Sudakov resummations at higher orders

Andreas Vogt

IPPP, Univ. of Durham

Collaboration with Sven Moch (Zeuthen), Jos Vermaseren (NIKHEF)

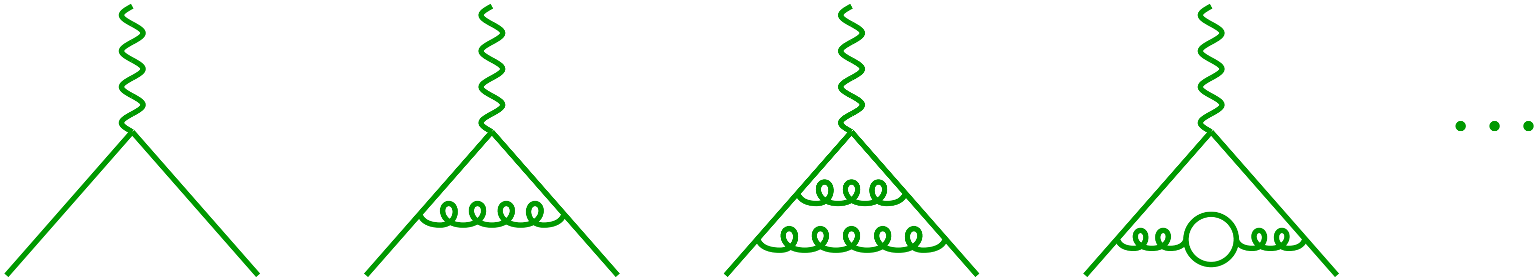
- **On-shell quark and gluon form factors at three loops and beyond**

  hep-ph/0507039 = JHEP 08 ('05) 049, hep-ph/0508055 = Phys. Lett. B625 ('05) 245

- **N^3LL threshold resummation, Higgs production at approx. N^3LO**

  hep-ph/0506288 = Nucl. Phys. B726 ('05) 317, hep-ph/0508265 $\longrightarrow$ Phys. Lett. B

Form factors of massless quarks and gluons



On-shell $m = 0$ quark form factor $\mathcal{F}_{\rm q}$: QCD corr's to $\gamma^* qq$ vertex

$$\Gamma_\mu \; = \; {\rm i} e_{\rm q} \, (\bar{u} \gamma_\mu u) \, \mathcal{F}_{\rm q}(\alpha_{\rm s}, Q^2)$$

Gauge invariant, divergent: dimensional regularization, $D = 4 - 2\varepsilon$

Gluon form factor $\mathcal{F}_{\rm g}$: effective Hgg vertex in heavy top-quark limit

$$\mathcal{L}_{\rm eff} \; = \; -\frac{1}{4} \, C_H \, H G^a_{\mu\nu} G^{a,\mu\nu}$$

Coefficient C_H known to N^{3}LO Chetyrkin, Kniehl, Steinhauser (97)

Renormalization of $G^a_{\mu\nu} G^{a,\mu\nu}$: $Z_{G^2} \; = \; \left[1 - \beta(a_{\rm s})/(a_{\rm s}\varepsilon)\right]^{-1}$

Exponentiation of the form factors

Evolution equation for renormalized (scale μ) form factor

$$Q^2 \frac{\partial}{\partial Q^2} \ln \mathcal{F}\left(\alpha_s, \frac{Q^2}{\mu^2}, \varepsilon\right) = \frac{1}{2} K(\alpha_s, \varepsilon) + \frac{1}{2} G\left(\frac{Q^2}{\mu^2}, \alpha_s, \varepsilon\right)$$

- K : scale independent, series of $1/\varepsilon$ poles in $\overline{\text{MS}}$
- G : dependence on Q^2, finite for $\varepsilon \longrightarrow 0$. RGEs : Collins (80), . . .

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s, \epsilon) \frac{\partial}{\partial \alpha_s}\right) \{ G \, , \, K \} = \{ A(\alpha_s) \, , \, -A(\alpha_s) \}$$

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Solution in terms of D-dim. coupling $\bar{a}$ with $\bar{a}(1, a_s, \varepsilon) = a_s \equiv \frac{\alpha_s}{4\pi}$

$$2 \ln \mathcal{F}\left(\alpha_s, \frac{Q^2}{\mu^2}, \varepsilon\right) = \int_0^{Q^2/\mu^2} \frac{d\xi}{\xi} \left[K(\alpha_s, \varepsilon) + \underbrace{G(1, \bar{a}(\xi, a_s, \varepsilon), \varepsilon)}_{G(\bar{a}, \varepsilon)} + \int_\xi^1 \frac{d\lambda}{\lambda} A(\bar{a}(\lambda, a_s, \epsilon)) \right]$$

Magnea, Sterman (90)

Perturbative expansions, integrations using nested-sum algorithms

[Vermaseren (98); Moch, Uwer, Weinzierl (02)] $\longrightarrow$ **higher-order predictions**

Bare form factors in terms of A_l and $G_l(\varepsilon)$

Expansion in powers of the bare coupling $a_s^b = \alpha_s^b/(4\pi)$

$$\mathcal{F}_b(\alpha_s^b, Q^2) \;=\; 1 + \sum_{l=1} (a_s^b)^l \left(Q^2/\mu^2\right)^{-l\varepsilon} \mathcal{F}_l$$

with

$$\begin{aligned}
\mathcal{F}_1 &= -\frac{1}{2\varepsilon^2} A_1 - \frac{1}{2\varepsilon} G_1 \\
\mathcal{F}_2 &= \frac{1}{8\varepsilon^4} A_1^2 + \frac{1}{8\varepsilon^3} A_1 (2G_1 - \beta_0) + \frac{1}{8\varepsilon^2} (G_1^2 - 2\beta_0 G_1 - A_2) - \frac{1}{4\varepsilon} G_2 \\
\mathcal{F}_3 &= -\frac{1}{48\varepsilon^6} A_1^3 + \ldots + \frac{1}{72\varepsilon^2} (9G_1 G_2 - 6\beta_1 G_1 - 24\beta_0 G_2 - 4A_3) - \frac{1}{6\varepsilon} G_3 \\
\mathcal{F}_4 &= \frac{1}{384\varepsilon^8} A_1^4 + \ldots + \frac{1}{96\varepsilon^2} (3G_2^2 + 8G_1 G_3 \ldots - 36\beta_0 G_3 - 3A_4) - \frac{1}{8\varepsilon} G_4
\end{aligned}$$

Two- and three-loop calculations: $\varepsilon^{-8} \ldots \varepsilon^{-3}$ terms of $\mathcal{F}_4$ predicted

$\mathcal{F}_2$: van Neerven et al (88) [q, ε^0]; Harlander (00) [g, ε^0]; MVV (05); Gehrmann et al (05)

$\mathcal{F}_3$: MVV (05) [ε^{-1}, $q : n_f \varepsilon^0$]; Bern, Dixon, Smirnov (05) [$\zeta_2\zeta_3\varepsilon^{-1}, \zeta_5\varepsilon^{-1} \leftrightarrow$ MSYM]

Extraction of $\mathcal{F}_3$ from (ϕ)DIS at third order

a_s expansion of the bare structure functions at large Bjorken-x

$$
\begin{aligned}
F_0^{\mathrm{b}} &= \delta(1-x) \\
F_1^{\mathrm{b}} &= 2\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1 \\
F_2^{\mathrm{b}} &= (2\,\mathcal{F}_2 + \mathcal{F}_1^{\,2})\delta(1-x) + 2\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2 \\
F_3^{\mathrm{b}} &= (2\,\mathcal{F}_3 + 2\,\mathcal{F}_1\mathcal{F}_2)\delta(1-x) + (2\,\mathcal{F}_2 + \mathcal{F}_1^{\,2})\mathcal{S}_1 + 2\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3
\end{aligned}
$$

$\mathcal{F}_l$: bare l-loop space-like q or g form factor. $\mathcal{S}_l$: soft real emissions

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\end{aligned}$$

$\mathcal{F}_l$: bare l-loop space-like q or g form factor. $\mathcal{S}_l$: soft real emissions

$$\begin{aligned}
\mathcal{S}_k &= \mathsf{S}_k(\varepsilon)\cdot\varepsilon[\,(1-x)^{-1-k\varepsilon}\,]_+ \\
&= \mathsf{S}_k(\varepsilon)\Big\{-\frac{1}{k}\,\delta(1-x) + \sum_{i=0}\frac{(-k\varepsilon)^i}{i\,!}\,\varepsilon\,\mathcal{D}_i\Big\}\;, \quad \mathcal{D}_i \equiv \left[\frac{\ln^i(1-x)}{(1-x)}\right]_+
\end{aligned}$$

Calculation of F_3^{b} to order ε^m $\Rightarrow$ $\mathcal{F}_3$ and $\mathcal{S}_3$ to order ε^{m-1}

MVV(04/05): coefficient fct's for (ϕ)DIS + dedicated n_f calc. to $\mathcal{O}(\varepsilon)$

The expansion coefficients A_l

$A_{p,l}$: $\ln N$ in splitting fct. $P_{pp}^{(l-1)}(N) \Leftrightarrow$ cusp anomalous dimension

$$A_{\mathrm{q},1} = 4\,C_F \;, \quad A_{\mathrm{q},2} = 8\,C_F\big[\big(67/18-\zeta_2\big)\,C_A - 5/9\,n_f\big]$$

Kodaira, Trentadue (82)

$$\begin{aligned} A_{\mathrm{q},3} = \; & 16\,C_F C_A^2\,\left[245/24 - 67/9\,\zeta_2 + 11/6\,\zeta_3 + 11/5\,\zeta_2^2\right] \\ & - 16\,C_F C_A n_f\,\left[209/108 - 10/9\,\zeta_2 + 7/3\,\zeta_3\right] \\ & - 16\,C_F^2 n_f\,\left[55/24 - 2\,\zeta_3\right] - 16/27\,C_F n_f^2 \end{aligned}$$

Gracey (94) [n_f^2]; C.Berger (02), MVV (02) [n_f]; MVV (04); Bern, Dixon, Smirnov (05) [ζ_2^2]

Numerical result for $n_f = 4$ flavours, incl. Padé estimate for A_4

$$A_{\mathrm{q}}(\alpha_s) \cong 0.424\,\alpha_s\left(1 + 0.638\,\alpha_s + 0.510\,\alpha_s^2 + 0.4_{[1/1]}\,\alpha_s^3 + \ldots\right)$$

Contribution of A_3: $\simeq 2\%$ for $\alpha_s = 0.2$ ($\simeq 1/6$ of A_2). $A_4 : < 1\%$

Maximally non-abelian, $A_{\mathrm{g}} = C_A/C_F\,A_{\mathrm{q}}$ Korchemsky (89)

The resummation functions $G(\alpha_s, \varepsilon)$

Recursive representation of ε^0 parts, using $\widetilde{F} = \varepsilon^{-1}\,[\,F - F(\varepsilon = 0)]$

$$
\begin{aligned}
G_1^p &= 2\left(P_{1,p}^{\delta} - \delta_{pg}\beta_0\right) + f_1^p + \varepsilon\widetilde{G_1^p} \\
G_2^p &= 2\left(P_{2,p}^{\delta} - 2\delta_{pg}\beta_1\right) + f_2^p + \beta_0\widetilde{G_1^p}(0) + \varepsilon\widetilde{G_2^p} \\
G_3^p &= 2\left(P_{3,p}^{\delta} - 3\delta_{pg}\beta_2\right) + f_3^p + \beta_1\widetilde{G_1^p}(0) + \beta_0\left[\widetilde{G_2^p}(0) - \beta_0\widetilde{\widetilde{G_1^p}}(0)\right] + \varepsilon\widetilde{G_3^p}
\end{aligned}
$$

$P_{l,p}^{\delta}$: $\delta(1-x)$ coeff. of $P_{pp}^{(l-1)}$.

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G_3^p &= 2\left(P_{3,p}^{\delta} - 3\delta_{pg}\beta_2\right) + f_3^p + \beta_1\widetilde{G_1^p}(0) + \beta_0\left[\widetilde{G_2^p}(0) - \beta_0\widetilde{\widetilde{G_1^p}}(0)\right] + \varepsilon\widetilde{G_3^p}
\end{aligned}
$$

$P_{l,p}^{\delta}$: $\delta(1\!-\!x)$ coeff. of $P_{pp}^{(l-1)}$. $f_l^{\,p}$: max. non-abelian, $C_F f_l^{\,g} = C_A f_l^{\,q}$

$$
f_1^q = 0 \;, \quad f_2^q = 2C_F\left\{-\beta_0\zeta_2 - 56/27\, n_f + C_A\,(404/27 - 14\zeta_3)\right\}
$$

Ravindran, Smith, van Neerven (04)

$$
\begin{aligned}
f_3^q &= C_F C_A^2\left(\frac{136781}{729} - \frac{12650}{81}\zeta_2 - \frac{1316}{3}\zeta_3 + \frac{352}{5}\zeta_2^2 + \frac{176}{3}\zeta_2\zeta_3 + 192\zeta_5\right) \\
&\quad - C_A C_F n_f\left(\frac{11842}{729} - \frac{2828}{81}\zeta_2 - \frac{728}{27}\zeta_3 + \frac{96}{5}\zeta_2^2\right) - C_F^2 n_f\left(\frac{1711}{27} - 4\zeta_2\right. \\
&\quad \left. - \frac{304}{9}\zeta_3 - \frac{32}{5}\zeta_2^2\right) - C_F n_f^2\left(\frac{2080}{729} + \frac{40}{27}\zeta_2 - \frac{112}{27}\zeta_3\right)
\end{aligned}
$$

MVV (05)

Ratio of time- and space-like form factors

Continuation from DIS case $q^2 = -Q^2 < 0$ to time-like $q^2 > 0$

$$\left(\frac{-q^2}{\mu^2}\right)^{-l\epsilon} = \left(\frac{q^2}{\mu^2}\right)^{-l\epsilon}\left(\frac{\Gamma(1-l\epsilon)\Gamma(1+l\epsilon)}{\Gamma(1-2l\epsilon)\Gamma(1+2l\epsilon)} - \mathrm{i}\,\frac{\pi l\epsilon}{\Gamma(1-l\epsilon)\Gamma(1+l\epsilon)}\right)$$

Absolute ratio: infrared finite, enters cross sections, known to $\mathcal{O}(a_s^{\,4})$

$$\left|\frac{\mathcal{F}(q^2)}{\mathcal{F}(-q^2)}\right|^2 = 1 + a_s\{3\zeta_2 A_1\} + a_s^{\,2}\{9/2\,\zeta_2^2 A_1^2 + 3\zeta_2(A_2 + \beta_0 G_1)\}$$
$$+ a_s^{\,3}\{\ldots + 3\zeta_2(A_3 + \beta_1 G_1 + 2\beta_0 G_2)\}$$
$$+ a_s^{\,4}\{\ldots + 3\zeta_2(A_4 + \beta_2 G_1 + 2\beta_1 G_2 + 3\beta_0 G_3)\} + \mathcal{O}(a_s^{\,5})$$

Magnea, Sterman (90) [q, a_s^2]; Harlander (00) [g, a_s^2]; MVV (05)

Numerical for $q\bar{q}\gamma^*$ ($n_f = 4$) and ggH ($n_f = 5$), uncertainty: A_4

$$q\bar{q}\gamma^* \;:\; 1 + 2.094\,\alpha_s + 5.613\,\alpha_s^{\,2} + 15.70\,\alpha_s^{\,3} + (48.63 \pm 0.43)\,\alpha_s^{\,4}$$
$$ggH \;:\; 1 + 4.712\,\alpha_s + 13.69\,\alpha_s^{\,2} + 25.94\,\alpha_s^{\,3} + (36.65 \pm 0.35)\,\alpha_s^{\,4}$$

Coefficient functions at large x / large N

Threshold: inhibited radiation

$\Rightarrow$ log^2-enhanced corrections

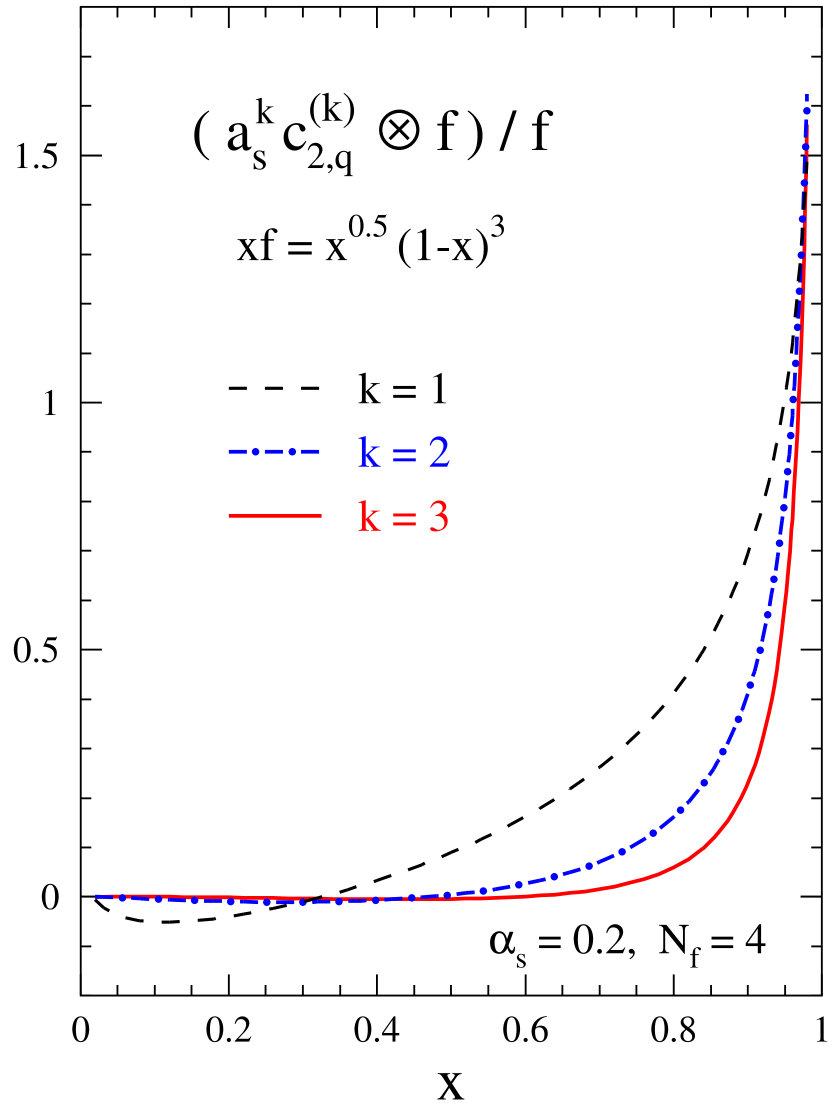
Scaling variable x, moments N

$$\alpha_s^k \left[\frac{\ln^{2l-1}(1-x)}{1-x} \right]_+ \ ,$$

$$\alpha_s^k \ \ln^{2l} N \quad , \quad l = 1, \ldots, k$$

α_s expansion spoiled for $x \to 1$, $N \to \infty$ $\Rightarrow$ resummation

Example: DIS, only x-dependent case known to N^{3}LO **MVV(05)**



Soft gluon exponentiation

$\overline{\text{MS}}$ coefficient functions for few-parton cases, large Mellin-N

$$C^N / C^N_{\text{LO}} \; = \; g_0 \cdot \exp \, \mathcal{G}^N \; + \; \mathcal{O}(N^{-1} \ln^n N)$$

g_0 : N^0 contributions, $\mathcal{G}^N$: resummation of $\ln^n N$ terms

Sterman (87); Catani, Trentadue (89); . . .

Drell-Yan, DIS

$$\mathcal{G}^N_{\text{DY}} \; = \; 2 \, \ln \Delta_q + \ln \Delta^{\text{int}}_{\text{DY}}$$

$$\mathcal{G}^N_{\text{DIS}} \; = \; \ln \Delta_q + \ln J_q + \underbrace{\ln \Delta^{\text{int}}_{\text{DIS}}}_{=\,0}$$

Forte, Ridolfi; Gardi, Roberts (02)

$gg \longrightarrow H$ for large m_{top} $\Leftrightarrow$ Drell-Yan

Direct photons in $pp\,,\, ab \longrightarrow c\,\gamma$ Catani, Mangano, Nason (98)

The radiative factors

- Soft collinear radiation off initial-state parton $p = q,\ g$

$$\ln \Delta_{\rm p} \; = \int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \int_{\mu_f^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} \, A_{\rm p}(\alpha_{\rm s}(q^2))$$

- Collinear emission off 'unobserved' final-state parton

$$\ln J_{\rm p} \; = \int_0^1 dz \, \frac{z^{N-1} - 1}{1 - z} \left[\int_{(1-z)^2 Q^2}^{(1-z) Q^2} \frac{dq^2}{q^2} \, A_{\rm p}(\alpha_{\rm s}(q^2)) + B_{\rm p}(\alpha_{\rm s}([1 - z] Q^2)) \right]$$

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- Large-angle soft gluons, process-dependent

$$\ln \Delta^{\mathrm{int}} \ = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z}\, D(\alpha_{\mathrm{s}}([1-z]^2 Q^2))$$

Integrands of Δ_{p}, J_{p}, Δ^{int} : power expansions in $a_{\mathrm{s}} = \dfrac{\alpha_{\mathrm{s}}}{4\pi}$

$$F(\alpha_{\mathrm{s}}) \ = \ \sum_{l=1} F_l\, a_{\mathrm{s}}^l \ , \quad F = A,\ B,\ D$$

The resummation exponents

Up to next-to-next-to-next-to-leading logarithmic (N^3LL) accuracy

$$\mathcal{G}^N \;=\; \ln N \, g_1(\lambda) + g_2(\lambda) + a_\mathrm{s}\, g_3(\lambda) + a_\mathrm{s}^2\, g_4(\lambda) \;, \qquad \lambda \;=\; \beta_0 a_\mathrm{s} \ln N$$

Integrations for g_3, g_4, $\ldots$: A.V. (00); Catani, de Florian, Grazzini, Nason (03)
MVV (05) [$\longleftarrow$ XSUMMER package: Moch, Uwer (05)]

Dependence on coefficients

$$\begin{aligned} g_1 &= g_1(\lambda, A_1, \beta_0) \\ g_2 &= g_1(\ldots, A_2, B_1, D_1, \beta_1) \\ &\vdots \\ g_k &= g_k(\ldots, A_k, B_{k-1}, D_{k-1}, \beta_{k-1}) \end{aligned}$$

N^nLO calculation (B_n, D_n) $\Rightarrow$ N^nLL resummation (mod. A_{n+1})

MVV (05) : N^3LL for inclusive DIS, lepton-pair and Higgs production

Expansion coefficients B_l

$B_{\mathrm{p},n}$ for $n \leq 3$: matching to 3-loop results for (ϕ)DIS MVV (05)

$$\mathbf{B}_{\mathrm{g},1} = -\frac{11}{3}\,C_A + \frac{2}{3}\,n_f = -\beta_0$$

$$\mathbf{B}_{\mathrm{g},2} = C_A^2\left[-\frac{611}{9} + \frac{88}{3}\,\zeta_2 + 16\,\zeta_3\right] + C_A n_f\left[\frac{428}{27} - \frac{16}{3}\,\zeta_2\right] + 2\,C_F n_f - \frac{20}{27}\,n_f^2$$

$$\begin{aligned}\mathbf{B}_{\mathrm{g},3} = {} & C_A^3\left[-\frac{1492081}{1458} + \frac{60875}{81}\,\zeta_2 + \frac{13796}{27}\,\zeta_3 - \frac{2596}{15}\,\zeta_2^2 - \frac{128}{3}\zeta_2\zeta_3 - 112\,\zeta_5\right] \\ & + C_A^2 n_f\left[\frac{498329}{1458} - \frac{21014}{81}\,\zeta_2 - \frac{296}{9}\,\zeta_3 + \frac{568}{15}\,\zeta_2^2\right] - C_F^2 n_f \\ & + C_A C_F n_f\left[\frac{8579}{54} - 16\,\zeta_2 - \frac{832}{9}\,\zeta_3 - \frac{32}{5}\,\zeta_2^2\right] + C_F n_f^2\left[-\frac{47}{3} + \frac{32}{3}\,\zeta_3\right] \\ & + C_A n_f^2\left[-\frac{48829}{1458} + \frac{716}{27}\,\zeta_2 - \frac{176}{27}\,\zeta_3\right] + n_f^3\left[\frac{200}{243} - \frac{8}{9}\,\zeta_2\right]\end{aligned}$$

Numerical ($n_f = 4$):

$$B_{\mathrm{g}}(\alpha_{\mathrm{s}}) \cong -0.663\,\alpha_{\mathrm{s}}\,(1 - 0.765\,\alpha_{\mathrm{s}} - 2.696\,\alpha_{\mathrm{s}}^2 + \ldots)$$

$$B_{\mathrm{q}}(\alpha_{\mathrm{s}}) \cong -0.318\,\alpha_{\mathrm{s}}\,(1 - 1.227\,\alpha_{\mathrm{s}} - 3.405\,\alpha_{\mathrm{s}}^2 + \ldots)$$

Pole structure of $q\bar{q} \longrightarrow \gamma^*$ and $gg \longrightarrow H$

a_s^n **expansion coefficients of bare partonic cross sections to** $n = 3$

cf. Matsuura, van Neerven (88)

$$
\begin{aligned}
W_0^{\mathrm{b}} &= \delta(1-x) \\
W_1^{\mathrm{b}} &= 2\,\mathrm{Re}\,\mathcal{F}_1\,\delta(1-x) + \mathcal{S}_1 \\
W_2^{\mathrm{b}} &= (2\,\mathrm{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\delta(1-x) + 2\,\mathrm{Re}\,\mathcal{F}_1\mathcal{S}_1 + \mathcal{S}_2 \\
W_3^{\mathrm{b}} &= (2\,\mathrm{Re}\,\mathcal{F}_3 + 2\,|\mathcal{F}_1\mathcal{F}_2|)\delta(1-x) + (2\,\mathrm{Re}\,\mathcal{F}_2 + |\mathcal{F}_1|^2)\mathcal{S}_1 + 2\,\mathrm{Re}\,\mathcal{F}_1\mathcal{S}_2 + \mathcal{S}_3
\end{aligned}
$$

$\mathcal{F}_l$ **: bare l-loop time-like q or g form factor,** $\mathcal{S}_l$ **: soft real emissions**

$$
\mathcal{S}_k \;=\; \mathsf{S}_k(\varepsilon)\cdot\varepsilon[\,(1-x)^{-1-2k\varepsilon}\,]_+ \;=\; \mathsf{S}_k(\varepsilon)\Big[-\frac{1}{2k}\,\delta(1-x) + \sum_{i=0}\frac{(-2k\varepsilon)^i}{i\,!}\,\varepsilon\,\mathcal{D}_i\Big]
$$

Poles in $\varepsilon = 2 - D/2$ **: KLN, renormalization, mass factorization**

$1/\varepsilon$ **pieces of** $\mathcal{F}_n$ **+** n**-loop splitting fct's** $\longrightarrow$ $1/\varepsilon$ **coefficients of** S_n

$\longrightarrow$ $\mathcal{D}_{2n,\dots,0}$ **terms of coefficient fct's** c_n $\longrightarrow$ **resummation coeff's** D_n

Coeff's D_l for Drell-Yan and Higgs production

Maximally non-abelian, $C_I = C_F$ (DY), $C_I = C_A$ (Higgs) with

$$\mathbf{D_1} = 0 \,, \qquad \mathbf{D_2} = C_I \left[C_A \left(-\frac{1616}{27} + \frac{176}{3}\,\zeta_2 + 56\,\zeta_3 \right) + n_f \left(\frac{224}{27} - \frac{32}{3}\,\zeta_2 \right) \right]$$

Catani, Trentadue (89) [D_1]; A.V. (00); Catani, de Florian, Grazzini, Nason (03)

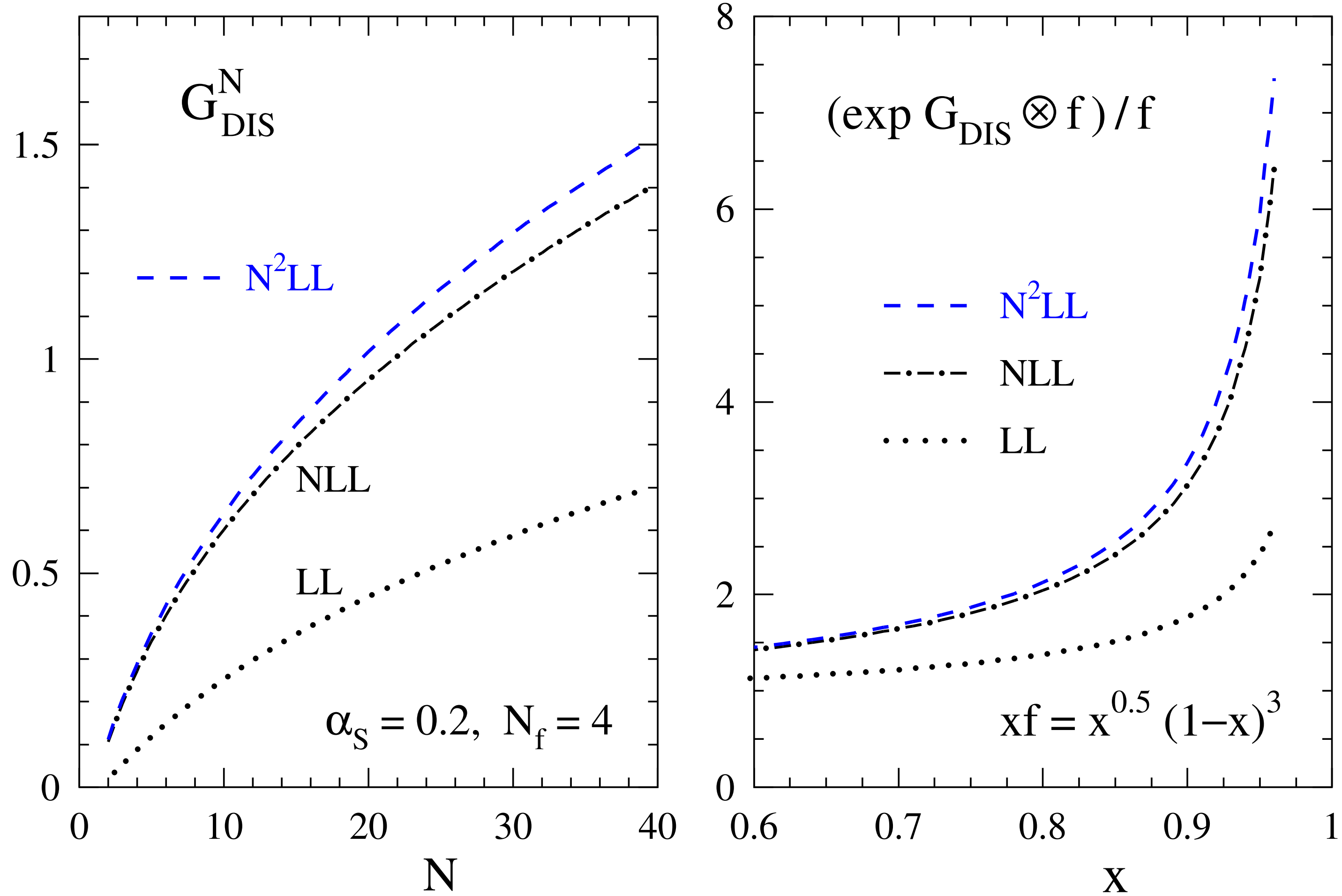
$$\begin{aligned}
\mathbf{D_3} \;=\; & C_I C_A^2 \left[-\frac{594058}{729} + \frac{98224}{81}\,\zeta_2 + \frac{40144}{27}\,\zeta_3 - \frac{2992}{15}\,\zeta_2^2 - \frac{352}{3}\,\zeta_2\zeta_3 - 384\,\zeta_5 \right] \\
& + C_I C_A n_f \left[\frac{125252}{729} - \frac{29392}{81}\,\zeta_2 - \frac{2480}{9}\,\zeta_3 + \frac{736}{15}\,\zeta_2^2 \right] \\
& + C_I C_F n_f \left[\frac{3422}{27} - 32\,\zeta_2 - \frac{608}{9}\,\zeta_3 - \frac{64}{5}\,\zeta_2^2 \right] - C_I n_f^2 \left[\frac{3712}{729} - \frac{640}{27}\,\zeta_2 - \frac{320}{27}\,\zeta_3 \right]
\end{aligned}$$

Moch, A.V. (05); Laenen, Magnea (05) [DY]

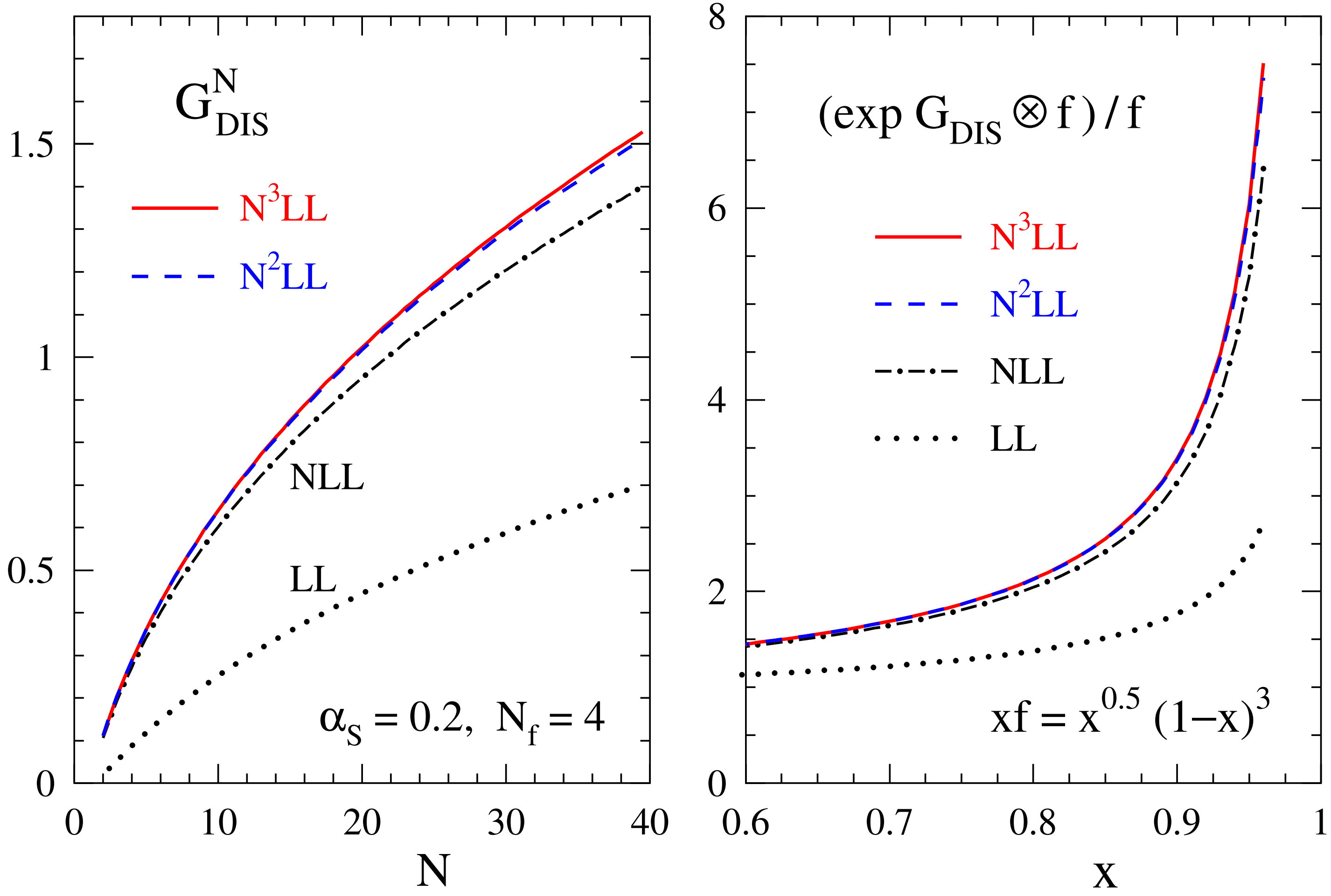
Relation to form-factor resummation coefficients f_n

$$\begin{aligned}
D_2 &= -2 f_2 + 2\beta_0\, \mathsf{S}_{1,0} \\
D_3 &= -2 f_3 + 2\beta_1\, \mathsf{S}_{1,0} - 4\beta_0^2\, \mathsf{S}_{1,1} + 4\beta_0 \left(\mathsf{S}_{2,0} - 36/5\, \zeta_2^2\, C_I^2 \right)
\end{aligned}$$

DIS resummation exponents up to N^3LL



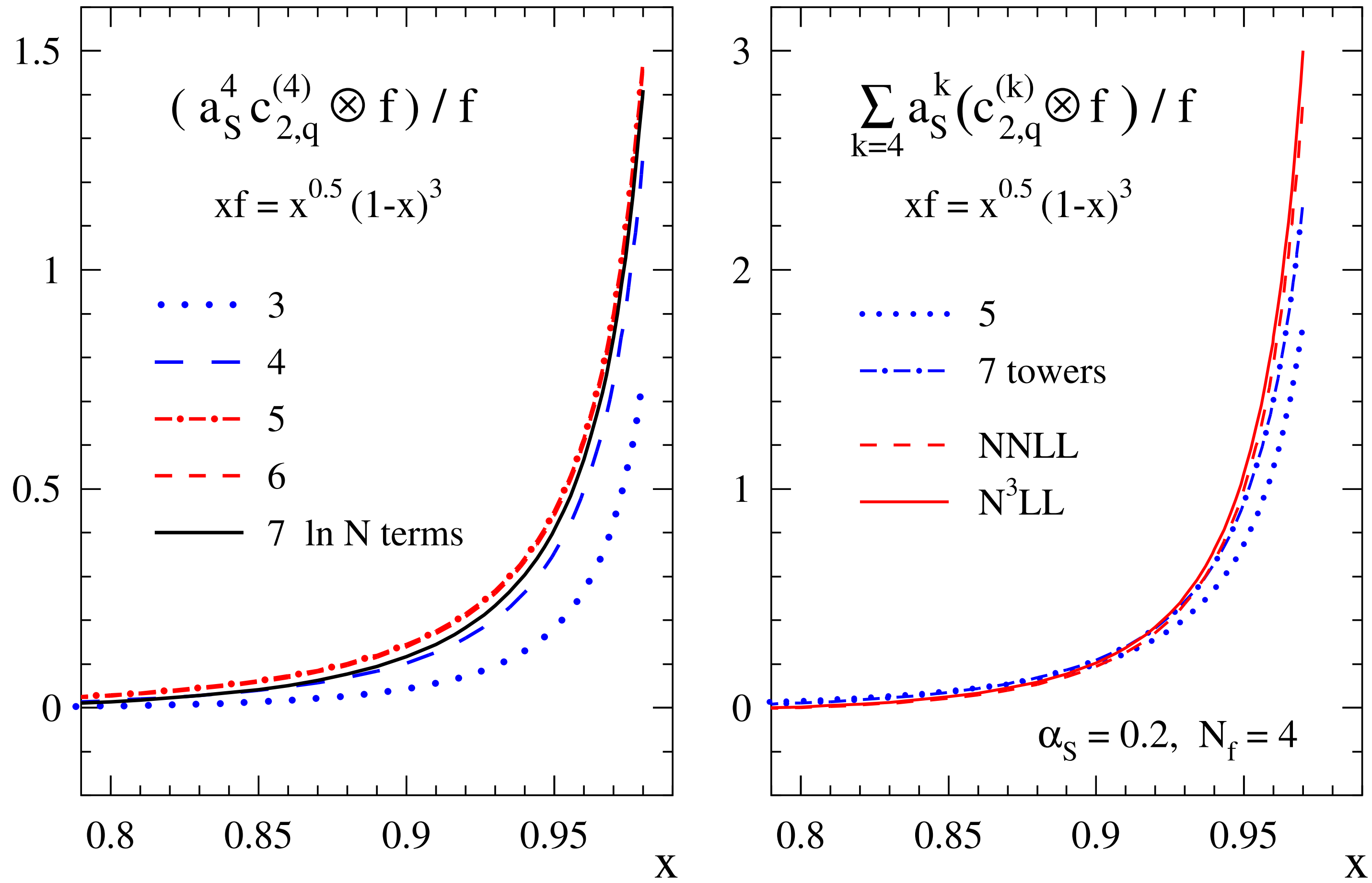
DIS resummation exponents up to N^{3}LL



For x-space soft-gluon enhancements up to almost a factor of ten:

N^{3}LL corr's $\ll$ N^{2}LL, $<$ 10% even at $\alpha_s = 0.3$, $<$ n_f dependence

DIS: four-loop and all-order predictions

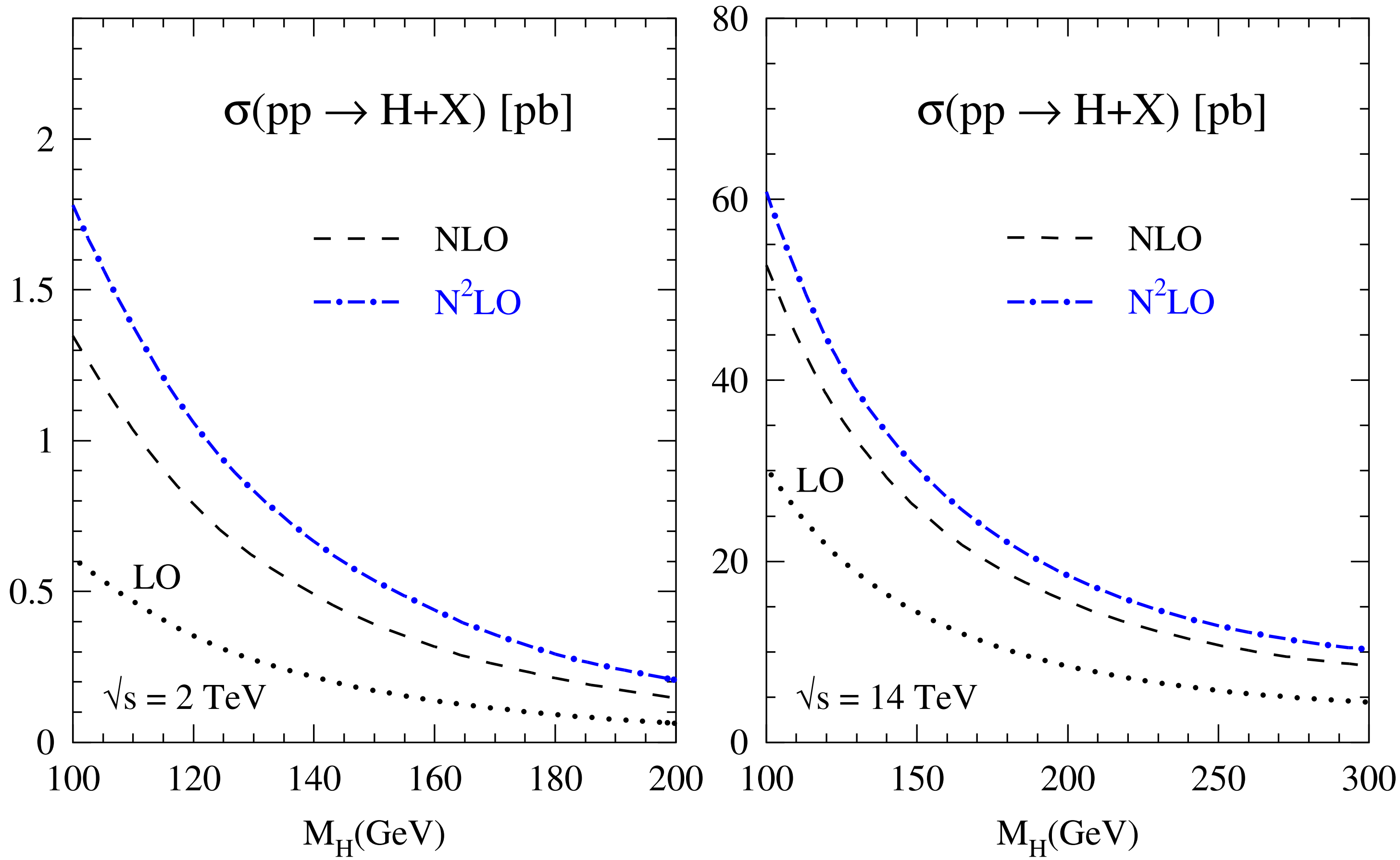


n towers: expand in α_s, keep leading n powers of $\ln N$ to all orders

$N^{2,3}$LL: 'minimal prescription' **Catani, Mangano, Nason, Trentadue (96)**

Higgs production at Tevatron and LHC

Parameters ($m_{\rm top} = 173.4$ GeV etc): Ravindran, Smith, van Neerven (03)



NNLO: Harlander, Kilgore; Anastasiou, Melnikov (02); RSvN (03)

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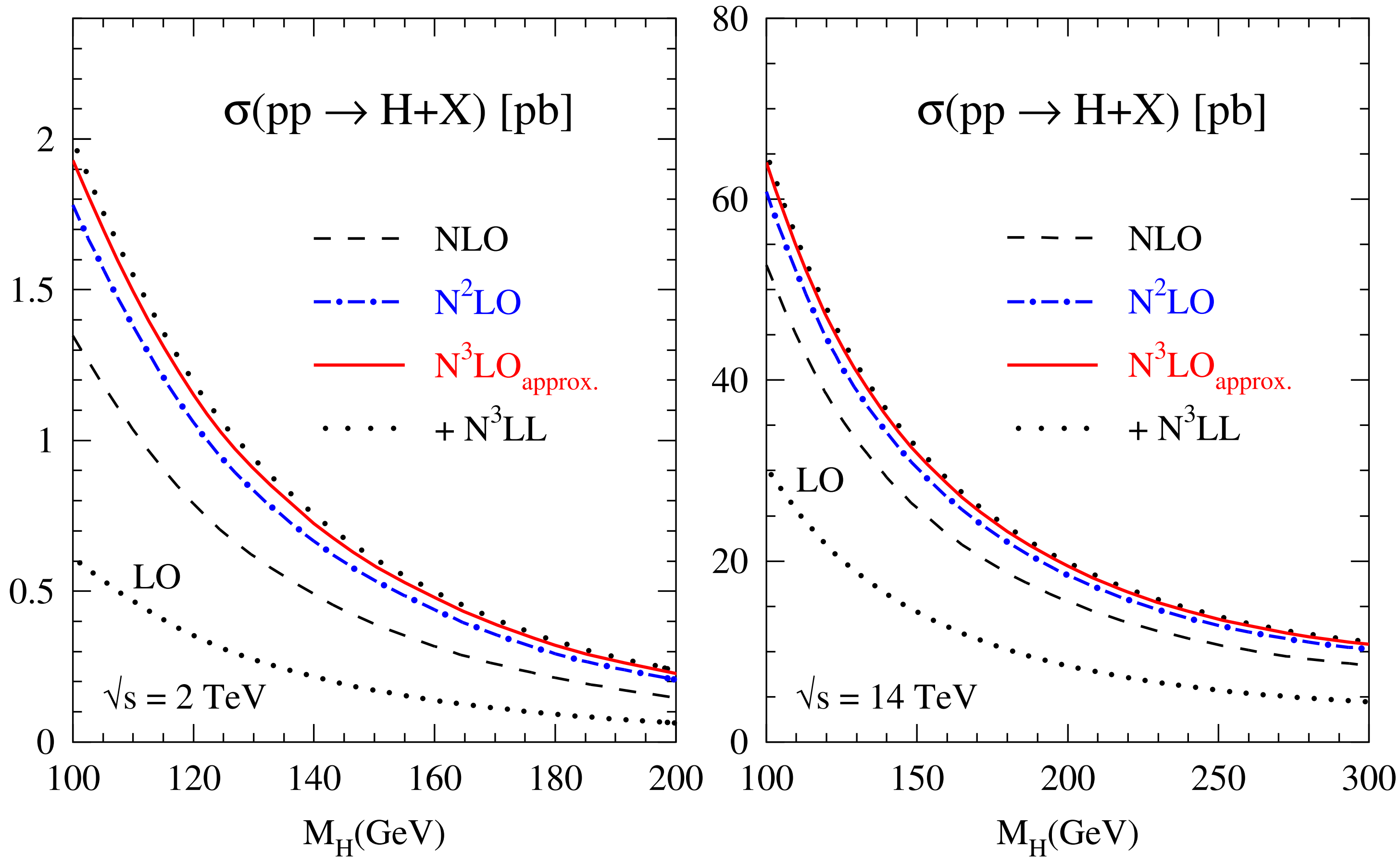
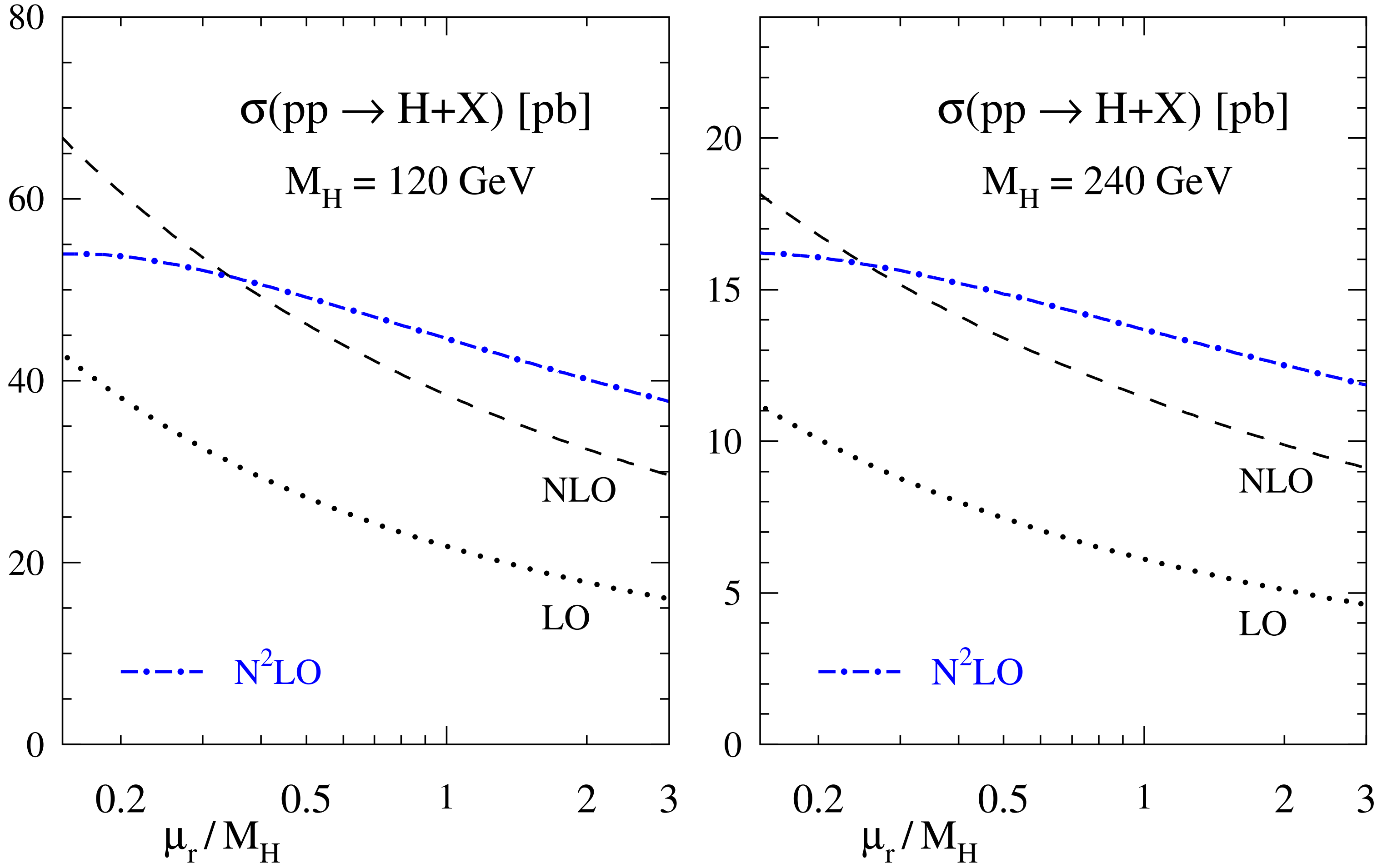


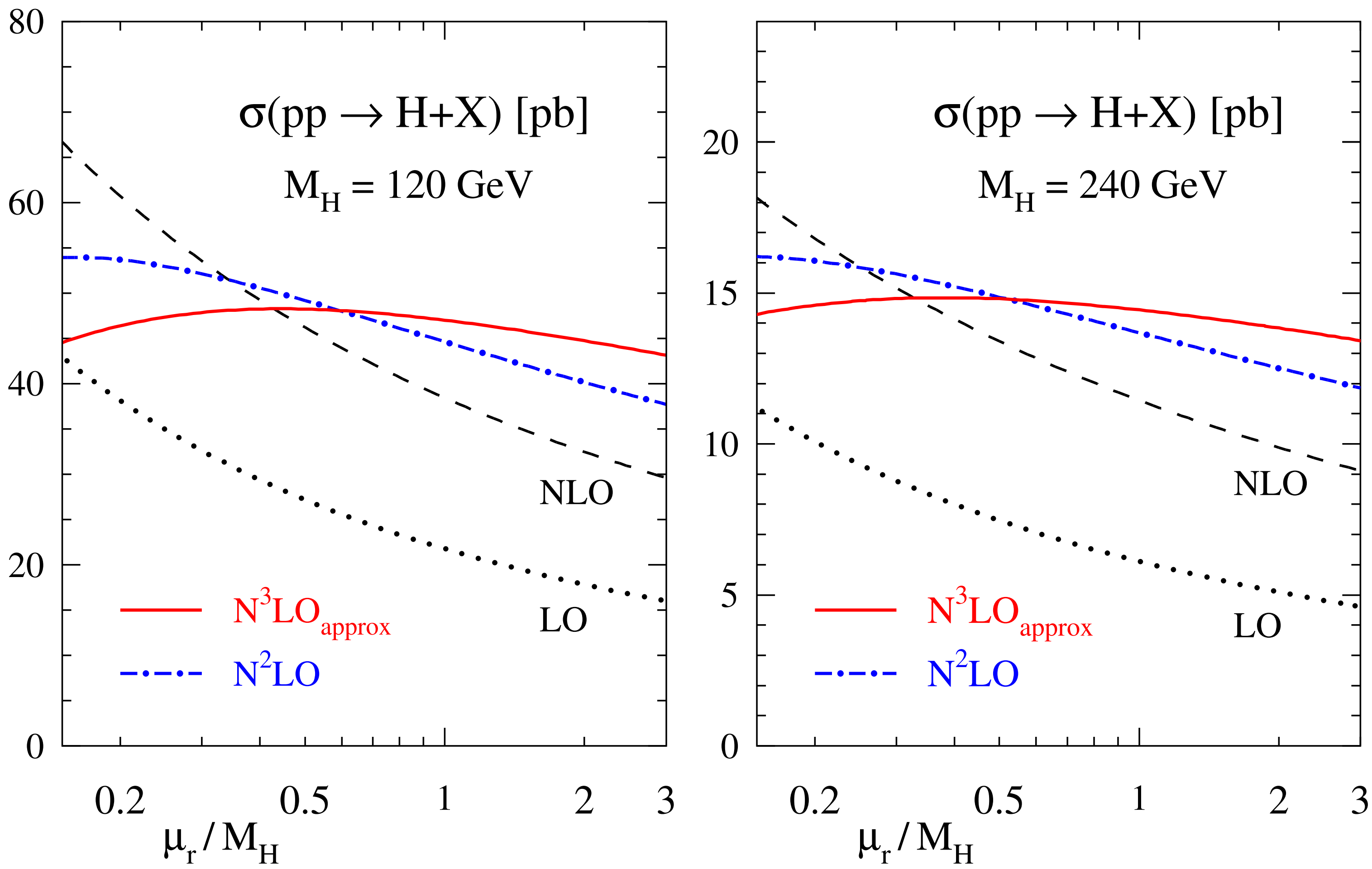
NNLO: Harlander, Kilgore; Anastasiou, Melnikov (02); RSvN (03); **N^3LO:** Moch, A.V. (05)

$N^3LO_{\text{approx.}}$: trf. $\mathcal{D}_k$ to N, drop $1/N$ terms (10% error for $N^{(2)}$LO corr.)

LHC Higgs production: renormalization scale



LHC Higgs production: renormalization scale



N^3LO increase at $\mu_r = M_H$: 5% (NNLO pdf's). μ_r variation: 4%

Estimated higher-order uncertainty: 5% for LHC, 7% for Tevatron

Summary

Resummation of the quark and gluon form factors

- works: $\varepsilon^{-6} \dots \varepsilon^{-2}$ at three loops verified by explicit calculation
- leads to higher-order predictions: $|\mathcal{F}(q^2)/\mathcal{F}(-q^2)|$ at order α_s^4
- helps to uncover structural features: universal contributions f_l

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Resummation of threshold-enhanced logarithms

- works: $\ln^6 \ldots \ln^2 N$ at N^{3}LO verified for DIS coefficient functions
- now extended to N^{3}LL accuracy for simplest (two-parton) cases
- universal coefficients $B_{2,3}$ known: ready for, e.g., N^{2}LO $\rightarrow D_2^{\mathrm{dir.}\gamma}$
- closely related to form factors by KLN and mass factorization: $D_3^{\mathrm{DY,H}} (\leftrightarrow f_l^{\mathrm{q,g}})$ known, sufficient approx. to N^{3}LO for $pp \rightarrow HX$