Holography in General Discrete Spacetimes: Cochain Quantum Fields on Simplicial Screens

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Introduction

We propose a discrete holographic model for general spacetimes in the spirit of screen holography: a bulk simplicial spacetime with Lorentzian geometric data, a preferred holographic screen with leaves, and a quantum theory on the screen whose special states reproduce a discrete HRT law exactly at the microscopic level.

- Natural regularization.
- A clear and explicit description of the state space.
- Compatibility with numerical tools.

In contrast to standard asymptotic AdS holography, here the fundamental geometric object is a *finite* holographic screen in the interior of a general spacetime, defined independently of any special boundary conditions.

- exploring holography in cosmological or black hole geometries,
- implementing numerical experiments on fully discrete quantum states,
- and connecting bulk geometry to microscopic entanglement exactly, without introducing continuum or large-N limits at the definition level.

Bulk simplicial spacetime

Let K be a causal d-dimensional simplicial pseudomanifold with discrete exterior calculus data

$$(C^\bullet(K), , d, , *_g).$$

The Lorentzian Hodge star determines the bulk geometric weights. For a codimension-2 chain $X \subset K$,

$$\mathcal{A}_g(X) = \sum *\tau \subset X, \ \dim \tau = d - 2a_g(\tau),$$

where $a_g(\tau)$ is the Hodge-star-induced area weight of τ.

In this way, all continuum notions that enter the holographic entropy formula (such as areas of extremal surfaces) are replaced by purely combinatorial expressions built from cochains and the discrete Hodge-star. The bulk metric and causal structure appear only through:

- the incidence relations in the simplicial complex,
- the Lorentzian signature data encoded in $*_g$,
- the induced area weights $a_g(\tau)$ to codimension-2 simplices.

This keeps the construction algebraic and topological at finite resolution, while still retaining enough structure to define a discrete analogue of extremal surfaces and maximin entropy on the one hand, and cochain based quantum theories on the other.

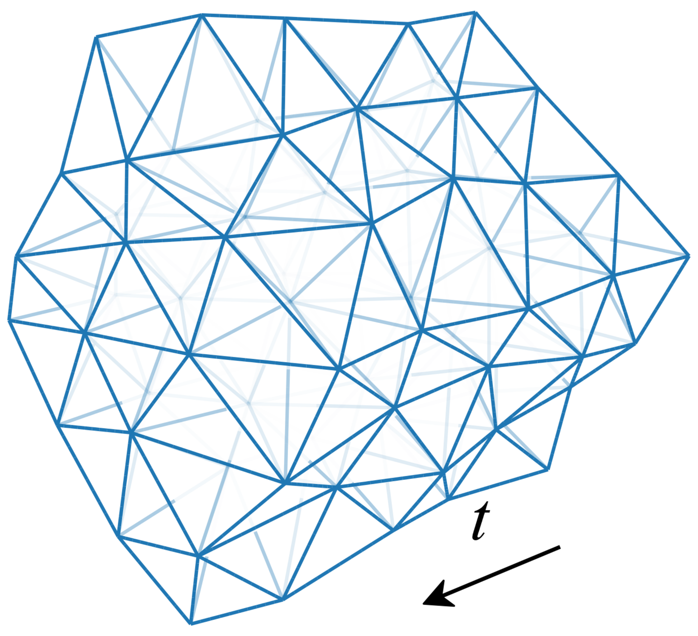

Figure 1. Example of a causal Lorentzian simplicial spacetime K. Nodes and simplices encode both the topology and the discrete geometric weights used to define areas of codimension-2 chains.

Discrete past holographic screen

A discrete screen is a pair

$$(H, \lambda),$$

where $H \subset K$ is a codimension-1 subcomplex and

$$\lambda : H \to I \subset \mathbb{Z}$$

is a rank function whose level sets

$$\sigma_n := \lambda^{-1}(n)$$

are the **leaves**.

We define the leaves by a discrete analogue of marginal anti-trappedness. Let N_n be a discrete null slice and let $\Sigma \subset N_n$ be a codimension-2 cross-section. For the two discrete null directions k, ℓ, define:

$$\Theta_k^{\mathrm{disc}}(\Sigma) = \max_{\Sigma' \in \mathcal{U}_k(\Sigma)} \big[\mathcal{A}_g(\Sigma') - \mathcal{A}_g(\Sigma)\big],$$

$$\Theta_\ell^{\mathrm{disc}}(\Sigma) = \min_{\Sigma'' \in \mathcal{U}_\ell(\Sigma)} \big[\mathcal{A}_g(\Sigma'') - \mathcal{A}_g(\Sigma)\big].$$

A **discrete past-screen leaf** satisfies

$$\Theta_k^{\mathrm{disc}}(\Sigma) = 0, \qquad \Theta_\ell^{\mathrm{disc}}(\Sigma) > 0.$$

The marginality condition is implemented by **local max/min conditions**, not by literal discrete derivatives. This is the discrete analogue of area-maximizing cross-sections along one null direction and area-increasing flow along the other.

The rank function λ induces a preferred discrete foliation of the screen by leaves $\{\sigma_n\}$, and the screen flow between leaves is represented by sequences of elementary moves along the null directions. This structure is the discrete counterpart of the unique foliation and fibration of continuum holographic screens.

For each leaf σ_n, one can choose an acausal filling S_n with $\partial S_n = \sigma_n$ and define a discrete domain of dependence $D_n := D(S_n)$. This gives the simplicial version of the causal region used in the maximin construction in the continuum.

Algebraic ambient quantum theory on the screen

For a finite screen subcomplex $(U \subset H)$, define the local ambient algebra

$$\mathcal{F}(U) = \Big(\bigotimes_{\ell \in U_1} M_D(\mathbb{C})\Big) \otimes \Big(\bigotimes_{v \in U_0} M_q(\mathbb{C})\Big).$$

- Edge degrees of freedom: 1-cochain/gauge variables.
- Vertex degrees of freedom: matter variables.

Gauge transformations are generated by 0-cochains $(\lambda \in C^0(U; \mathbb{Z}_D))$, acting by

$$\gamma_\lambda(Z_\ell) = \omega^{(d\lambda)(\ell)} Z_\ell, \qquad \gamma_\lambda(X_\ell) = X_\ell, \qquad \omega = e^{2\pi i/D}.$$

The physical local algebra is the gauge-invariant subalgebra

$$\mathcal{A}_{\mathrm{amb}}(U) = \mathcal{F}(U)^\gamma.$$

Thus

$$U \subset V \implies \mathcal{A}_{\mathrm{amb}}(U) \subset \mathcal{A}_{\mathrm{amb}}(V),$$

so $(U \mapsto \mathcal{A}_{\mathrm{amb}}(U))$ is a local net on the screen.

HRT Subalgebra

Choose a **screen skeleton**

$$\mathcal{S} \subset H,$$

a branched ribbon complex transverse to the preferred foliation.

Its intersection with each leaf is a weighted graph/tree

$$T_n = \mathcal{S} \cap \sigma_n.$$

For each ribbon $r \subset \mathcal{S}$, define collective ribbon operators inside the ambient cochain gauge algebra:

$$U_r = \prod_\ell X_\ell^{\chi_r(\ell)}, \qquad V_r = \prod_\ell Z_\ell^{\eta_r(\ell)},$$

where χ_r, η_r are the characteristic cochains of the ribbon and its transverse dual.

At a branch point b, impose a flux-conservation constraint

$$G_b := U_{e_1} U_{e_2} U_{e_3}^{-1} = 1.$$

Define the holographic subalgebra by

$$\mathcal{O}_{\mathrm{HRT}} = C^*\big(\{U_r, V_r, G_b\} \mid r \subset \mathcal{S}\big) \subset \mathcal{A}_{\mathrm{amb}}(H).$$

For a region $A \subset \sigma_n$, the **leaf-region HRT algebra**

$$\mathcal{M}_n(A) \subset \mathcal{O}_{\mathrm{HRT}}$$

is generated by the ribbon endpoint modes whose support on the leaf is associated with the atomic cells of A.

This is the key structural point: the HRT sector is **not appended externally**; it is a **ribbon-flux subnet** of the ambient simplicial cochain theory. The ribbon data are mapped to the leaves by a structure-preserving map, not by direct identification with simplices.

Nonlocal geometric state

For each leaf n, compute the geometric entropy vector

$$S_{\mathrm{geo}}^{(n)}(A)$$

from the bulk geometry. If this vector is **simple-tree realizable**, then there exists a weighted tree T_n reproducing it exactly. This is the exact class for which our finite-dimensional microscopic construction works.

Let the tree edge weights be

$$w_e = m_e \log D.$$

Define the distinguished state by Bell pairing internal bonds and projecting onto flux conserving intertwiners. The resulting nonlocal state ω_g lives inside the ambient screen theory and encodes the bulk simplicial geometry in its entanglement structure.

Discrete maximin HRT law

For a region $(A \subset \sigma_n)$, let D_n be the discrete causal domain associated with the leaf. Define

$$S_{\mathrm{geo}}^{(n)}(A) = \frac{1}{4G} \max_{\Sigma \in \mathfrak{C}_n} \min_{X \subset \Sigma} \min_{\partial X = \partial A} \min_{X \sim A} \mathcal{A}_g(X),$$

where:

- $\mathfrak{C}_n$: admissible acausal fillings of the leaf inside D_n,
- X: codimension-2 chains homologous to A in Σ.

This is the discrete **maximin HRT functional**.

The exact statement of the model is:

$$S_{\mathrm{alg},\omega_g}\big(\mathcal{M}_n(A)\big) = S_{\mathrm{geo}}^{(n)}(A).$$

If the region algebras factorize, the left side reduces to ordinary von Neumann entropy, if gauge constraints produce centers or non-factorization, one should use **algebraic entropy**.

A simplicial screen theory contains a natural holographic subnet whose special states reproduce discrete HRT entropies exactly.

The discrete HRT law has been realized in examples such as a discrete FLRW expanding universe, but **the fully general case remains open**.

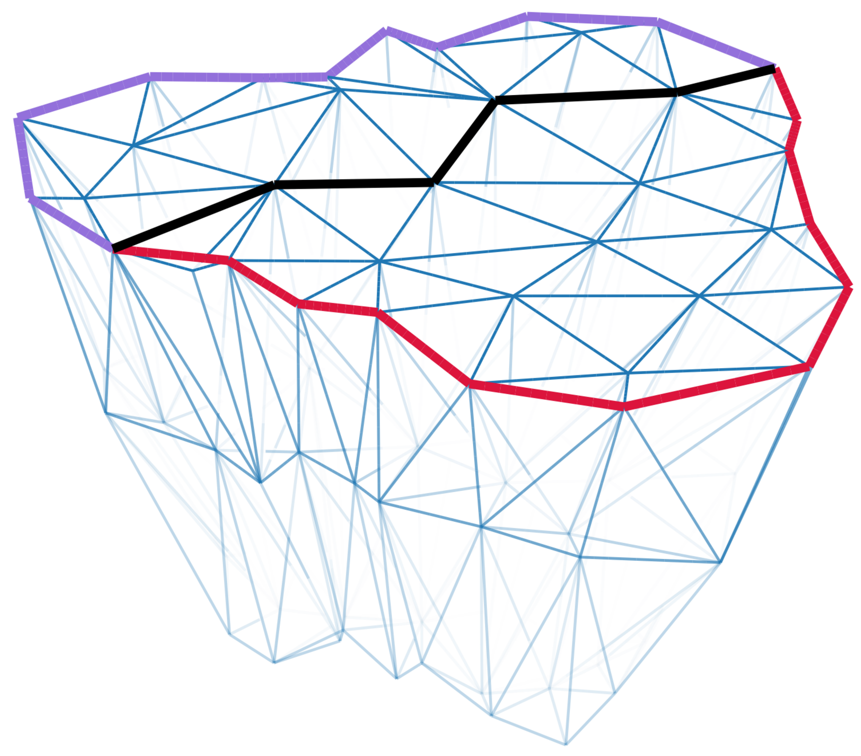

Figure 2. Example of a bulk simplicial spacetime with a leaf (purple), a leaf region A (red), and an interior extremal chain X (black) anchored to ∂A inside the causal region.

Bulk overlap and shared information

Different observers may have different screens $H^{(a)}$, $H^{(b)}$, with different foliations and skeletons. Information sharing is fundamentally about **bulk overlap**.

Attach an abstract algebra $\mathcal{R}(R)$ to each bulk region $R \subset K$. For each screen $H^{(a)}$, one has a representation (encoding map)

$$\Phi_R^{(a)} : \mathcal{R}(R) \to \mathcal{A}_{\mathrm{amb}}^{(a)}(H^{(a)}),$$

and, more locally,

$$\Phi_{R,A}^{(a)} : \mathcal{R}(R) \to \mathcal{M}_n^{(a)}(A),$$

when bulk region R is encoded on leaf region $A \subset \sigma_n^{(a)}$.

If two observers a, b have overlapping bulk regions

$$R = R^{(a)} \cap R^{(b)},$$

then they share the bulk algebra $\mathcal{R}(R)$. On their screens, this appears as two (possibly different) subalgebras

$$\Phi_R^{(a)}\big(\mathcal{R}(R)\big) \subset \mathcal{A}_{\mathrm{amb}}^{(a)}, \qquad \Phi_R^{(b)}\big(\mathcal{R}(R)\big) \subset \mathcal{A}_{\mathrm{amb}}^{(b)}.$$

Two observers agree on all shared information in R precisely when their states agree on $\mathcal{R}(R)$, i.e.

$$\omega^{(a)} \circ \Phi_R^{(a)} = \omega^{(b)} \circ \Phi_R^{(b)} \quad \text{on} \quad \mathcal{R}(R).$$

Screens are different boundary realizations of these bulk algebras. This allows us to compare observers via shared bulk-region algebras, not just by geometric overlap. Future work will explore how this can be implemented on a dynamic bulk theory, with physically meaningful bulk observables.

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