KEK Cosmophysics Group Inaugural Conference

AIU 08 @ KEK 2008. 3. 13

Primordial non-Gaussianities in new ekpyrotic cosmology

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CQG 24, 3919 (2007) JCAP11, 024 (2007)

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1.Introduction

Observations on non-Gaussianities

deviation from Gaussian distribution

- conventional parametrisation Komatsu and Sperbel 2001

$$\zeta = \zeta_L - \frac{3}{5} f_{NL} \zeta_L^2$$

Curvature perturbations

ζ_L obeys Gaussian statistics

$f_{NL} \sim 0$ for almost free theories like standard inflation

- Constraints on f_{NL} from WMAP 5-year

$$-9 < f_{NL} < 111$$

Komatsu et al. 2008

favoring relatively large non-Gaussianity

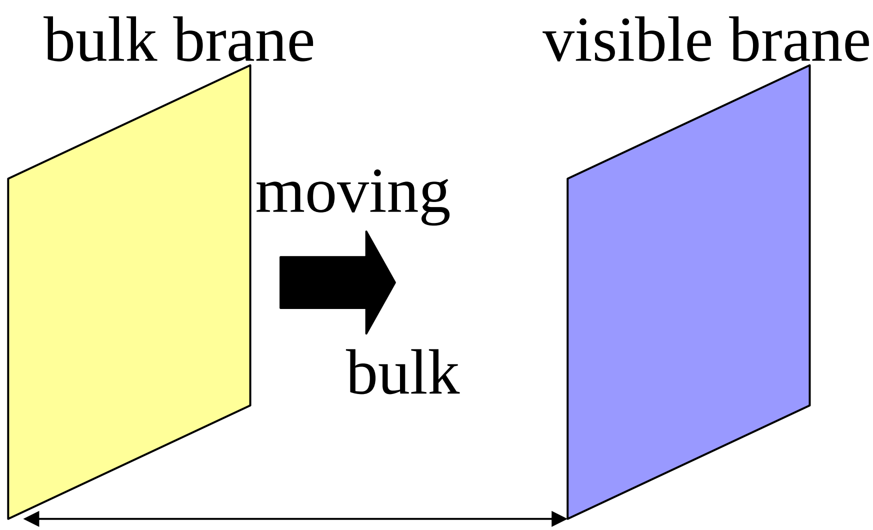
→ Need to consider the early universe scenarios other than the standard inflationary screario?

Old ekpyrotic scenario

- Set-up

Khoury, Ovrut, Steinhardt, Turok `01

Hot big-bang universe is produced
by the collision of branes!



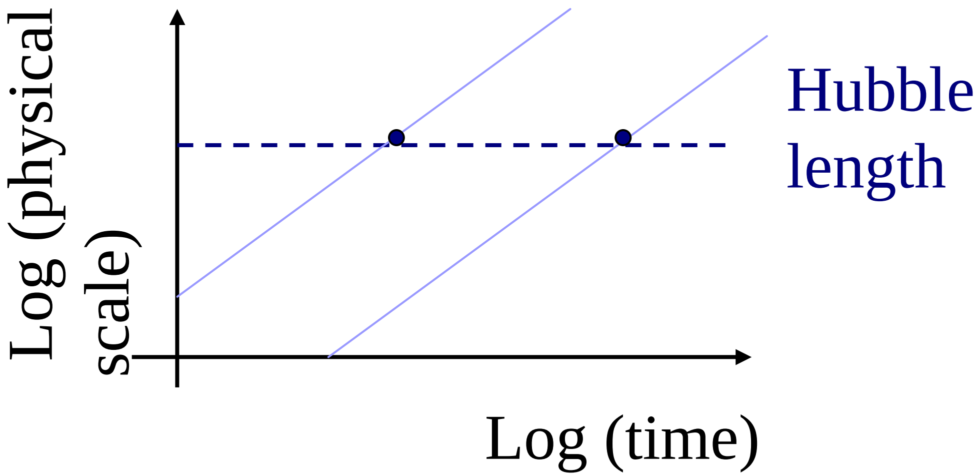
separation is parametrized
by a scalar field Φ

- Set up is based on heterotic M theory
  Lucas, Ovrut, Waldram `98
- Fluctuation is generated in the contracting phase

  bulk brane moves
  toward visible brane
- Collision of two branes thermalise the visible brane

Evolution of scales

- Slow roll inflation



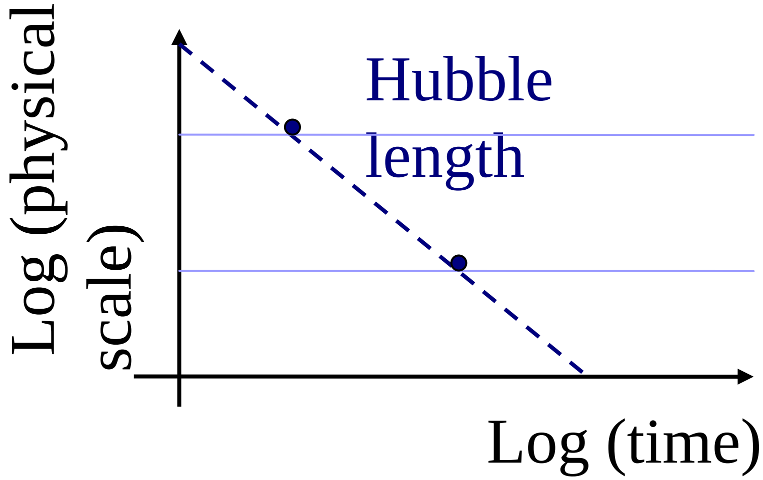
$$a = t^p$$

$$H = \frac{p}{t}$$

with $p \gg 1$

$$\tau = -1/(aH)$$

- Ekpyrotic collapse



$$a = (-t)^p$$

$$H = \frac{p}{-t}$$

with $p \ll 1$

$$\tau \simeq t/a$$

Primordial curvature perturbations

In contracting phase, cosmology on the brane is described by 4d effective scalar field theory with potential $V(\phi) = -V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{M_p}\right)$ $p \ll 1$

- Spectrum Lyth `02

$$n = 1 + \frac{2}{1-p} \underset{p \to 0}{\longrightarrow} 3$$

strongly blue tilted

- comoving curvature perturbation remains constant for adiabatic perturbations on large scales

⇒ strongly blue tilted spectrum for single field model in which 4d effective theory is valid

Is it really impossible to obtain scale-invariant spectrum in the context of the ekpyrotic scenario?

2. New ekpyrotic cosmology (1)

generation of nearly scale-invariant spectrum

SM with K. Koyama and D. Wands CQG 24 (2007) 3919

2.New ekpyrotic cosmology (1)

Lehners, McFadden, Turok, Steinhardt `07
Buchbinder, Khoury, Ovrut `07,   Creminelli, Senatore `07

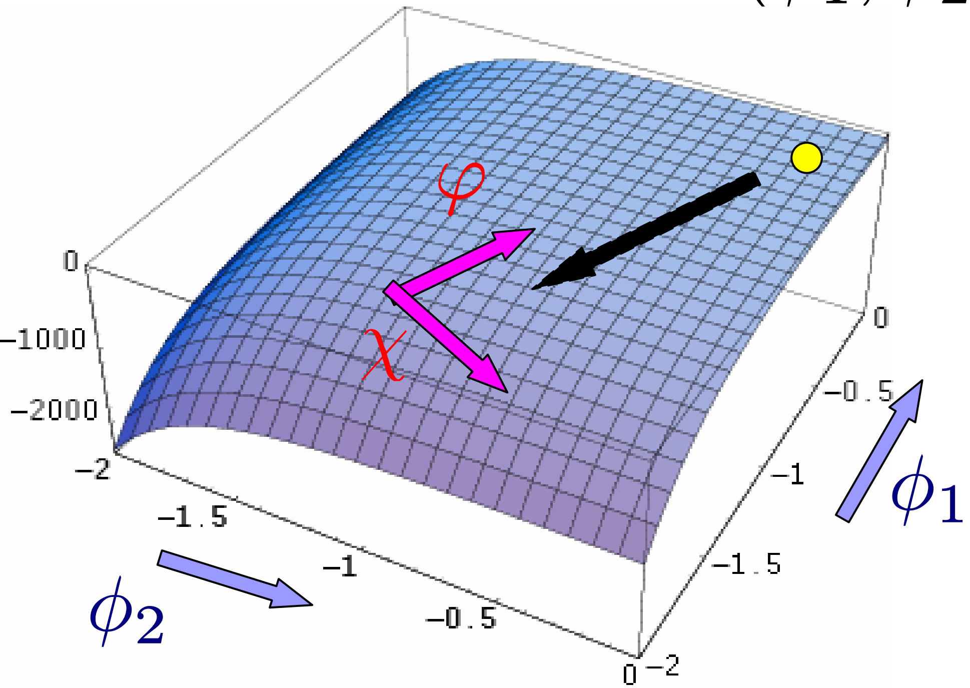
- Model   Considering the coupling with moduli
  During the phase much before the collision

$$V(\phi_1,\phi_2) = -V_1 e^{-c_1\phi_1} - V_2 e^{-c_2\phi_2}$$

$$= -\underline{U(\chi)} e^{-c\varphi} \qquad c_1, c_2 \gg 1$$

$$U_0\left[1+\frac{c^2}{2}(\chi-\chi_0)^2+\cdots\right]$$

$$c = \frac{c_1 c_2}{\sqrt{c_1^2+c_2^2}} \gg 1$$



Background dynamics

Scaling solutions play important roles in the system with exponential potentials

- Scaling solution supported by a single field $\phi_i \; (i=1,2)$

$$a=(-t)^{p_i}, \quad \phi_i = \frac{2}{c_i}\ln(-t) - \frac{1}{c_i}\ln\left(\frac{p_i(1-3p_i)}{V_i}\right)$$

with $p_i = \frac{2}{c_i^2}$

cf. old ekpyrotic model

- Multi-field scaling solution

assisted contraction

Finelli `02

$$a=(-t)^{p}, \quad \phi_i = \frac{2}{c_i}\ln(-t) - \frac{1}{c_i}\ln\left(\frac{2(1-3p)}{c_i^2 V_i}\right)$$

with $p=\frac{2}{c^2}, \quad c=\frac{c_1 c_2}{\sqrt{c_1^2+c_2^2}}$ if $c^2>6$

Adiabatic and entropy perturbations

scalar field perturbations with multi fields can be decomposed into the instantaneous adiabatic and entropy perturbations

Sasaki, Tanaka `98 Gordon, Wands, Bassett, Maartens `00

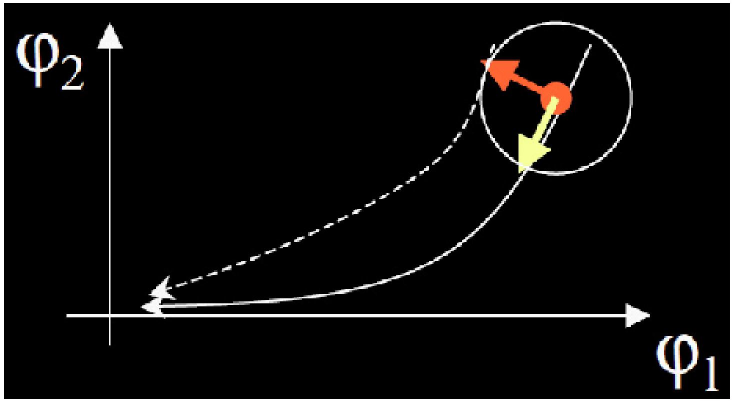
- adiabatic perturbation

$$\delta r = \sin\theta\delta\phi_1 + \cos\theta\delta\phi_2$$

- entropy perturbation

$$\delta s = \cos\theta\delta\phi_1 - \sin\theta\delta\phi_2$$

with $\theta = \arctan \frac{\dot\phi_1}{\dot\phi_2}$



δr : along the direction of the background field's evolution

δs : orthogonal to the background trajectory

- curvature perturabtions

$$\mathcal{R}_c = \frac{H\delta r}{\dot r}$$

Generation of entropy perturbations

- evolution eq. for entropy field

$$\ddot{\delta s} + 3H\dot{\delta s} + \frac{k^2}{a^2}\delta s + (V_{,ss} - \dot{\theta}^2)\delta s = \underline{-2\frac{\dot{\theta}}{\dot{r}}}\left[\dot{r}\dot{\delta r} - \left(\frac{\dot{r}^3}{2H} + \ddot{r}\right)\delta r\right]$$

coupling with adiabatic field

If we choose the background as

- multi-field scaling solution (B) $\theta = \arctan\frac{c_2}{c_1}$
- single field scaling solution (B1, B2) $\theta = \frac{\pi}{2}, \quad 0$

→
- adiabatic and entropy fields are decoupled
- possible to quantise the independent fluctuations

- spectral index

B: $n_{\delta s} = 2p$ scale-invariant for $p \to 0$ B1,B2: $n_{\delta s} = 2$ blue

Stability analysis

Koyama, Wands `07

- Phase space variables

$$x_i \equiv \frac{\dot{\phi}_i}{\sqrt{6}H}, \quad y_i \equiv \frac{\sqrt{V_i e^{-c_i\phi_i}}}{\sqrt{3}H}$$

$i = 1, 2$

constrained by $\sum_j x_j^2 - \sum_j y_j^2 = 1$

- FIxed points

A: $\sum_j x_j^2 = 1, \quad y_j = 0$ (kinetic term dominant solution)

Bi: $x_i = \frac{c_i}{\sqrt{6}}, y_i = -\sqrt{\frac{c_i^2}{6} - 1}, x_j = y_j = 0, \ (\text{for} j \neq i)$

(single field dominant scaling solution)

B: $x_j = \frac{\sqrt{6}}{3p}\frac{1}{c_j}, y_j = -\sqrt{\frac{2}{c_j^2 p}\left(\frac{1}{3p} - 1\right)}$

(multi-field scaling solution)

- linear analysis

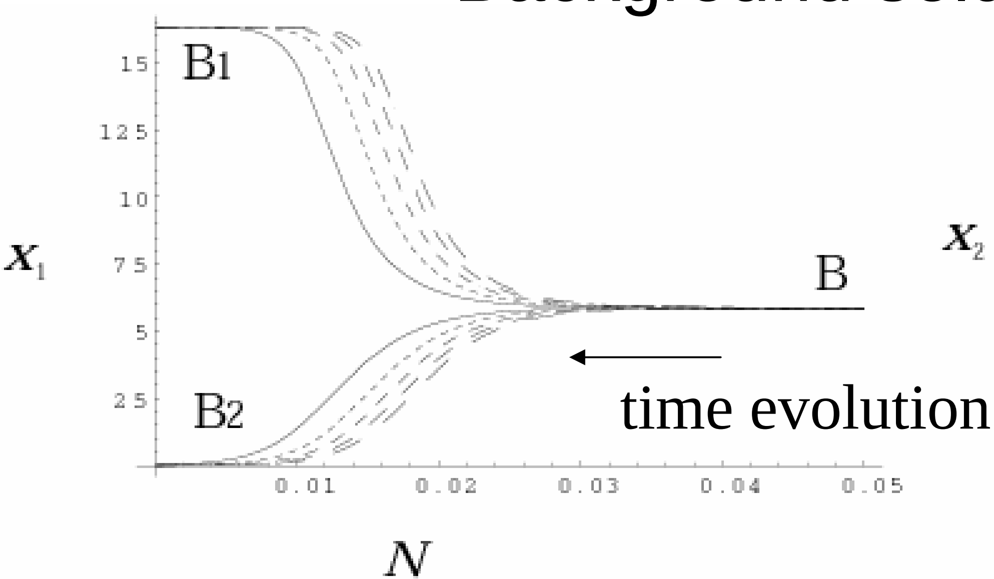
A: unstable Bi: stable B: saddle for $\begin{cases} c_i^2 > 6 \\ \sum c_i^{-2} < 1/6 \end{cases}$

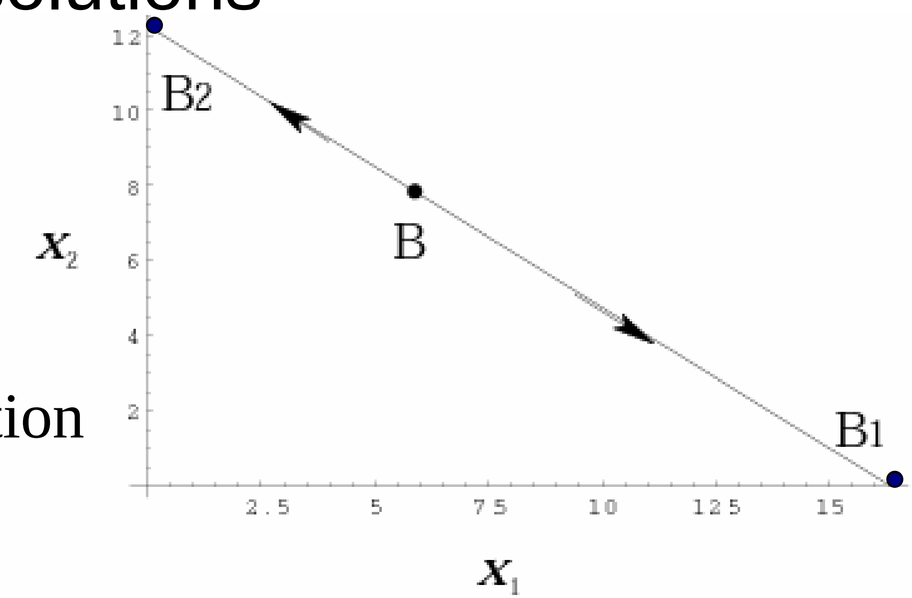
Desirable background

time →

	$\dot{\theta}=0$	$\dot{\theta}\neq 0$	$\dot{\theta}=0$		
	B (saddle)	transit	Bi (attractor)	non-singular bounce	expanding Universe
curvature	blue	mixing	blue		
isocurvature	scale-invariance		blue		

- Background solutions





Predictions from linear perturbation

SM with Koyama, Wands, CQG 24, 3919 (2007)

- Primordial power spectrum

$$\mathcal{R}_c = \frac{c^2}{2\sqrt{c_1^2+c_2^2}} \left|\frac{H}{2\pi}\right|_T \qquad c = \frac{c_1 c_2}{\sqrt{c_1^2+c_2^2}}$$

- teosor-scalar ratio

T : transition time from the scaling sol.

$r \simeq 0$ ⟵ spectrum of tensor is blue tilted in scaling solutions

⇑

Assuming the followings:

- non-singular bounce is realised
  comoving curvature perturbation is conserved
- matter comes from the adiabatic field at the bounce
  no isocurvature perturbations in the expanding universe

3. New ekpyrotic cosmology (2)

esitimate of primordial non-Gaussianities

SM with Koyama, Vernizzi, Wands JCAP 11 (2007) 024

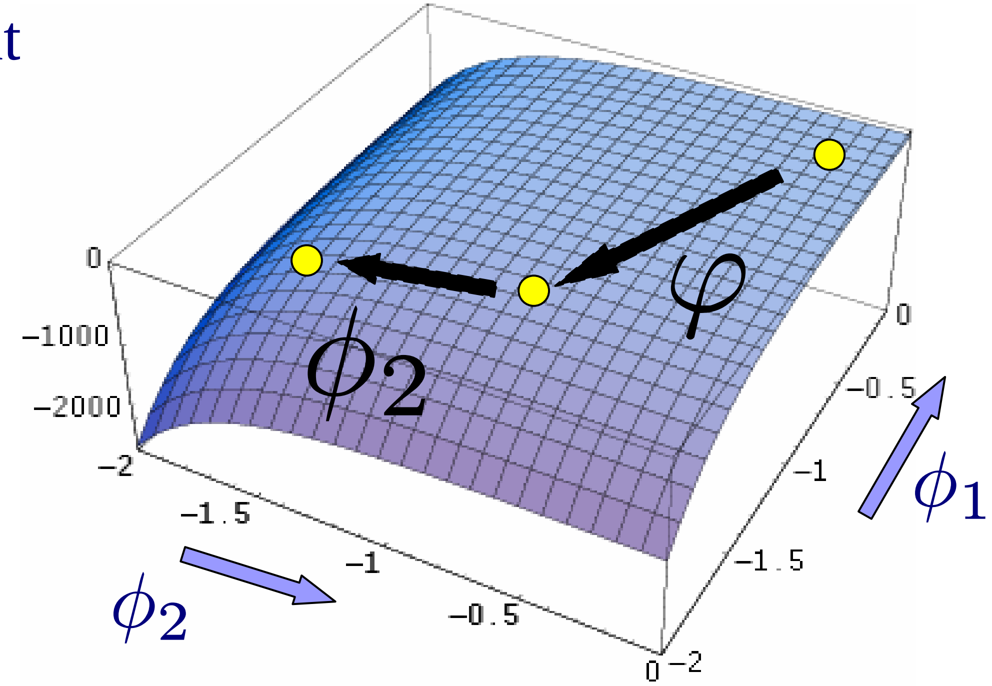
3. Primordial Non-Gaussianities

- assumptions

Koyama, SM, Vernizzi, Wands, JCAP11 024 (2007)

  - Initial condition are set to realise multi-field scaling solution (B).
  - Final stage of the ekpyrotic collapse is descrebed by single-field dominated scaling solution (B2).
  - Transition from B to B2 occurs instantaneously.

valid in fast-roll limit



Statistics of scalar field fluctuations

origin of the perturbations is scalar field fluctuation $\delta\chi$

- perturbative expansion

$$\delta\chi = \underline{\delta\chi_1} + \frac{1}{2}\delta\chi_2 + \frac{1}{6}\delta\chi_3 + \dots\,,$$

free field

- power spectrum

$$\langle \delta\chi_{\mathbf{k}_1}\delta\chi_{\mathbf{k}_2}\rangle \equiv P(k)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\,,$$

$\longrightarrow$ leading order $\mathcal{P}(k) \equiv \frac{4\pi k^3}{(2\pi)^3}P(k) = \frac{1}{p^2}\left(\frac{H}{2\pi}\right)^2$

- bispectrum

$$\langle \delta\chi_{\mathbf{k}_1}\delta\chi_{\mathbf{k}_2}\delta\chi_{\mathbf{k}_3}\rangle \equiv B(k_1,k_2,k_3)(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,,$$

$\longrightarrow$ leading order $\sim \langle \delta\chi_1\delta\chi_1\delta\chi_2\rangle$

δ N formalism

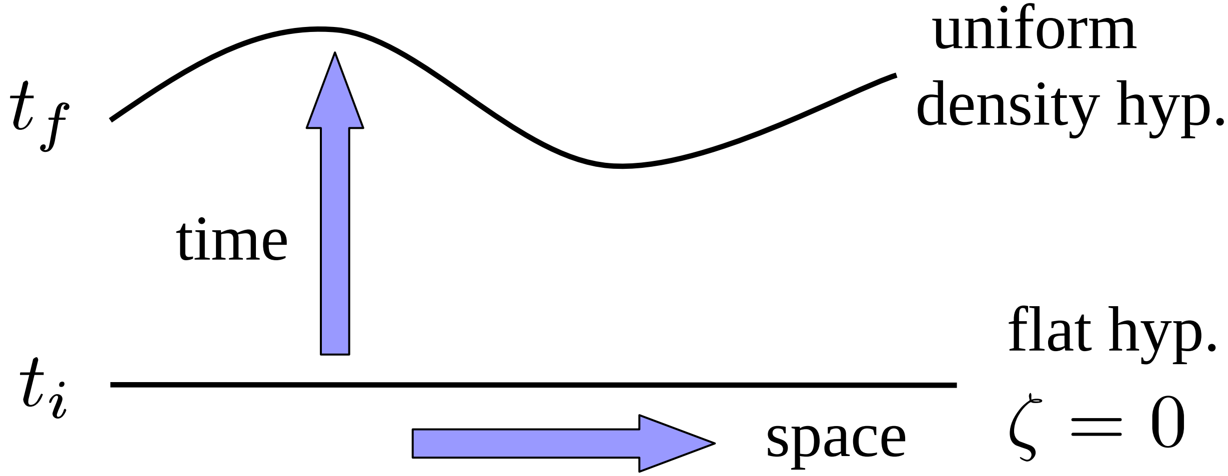
(e.g. Sasaki and Stewart, 1996)

- idea

(curvature perturbation) = (perturbed expansion)

$$g_{ij} = a^2(t)e^{2\zeta(t,x)}\gamma_{ij}(t,x) = \tilde{a}^2(t,x)\gamma_{ij}(t,x)$$

$$\det\gamma = 1$$



$$\zeta(t_f, x) = N(t_f, x) - N_0(t_f)$$

$$N \equiv \log\frac{\tilde{a}(t_f, x)}{a(t_i)}$$

Separate universe assumption

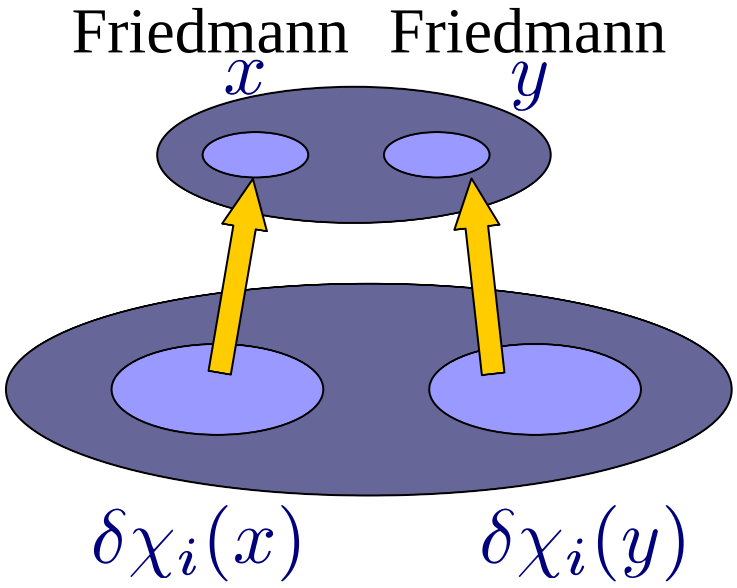
(e.g. Salopek and Bond, `90

Wands, Malik, Lyth, Liddle, `00)

- assumption

local expansion behaves like a locally homogeneous and isotropic universe

justified on sufficiently large scales



- local expansion

$$N(t_f, x) = N(t_f, \delta\chi(t_i, x))$$

- curvature perturbations

$$\zeta(x) = \delta N(x) = N_{,\chi_i}\delta\chi_i + \frac{1}{2}N_{,\chi_i\chi_i}(\delta\chi_i)^2$$

Statistics of curvature perturbations

- nonlinear parameter

$$f_{NL} \equiv \frac{5}{6} \frac{\prod_j k_j^3}{\sum_j k_j^3} \frac{B_\zeta}{4\pi^4 \mathcal{P}_\zeta^2}$$ ← bispectrum

- three-point function
  - δN formalism
  - separate universe assumption

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = N_{,\chi_i}^3 \langle \delta\chi_{i\mathbf{k}_1} \delta\chi_{i\mathbf{k}_2} \delta\chi_{i\mathbf{k}_3} \rangle$$ intrinsic non-Gaussianity

$$+ \frac{1}{2} N_{,\chi_i}^2 N_{,\chi_i\chi_i} \langle \delta\chi_{i\mathbf{k}_1} \delta\chi_{i\mathbf{k}_2} (\delta\chi_i \star \delta\chi_i)_{\mathbf{k}_3} \rangle + \text{perms.}$$

by nonlinear dynamics on super Hubble scale

N : local expansion $\delta\chi_i$: initial entropy field fluctuation

Non-Gaussianity by nonlinear dynamics

- nonlinear parameter

$$f_{NL}^{(4)} = \frac{5}{6}\frac{N_{,\chi_i\chi_i}}{N_{,\chi_i}^2} \quad \longleftarrow \quad \delta\chi_{\mathbf{k}} \text{ is free field at leading order}$$

- background expansion

$$N = \int_{t_i}^{t_T} H dt + \int_{t_T}^{t_c} H dt \qquad \text{(instantaneous transition)}$$

multi-field scaling solution — single-field scaling solution

$$= \frac{2}{c_1^2}\log|H_T| + const. = -\frac{2}{c_1^2}\log|\delta\chi_i| + const.$$

$$\Rightarrow \quad f_{NL}^{(4)} = \frac{5}{12}c_1^2$$

Non-Gaussianity by intrinsic scalar field

- three-point correlator of scalar field

$$\langle \delta\chi^3(t_i) \rangle = -i \int_{-\infty}^{t_i} dt \langle [\delta\chi^3(t_i), H_{\rm int}(t)] \rangle$$

Maldacena, 2003

- interaction Hamiltonian

$$H_{\rm int}(t) = \int dx a^3 \frac{1}{3!} \frac{d^3 V(t)}{d\chi^3} \delta\chi^3$$

- considering only the cubic interaction
- neglect interaction with graviton

$$\Rightarrow \quad f_{NL}^{(3)} = \frac{5}{12}(c_1^2 - c_2^2)\left(\frac{H_i}{H_T}\right)$$

← suppressed by $\frac{H_i}{H_T}$

$$\Rightarrow \quad f_{NL} \simeq \frac{5}{12} c_1^2$$

4.Conclusion

$$V = -V_1 e^{-c_1\phi_1} - V_2 e^{-c_2\phi_2}$$

- Ekpyrotic collapse with multiple fields

- Scale-invariant isocurvature perturbations are generated during multi-field scaling solutions

- Unstable modes drives the multi-field scaling solutions to single field dominated scaling solutions

- Transition automatically converts the initial isocurvature perturbations to curvature perturbations

$$r \simeq 0, \qquad \mathcal{R}_c = \frac{c^2}{2\sqrt{c_1^2+c_2^2}} \left|\frac{H}{2\pi}\right|_T \quad \text{with} \quad c = c_1^2 c_2^2/\sqrt{c_1^2+c_2^2}$$

- Large non-Gaussianities are generated during transition

$$f_{NL} = \frac{5}{12}c_j^2 \qquad \text{cf,} \quad n_s = \frac{4}{c^2}$$

Discussions

- Spectrum is slightly blue for pure exponential potentials $n = 4/c^2, \quad c \gg 1$

  -deviation from exponential potential

- Non-Gaussianities from other mechanisms

  Lehners and Steinhardt (arXive: 0712.3779)

- How to realise the non-singular bounce?

  -relying on ghost condensation

- How to realise initial to be near the saddle point

  Buchbinder, Khoury, Ovrut (arXive: 0706.3903)