Two-loop SUSY QCD correction to the gluino pole mass

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- Gluino mass:

  SUSY breaking parameters

  precision measurement vs. radiative correction

- $O(\alpha_s^2)$ correction to the gluino pole mass

- Numerical results

The mass of the Higgs boson in the standard model, $m_H^2 = O(100)$ GeV, generally receives very large radiative correction from unknown physics in the unified theory around $M_P = O(10^{19})$ GeV.

$$v^2 \sim m_H^2 = m_H^2(\text{bare}) + O(g^2 M_P^2)$$

$m_H^2 \lll\lll m_H^2(\text{bare}) \sim M_P^2$:
Fine tuning problem

The most popular solution: introduce supersymmetry (SUSY) in the unified theory. Forbid $O(M_P^2)$ corr.

In SUSY theories, each particle has its "superpartner" with common mass and gauge charges, but different spin by 1/2.

An example: MSSM (minimal supersymmetric standard model)

Chiral supermultiplets

fermion ($s = 1/2$)		scalar ($s = 0$)	
$q_L = (u_L, d_L)$ $u_R,\ d_R$ $l_L = (\nu_L, e_L)$ e_R	quarks leptons 	$\tilde{q}_L = (\tilde{u}_L, \tilde{d}_L)$ $\tilde{u}_R,\ \tilde{d}_R$ $\tilde{l}_L = (\tilde{\nu}_L, \tilde{e}_L)$ $\tilde{e}_R$	squarks sleptons
$\tilde{H}_D,\ \tilde{H}_U$	higgsino	$H_D,\ H_U$	Higgs bosons

Vector supermultiplets

fermion ($s = 1/2$)		gauge boson ($s = 1$)	
$\tilde{g}$ $\tilde{W}$ $\tilde{B}$	gluino wino bino	g_μ W_μ B_μ	SU(3) boson (gluon) SU(2) boson U(1) boson

The superpartners must be heavier than current experimental reach.

$\Rightarrow$ Breaking of SUSY

Supersymmetry in the unified theory must be broken in our world.

Unified Theory

spontaneous SUSY breaking

$\Downarrow$

Low-energy effective theory (here assume MSSM)

soft SUSY breaking parameters

scalar masses m_ϕ^2, ϕ^3 couplings A_f, gaugino masses $M_{3,2,1}$

Important clue to the SUSY breaking mechanism

$\delta m_H^2 = O(g^2 M_{\text{SUSYbreaking}})$:

SUSY solves hierarch problem if $M_{\text{SUSYbreaking}} < O(\text{TeV})$

Gaugino masses in the MSSM

Gauginos in MSSM

SU(3) gluino $\tilde{g}$, SU(2) wino $\widetilde{W}$, U(1) bino $\tilde{B}$

In many candidates of the unified theories, the masses (M_3, M_2, M_1) of these gauginos unify at the GUT/Planck scale

- SUSY GUT with gauge-symmetric SUSY breaking
- Universal gaugino mass at M_P (minimal SUGRA etc.)

$\Rightarrow M_3(Q)/\alpha_s(Q) \sim M_2(Q)/\alpha_2(Q) \sim 3M_1(Q)/5\alpha_Y(Q)$ at low Q

In other theories without unification of $M_{3,2,1}$, their ratios are predicted.

Ex. anomaly mediation $(M_3, M_2, M_1)(Q) \propto (-3\alpha_s, \alpha_2, 11\alpha_Y)(Q)$

To test these models, it is important to obtain precise values of (M_3, M_2, M_1) in future studies at LHC and ILC

$M_{1,2,3}$ also contribute to the running of other SUSY breaking paraters (m_ϕ^2, A_f)

To probe the SUSY breaking mechanism by precision measurements, we need precise formulas of the relations between physical observables and lagrangian parameters.

Physical observables	$\Longleftrightarrow$	Lagrangian parameters
masses, cross-sections. . . of SUSY particles		$M_{1,2,3}(Q \sim 1$ TeV$)\ldots$

Ref. Supersymmetry Parameter Analysis (SPA) Project

Two-loop mass corrections by Bednyakov et al. (t, b), Heinemeyer et al. (Higgs), Martin (scalars), . . .

Here we consider the relation between the physical mass of the gluino $m_{\tilde{g}}$ and the tree-level mass M_3 in the lagrangian, to the two-loop order.

Precision measurement vs. loop correction for the gluino mass

Gluino $\tilde{g}$ is expected to be copiously produced at the LHC, then produce decay chains such as

$$\tilde{g} \to q\tilde{q} \to qq\tilde{\chi}_2^0 \to qql l\tilde{\chi}_1^0$$

Combined analysis of various decay chains of SUSY particles may determine $m_{\tilde{g}}$ ($= M_3$ at tree-level) quite precisely.

A simulation (Chiorboli et al. (2004)):
for the parameter set SPS1a with $m_{\tilde{g}} \sim 600$ GeV
$\delta m_{\tilde{g}} = \pm 8$ GeV from the LHC ($\mathcal{L} \sim 300$ fb^{-1})
$\to$ 6.5 GeV by combining with the ILC data
[$\sqrt{s} \leq 1$ TeV, $\mathcal{L} \sim 1000$ fb^{-1}]

On the other hand, $m_{\tilde{g}}$ receives large radiative corrections.

$O(\alpha_s)$ corr. $\sim O(10)$ %
$\Downarrow$
Naive expectation: $O(\alpha_s^2)$ corr. $\sim O(1)$ %
Comparable to experimental uncertainty?

We need explicit calculation of the two-loop correction to $m_{\tilde{g}}$.

Gluino pole mass $m_{\tilde{g}}$ at $O(\alpha_s^2)$

Given by the complex pole of the gluino propagator $s_p = (m_{\tilde{g}} - i\Gamma_{\tilde{g}}/2)^2$

$$m_{\tilde{g}} = M_3(Q) + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2)}$$

$M_3(Q)$: running mass in the lagrangian

$\delta m_{\tilde{g}}$: Calculated from the self energy $i(\Sigma_K(p^2)\not{p} + \Sigma_M(p^2))$

For simplicity, we assume

$\diamond$ degenerate squark mass $m_{\tilde{q}}$

$\diamond$ $m_q \ll (m_{\tilde{g}}, m_{\tilde{q}})$

$\rightarrow$ ignore m_q and $\tilde{q}_L$-$\tilde{q}_R$ mixing in the loops.

One-loop correction (Martin, Vaughn; Pierce, Papadopoulos; . . .)

$$\begin{aligned}\delta m_{\tilde{g}}^{(1)} &= -\mathrm{Re}[M_3\Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)] \\ &= \frac{C_V\alpha_s(Q)}{4\pi}M_3(Q)\left(5 - 6\log\frac{M_3(Q)}{Q}\right) \\ &\quad + \frac{\alpha_s(Q)}{\pi}N_q T_F M_3(Q) B_1(M_3(Q)^2, 0, m_{\tilde{q}}(Q)) + O(\alpha_s m_q^2/m_{\tilde{q}}^2)\end{aligned}$$

$C_V = 3$, $T_F = 1/2$, $N_q = 6$

typically $\delta m_{\tilde{g}}^{(1)}/m_{\tilde{g}} = O(10)$ %

Enhanced by

- large α_s
- large SU(3) representation (C_V(octet) $\gg T_F$(doublet))

(α_s, M_3, $m_{\tilde{q}}$) in $\delta m_{\tilde{g}}^{(1)}$: need precise definition to define $\delta m_{\tilde{g}}^{(2)}$

Here we use $\overline{\mathrm{DR}}'$ parameters at $Q \sim M_3$.

Two-loop $O(\alpha_s^2)$ correction to $m_{\tilde{g}}$

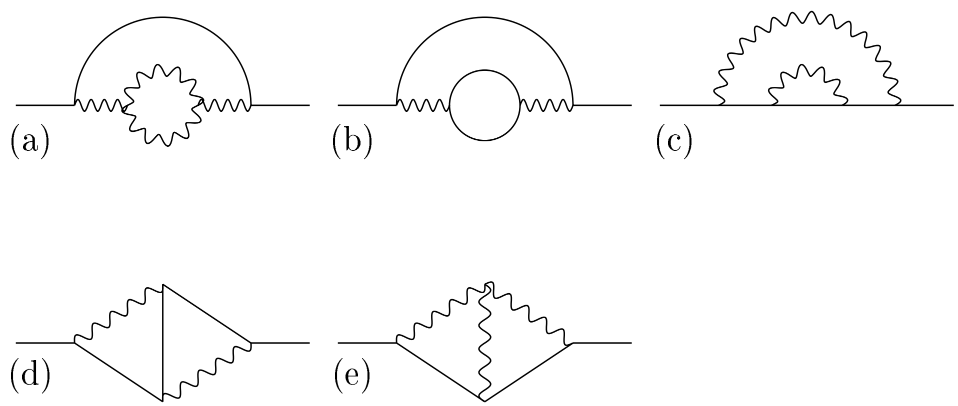
$$
\begin{aligned}
\delta m_{\tilde{g}}^{(2)} &= -\mathrm{Re}[M_3 \Sigma_K^{(2)}(M_3^2) + \Sigma_M^{(2)}(M_3^2)] \\
&\quad + \mathrm{Re}\left[\{M_3 \Sigma_K^{(1)}(M_3^2) + \Sigma_M^{(1)}(M_3^2)\}\{\Sigma_K^{(1)}(M_3^2) + 2M_3^2 \dot{\Sigma}_K^{(1)}(M_3^2) + 2M_3 \dot{\Sigma}_M^{(1)}(M_3^2)\}\right]
\end{aligned}
$$

$\delta m_{\tilde{g}}^{(2)} = \delta m_{\tilde{g}}^{(2,1)} + \delta m_{\tilde{g}}^{(2,2)}$: function of $(M_3, \alpha_s, m_{\tilde{q}})$

$\delta m_{\tilde{g}}^{(2,1)}$: loops with only gluons and gluinos ($m_{\tilde{q}}$ indep.)

$\delta m_{\tilde{g}}^{(2,2)}$: loops including quarks and squarks

Correction with only gluinos and gluons

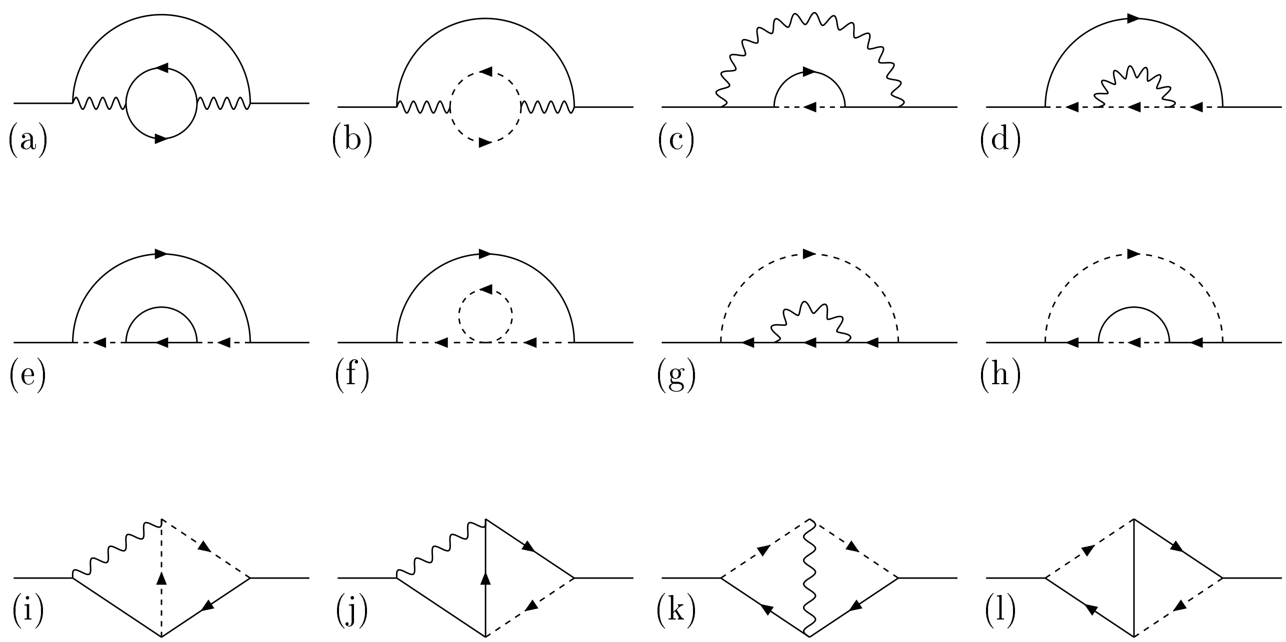


$$\delta m_{\tilde{g}}^{(2,1)} = \left(\frac{C_V \alpha_s}{4\pi}\right)^2 M_3 \left(-48 \log \frac{M_3}{Q} + 36 \log^2 \frac{M_3}{Q} + 26 + 5\pi^2 - 4\pi^2 \log 2 + 6\zeta_3\right)$$

At $Q = M_3$, $\delta m_{\tilde{q}}^{(2,1)}/M_3 \sim 31(\alpha_s/\pi)^2 \sim 0.03$.

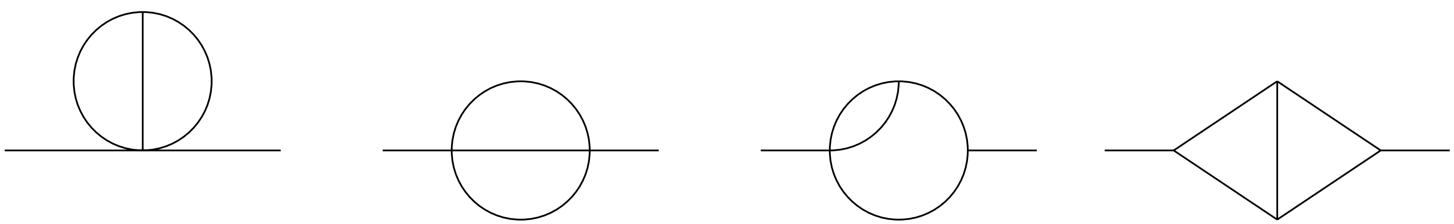
cf. $\delta m_{\tilde{g}}(\text{exp.})/m_{\tilde{g}} \sim 1.3$ % for $m_{\tilde{g}} \sim 600$ GeV (SPS1a)

Correction including quarks/squarks



solid line with an arrow: quark, dashed line with an arrow: squark

$\delta m_{\tilde{g}}^{(2,2)}(M_3, \alpha_s, m_{\tilde{q}})$: Expressed in terms of scalar intergral basis (numerical calculation by TSIL package)



$I(m_{1,2,3})$ $S(m_{1,2,3}), T(m_{1,2,3})$ $U(m_{1,2,3,4})$ $M(m_{1,2,3,4,5})$

$\delta m_{\tilde{g}}^{(2,2)} < 0$ for $m_{\tilde{q}} \sim m_{\tilde{g}}$: partially cancel $\delta m_{\tilde{g}}^{(2,1)} > 0$

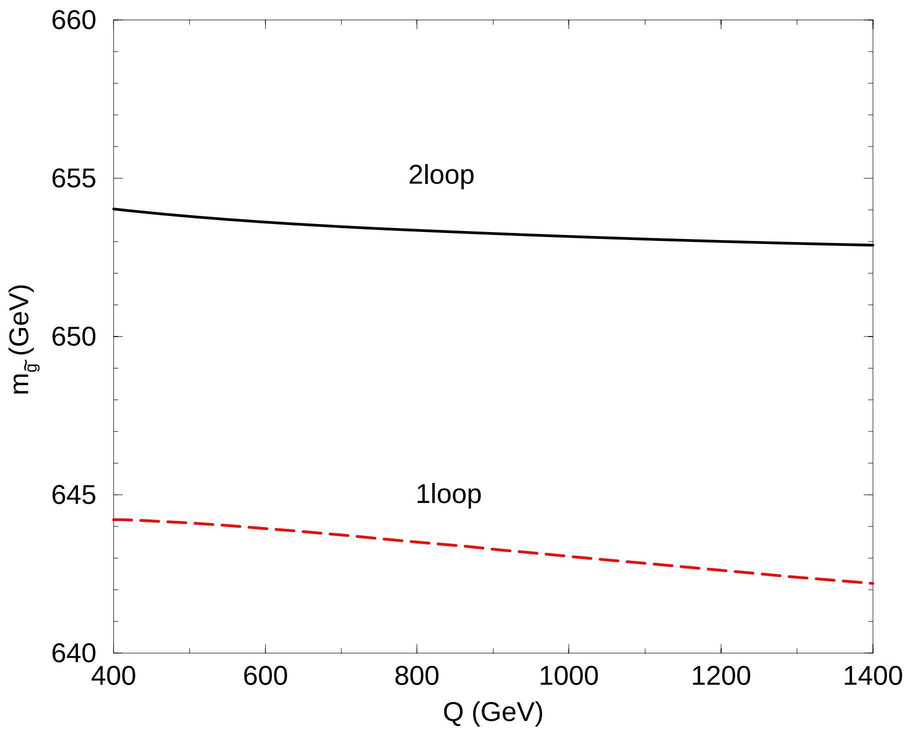
$m_{\tilde{q}} \gg M_3$ limit:

$$\delta m_{\tilde{g}}^{(2,2)}(m_{\tilde{q}} \gg M_3) = \frac{\alpha_s^2 M_3}{(4\pi)^2}\left[72\log^2\frac{m_{\tilde{q}}}{Q} + 242\log\frac{m_{\tilde{q}}}{Q} + \log\frac{M_3}{Q}\left(54 - 288\log\frac{m_{\tilde{q}}}{Q}\right) - 172 + \frac{14}{3}\pi^2\right]$$
$$+\frac{\alpha_s^2 M_3}{(4\pi)^2}N_q C_V T_F\left(-8\log^2\frac{M_3}{Q} + \frac{52}{3}\log\frac{M_3}{Q} - \frac{37}{3} - \frac{4}{3}\pi^2\right)$$ [diagram without $\tilde{q}$].

Residual dependence of $m_{\tilde{g}}$ on the renormalization scale

$M_3(580\text{GeV}) = 580$ GeV, $m_{\tilde{q}}(580\text{GeV}) = 800$ GeV,

cf. tree-level mass: $M_3(400) = 589$ GeV $\rightarrow M_3(1400) = 559$ GeV

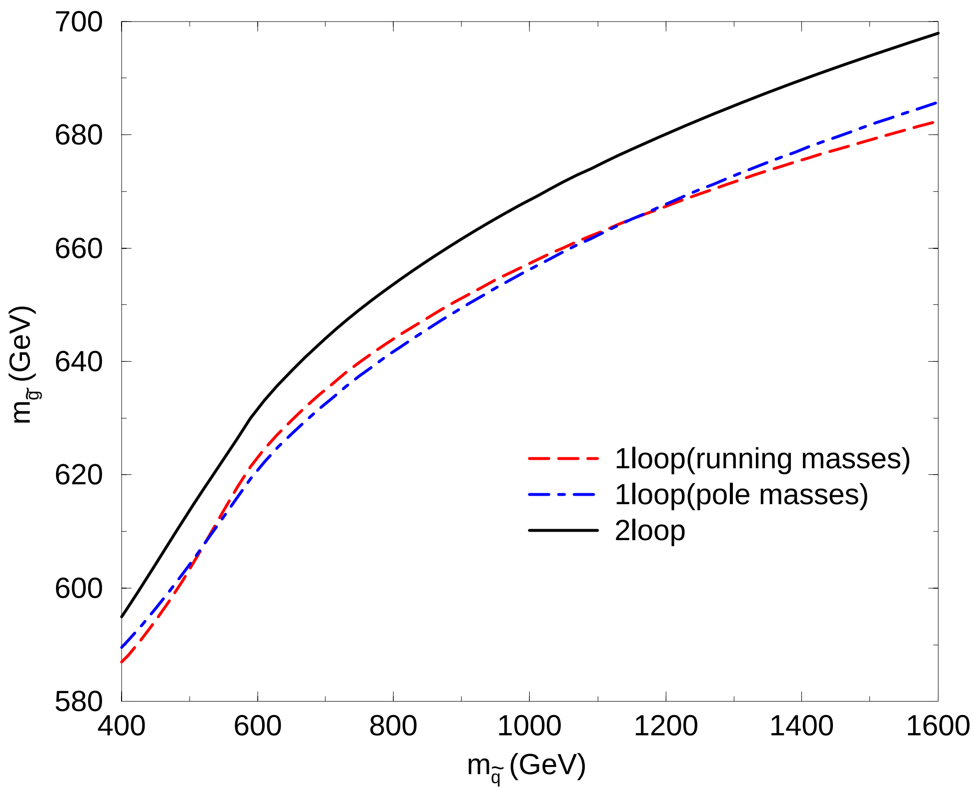


slight improvement by $\delta m_{\tilde{g}}^{(2)}$

$|\delta m_{\tilde{g}}^{(2)}| \gg$ (Q dependence of $m_{\tilde{g}}^{(1)}$)

Gluino pole mass at one- and two-loops

(tree: $M_3(M_3) = 580$ GeV)



$1-2$ % increase of $m_{\tilde{g}}$ by $O(\alpha_s^2)$ corr. $\geq \delta m_{\tilde{g}} \sim 1$ % at LHC/ILC(expected)

$\delta m_{\tilde{g}}^{(2)} > |\delta m_{\tilde{g}}^{(1)}(m_{\overline{\mathrm{DR}}}(Q = M_3)) - \delta m_{\tilde{g}}^{(1)}(m_{\mathrm{pole}})|$

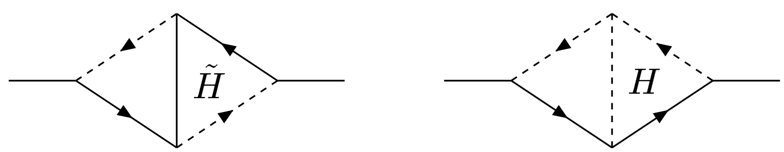
Remaining issues

* contribution of m_q and $\tilde{q}_L - \tilde{q}_R$ mixing to $\delta m_{\tilde{g}}^{(2)}$

suppressed by $m_q^2/(m_{\tilde{g}}^2, m_{\tilde{q}}^2)$

may be important for light gluino/squarks

* $O(\alpha_s h_q^2)$ corrections involving Higgs bosons/higgsinos



Cf. General formulas for two-loop corrections to fermion masses:
(S.P. Martin, hep-ph/0509115)

* Installation to the computer codes for calculating SUSY particle masses

Conclusion

* The pole mass of the gluino $m_{\tilde{g}}$ has been calculated as a function of the lagrangian parameters ($M_3(Q)$, $m_{\tilde{q}}(Q)$, $\alpha_s(Q)$) to $O(\alpha_s^2)$.

* The two-loop corrrection to $m_{\tilde{g}}$ for a given $M_3(Q)$ is typically 1–2 %, which may be larger than the expected uncertainty in precision mass determination at future colliders.