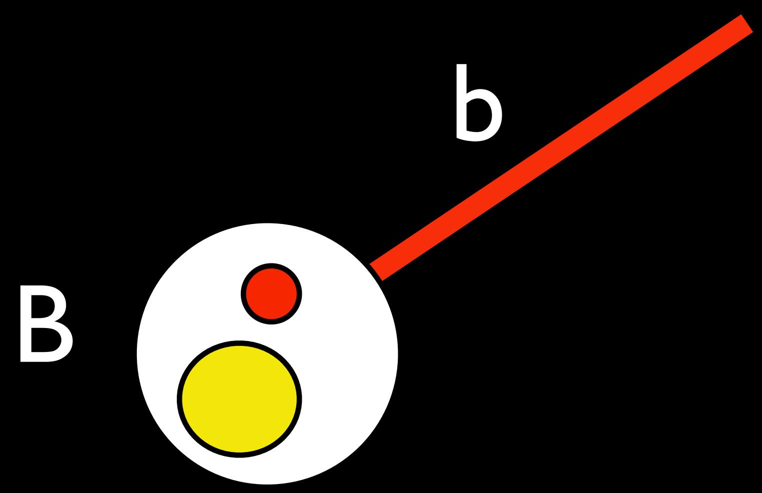
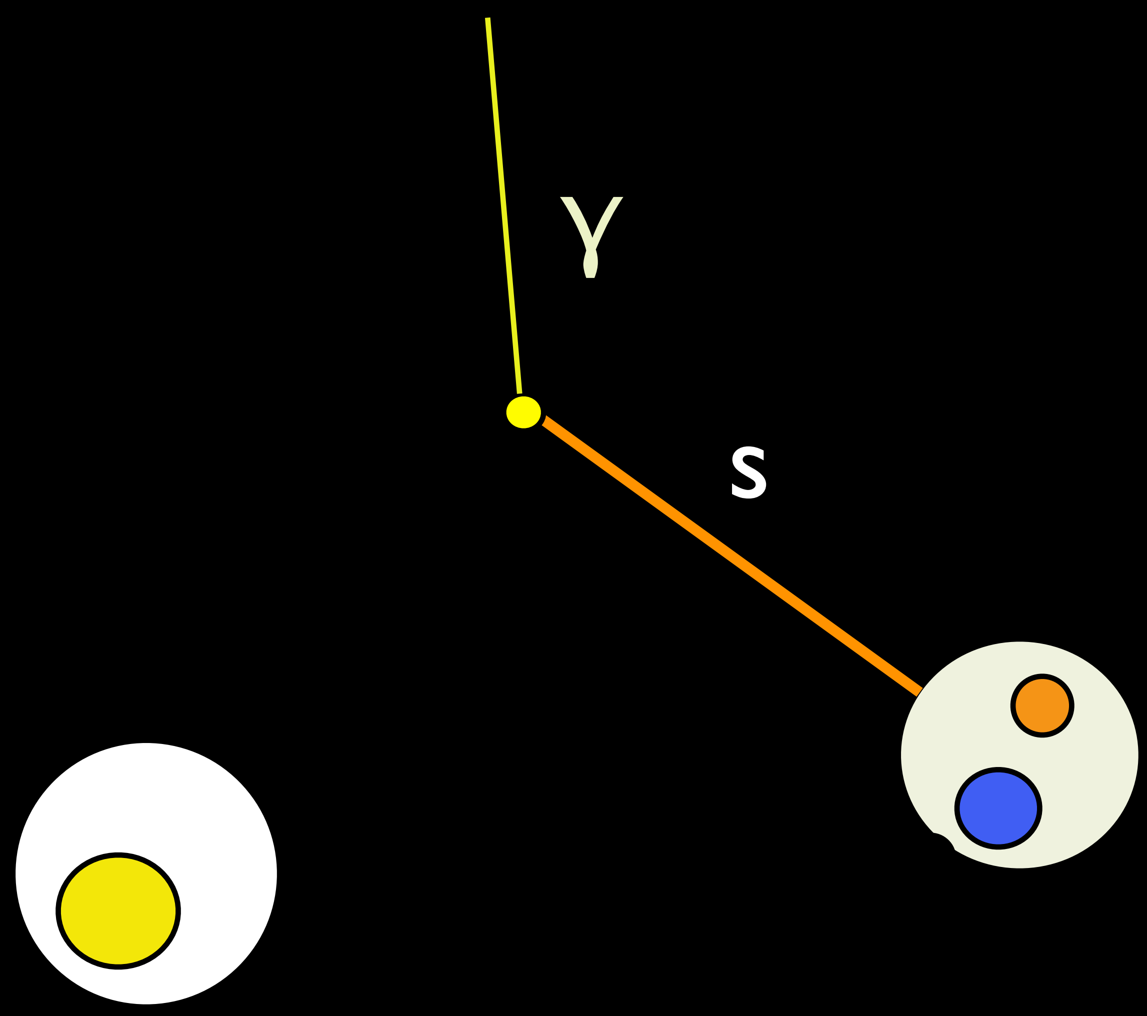
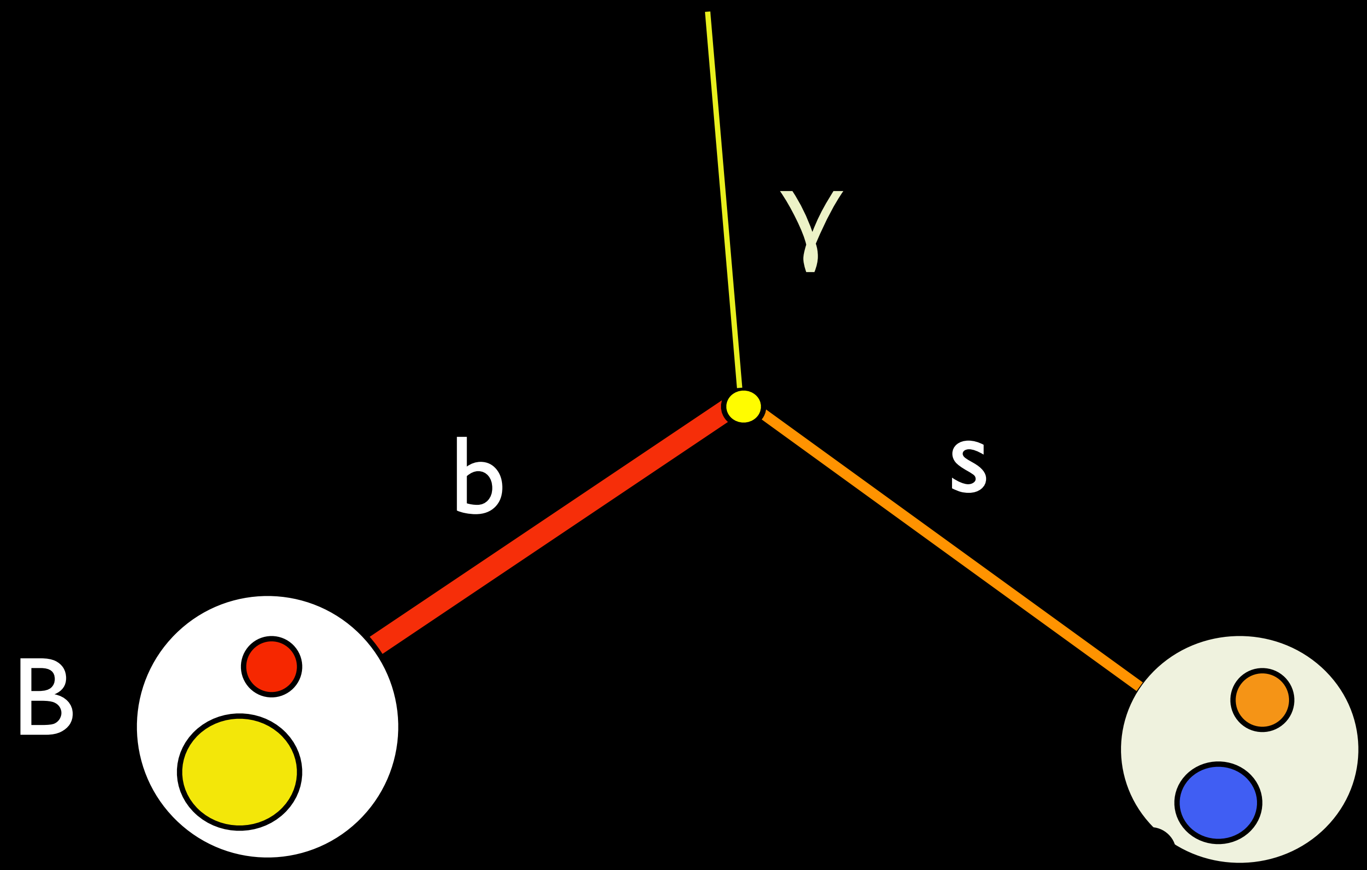

b
B


γ
s


γ
b
s
B

Perturbative QCD corrections

to

$$b \longrightarrow s \ \gamma$$

- outlook and motivations
- transverse momentum distribution
- complete first order

Motivations
for a perturbative approach

$\alpha_S(m_b) \cong 0.22.$ $\alpha_S(M_{Z^0}) = 0.12.$

$\Gamma \propto m_b^5$ large mass effects and unknown CKM ⟶ $B_{SL} = \frac{\Gamma_{SL}}{\Gamma_{TOT}}$

in general uncertainties in these parameters may affect QCD predictions

$$B_{inclusive} = \sum_{n=0}^{\infty} c_n \, \alpha_S^n(m_B). \qquad B_{semi-inclusive} = \sum_{n=0}^{\infty} \sum_{k=0}^{2n} c_{n,k} \, \alpha_S^n \, \log^k x,$$

resummed perturbative series (n,k) $\log^k x$ LL - NLO - NNLO

- QCD next-to-leading resummation with fixed order hard emission

why $b \rightarrow s \, \gamma$

small branching ratio $BR(B \rightarrow X_s \gamma) = (3.34 \pm 0.38) 10^{-4}$

several experimental difficulties

virtues

two body decay - simple kinematics - direct access to the final state photon

widely studied - new physics potential

Transverse momentum distributions

perturbative contributions

double log $$-\frac{\alpha_S C_F}{4\pi}\log^2\frac{p_t^2}{m_b^2}\sim -0.7$$

$$p_t \sim \Lambda_{QCD} = 300 \ \text{MeV}.$$

single log $$-\frac{5\alpha_S C_F}{4\pi}\log\frac{p_t^2}{m_b^2}\sim 0.6.$$

if $$\alpha_S(m_b)\to\alpha_S(p_t)=0.45 \qquad \text{for} \qquad p_t = 1 \ \text{GeV},$$

the logarithmic terms have sizes of order:

$$-\frac{\alpha_S(p_t)C_F}{4\pi}\log^2\frac{p_t^2}{m_b^2}\sim -0.5$$

and

$$-\frac{5\alpha_S C_F}{4\pi}\log\frac{p_t^2}{m_b^2}\sim 0.8.$$

The main difference with respect to resummation in Z^0 decays is a hard scale smaller by over an order of magnitude, i.e. a coupling larger by a factor 2 and infrared logarithms smaller by a factor 3.

b → s γ effective Hamiltonian

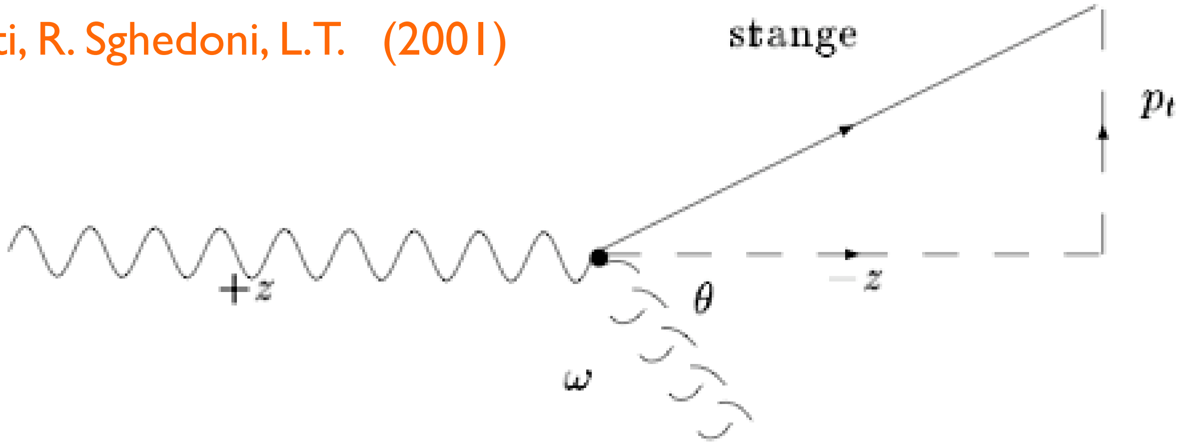
$$\mathcal{H}_{eff}(x) = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{j=1}^{8} C_j(\mu_b)\, \hat{\mathcal{O}}_j(x;\mu_b).$$

$$\begin{aligned}
\hat{\mathcal{O}}_1 &= (\overline{s}_L \gamma_\mu T^a c_L)(\overline{c}_L \gamma^\mu T^a b_L) \\
\hat{\mathcal{O}}_2 &= (\overline{s}_L \gamma_\mu c_L)(\overline{c}_L \gamma^\mu b_L) \\
\hat{\mathcal{O}}_3 &= (\overline{s}_L \gamma_\mu b_L)(\sum_q \overline{q} \gamma^\mu q) \\
\hat{\mathcal{O}}_4 &= (\overline{s}_L \gamma_\mu T^a b_L)(\sum_q \overline{q} \gamma^\mu T^a q_L) \\
\hat{\mathcal{O}}_5 &= (\overline{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L)(\sum_q \overline{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q) \\
\hat{\mathcal{O}}_6 &= (\overline{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L)(\sum_q \overline{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q) \\
\hat{\mathcal{O}}_7 &= \frac{e}{16\pi^2} m_{b,\overline{MS}}(\mu_b) \overline{s}_{L,\alpha} \sigma^{\mu\nu} b_{R,\alpha} F_{\mu\nu} \\
\hat{\mathcal{O}}_8 &= \frac{g}{16\pi^2} m_{b,\overline{MS}}(\mu_b) \overline{s}_{L,\alpha} \sigma^{\mu\nu} T^a_{\alpha\beta} b_{R,\alpha} G^a_{\mu\nu},
\end{aligned}$$

$$\Gamma_0 \simeq \frac{\alpha_{em}}{\pi} \frac{G_F^2\, m_b^3 m_{b,\overline{MS}}^2(m_b)\, |V_{tb} V_{ts}^*|^2}{32\pi^3} C_7^2(\mu_b),$$

K.Chetyrkin, M.Misiak, M.Munz, Phys. Lett. B400, 206 (1997) single gluon emission - transverse momentum distribution

U. Aglietti, R. Sghedoni, L.T. (2001)



$$b \longrightarrow s + \gamma + g$$

$$x \equiv \frac{p_t^2}{Q^2} \quad , \quad Q = m_b$$

$$d(x) = \frac{1}{\Gamma_0} \frac{d\Gamma}{dx}.$$

$$\omega = \frac{2P \cdot k}{m_b^2} = \frac{2E_g}{m_b}, \qquad t = \frac{1 - \cos\theta}{2},$$

$$p_t^2 = k_t^2 = m_b^2 \left(\omega^2 t(1-t)\right).$$

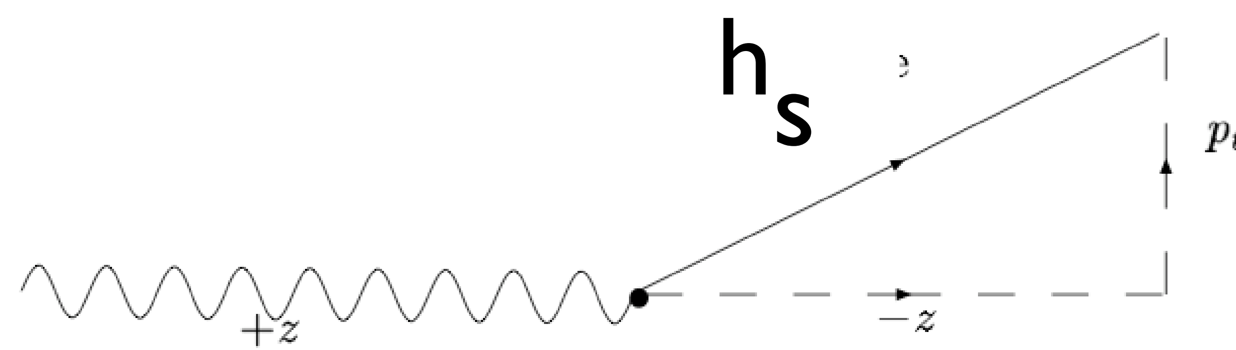
$$d(x) = \delta(x) + \alpha_S \int_0^1 d\omega \int_0^1 dt \left[\frac{A_1}{\omega t} + \frac{S_1(t)}{\omega} + \frac{C_1(\omega)}{t}\right] \left[\delta(x - \omega^2 t) - \delta(x)\right].$$

$$B_1 = C_1 + \frac{S_1}{2} = -\frac{5}{4} \frac{C_F}{\pi}$$

sources of transverse momentum p_t

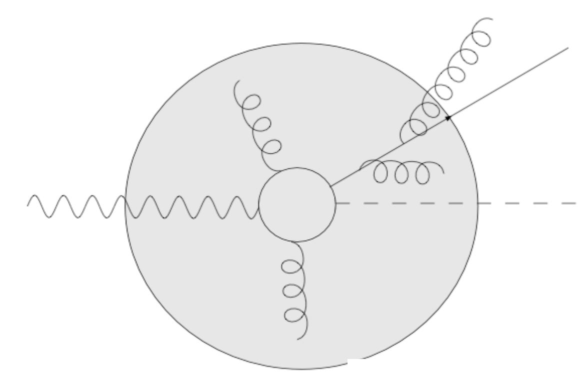
$$B \rightarrow h_s + X + \gamma$$

$$b \rightarrow s + \gamma$$

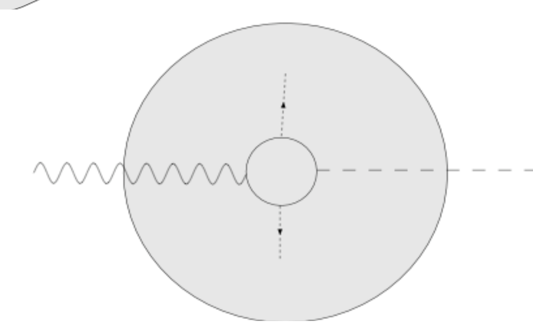


- p_p multiple gluon emissions

$$b \rightarrow s\gamma g_1 \ldots g_n$$

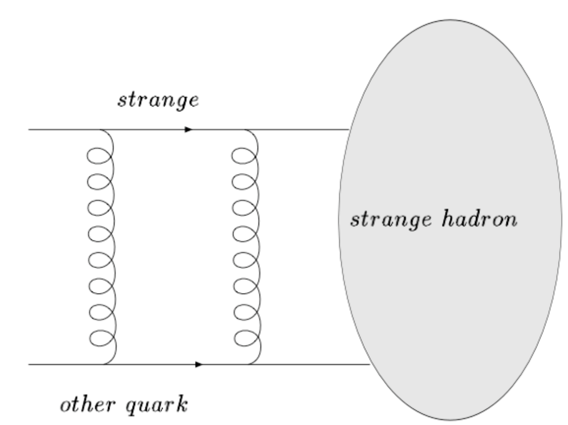


- p_f initial state effects (Fermi motion)



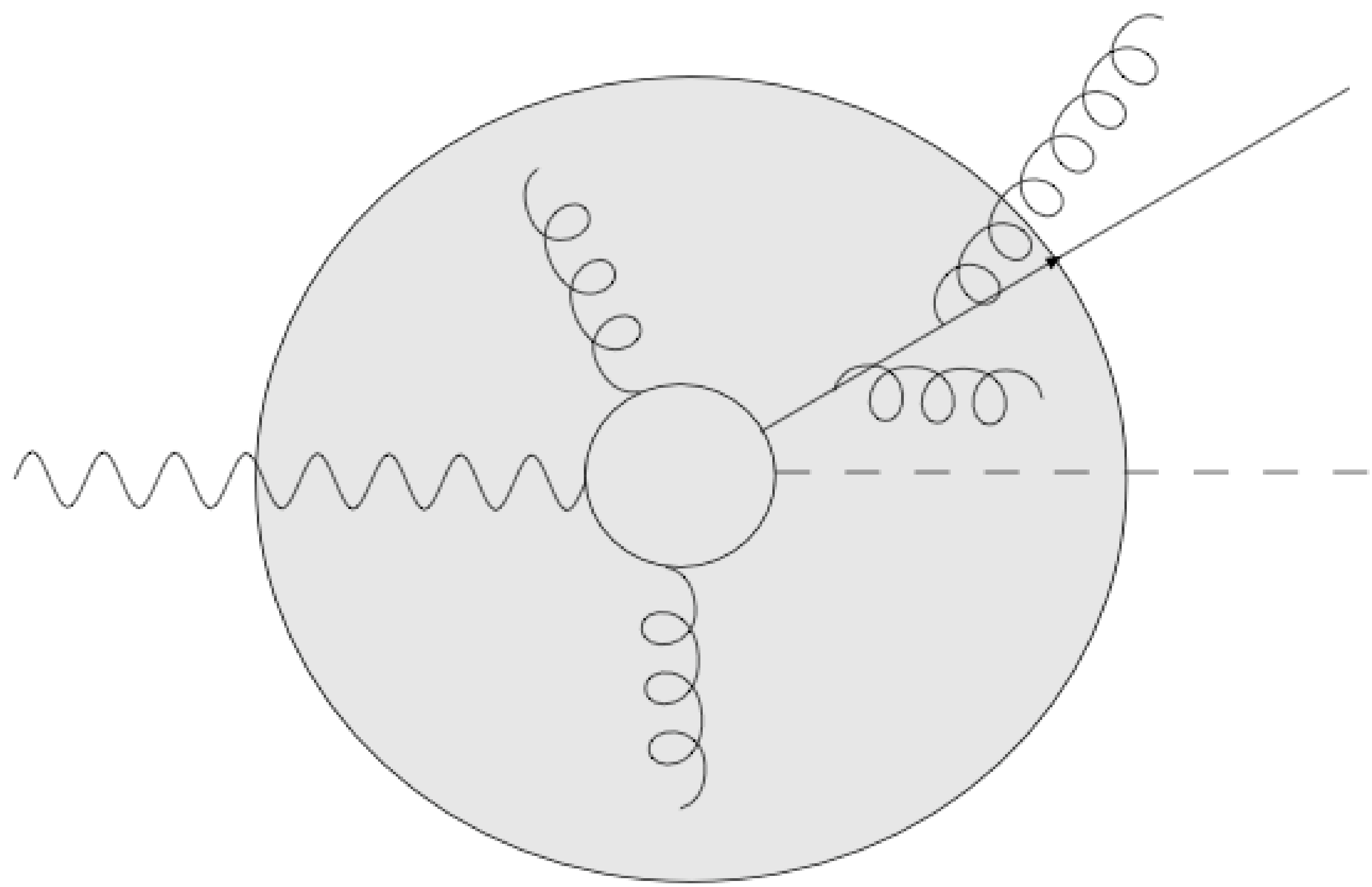
$$p_b = m_B v + p_f .$$

- p_h final state hadronization effects

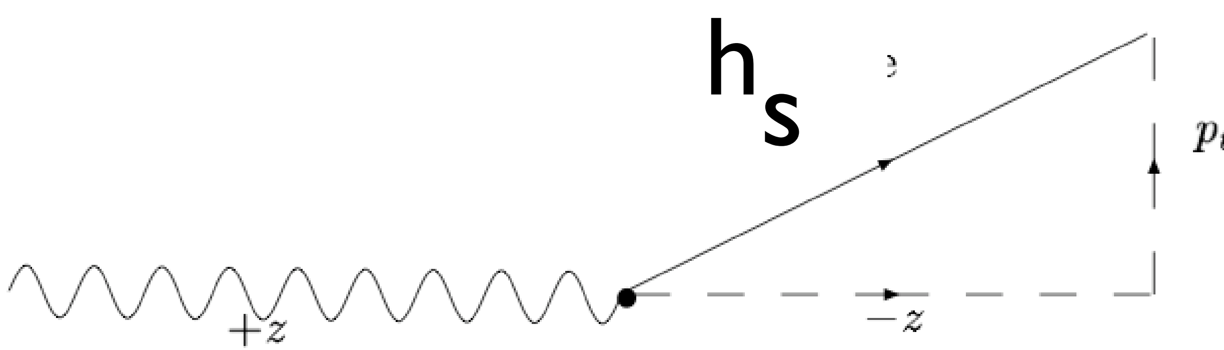


$$p_t = p_p + p_f + p_h$$

P_P

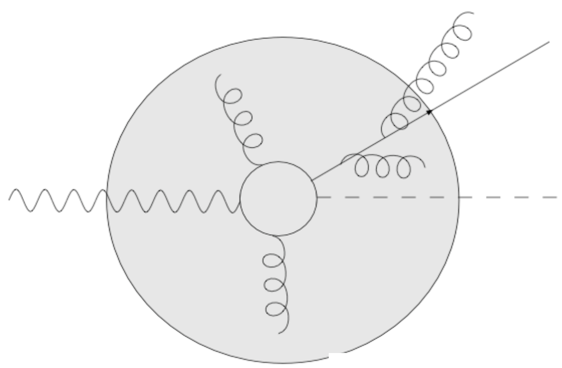


$$B \longrightarrow h_s + X + \gamma \qquad b \longrightarrow s + \gamma$$

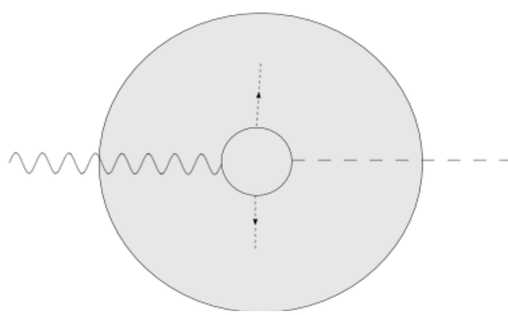


- p_p multiple gluon emissions

$b \longrightarrow s\gamma g_1 \ldots g_n$

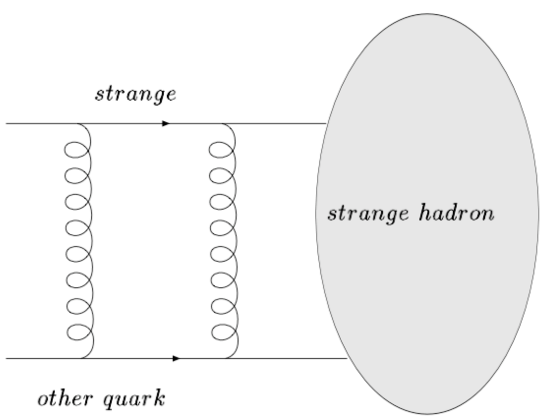


- p_f initial state effects (Fermi motion)



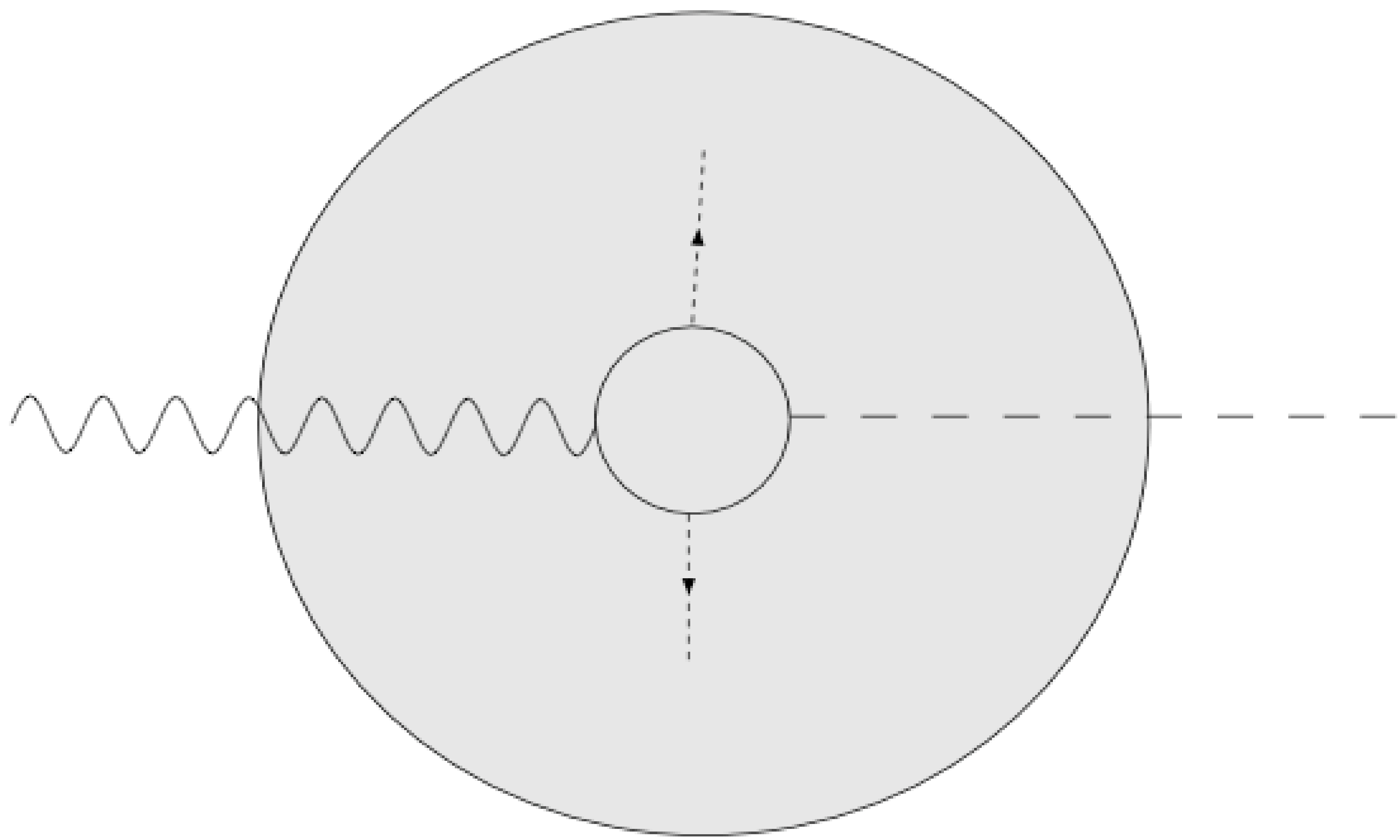
$p_b = m_B v + p_f\,.$

- p_h final state hadronization effects



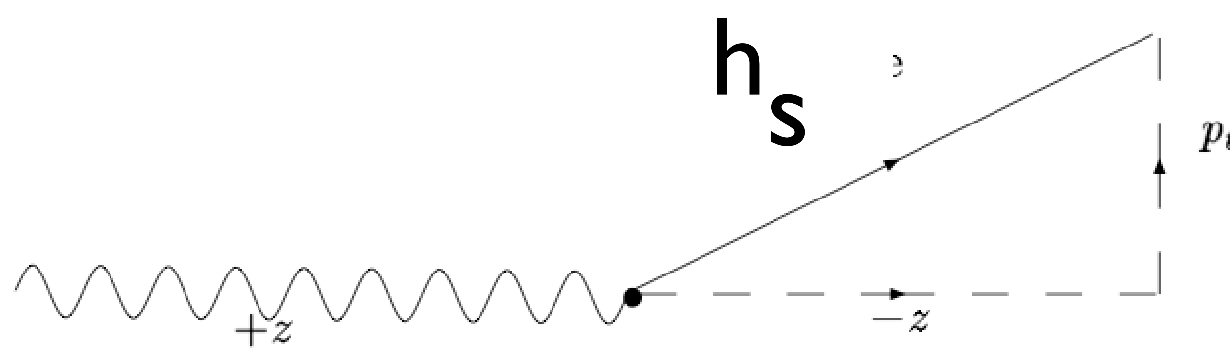
$p_t = p_p + p_f + p_h$

P_f



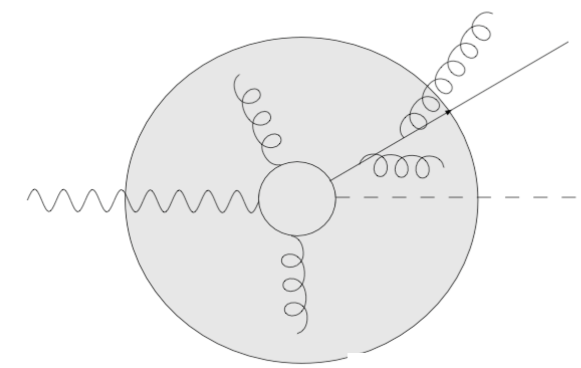
$$B \to h_s + X + \gamma$$

$$b \to s + \gamma$$

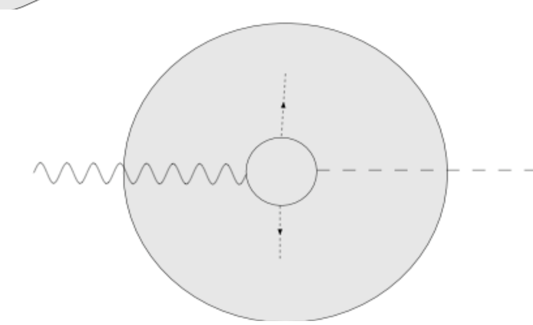


- p_p multiple gluon emissions

$$b \to s\gamma g_1 \ldots g_n$$



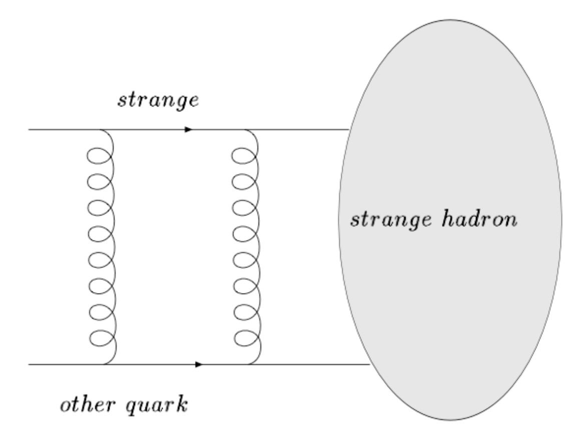
- p_f initial state effects (Fermi motion)



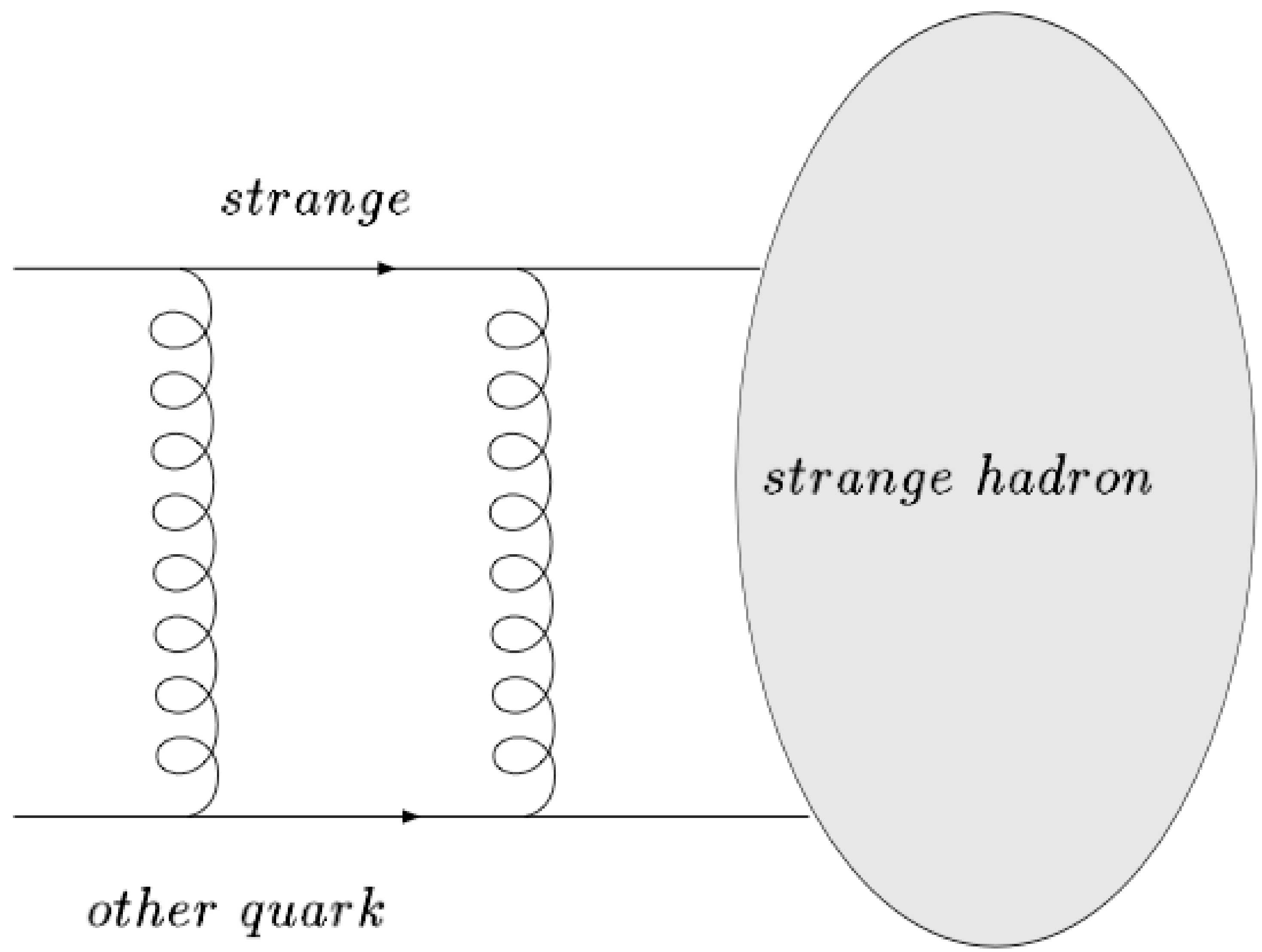
$$p_b = m_B v + p_f .$$

- p_h final state hadronization effects

$$p_t = p_p + p_f + p_h$$

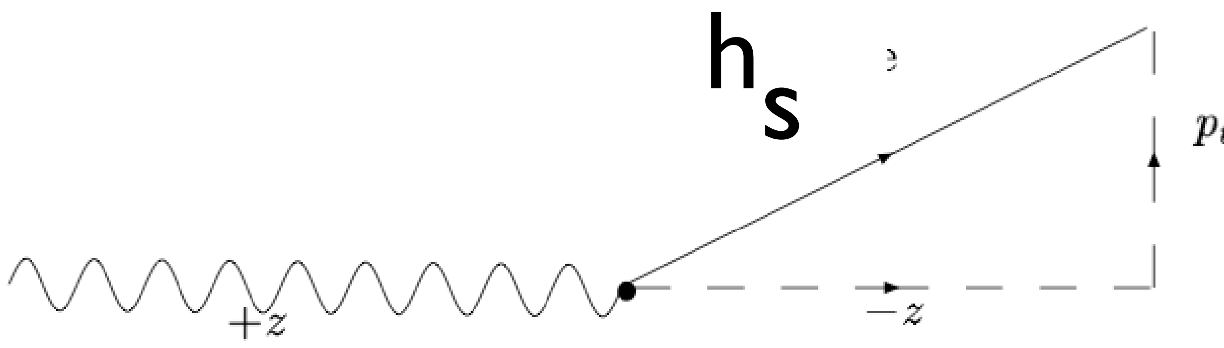


p_h



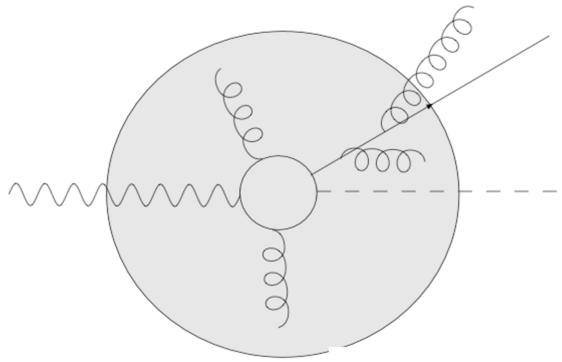
$$B \longrightarrow h_s + X + \gamma$$

$$b \longrightarrow s + \gamma$$

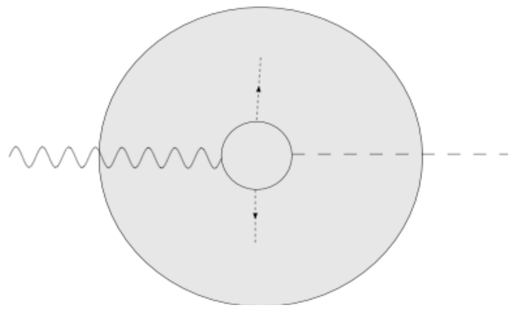


- p_p multiple gluon emissions

$b \longrightarrow s\gamma g_1 \ldots g_n$



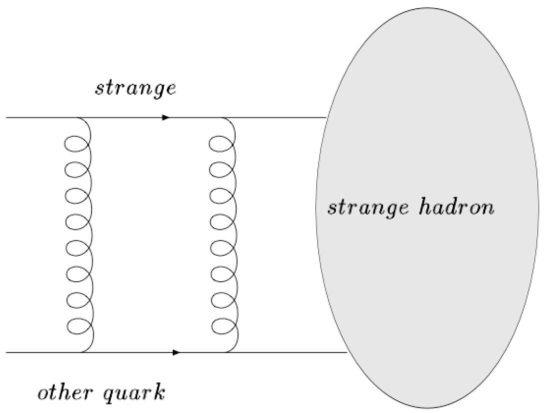
- p_f initial state effects (Fermi motion)



$$p_b = m_B v + p_f \, .$$

- p_h final state hadronization effects

$$p_t = p_p + p_f + p_h$$



multiple gluon factorization

$$b \longrightarrow s + \gamma + g_1 + g_2 \qquad\qquad p_t = -k_{t1} - k_{t2}.$$

$$\frac{1}{\Gamma_B}\frac{d^2\Gamma_2}{dk_{t1}dk_{t2}}\left(k_{t1},k_{t2}\right) \simeq \frac{1}{2}\frac{1}{\Gamma_B}\frac{d\Gamma_1\left(k_{t1}\right)}{dk_{t1}}\frac{1}{\Gamma_B}\frac{d\Gamma_1\left(k_{t2}\right)}{dk_{t2}}.$$

$$\begin{aligned}\frac{1}{\Gamma_0}\frac{d\Gamma}{dp_t}\left(p_t\right) &= \delta\left(p_t\right) + \int dp_{t1}\delta\left(p_t + k_{t1}\right)\frac{1}{\Gamma_0}\frac{d\Gamma_1\left(k_{t1}\right)}{dk_{t1}} + \\ &+ \frac{1}{2}\int dk_{t1}k_{t2}\delta\left(p_t + k_{t1} + k_{t2}\right)\frac{1}{\Gamma_0}\frac{d\Gamma_1\left(k_{t1}\right)}{dk_{t1}}\frac{1}{\Gamma_0}\frac{d\Gamma_1\left(k_{t2}\right)}{dk_{t2}} + \cdots\end{aligned}$$

$$\frac{1}{\Gamma_0}\frac{d\widetilde{\Gamma}}{db}\left(b\right) \equiv \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dp_t \exp\left[ip_t \cdot b\right]\,\frac{1}{\Gamma_0}\frac{d\Gamma\left(p_t\right)}{dp_t}.$$

$$\frac{d\Gamma^{(n)}}{dp_t}\left(p_t\right) = \int_{-\infty}^{+\infty}\prod_{l=1}^{n} dk_{tl}\frac{d^n\Gamma\left(k_{t1},k_{t2},\cdots k_{tn}\right)}{dk_{t1}dk_{t2}\cdots dk_{tn}}\delta\left(p_t + k_{t1} + k_{t2} + \cdots + k_{tn}\right)$$

$$\frac{1}{\Gamma_0}\frac{d^n\Gamma_n\left(k_{t1},k_{t2},\cdots k_{tn}\right)}{dk_{t1}dk_{t2}\cdots dk_{tn}} \simeq \frac{1}{n!}\prod_{l=1}^{n}\frac{1}{\Gamma_0}\frac{d\Gamma_1}{dk_{tl}}\left(k_{tl}\right).$$

Higher Orders

$$A_1\alpha_S \to A_1\alpha_S + A_2\alpha_S^2.$$

$$A_2 = \frac{C_F}{2\pi^2}[C_A(\frac{67}{18} - \frac{\pi^2}{6}) - \frac{5}{9}\, n_f].$$

$$A\left(\alpha_S\right) = \sum_{n=1}^{\infty} A_n\, \alpha_S^n = A_1\alpha_S + A_2\alpha_S^2 + \cdots,$$

$$B\left(\alpha_S\right) = \sum_{n=1}^{\infty} B_n\, \alpha_S^n = B_1\alpha_S + \cdots,$$

$$d(x) = \delta(x) + \alpha_S^{NLO}(Q^2 x)\int_0^1 d\omega \int_0^1 dt \left[\frac{A_1 + \alpha_S^{NLO}(Q^2 x) A_2}{\omega t} + \right.$$
$$\left. + \frac{S_1(t)}{\omega} + \frac{C_1(\omega)}{t}\right]\left[\delta(x - \omega^2 t) - \delta(x)\right]$$

The impact parameter b

$$\frac{1}{\Gamma_0}\frac{d\tilde{\Gamma}}{db} = K\left(\alpha_S\right)\Sigma\left(b;\alpha_S\right) + R\left(b;\alpha_S\right)$$

$$\Sigma(b) = \exp\left\{\int_0^1 dx\; d_R(x)\left[J_0(Qb\sqrt{x}) - 1\right]\right\}$$

$$\Sigma(b) = \exp\left[-\frac{A_1}{4}\alpha_S \log^2\left(\frac{Q^2 b^2}{b_0^2}\right) - B_1\alpha_S \log\left(\frac{Q^2 b^2}{b_0^2}\right)\right]$$

$$\Sigma(b) = \exp\left[L\, g_1(\beta_0 \alpha_S L) + g_2(\beta_0 \alpha_S L) + \alpha_S\, g_3(\beta_0 \alpha_S L) + \cdots\right]$$

$$\begin{aligned}
g_1(\omega) &= \frac{A_1}{2\beta_0}\frac{1}{\omega}\left[\omega + \log(1-\omega)\right], \\
g_2(\omega) &= -\frac{A_2}{2\beta_0^2}\left[\frac{\omega}{1-\omega} + \log(1-\omega)\right] + \\
&+ \frac{A_1\beta_1}{2\beta_0^3}\left[\frac{\omega}{1-\omega} + \frac{\log(1-\omega)}{1-\omega} - \frac{1}{2}\log^2(1-\omega)\right] - \frac{B_1}{\beta_1}\log(1-\omega).
\end{aligned}$$

$$\log\Sigma(b;\alpha_S) = -\frac{1}{4}A_1\alpha_S L^2 - B_1\alpha_S L - \frac{1}{6}A_1\beta_0\alpha_S^2 L^3 - \frac{1}{4}A_2\alpha_S^2 L^2 - \frac{1}{2}B_1\beta_0\alpha_S^2 L^2$$

Treshold distribution

$$f(z) = \frac{1}{\Gamma_0}\frac{d\Gamma}{dz}$$

$$z \equiv \frac{2E_\gamma}{m_B} = 1 - \frac{m_X^2}{m_B^2}.$$

$$f(z) = \delta(1-z) + \alpha_S \int_0^1 d\omega \int_0^1 dt \left[\frac{A_1}{\omega t} + \frac{S_1(t)}{\omega} + \frac{C_1(\omega)}{t}\right]\left[\delta(1-z-\omega t) - \delta(1-z)\right].$$

$$\frac{1}{\Gamma_0}\,\Gamma_N = \int_0^1 \, dx\; x^{N-1}\frac{1}{\Gamma_0}\,\frac{d\Gamma}{dx}$$

$$L \equiv \log\frac{N}{N_0}$$

$$\frac{1}{\Gamma_0}\,\Gamma_N = \mathcal{C}\left(\alpha_S\right)\, f_N\left(\alpha_S\right) + R_N\left(\alpha_S\right),$$

$$N_0 \equiv \exp\left[-\gamma_E\right].$$

$$\begin{aligned} g_1(\lambda) &= -\frac{A_1}{2\beta_0}\frac{1}{\lambda}\left[(1-2\lambda)\log(1-2\lambda) - 2(1-\lambda)\log(1-\lambda)\right], \\ g_2(\lambda) &= \frac{\beta_0 A_2 - \beta_1 A_1}{2\beta_0^3}\left[\log(1-2\lambda) - 2\log(1-\lambda)\right] - \frac{\beta_1 A_1}{4\beta_0^3}\left[\log^2(1-2\lambda) - 2\log^2(1-\lambda)\right] + \\ &\quad + \frac{S_1}{2\beta_0}\log(1-2\lambda) + \frac{C_1}{\beta_0}\log(1-\lambda). \end{aligned} \quad (6.17)$$

singularities

treshold

$$\frac{1}{2} = \lambda \approx \frac{\log Q^2/m_X^2}{\log Q^2/\Lambda^2},$$

$$\lambda = 1,$$

$$m_X^2 \approx \Lambda^2$$

$$m_X^2 \approx \Lambda Q,$$

Fermi motion

hadronization

transverse momentum

$$\omega \to 1^-$$

$$\omega = \beta_0 \alpha_S L \approx \frac{\log Q^2 b^2}{\log Q^2/\Lambda^2},$$

$$p_\perp \approx \frac{1}{b} \approx \Lambda.$$

Complete $O(\alpha_S)$ evaluation in analytic form

U. Aglietti, R. Sghedoni, L.T. (2004)

$$D(x) = K(\alpha_S)\Sigma(x;\alpha_S) + R(x;\alpha_S).$$

process dependent — universal — process dependent

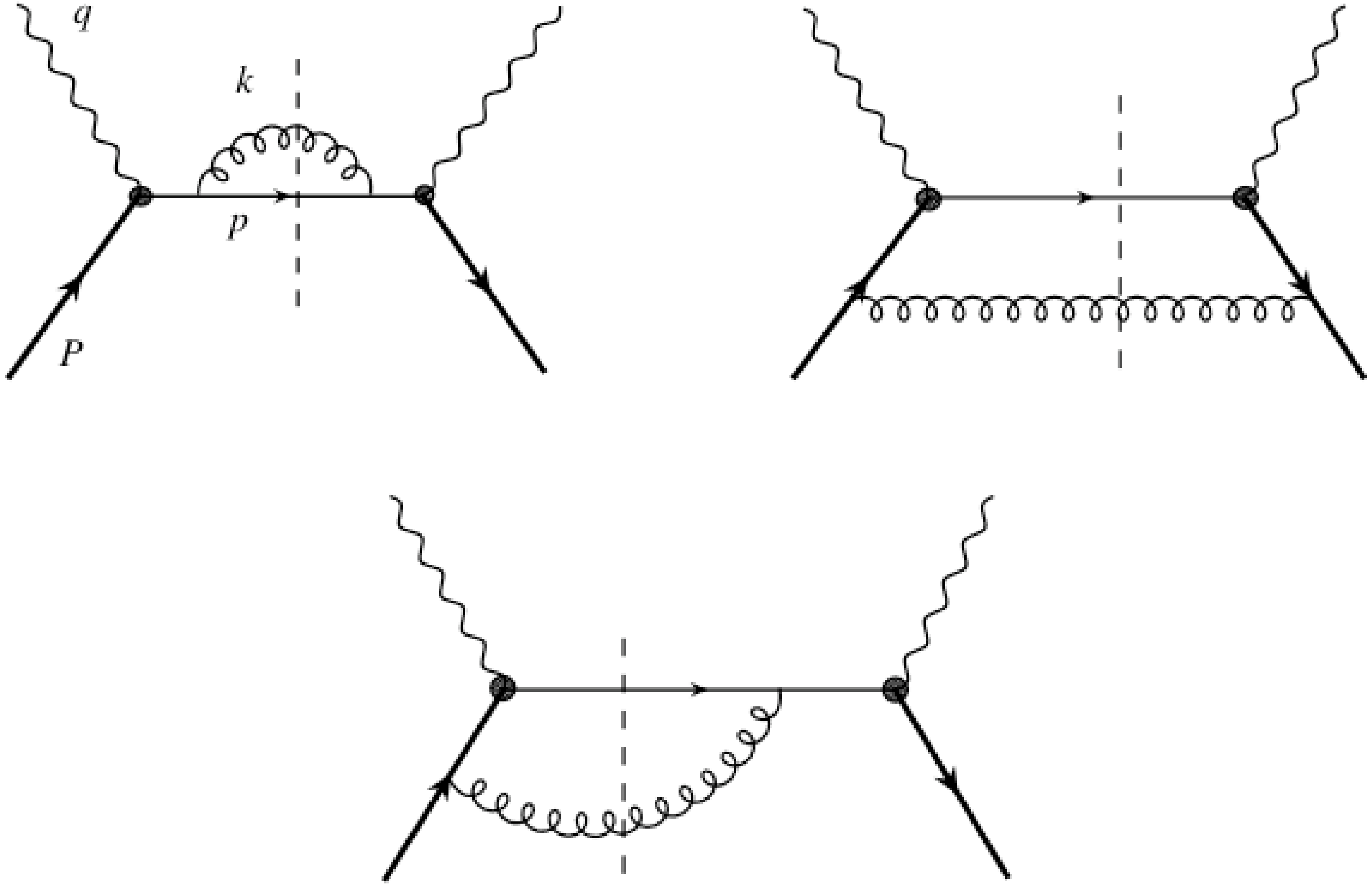
Coefficient function

$$K(\alpha_S) = 1 + \frac{\alpha_S C_F}{\pi} k_1 + O\left(\alpha_S^2\right).$$

Remainder function - hard contributions

$$R(x;\alpha_S) = \frac{\alpha_S C_F}{\pi} r_1(x) + O\left(\alpha_S^2\right).$$

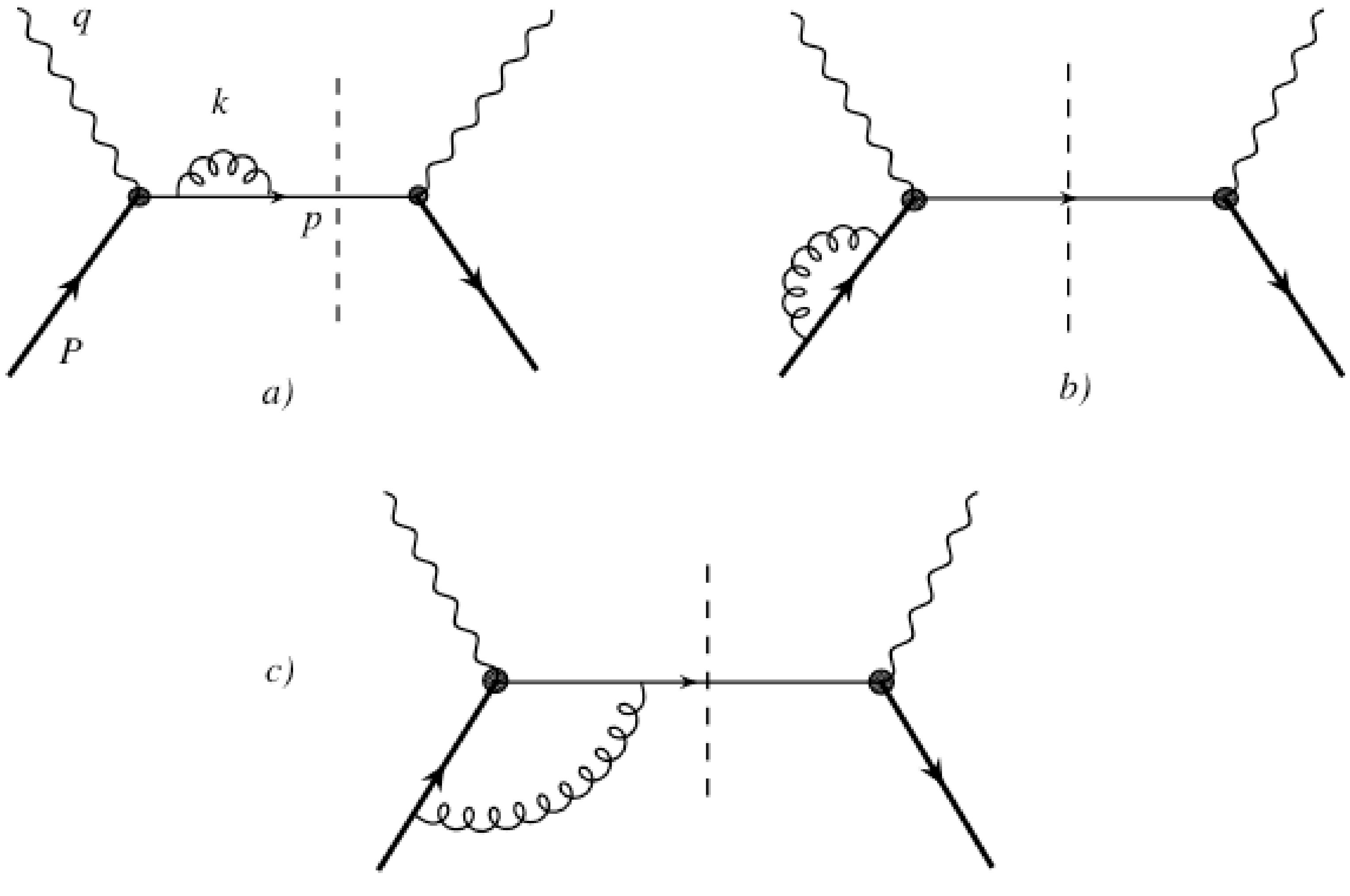
$$R(x;\alpha_S) \to 0 \quad \text{for} \quad x \to 0.$$



$$\frac{d\Gamma}{\Gamma_0} = \frac{M(\omega,t;\epsilon)}{\omega^{1-\epsilon}t^{1-\epsilon/2}}\ dt\ d\omega = \left[\frac{A_1(\omega,t;\epsilon)}{\omega^{1-\epsilon}t^{1-\epsilon/2}} + \frac{S_1(t;\epsilon)}{\omega^{1-\epsilon}} + \frac{C_1(\omega;\epsilon)}{t^{1-\epsilon/2}} + F_1(\omega,t;\epsilon)\right]\ dt\ d\omega$$

$$\begin{aligned}
A_1 &\equiv M(0,0;\epsilon) \\
S_1(t) &\equiv \frac{M(0,t;\epsilon) - M(0,0;\epsilon)}{t^{1-\epsilon/2}} \\
C_1(\omega) &\equiv \frac{M(\omega,0;\epsilon) - M(0,0;\epsilon)}{\omega^{1-\epsilon}} \\
F_1(\omega,t;\epsilon) &\equiv \frac{M(\omega,t;\epsilon) - M(0,t;\epsilon) - M(\omega,0;\epsilon) + M(0,0;\epsilon)}{\omega^{1-\epsilon}\, t^{1-\epsilon/2}}
\end{aligned}$$

$$\omega = \frac{2P\cdot k}{m_b^2} = \frac{2E_g}{m_b}, \qquad t = \frac{1-\cos\theta}{2},$$



$$D_V = C_F \frac{\alpha_S}{\pi} \left(\frac{m_b^2}{4\pi\mu^2}\right)^{\epsilon/2} \Gamma\left(1-\frac{\epsilon}{2}\right) \left[-\frac{2}{\epsilon^2}+\frac{5}{2\epsilon}+4\log\frac{m_b}{\mu_b}-3\right].$$

$$D(x) = 1 \ + \ C_F \frac{\alpha_S}{\pi} \ \left[-\frac{1}{4} \ \log^2 x - \ \frac{5}{4} \ \log x + f + \ d(x) \right].$$

$r_1(x)$ ~ 10-20 % for $x > 0.1$

$$\begin{aligned} D(x) &= \left(1 + \frac{C_F \alpha_S}{\pi} \kappa_1\right) \left(1 - \frac{A_1}{4} \alpha_S \log^2 x + B_1 \alpha_S \log x\right) + \frac{C_F \alpha_S}{\pi} r_1(x) \\ &= 1 - \frac{A_1}{4} \alpha_S \log^2 x + B_1 \alpha_S \log x + \frac{C_F \alpha_S}{\pi} \kappa_1 + \frac{C_F \alpha_S}{\pi} r_1(x) + O(\alpha_S^2). \end{aligned}$$

$$k_1 \ = f \ = \ - \ \frac{11}{4} - \ \frac{\pi^2}{12} + \ 4 \log \frac{m_b}{\mu},$$

$$\tau = \frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}}.$$

$$\begin{aligned} r_1(\tau) &= \frac{(\tau - 1)(49\tau^8 + 468\tau^7 + 1797\tau^6 + 3642\tau^5 + 4450\tau^4 + 3642\tau^3 + 1797\tau^2 + 468\tau + 49)}{12(\tau + 1)^5(\tau^2 + 3\tau + 1)^2} \\ &+ \frac{-5 - 61\tau - 317\tau^2 - 912\tau^3 - 1622\tau^4 - 1934\tau^5 - 1622\tau^6 - 912\tau^7 - 317\tau^8 - 61\tau^9 - 5\tau^{10}}{4(\tau + 1)^6(\tau^2 + 3\tau + 1)^2} \log \tau \\ &- J[0, -3, \tau] + J[0, -3, 1/\tau] - 2J[0, -1, \tau] + J[-1, 0, \tau] + J[-1, -3, \tau] - J[-1, -3, 1/\tau] \\ &- 2\sqrt{\tau} \arctan(\sqrt{\tau}) \frac{(\tau + 1)(2\tau^2 + 7\tau + 2)}{(\tau^2 + 3\tau + 1)^2} + \frac{\pi}{2} \sqrt{\tau} \frac{(\tau + 1)(2\tau^2 + 7\tau + 2)}{(\tau^2 + 3\tau + 1)^2} + \frac{49}{12} \\ &+ \frac{5}{4} \log \tau - \frac{5}{2} \log(\tau + 1) + \log^2(\tau + 1). \end{aligned}$$

update

THRESHOLD RESUMMED SPECTRA IN $B \to X_u l\nu$ DECAYS IN NLO

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(I , II)

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We evaluate threshold resummed spectra in $B \to X_u l\nu$ decays in next-to-leading order. We present results for the distribution in the hadronic variables E_X and m_X^2/E_X^2, for the distribution in E_X and for the distribution in E_X and E_l, where E_X and m_X are the total energy and the invariant mass of the final hadronic state X_u respectively and E_l is the energy of the charged lepton. We explicitly show that all these spectra (where there is no integration over the hadronic energy) can be directly related to the photon spectrum in $B \to X_s\gamma$ via short-distance coefficient functions.

We resum to next-to-leading order the distribution in the ratio of the invariant hadron mass m_X to the total hadron energy E_X and the distribution in m_X in the semileptonic decays $B \to X_u l\nu$. By expanding our formulas, we obtain the coefficients of all the infrared logarithms at $O(\alpha_S^2)$ and of the leading ones at $O(\alpha_S^3)$. We explicitly show that the relation between these semileptonic spectra and the photon spectrum in the radiative decay $B \to X_s\gamma$ is not a purely short-distance one. There are long-distance effects in the semileptonic spectra which are not completely factorized by the structure function as measured in the radiative decay and have to be modelled in some way.

Conclusions

- A complete QCD calculation in the range of scales of m_b

- Resummation of large contributions to next-to-leading accuracy

- Detailed tests of pQCD are possible for the the decay

- The transverse momentum distribution "clean" final state gives a complementary information w.r.t. threshold distribution

- Numerical tests do show that $r(x)$ function represents the 10-15% to the distribution

Future

further higher order contributions

phenomenological tests possible with large statistics data

Perturbative QCD corrections
to

$$b \to s\ \gamma$$