{(\nu_1 - 1)!(\nu_2 - 1)!} \frac{\partial^{\nu_1}}{\partial (m_i^2)^{\nu_1}} \frac{\partial^{\nu_2}}{\partial (m_j^2)^{\nu_2}}$$
$$\times \int \frac{d^d k_1 \ldots d^d k_L}{\ldots (k_1^2 - m_i^2)((k_1 - p_1)^2 - m_j^2) \ldots} \Big|_{m_i^2 = m_1^2, m_j^2 = m_2^2},$$

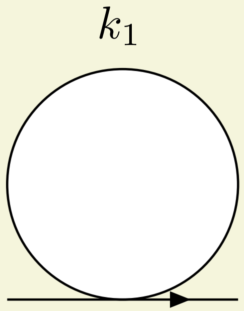
- For scalar integrals with different number of lines write down generalized recurrence relations and transform them into a system of differential equations
- Find differential Gröbner basis for this overdetermined system
- Use relations from the Gröbner basis to reduce all possible integrals (i.e. higher order derivatives) in terms of fixed finite number of basic integrals (i.e. lower order derivatives)
- To reduce integrals $I^{(d+2j)}$ with shifted space-time dimension use relation:

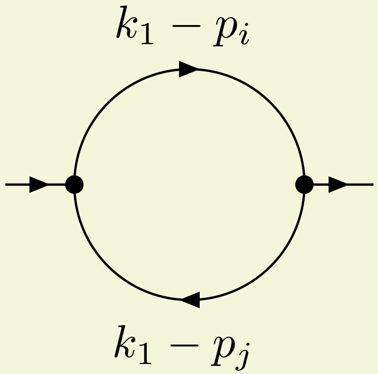
$$I^{(d-2)} = D(\partial_j) I^{(d)}$$

A very similar technique can be formulated without transformation to differential representation and introduction of different masses i.e. for integrals with different powers of propagators and with particular fixed masses.

In general case Gröbner basis for systems of recurrence relations has more parameters than differential Gröbner basis: additionally to masses, powers of propagators must be kept as parameters. Also number of terms in recurrence relations from the Gröbner basis is more than in differential Gröbner basis. It may be more effective for special kinematical configurations.

Example of DGB for one-loop integrals





The basis for tadpole integral consists of two relations:

$$\partial_i T_i^{(d)} = \frac{d-2}{2m_i^2} T_i^{(d)}, \qquad T_i^{(d+2)} = -\frac{2m_i^2}{d} T_i^{(d)}$$

where

$$\partial_j = \frac{\partial}{\partial m_j^2} \qquad \text{and} \qquad T_i^{(d)} = \frac{1}{i\pi^{(d/2)}} \int \frac{d^d k_1}{k_1^2 - m_i^2}.$$

The Gröbner basis for propagator type integral consists of three relations:

$$2\lambda_{ij} \partial_i I_{2,ij}^{(d)} = (3-d)(\partial_i \lambda_{ij}) I_{2,ij}^{(d)} - \frac{\partial \lambda_{ij}}{\partial p_{ij}} \frac{(d-2)}{2m_i^2} T_i^{(d)} + 2(d-2) T_j^{(d)},$$

$$2\lambda_{ij} \partial_j I_{2,ij}^{(d)} = (3-d)(\partial_j \lambda_{ij}) I_{2,ij}^{(d)} - \frac{\partial \lambda_{ij}}{\partial p_{ij}} \frac{(d-2)}{2m_j^2} T_j^{(d)} + 2(d-2) T_i^{(d)},$$

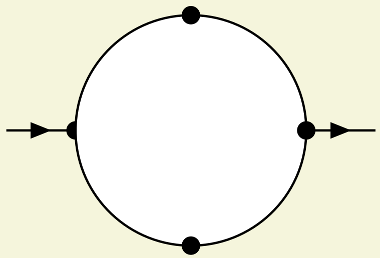
$$(d-1)g_{ij}I^{(d+2)}_{2,ij} = 2\lambda_{ij}I^{(d)}_{2,ij} + (\partial_i\lambda_{ij})T^{(d)}_j + (\partial_j\lambda_{ij})T^{(d)}_i,$$

where

$$\lambda = -p^2_{ij} - m^4_i - m^4_j + 2p_{ij}m^2_i + 2p_{ij}m^2_j + 2m^2_i m^2_j,$$

$$g_{ij} = -4p_{ij} = -4(p_i - p_j)^2.$$

Application of DGB to an integral



$$\frac{1}{i\pi^{d/2}}\int\frac{d^dk_1}{(k_1^2-m^2)^2((k_1-p_1)^2-m^2)^2}=\partial_i\partial_j\ I^{(d)}_{2,ij}\Big|_{m_i^2=m_j^2=m^2},$$

$$I^{(d)}_{2,ij}=\frac{1}{i\pi^{d/2}}\int\frac{d^dk_1}{(k_1^2-m_i^2)((k_1-p_1)^2-m_j^2)}.$$

$$\partial_i I^{(d)}_{2,ij}=f_1(m_i^2,m_j^2)I^{(d)}_{2,ij}+r_1(m_i^2,m_j^2),$$

$$\partial_j I^{(d)}_{2,ij}=f_2(m_i^2,m_j^2)I^{(d)}_{2,ij}+r_2(m_i^2,m_j^2),$$

$$\partial_j T^{(d)}_j=t_jT^{(d)}_j.$$

$$\begin{aligned}\partial_j\partial_i I^{(d)}_{2,ij} &= \partial_j[f_1(m_i^2,m_j^2)I^{(d)}_{2,ij}+r_1(m_i^2,m_j^2)]\\ &= (\partial_j f_1)I^{(d)}_{2,ij}+f_1\partial_j I^{(d)}_{2,ij}+\partial_j r_1\\ &= [\partial_j f_1+f_1f_2]I^{(d)}_{2,ij}+f_1r_2+\partial_j r_1\end{aligned}$$

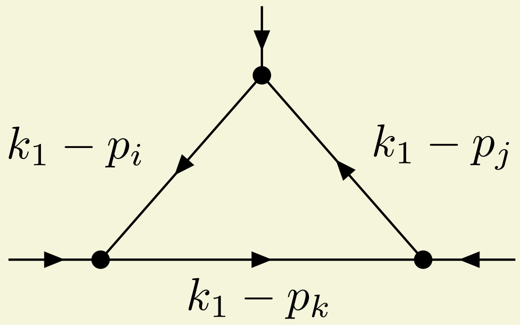
$$r_1 = \frac{1}{2\lambda_{ij}}\left[-\frac{\partial\lambda_{ij}}{\partial p_{ij}}\frac{(d-2)}{2m_i^2}T_i^{(d)} + 2(d-2)T_j^{(d)}\right] = f_3 T_j^{(d)} + r_3 T_i^{(d)},$$

$$r_2 = f_4 T_i^{(d)} + r_4 T_j^{(d)}.$$

$$\begin{aligned}\partial_j r_1 &= \partial_j[f_3 T_j^{(d)} + r_3 T_i^{(d)}] \\ &= (\partial_j f_3)T_j^{(d)} + f_3\partial_j T_j^{(d)} + (\partial_j r_3)T_i^{(d)} \\ &= [(\partial_j f_3) + f_3 t_j]T_j^{(d)} + T_i^{(d)}\partial_j r_3.\end{aligned}$$

$$\begin{aligned}\partial_j\partial_i I_{2,ij}^{(d)} = [\partial_j f_1 + f_1 f_2]I_{2,ij}^{(d)} + [f_1 f_4 + \partial_j r_3]T_i^{(d)} \\ +[r_4 f_1 + (\partial_j f_3) + f_3 t_j]T_j^{(d)}\end{aligned}$$

Vertex type integrals



The GB for 1-loop vertex integrals consists of 3 differential relations:

$$2\lambda_{ijk}\partial_i I^{(d)}_{3,ijk} = \frac{4-d}{2}(\partial_i\lambda_{ijk}) I^{(d)}_{3,ijk} - 2(d-3)\left[\frac{p_{ij}}{\lambda_{ij}}\frac{\partial\lambda_{ijk}}{\partial y_{ik}} I^{(d)}_{2,ij} + \frac{p_{ik}}{\lambda_{ik}}\frac{\partial\lambda_{ijk}}{\partial y_{ij}} I^{(d)}_{2,jk} + \frac{2p_{jk}}{\lambda_{jk}}\frac{\partial\lambda_{ijk}}{\partial y_{ii}} I^{(d)}_{2,jk}\right] + (d-2)\left[\frac{(\partial_j\lambda_{ijk})}{8m_k^2\lambda_{ik}} T_k^{(d)} + \frac{1}{4m_i^2}\left(\frac{\partial_k\lambda_{ik}}{\lambda_{ik}}\frac{\partial\lambda_{ijk}}{\partial y_{ij}} + \frac{\partial_j\lambda_{ij}}{\lambda_{ij}}\frac{\partial\lambda_{ijk}}{\partial y_{ik}}\right) T_i^{(d)} + \frac{(\partial_k\lambda_{ijk})}{8m_j^2\lambda_{ij}} T_j^{(d)}\right],$$

$+2$ other relations by cyclic permutations

One dimensional recurrency relation

$$(d-2)g_{ijk}I^{(d+2)}_{3,ijk} = 2\lambda_{ijk}I^{(d)}_{3,ijk} + (\partial_i\lambda_{ijk})I^{(d)}_{2,jk} + (\partial_j\lambda_{ijk})I^{(d)}_{2,ik} + (\partial_k\lambda_{ijk})I^{(d)}_{2,ij}.$$

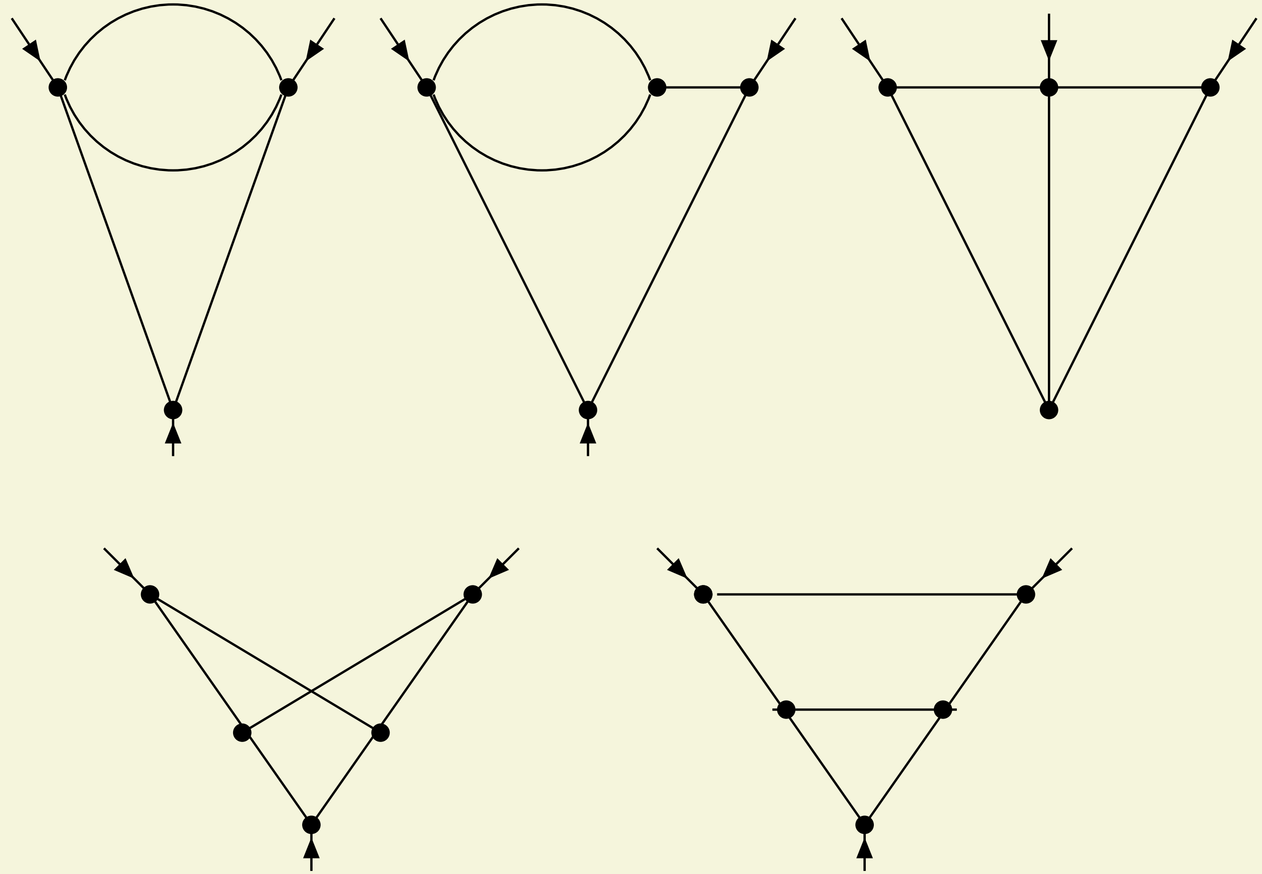
where

$$\begin{aligned}
\lambda_{ijk} = & \quad 2(p_{jk}+p_{ik}-p_{ij})(m_i^2 m_j^2 + p_{ij}m_k^2) \\
& + \quad 2(p_{ik}+p_{ij}-p_{jk})(p_{jk}m_i^2 + m_k^2 m_j^2) \\
& + \quad 2(p_{jk}+p_{ij}-p_{ik})(m_k^2 m_i^2 + p_{ik}m_j^2) \\
& - \quad 2m_j^4 p_{ik} - 2p_{ij}m_k^4 - 2m_i^4 p_{jk} - 2p_{ij}p_{ik}p_{jk},
\end{aligned}$$

$$g_{ijk} = 2p_{ij}^2 - 4(p_{ik}+p_{jk})p_{ij} + 2(p_{ik}-p_{jk})^2.$$

Gröbner basis for two-loop vertex integrals with arbitrary external momenta and masses was constructed. Huge polynomial expressions were calculated once and forever. In real calculations one don't need to manipulate with these polynomials. Only at the final stage particular values of masses and external momenta should be substituted in polynomials and their derivatives.

$\mathrm{Two-loop\ \ vertex\ \ \ type\ \ \ integrals}$



All two-loop diagrams were reduced to sums over 15 different combinations of 6 basis integrals

$$D_j = \sum_{k=1}^{15} M_k \frac{P_k(q_r^2, d)}{Q_k(q_s^2, d)}$$

with P and Q being polynomials in momenta squared and d. Momentum dependence of denominators is simple:

$$Q_k(q_s^2, d) = Q(d)(q_1^2)^{a_k}(q_2^2)^{b_k}(q_3^2)^{c_k}\Delta^{e_k}$$

where a_k, b_k, c_k, e_k, are some numbers, $Q(d)$ - is a polynomial in d and

$$\Delta = q_1^4 + q_2^4 + q_3^4 - 2q_1^2q_2^2 - 2q_1^2q_3^2 - 2q_2^2q_3^2.$$

Definitions of master integrals:

$$I_2^{(d)}(q_j^2) = \int \frac{d^d k_1}{[i\pi^{d/2}]} \frac{1}{k_1^2(k_1 - q_j)^2}$$

$$I_3^{(d)}(q_1^2, q_2^2, q_3^2) = \int \frac{d^d k_1}{[i\pi^{d/2}]} \frac{1}{k_1^2(k_1 - q_1)^2(k_1 - q_2)^2}$$

$$J_3^{(d)}(q_j^2) = \int\int \frac{d^d k_1 d^d k_2}{[i\pi^{d/2}]^2} \frac{1}{k_1^2(k_1 - k_2)^2(k_2 - q_j)^2},$$

$$R_1 = \int\int \frac{d^d k_1 d^d k_2}{[i\pi^{d/2}]^2} \frac{1}{k_1^2 (k_1-k_2)^2 (k_2-q_1)^2 (k_2+q_2)^2},$$

$$R_2 = \int\int \frac{d^d k_1 d^d k_2}{[i\pi^{d/2}]^2} \frac{1}{k_1^4 (k_1-k_2)^2 (k_2-q_1)^2 (k_2+q_2)^2},$$

$$P_5 = \int\int \frac{d^d k_1 d^d k_2}{[i\pi^{d/2}]^2} \frac{1}{k_1^2 k_2^2 (k_1-k_2)^2 (k_1-q_1)^2 (k_2+q_2)^2},$$

Epsilon expansion of master integrals to the order needed in calculations was given the paper by Davydychev and Ussyukina.

To maximally avoid problems with γ_5 the following procedure was adopted:

- We write down all fermion loops starting with the axial-vector vertex
- perform Feynman integrals and Dirac algebra without assuming any property of γ_5 at all.

Each diagram was reduced to 10 combinations of γ matrices:

$$
\begin{aligned}
4iA_1 &= \gamma_\rho\gamma_5\gamma_\mu\gamma_\nu\hat{q}_2, & 4iA_2 &= \gamma_\rho\gamma_5\hat{q}_1\gamma_\mu\gamma_\nu, \\
4iA_3 &= q_2^\mu\gamma_\rho\gamma_5\hat{q}_1\gamma_\nu\hat{q}_2, & 4iA_4 &= q_1^\mu\gamma_\rho\gamma_5\hat{q}_1\gamma_\nu\hat{q}_2, \\
4iA_5 &= -q_2^\nu\gamma_\rho\gamma_5\hat{q}_1\gamma_\mu\hat{q}_2, & 4iA_6 &= -q_1^\nu\gamma_\rho\gamma_5\hat{q}_1\gamma_\mu\hat{q}_2, \\
4iA_7 &= \gamma_\rho\gamma_5\hat{q}_1, & 4iA_8 &= \gamma_\rho\gamma_5\hat{q}_2, \\
4iA_9 &= \gamma_\rho\gamma_5\gamma_\mu, & 4iA_{10} &= \gamma_\rho\gamma_5\gamma_\nu
\end{aligned}
$$

The prescription is sufficient to achieve finite expressions in front of $A_1, \ldots A_{10}$ as $d \to 4$.

After this the usual formulas

$$
\mathrm{Tr}[\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu] = 4i\epsilon_{\alpha\beta\mu\nu},
$$

$$
\mathrm{Tr}[\gamma_5\gamma_\alpha\gamma_\beta] = 0
$$

were used.

We use arbitrary gauge parameter throughout the calculation. The sum of all relevant diagrams is gauge parameter independent.

We write formfactors as

$$w_L(q_1^2,q_2^2,q_3^2) = w_{1,L}(q_1^2,q_2^2,q_3^2) + \alpha_s C_2(R)\, w_{2,L}(q_1^2,q_2^2,q_3^2)$$

$$w_T^{(\pm)}(q_1^2,q_2^2,q_3^2) = w_{1T}^{(\pm)}(q_1^2,q_2^2,q_3^2) \, + \, \alpha_s\, C_2(R) w_{2T}^{(\pm)}(q_1^2,q_2^2,q_3^2)$$

$$\widetilde{w}_T^{(-)}(q_1^2,q_2^2,q_3^2) = \widetilde{w}_{1T}^{(-)}(q_1^2,q_2^2,q_3^2) \, + \, \alpha_s C_2(R)\, \widetilde{w}_{2T}^{(\pm)}(q_1^2,q_2^2,q_3^2)$$

where $\alpha_s = \frac{g_s^2}{16\pi^2}$

Results of our two-loop calculations:

$$
\begin{aligned}
q_3^2 w_{2,L}(q_1^2,q_2^2,q_3^2) &= 1 \\
\widetilde{w}_{2T}^{(-)}(q_1^2,q_2^2,q_3^2) &= -w_{2T}^{(-)}(q_1^2,q_2^2,q_3^2), \\
2q_3^2\Delta^2 w_{2T}^{(-)}(q_1^2,q_2^2,q_3^2) &= -2(x-y)\Delta - 2(x-y)(6xy+\Delta)\Phi^{(1)}(x,y) \\
&+ [18xy+6x^2-6x+(1+x+y)\Delta)]L_x \\
&- [18xy+6y^2-6y+(1+x+y)\Delta)]L_y
\end{aligned}
$$

$$
\begin{aligned}
2q_3^2\Delta^2 w_{2T}^{(+)}(q_1^2,q_2^2,q_3^2) &= -2(6xy+(x+y)\Delta)\Phi^{(1)}(x,y) - 2\Delta \\
&+ [6x+\Delta(x-y-1)]L_x \\
&- [6y+\Delta(x-y+1)]L_y
\end{aligned}
$$

$$
x = \frac{q_1^2}{q_3^2} \;\; y = \frac{q_2^2}{q_3^2}, \quad L_x = \ln x, \;\; L_y = \ln y,
$$

The result in terms of dilogarithms is:

$$\Phi^{(1)}(x,y) = \frac{1}{\lambda}\left\{ 2\left(\mathrm{Li}_2\left(-\rho x\right) + \mathrm{Li}_2\left(-\rho y\right)\right) \right.$$
$$\left. + \ln\frac{y}{x}\ln\frac{1+\rho y}{1+\rho x} + \ln(\rho x)\ln(\rho y) + \frac{\pi^2}{3} \right\},$$

where

$$\lambda(x,y) \equiv \sqrt{\Delta} \quad , \quad \rho(x,y) \equiv 2\left(1-x-y+\lambda\right)^{-1}, \;\; \Delta = (1-x-y)^2 - 4xy.$$

Our explicit expressions satisfy nonrenormalization theorems by Knecht et al.

Comparison with the results of one-loop calculation reveal that

$$\mathcal{W}_{\mu\nu\rho}(q_1, q_2)\,|_{two-loop} = 4C_2(R)\alpha_s \mathcal{W}_{\mu\nu\rho}(q_1, q_2)\,|_{one-loop}$$

Knecht's et al nonrenormalization theorems are not enough to explain this relation.

Taking into account rather nontrivial momentum dependence of formfactors it is very tempting to suggest that in all orders of perturbation theory

$$\mathcal{W}_{\mu\nu\rho}(q_1, q_2) = f(\alpha_s)\mathcal{W}_{\mu\nu\rho}(q_1, q_2)\,|_{one-loop}$$

where as usual momentum independent factor $f(\alpha_s)$ can be absorbed into the redefi nition of axial vector current

Conclusions

- First application of Gröbner basis technique to the nontrivial two-loop calulations demonstrated it's effectivness
- Surprising relation was possible to descover only due to the nontrivial momentum dependence
- Anomalous three point correlator demonstrate unusually simple structure. One can expect similar effects for other anomalous correlators.