Decoherence of Primordial Perturbations in the View of a Local Observer

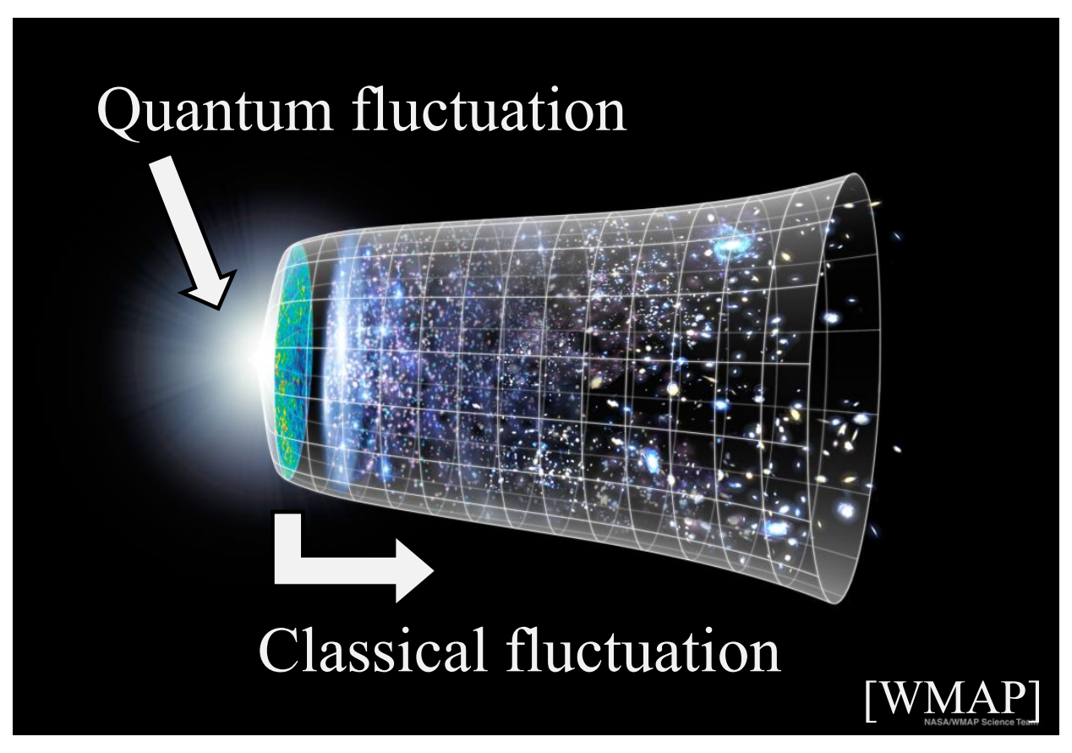
Fumiya Sano Institute of Science Tokyo, Japan
Based on **2504.10472** with Junsei Tokuda at McGill University, Canada
sanof.cosmo@gmail.com / https://sites.google.com/view/sanof-cosmo/home

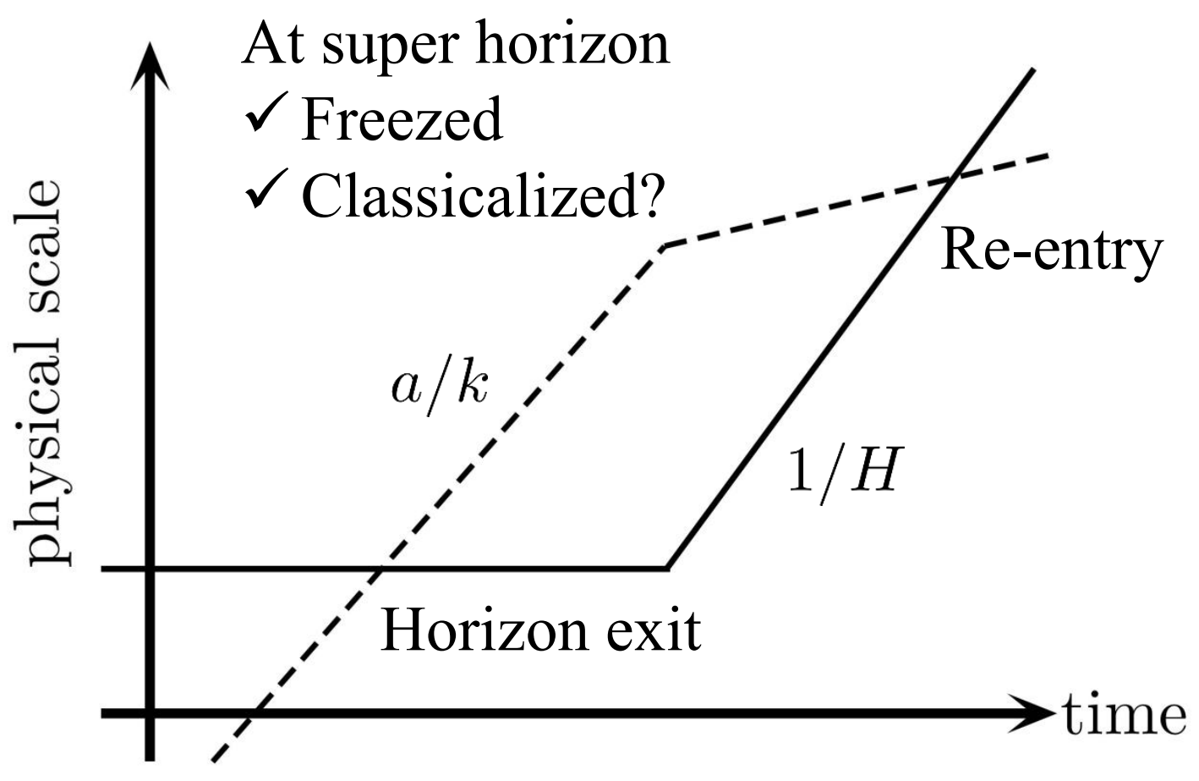
InspireHEP

My webpage

SCIENCE TOKYO

Background and Motivation — Inflation as a probe of quantum gravity





- Ultimate goal: *Quantum gravity theory!*
  - We quantize metric perturbations during inflation $h_{ij} = (e^{\zeta(x)}a(t))^2(\delta_{ij}+\gamma_{ij})$. ⬅ perturbative QG
  - Inflation may have the highest energy scale $\rho \lesssim 10^{15}$ GeV among observable phenomena known so far.

➡ *Bottom-up / phenomenological approaches to quantum gravity may be realized.*

- Important step: Is the gravity really quantum? ➡ Bell-like tests for ζ, γ_{ij} provides sufficient conditions that the fields are quantized [Campo and Parentani astro-ph/0505376, Maldacena 1508.01082, Choudhury et al. 1607.00237].

- An open question of inflation: When, how, or how much are quantum perturbations classicalized?
  - Decoherence of curvature perturbations due to non-linear interactions [Polarski and Starobinsky gr-qc/9504030, Nelson 1601.03734, Burgess et al. 2211.11046].
  - Objectivity and classical branching: We are in one of the branches of the density matrix, where one-point function is realized [Nelson and Riedel 1711.05719, Touil et al. 2208.05497, etc.].

Free propagation

Gravitational non-linearity.
Interactions among different scales induces decoherence.

$$\left(S = \int d^4x a^2\left[\epsilon(\zeta'^2 - \partial^2\zeta^2) + \epsilon^2\zeta(\zeta'^2+\partial^2\zeta^2) - 2\epsilon^2\zeta'\partial_i\zeta\partial_i^{-1}\zeta' + \frac{\epsilon\eta}{2a^2}\partial_\tau(a^3\zeta^2\zeta') + \cdots\right]\right)$$

- Short summary of previous works about decoherence effects from gravitational non-linearities: Decoherence happens right after the horizon crossing.
  - Only few interactions are considered, e.g., $a^2\epsilon^2\zeta\partial^2\zeta$ [Nelson 1601.03734 etc.] among bulk interactions and $-9\partial_\tau(a^3H\zeta^3)$ [Sou et al. 2207.04435] among boundary ones.
  - UV and IR divergences are not fully regularized. (See also refs. on divergences in wave fn. and open quantum system of QFT [Balasubramanian et al. 1108.3568, Cespedes et al. 2311.17990, Bucciotti 2410.01903, Burgess et al. 2411.09000])

Our work: *Making use of Ward identity in order to*
- *Take into account whole cubic order Lagrangian and clarify the symmetric structure.*
- *Define genuine observable quantities within standard quantum theory to show the absence of divergences.*

Setup — Wavefunction, Ward identity, Coherence

- Wavefunction of curvature perturbations

$$\Psi[\zeta] \equiv \langle\zeta;\tau|\Omega\rangle = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^6}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

  - Correlation functions

$$\langle\Omega|\widehat{\zeta}_{k_1}\widehat{\zeta}_{k_2}(\tau)|\Omega\rangle = \int\mathcal{D}\zeta(\tau)\langle\Omega|\zeta;\tau\rangle\langle\zeta;\tau|\Omega\rangle\zeta_{k_1}\zeta_{k_2} \equiv \int\mathcal{D}\zeta(\tau)|\Psi[\zeta(\tau)]|^2\zeta_{k_1}\zeta_{k_2}$$

  - Semi-classical approximation of wavefunction coefficients

$$\Psi \simeq e^{iS_{\rm cl}[\zeta]} \Longrightarrow \psi'_2 \simeq -i\int_{-\infty}^{\tau_0} d\tau\mathcal{L}_{\rm cl}^{(2)} = -9ia^3H(1+\epsilon) - i\frac{ak^2}{H}(1-\epsilon) + ia^2\epsilon k\frac{H^{(2)}_{1/2+\epsilon+\eta/2}(k/aH)}{H^{(2)}_{3/2+\epsilon+\eta/2}(k/aH)} \quad \text{etc.}$$

ψ'_n : $(2\pi)^3\delta\left(\sum\mathbf{k}_i\right)$ is omitted

- Ward identity [Maldacena astro-ph/0210603, Pimentel 1309.1793, etc.]

Under the gauge transf. $x^i \to x^i + \xi^i,\ h_{ij} \to h_{ij} - a^2D_{\{i}\xi_{j\}}$ where $h_{ij} = e^{2\zeta}a^2\delta_{ij}$,

$$\Psi[h_{ij} - a^2D_{\{i}\xi_{j\}}] = \Psi[h_{ij}] \Longrightarrow \lim_{k_1\to 0}\psi'_3(k_1,k_3) = \left(3 - k_3\frac{\partial}{\partial k_3}\right)\psi'_2(k_3)$$

  - Maldacena's consistency conditions

$$\langle\zeta_1\zeta_2\zeta_3\rangle' = -\frac{2\,\mathrm{Re}[\psi'_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi'_2(k_i)]} \xrightarrow[k_1\to 0]{} -\frac{1}{2\,\mathrm{Re}[\psi'_2(k_1)]}\left(3 + k_3\frac{\partial}{\partial k_3}\right)\frac{1}{2\,\mathrm{Re}[\psi'_2(k_3)]} = (1-n_s)\langle\zeta_1\zeta_1\rangle'\langle\zeta_3\zeta_3\rangle'$$

- Decoherence
  - Mode decomposition $\mathcal{H}_\zeta = \bigotimes_{\mathbf{k}}\mathcal{H}_{\mathbf{k}} = \bigotimes_{\mathbf{k}_{\rm S}}\mathcal{H}_{\mathbf{k}_{\rm S}}\bigotimes_{\mathbf{k}_{\rm E}}\mathcal{H}_{\mathbf{k}_{\rm E}}$ $\qquad \pm\mathbf{k}_{\rm S} \in \{\mathbf{k}_{\rm CMB}\}$
  - Non-unitary evolution in Schwinger-Keldysh formalism

$$\begin{aligned}\langle 0_{\rm int}|U^\dagger\zeta^2_{\mathbf{k}_{\rm S}}(t)U|0_{\rm int}\rangle &= \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{t_0}^t dt' H_{\rm I}}\right)\zeta^2_{\mathbf{k}_{\rm S}}(t)\left(\mathrm{T}e^{-i\int_{t_0}^t dt' H_{\rm I}}\right)\right|0\right\rangle\\ &= \langle 0|\zeta^2_{\mathbf{k}_{\rm S}}|0\rangle - i\int_{t_0}^t dt'\,\langle 0|\zeta^2_{\mathbf{k}_{\rm S}}(t)H_{\rm I}(t')|0\rangle + \text{c.c.}\\ &\quad + \int_{t_0}^t dt'dt''\,\langle 0|H_{\rm I}(t')\zeta^2_{\mathbf{k}_{\rm S}}(t)H_{\rm I}(t'')|0\rangle - \int_{t_0}^t dt'dt''\,\langle 0|\zeta^2_{\mathbf{k}_{\rm S}}(t)\mathrm{T}[H_{\rm I}(t')H_{\rm I}(t'')]|0\rangle + \text{c.c.}\end{aligned}$$

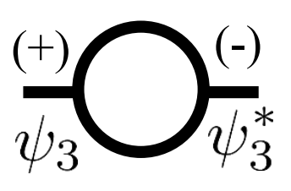
Unitary (*)

Non-unitary (**)

$$\Longleftrightarrow \rho_{\rm S}[\zeta,\widetilde{\zeta};t] = \int_\Omega^{\zeta,\widetilde{\zeta}}\mathscr{D}\zeta_\pm e^{\overset{(*)}{iS_+ - iS_-}+\overset{(**)}{iS_{\rm IF}[\zeta_+,\zeta_-]}} \equiv Ne^{\overset{(*)}{-A_{k_{\rm S}}\zeta^2_{\mathbf{k}_{\rm S}} - A^*_{k_{\rm S}}\widetilde{\zeta}^2_{\mathbf{k}_{\rm S}}}\overset{(**)}{-C_{k_{\rm S}}\zeta_{\mathbf{k}_{\rm S}}\widetilde{\zeta}_{\mathbf{k}_{\rm S}}}}$$

  - Trace out the environment to obtain influence functional [Nelson 1601.03734, Sou et al. 2207.04435]

$$\rho_{\rm S}[\zeta_{\mathbf{k}_{\rm S}},\widetilde{\zeta}_{\mathbf{k}_{\rm S}}] = \int\mathcal{D}\zeta_{\mathbf{k}_{\rm E}}(t)\Psi[\zeta_{\mathbf{k}_{\rm S}},\zeta_{\mathbf{k}_{\rm E}}]\Psi^*[\widetilde{\zeta}_{\mathbf{k}_{\rm S}},\zeta_{\mathbf{k}_{\rm E}}]$$

✓ Off-diag. $C_{k_{\rm S}} =$  $+ \cdots$ ✓ Purity $\mathcal{P} = \mathrm{Tr}\left[\rho^2\right] \simeq \frac{1}{1+\Gamma}$, $\quad \Gamma = 4P_{k_{\rm S}}C_{k_{\rm S}}$

$$\Longrightarrow \Gamma \sim \frac{H^2}{M_{\rm pl}^2}\left[\left(\underset{-9\partial_\tau(a^3H\zeta^3)}{\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6} + \underset{a^2\epsilon^2\zeta\partial^2\zeta}{\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3}\right)(1 + \log(k_{\rm IR}/k_{\rm S})) + \left(\frac{\Lambda_{\rm phys}}{H}\right)^\#\right]$$

IR/UV divergence: not fully resolved

IR divergence — Fermi normal coordinate

- Observer's coordinate (comoving observer) [Tanaka and Urakawa 1103.1251, Pajer et al. 1305.0824]

$$ds^2 = a^2(-d\tau^2 + e^{2\zeta}d\mathbf{x}^2) = a^2(-d\tau^2 + d\mathbf{x}_{\rm F}^2) + \cdots$$

* Geodesic distance $\mathbf{x}_{\rm F}$

$$\Longrightarrow \zeta_{\rm F,\mathbf{k}} \simeq \zeta_{\mathbf{k}} + \zeta(0)(3 + \mathbf{k}\cdot\partial_{\mathbf{k}})\zeta_{\mathbf{k}}$$

Gauge transf.
Fourier space

where $\zeta(0)$ is $\zeta(\mathbf{x}_{\rm F})$ at origin, and is super-horizon mode $\lim_{k\to 0}\zeta_{\mathbf{k}}$.

  - Gauge is fully fixed including large gauge transformation $\zeta \longrightarrow \zeta + c$
  - Vanishing squeezed correlator $\lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle'_{\rm F} = \lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle' + (n_s - 1)\langle\zeta_1\zeta_1\rangle'\langle\zeta_3\zeta_3\rangle' = 0$

Correlation with IR is artefact when the observable quantities are properly defined.

- Generalization to wavefunction [Sano and Tokuda 2504.10472]

$$\Psi[\zeta_{\rm F}] = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^6}\psi_{{\rm F},2}\zeta_{{\rm F},k_1}\zeta_{{\rm F},k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^9}\psi_{{\rm F},3}\zeta_{{\rm F},k_1}\zeta_{{\rm F},k_2}\zeta_{{\rm F},k_3} - \cdots\right]$$

$$\Longrightarrow \lim_{k_1\to 0}\psi'_{{\rm F},3} = \lim_{k_1\to 0}\psi'_3 - \left(3 - k_3\frac{\partial}{\partial k_3}\right)\psi'_2 = 0$$

IR divergence of Γ vanishes.

$\zeta_{\rm F,\mathbf{k}} \simeq \zeta_{\mathbf{k}} + \zeta(0)(3 + \mathbf{k}\cdot\partial_{\mathbf{k}})\zeta_{\mathbf{k}}$

UV divergence — Time-averaged observables

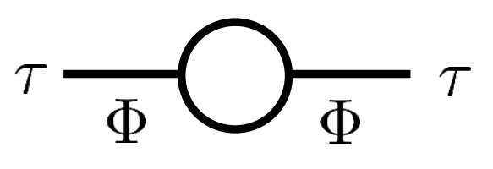
- UV divergence of equal time correlation functions
  - Composite operators: Counter terms to operators rather than Lagrangian [e.g., Ch.6, "Renormalization", Collins 2023]

Redefining observables $\phi^2_R(x) = Z_a\phi^2(x) + \mu^{-1}Z_bm^2\phi(x) + \mu^{-1}Z_c\Box\phi(x)$

  - Divergence at external momenta: Equal time is beyond IR effective theory [e.g., Balasubramanian et al. 1108.3568, Bucciotti 2410.01903]

Possible divergences even *at tree level* $\langle\mathcal{O}_1^{\mathbf{k}}\mathcal{O}_2^{-\mathbf{k}}(t)\rangle \sim \int_{\sim 0} d^3(\mathbf{x}_1 - \mathbf{x}_2)\frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}}$

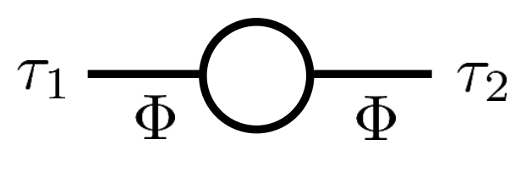
- Classifying the divergences

$\langle\Phi_1\Phi_2(\tau)\rangle \supset$ 

$\Phi = \zeta$ or π
$d\tau = dt/a$

➡ Time averaging

$$\langle\overline{\Phi}_1\overline{\Phi}_2(\tau)\rangle = \int d\tau_1 d\tau_2 W_\tau(\tau_1)W_\tau(\tau_2)\langle\Phi_1(\tau_1)\Phi_2(\tau_2)\rangle$$

$\supset \int d\tau_1 d\tau_2 W_\tau(\tau_1)W_\tau(\tau_2)\Big[$  $\Big]$

$\int^\Lambda k^\# dk \longrightarrow \Lambda^\#$

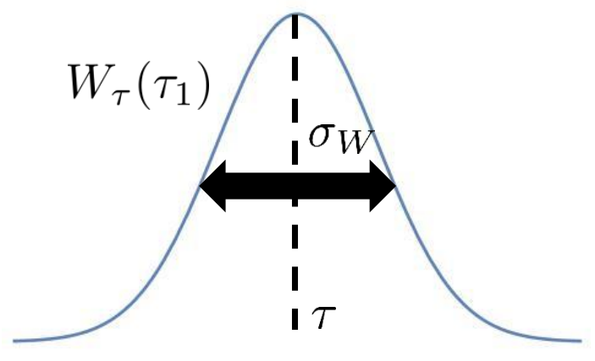
$\frac{1}{|\tau_1 - \tau_2|^\#}$ This is (expected to be) renormalized.

$\frac{e^{-ik(\tau_1-\tau_2)}}{|\tau_1 - \tau_2|^\#}$ Included in Wightman function
No counter term to cancel the divergence
(* absent when "IR" limit is taken first $\lim_{k\tau\to 0}\langle\zeta_k^2(\tau)\rangle$)

- Averaging out the unrenormalizable divergence

$$W_\tau(\tau_1) = \frac{e^{-(\tau_1-\tau)^2/2\sigma_W^2}}{\sqrt{2\pi\sigma_W^2}},$$

$$G(k;\tau_1,\tau_2) \propto e^{-i\underline{k}(\tau_1-\tau_2)}$$



$$\Longrightarrow \Gamma_{\rm UV} \sim \int_{k>aH} dk\ k^\# e^{-\underline{k^2}\sigma_W^2}$$

Exponential decay in sub-horizon

Discussions — Definition of observables can be crucial in quantum theory

- We made use of Ward identity to calculate the decoherence rate and its divergences. We confirmed that IR divergence is absent thanks to the symmetric structure.
  - ✓ (Conformal) Fermi normal coordinate provides us perfect gauge fixing specified to free-falling observer. IR divergence is absent in the coordinate and thus it is merely a gauge artefact.
- UV behavior is not protected by symmetry. Furthermore, the equal time correlations, or wavefunction, include UV divergence at external momenta, which is treated by redefining observables.
  - ✓ The UV divergences of time-smeared quantity are classified to those which can be removed by counter terms and those cannot. Average in time smear-out exactly the unrenormalizable ones.
- Outlook: How can we model the true observable quantities in our universe? CMB photons coupling to gravity, detector model on the Earth, role of objectivity in cosmology, …