IMPACT OF MASSIVE NEUTRINOS ON NONLINEAR MATTER POWER SPECTRUM

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MASSIVE NEUTRINOS & COSMOLOGY

Cosmology provides stringent constraints on total neutrino masses

- Terrestrial Experiment

Neutrino oscillations : mass square difference → $0.05\ \mathrm{eV} \lesssim \sum m_\nu \lesssim 6.0\ \mathrm{eV}$

Beta decay : Majorana mass

McKeown & Vogel (2004)

- Cosmological Observation : total masses

Distance test
- WMAP5 only $\sum m_\nu \lesssim 1.3\ \mathrm{eV}$
- WMAP5 + BAO + SN $\sum m_\nu \lesssim 0.6\ \mathrm{eV}$

Komatsu et al (2008)

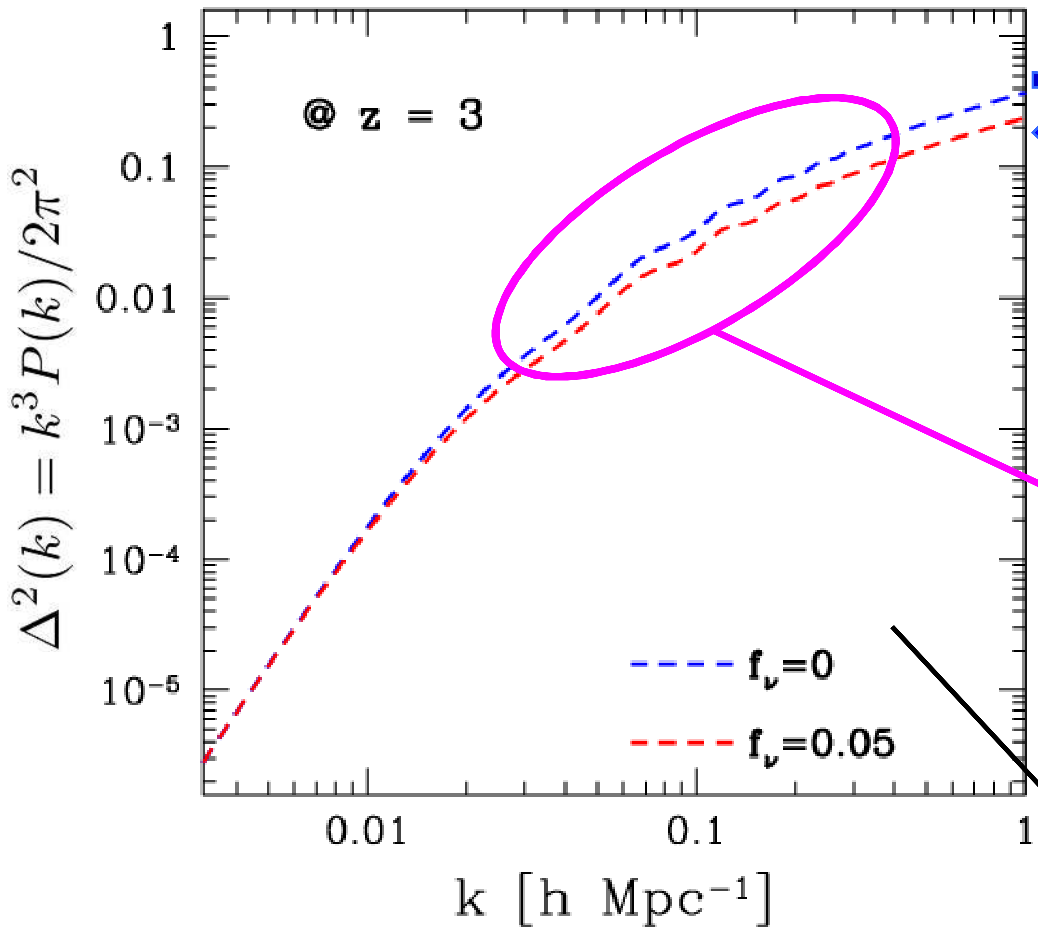
Suppression of growth WMAP3 + SDSS $\sum m_\nu \lesssim 0.6\ \mathrm{eV}$

Tegmark et al (2006), Fukugita et al (2006)

Next generation experiments aim the detection of $\sum m_\nu \sim \mathcal{O}(0.1\mathrm{eV})$

SUPPRESSION OF MATTER PERTURBATION

Neutrino perturbations cannot stay at smaller scale than neutrino free-streaming ➜ weaken gravity via Poisson Eq.



suppression !!

scale-dependent growth

$$\frac{P(k)_{f_\nu \neq 0}}{P(k)_{f_\nu = 0}} - 1 \approx -8 f_\nu \geq 4\%$$

Baryon Acoustic Oscillations (BAOs)

calculated by **Linear** theory

Non-linear regime !!

NON-LINEAR THEORY IMPROVEMENT?

- ★ The most realistic cosmology is not ΛCDM but ΛCDM with massive neutrinos in the sense that neutrino effect cannot be neglected.
- ★ The planned galaxy high-z surveys such as WFMOS aim the **BAOs**.
- ★ **Neutrinos' free-streaming scale** is comparable to **BAO** scale. Good chance to probe neutrino effect !!
- ★ But we cannot neglect **nonlinear gravitational clustering**.

Questions

Can we develop the theory to calculate the nonlinear P(k) with massive neutrinos having mass of ~0.1eV?

If we have such nonlinear theory, how is the constraint on neutrino masses improved for WFMOS-like survey?

METHODOLOGY

Perturbation Theory : natural extension of linear theory

multi-fluid component of baryon + mixed dark matter (CDM + Neutrinos)

$$\delta_{\rm m} = f_{\rm cb}\delta_{\rm cb} + f_\nu \delta_\nu \quad \left[f_{\rm cb} \equiv \frac{\Omega_{\rm c}+\Omega_{\rm b}}{\Omega_{\rm m}},\ f_\nu \equiv \frac{\Omega_\nu}{\Omega_{\rm m}} = \frac{\sum m_\nu}{94.1\Omega_{\rm m}h^2} \right]$$

$$\Rightarrow \quad P(k) = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

⇓

Perturbative expansion of non-linear Continuity & Euler equations

* Contrasted to only CDM case, some difficulties are involved:

① Neutrinos cannot be treated as fluid-component.

② Nonlinear growth functions are also scale-dependent,
which complicates the calculation of nonlinear correction.

① Neutrino fluctuations

Tiny contributions from neutrinos perturbation to total P(k) for $f_\nu \lesssim 0.06$

$$P(k) = f_{\rm cb}^2 P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f_\nu^2 P_\nu$$

We assume **neutrino perturbations stay at linear level**

and add nonlinear corrections **only for Pcb** term.

Does neutrinos really stay at linear level? **S.S, Takada, Taruya in prep (2008)**

- Neutrino dynamics are controlled by nonlinear Newton potential which is supported by CDM + baryon fluctuations.

- Even if nonlinear CDM + baryon fluctuations are included in linear Boltmann equations, less than 0.01% change of P(k).

② ONE-LOOP CORRECTION

calculate next-to-leading order correction for Pcb(k)

From standard perturbation theory Makino,Sasaki,Suto (1992)

$$P_{\rm cb}^{\rm Approx}(k) = P_{\rm cb}^{L} + P_{\rm cb}^{(22)} + P_{\rm cb}^{(13)}$$

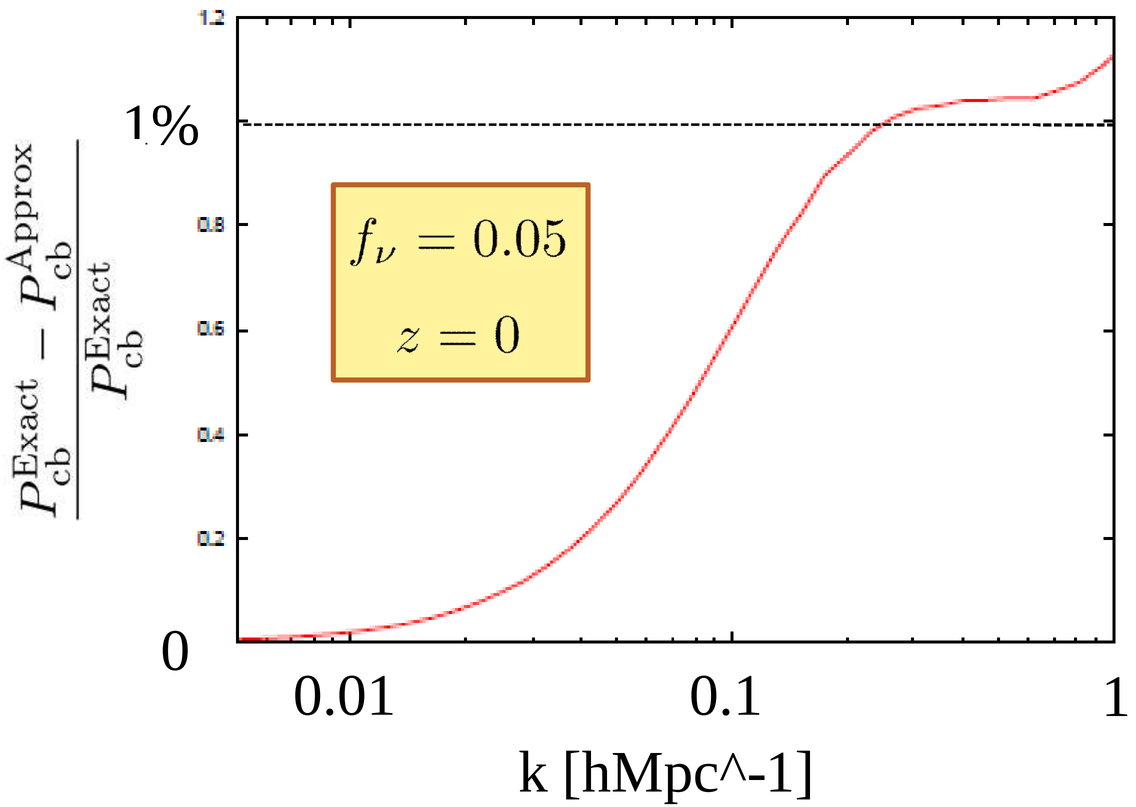
$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z)\int_{-1}^{1} d\mu P_{\rm cb}^L(k\sqrt{1+r^2-2\mu r};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2}$$

$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3}{252(2\pi)^2}P_{\rm cb}^L(kr;z)\int_0^\infty dr P_{\rm cb}^L(kr;z) \times\left[\frac{12}{r^2}-158+100r^2-42r^4+\frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right]$$

However, this is approximation in the sense that scale-dependency of growth functions are neglected.

② ONE-LOOP CORRECTION

difference between exact and approximated $P_{\rm cb}(k) = P^L_{\rm cb} + P^{(22)}_{\rm cb} + P^{(13)}_{\rm cb}$



Numerically

solve the 2nd & 3rd order equations

⇩

integrate mode-coupling

⇩

$P^{\rm Exact}_{\rm cb}(k)$

The fractional difference is less than ~ 1%.
Because only nearby mode-coupling contributes to one-loop integration.

RECIPE FOR NONLINEAR P(K)

Can we develop the theory to calculate the nonlinear P(k) with massive neutrinos having mass of ~0.1eV? ➡ **YES!!**

Recipe to calculate the nonlinear P(k;z)

① calculate **linear** power spectra $P^L_{\rm cb}(k;z),\ P^L_{{\rm cb}\nu}(k;z),\ P^L_{\nu}(k;z)$

for redshift z from CAMB or CMBFAST

② add **one-loop correction** for CDM + baryon term

$$P^{\rm Approx}_{\rm cb}(k) = P^L_{\rm cb} + P^{(22)}_{\rm cb} + P^{(13)}_{\rm cb}$$

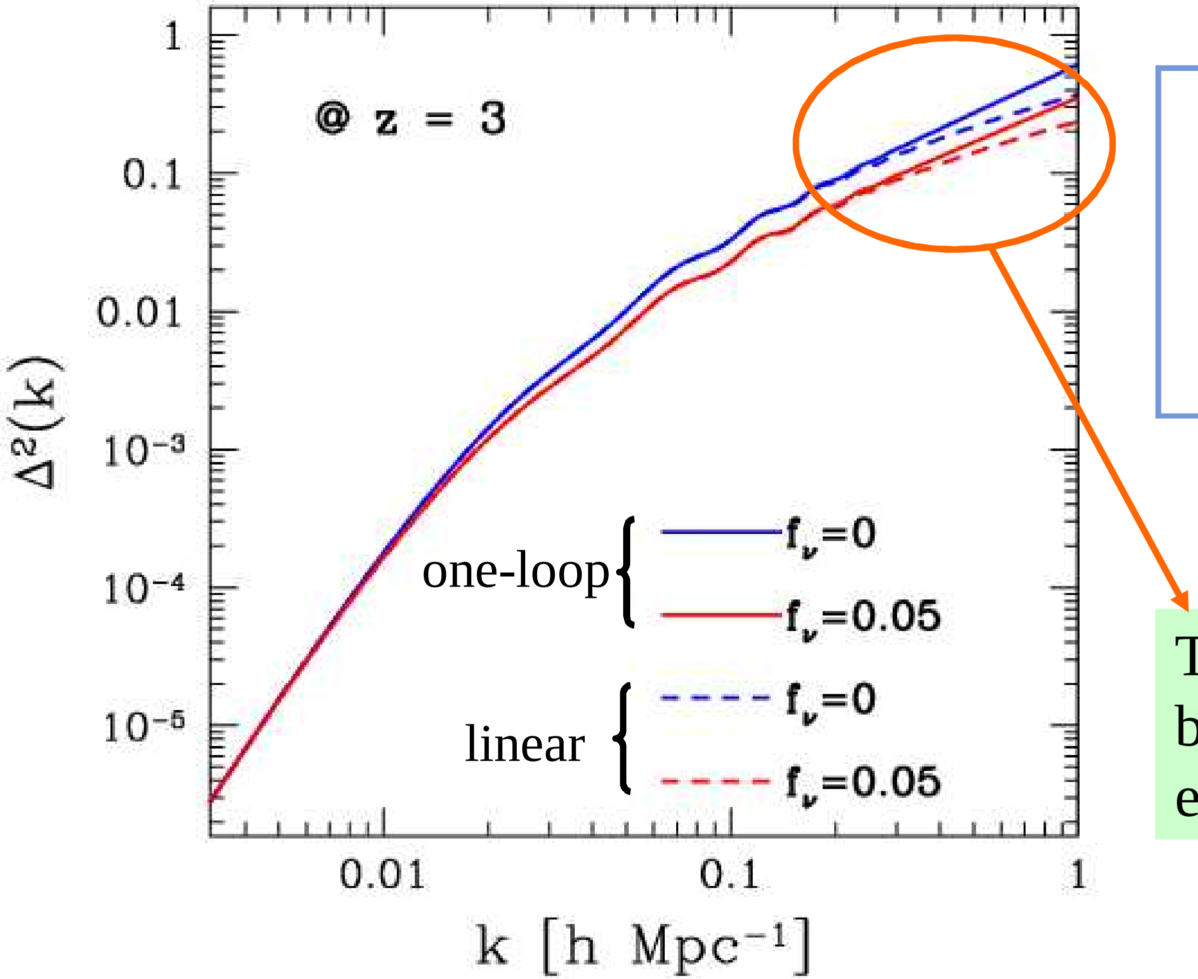
③ sum up all components $P(k) = f^2_{\rm cb} P_{\rm cb} + 2 f_{\rm cb} f_\nu P_{{\rm cb},\nu} + f^2_\nu P_\nu$

One-loop corrections are roughly proportional to the square of $P^L_{\rm cb}(k;z)$

➜ Neutrino suppression effect is expected to be enhanced.

NONLINEAR P(K)

dimensionless power $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$



fiducial cosmology

$\Omega_b h^2 = 0.0223$

$\Omega_m h^2 = 0.1277$

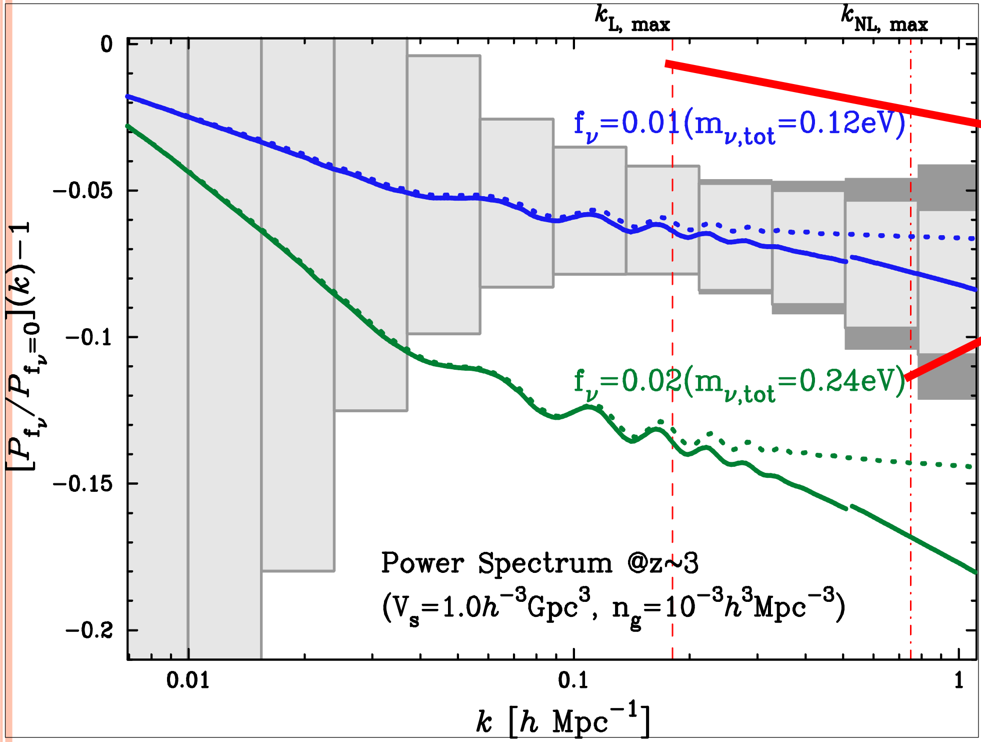
$h = 0.73, \ w_0 = -1$

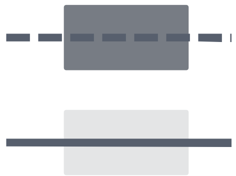
$\Delta^2_{\mathcal{R}} = 2.35 \times 10^{-9}, \ n_S = 1$

The amplitude is enhanced by non-linear gravitational evolution.

NEUTRINO SUPPRESSION EFFECT

$P_{f_\nu \neq 0}/P_{f_\nu = 0} - 1$





Linear

Perturbation Theory

Limitation of linear theory

applicapable range of PT？

$k_{\rm NLmax}$: 0.69 hMpc^-1

$$\Delta^2 = k^3 P^{NL}_{f_\nu=0}(k)/2\pi^2 < 0.4$$

Jeong,Komatsu (2006)

True criterion should be derived from simulations with neutrinos

Neutrino suppression effect is enhanced in weakly nonlinear regime.

The larger amplitude leads to less shot noise error.

FORECAST

Does our non-linear theory improve the constraint on neutrino masses for future galaxy redshift survey?

Fisher information matrix formalism Takada,Futamase,Komatsu(2006)

$$F_{\alpha\beta}^{\rm LSS} = \frac{V_s}{8\pi^2}\int_{-1}^{1} d\mu \int_{k_{\rm min}}^{k_{\rm max}} k^2 dk \frac{\partial \ln P_s}{\partial p_\alpha}\frac{\partial \ln P_s}{\partial p_\beta}\left[\frac{\bar{n}_g P_s}{\bar{n}_g P_s + 1}\right]^2$$

where $P_s(k,\mu) = [1+\beta\mu^2]^2 b_1^2 P(k;z)$

$$\mathbf{F} = \mathbf{F}^{\rm PLANCK} + \mathbf{F}^{\rm WFMOS}$$

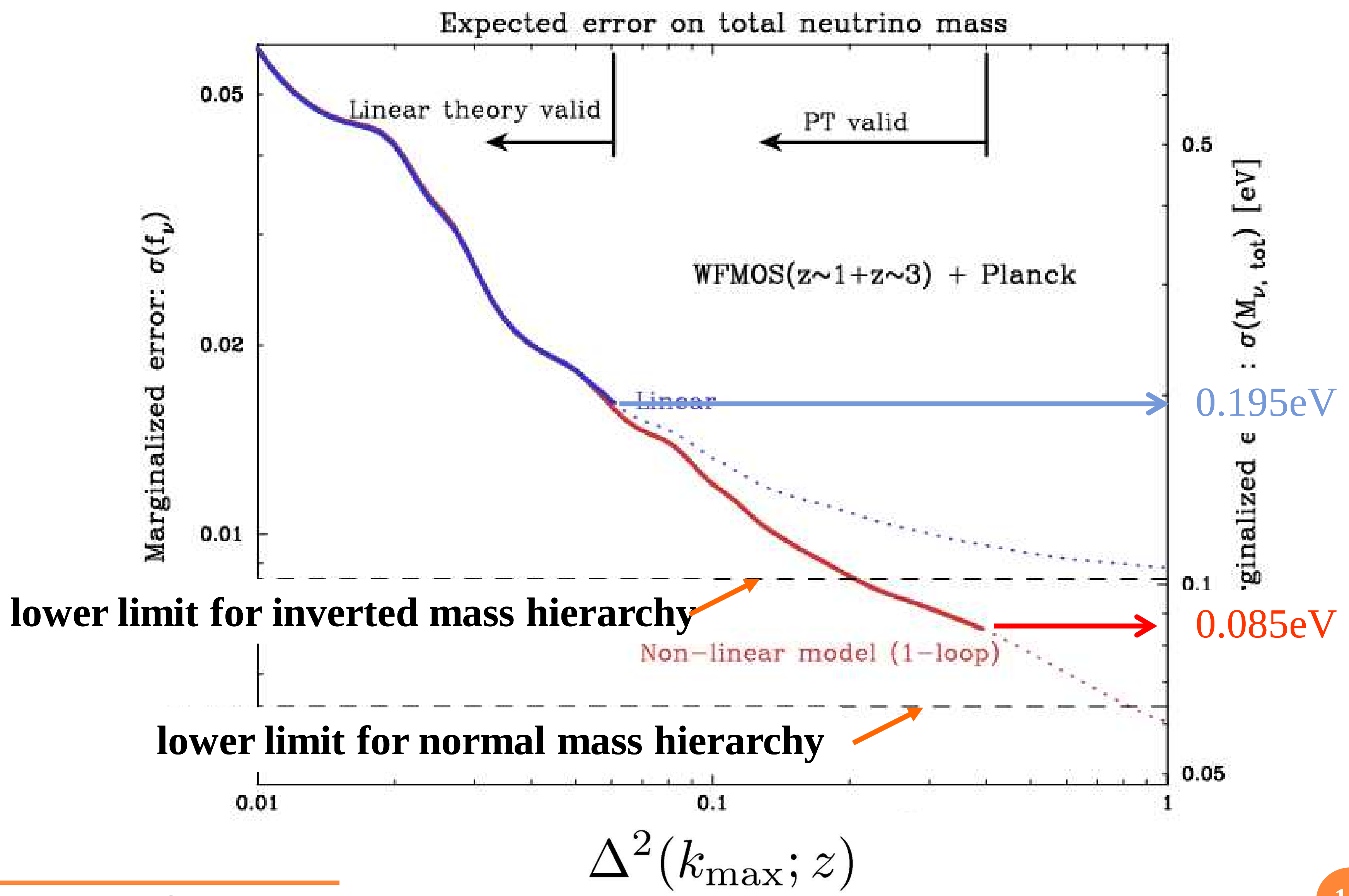
$\sigma^2(p_\alpha) = (\mathbf{F}^{-1})_{\alpha\alpha}$: marginalized 1σ error on certain parameter

free parameters: $\mathbf{p} = (\Omega_{\rm b}h^2, \Omega_{\rm c}h^2, \Omega_m, \Delta_{\mathcal{R}}^2, n_S, \alpha, w_0, f_\nu, \beta)$

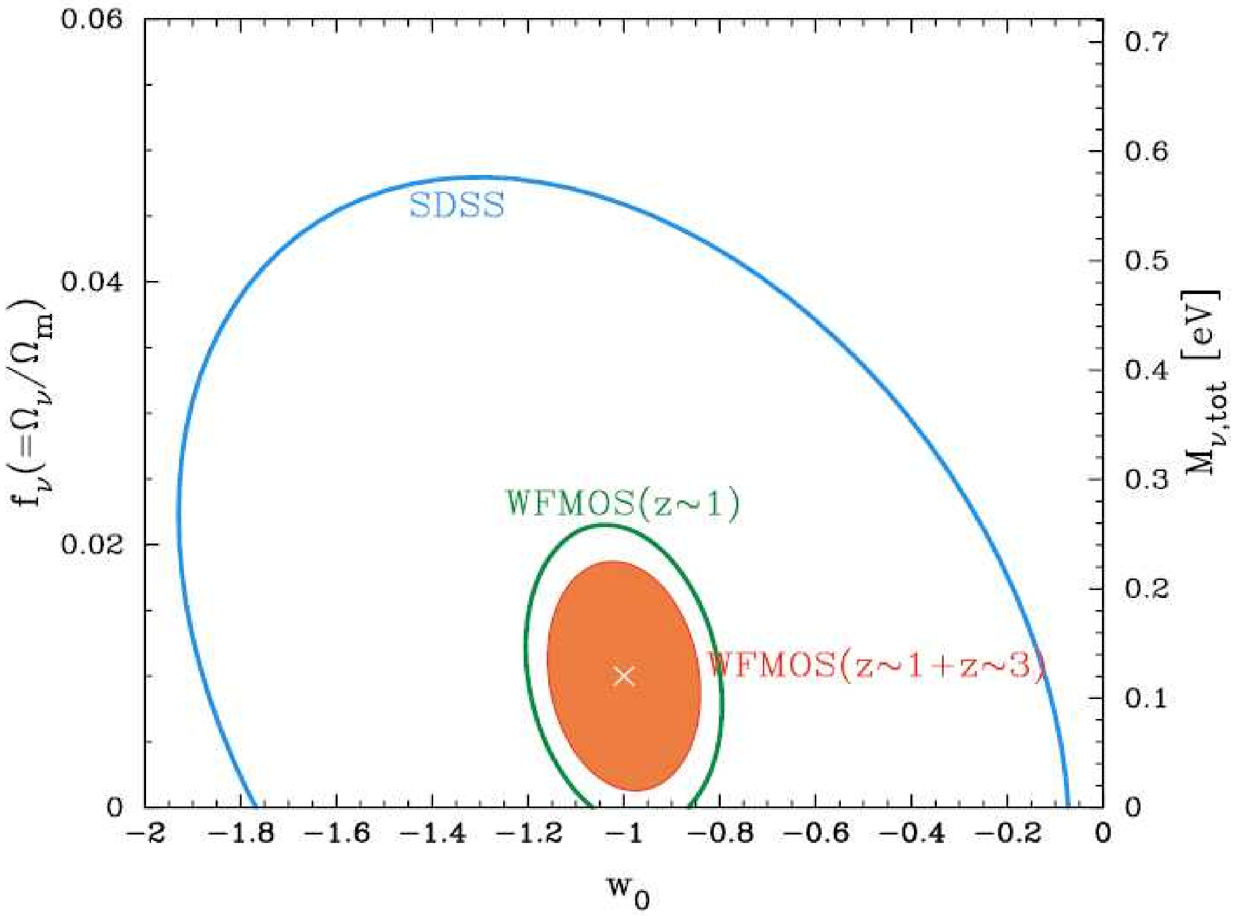
assume

flat geometry, linear bias, Gaussian statistics

NEUTRINO MASS CONSTRAINT



DEGENERACY WITH ` **w** '



Neutrino effect does not shift the BAO peaks.

➜ Break the degeneracy between w & f_nu

SUMMARY

* Neutrino effect cannot be neglected for future galaxy redshift survey.
* We carefully establish the approach to calculate nonlinear P(k) with massive neutrinos based on perturbation theory.
* We show that neutrino effect is enhanced in weakly nonlinear regime and this refined model description leads to improve the constraint on neutrino masses for WFMOS-like survey.

<u>Discussion</u>

S.S, Takada, Taruya in prep

* include the other nonlinearities, biasing, redshift distortion.
* compare N-body simulation with neutrinos Brandenbyge et al (2008)
* possibility to improve the theory, e.g. renormalized perturbation theory

PARAMETER DEGENERACY

correlation coefficient $$r \equiv \frac{(\mathbf{F}^{-1})_{p_\alpha p_\beta}}{\sqrt{(\mathbf{F}^{-1})_{p_\alpha p_\alpha}(\mathbf{F}^{-1})_{p_\beta p_\beta}}}$$

- $|r| \sim 1$: **strong degeneracy**
- $|r| \sim 0$: **independent**

	$\ln(\Omega_{de0})$	$\ln A_s$	w_0	$f_\nu\,(m_{\nu,\mathrm{tot}}\,[\mathrm{eV}])$	n_s	α	$\ln(\omega_b)$	$\ln(\omega_m)$	$b_1(z\sim 1)$	$\beta(z\sim 1)$
Linear: Errors	0.046	0.020	0.16	0.017 (0.20)	0.014	0.0043	0.0032	0.0099	0.099	0.055
Non-linear: Errors	0.012	0.015	0.008	0.0071 (0.085)	0.0091	0.0028	0.0031	0.0091	0.041	0.018
Linear: r	0.26	0.065	0.10		0.023	0.0042	0.045	0.35	0.95	0.033
Non-linear: r	0.33	0.30	-0.014	–	-0.51	0.60	-0.20	0.69	0.86	0.064

- Degeneracy between w0 & f_nu is resolved by BAOs
- Strong degeneracy with b1
  ⇒ necessity of non-linear biasing analysis

LINEAR THEORY REVISITED

2 suppression of matter perturbation More crucial !!

Massive neutrinos contribute naturally to matter perturbation, but do not contribute less than free-streaming scale due to large velocity dispersion.

free-streaming scale

$$k_{\rm fs} = 0.239 h{\rm Mpc}^{-1} \left(\frac{m_\nu}{1{\rm eV}}\right) \left(\frac{\Omega_m}{0.25}\frac{2}{1+z}\right)^{1/2}$$

~ BAO scale

Linear Theory

Scale-dependent linear growth rate

$$\begin{cases} D_{{\rm cb}\nu}(k,z) \propto D(z) & k \ll k_{\rm fs}(z), \\ D_{{\rm cb}\nu}(k,z) \propto (1-f_\nu)[D(z)]^{1-0.6 f_\nu} & k \gg k_{\rm fs}(z) \end{cases}$$

Ω_m =constant, $f_\nu \equiv \Omega_\nu/\Omega_m = 0$

slow down the growth of small-scale structure