Intertwiner for Topological Chern-Simons Theory

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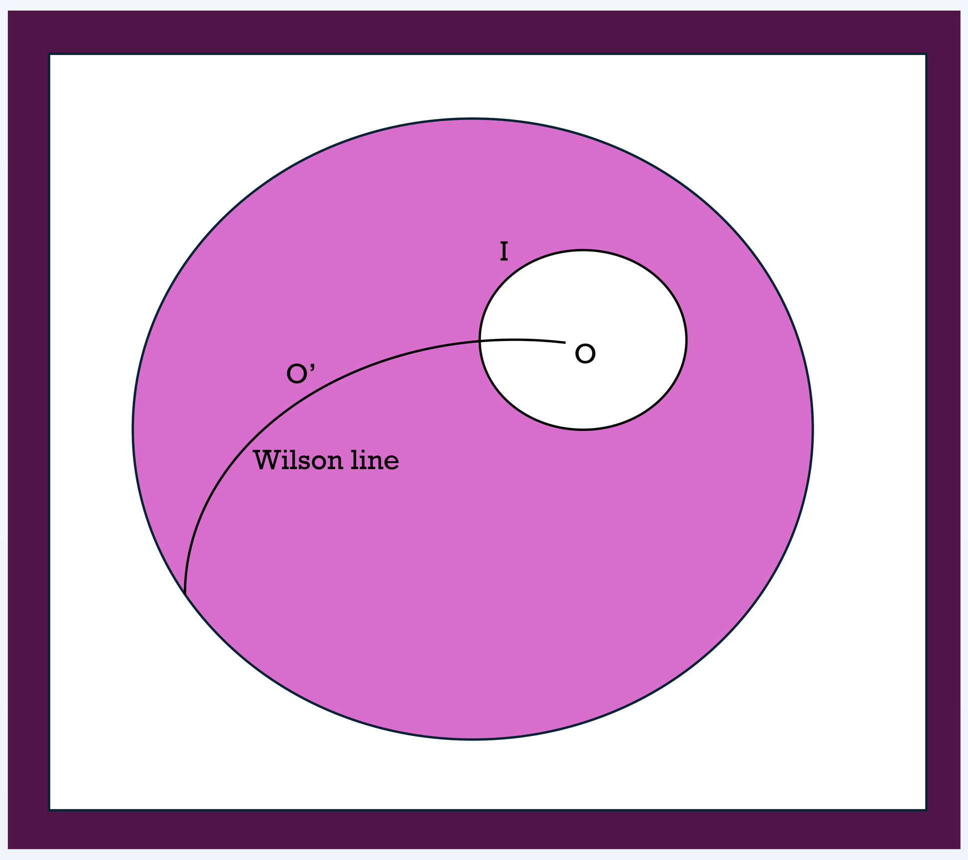
Some conceptual problems

- In a gauge theory, the physical operators are gauge invariant.
- The more natural way to compute a gauge-invariant charged field is to take the local operator and to dress it with a Wilson line that extends to infinity

$$\psi^{inv} = W_\gamma(x, x_0)\psi(x) \qquad (1)$$

- It cannot be used to define states in a bounded subregion of a Cauchy surface (an "island") commuting with all physical operators outside the region: some of the outside operators are going to "intercept" the Wilson line and generically not commute with it.



One possible resolution: the Intertwiner

- Define an automorphism of algebras that maps local operators into gauge-invariant ones

$$\boldsymbol{O' = \Omega^\dagger O \Omega \qquad with \qquad \Omega^\dagger\Omega = 1} \qquad (2)$$

- This restore part of our locality since the dressed operator will have the same algebra as the local one

$$[\boldsymbol{O_1', O_2'}] = \boldsymbol{\Omega^\dagger [O_1, O_2] \Omega} \qquad (3)$$

Intertwiner, how can be constructed?

The assumptions are the following:

1. The gauge theory we took into consideration is non-anomalous.
2. The BRST charge admits the following decomposition* with respect to the grading operator S

$$\boldsymbol{Q_{BRST} = Q_0 + \textstyle\sum_{0<n\le N} Q_n \qquad [S, Q_n] = nQ_n} \qquad (4)$$

- It is defined the following operator that interpolates between Q_{BRST} and Q_0 as follows:

$$\boldsymbol{Q(t) = Q_0 + \sum_{0<n\le N} e^{-nt} Q_n = Q_0 + Q_I(t)} \qquad (5)$$

- Since $\boldsymbol{\Omega}$ is an intertwiner it holds that the following quantity is conserved

$$\boldsymbol{\frac{d}{dt}\left(\Omega(t)Q(t)\Omega^{-1}(t)\right) = 0.} \qquad (6)$$

- $\boldsymbol{\Omega}$ shall solve the following differential equation

$$\boldsymbol{\frac{d\Omega(t)}{dt} = i\Omega(t)[Q_I(t), R]_+} \text{ where } \boldsymbol{[Q_0, R]_+ = iS} \qquad (7)$$

- The law between the "free" and the total BRST chage is therefore

$$\boldsymbol{Q_0 = \Omega(0) Q_{BRST} \Omega(0)} \qquad (8)$$

(*) This condition is not trivial; for instance if we add a cosmological constant to Chern-Simons or Einstein-Hilbert action this requirement is no longer satisfied. The construction still holds with some modifications.

How does it work? Holomorfic quantisation of CS

The easier way to construct an intertwiner is through holomorphic quantisation.

Split the Chern-Simons BRST charge in $Q_0 + Q_I$

$$Q_0 = \int_\Sigma c^a \partial_z \tilde{A}^a_{\bar{z}} + \rho^a \ \pi^b_a$$

$$Q_I = \int_\Sigma -c^a(\partial_{\bar{z}} \tilde{A}^a_z - f^a_{bc} \tilde{A}^b_z \tilde{A}^c_{\bar{z}}) + \frac{1}{2} f^a_{bc} \pi^c_a c^b c^c$$

Compute R and S operators

$$S = \int_\Sigma \left(- b^a \pi^b_a + c^a \pi^c_a + i \rho_a A^a_0 - i g_{ab} \tilde{A}^a_z \tilde{A}^b_{\bar{z}} \right)$$

$$R = \int_\Sigma d^2 z(\ b_a(z) A^a_0(z) + \pi^c_a(z)) \int_{\Sigma'} d^2 w \, G(z-w) \partial_{\bar{w}} \tilde{A}^a_w (w)$$

Solve the differential equation and compute the Intertwiner

$$\boldsymbol{\Omega(0) = e^{i\Gamma t}}$$

Γ is non-local and extend to infinity

$$\Gamma = \int_\Sigma (f_{abc} \tilde{A}^b_z \tilde{A}^c_{\bar{z}} + \pi^{c,b} c^c) \int_\Sigma \frac{1}{\bar{z}-\bar{w}} A^a_w(w) - \int_\Sigma \pi^{c,a} \int_\Sigma \frac{1}{\bar{z}-\bar{w}} f_{abc} c^b A^c_w(w) + \frac{1}{2} g_{ab} \int_\Sigma \partial_{\bar{z}} A^a_z \int_\Sigma \frac{1}{\bar{z}-\bar{w}} A^b_z(w)$$

Triviality of the BRST local cohomology

- In the canaonical quantization, the field $\tilde{A}$ is canonically conjugated to itself

$$\boldsymbol{[\tilde{A}^a(\vec{x}t), \tilde{A}^b(\vec{y}t)] = i g^{ab} \delta^2(\vec{x} - \vec{y}) Vol_\Sigma}$$

- The grading is inconsistent. We therefore split $\tilde{A}^a$ into its longitudinal and trasversal component

$$\boldsymbol{\tilde{A}^a = \partial_\mu L^a + \epsilon_{\mu\nu} \partial^\nu T^a}$$

- The longitudinal mode is not BRST invariant while the trasversal mode is BRST exact. So every local field built with L^a or T^a is either non-BRST invariant or trivial in the cohomology.
- It is known that Chern-Simons cohomology is not empty. This contradiction arise because Chern-Simons cohomology is non-local. For instance

$$\boldsymbol{Y^a = c^a \delta(L^a)}$$

is in the cohomology but it is non-local.

Future develops

- Build the observables of TJ gravity
- Study in this framework the island problems for black holes

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