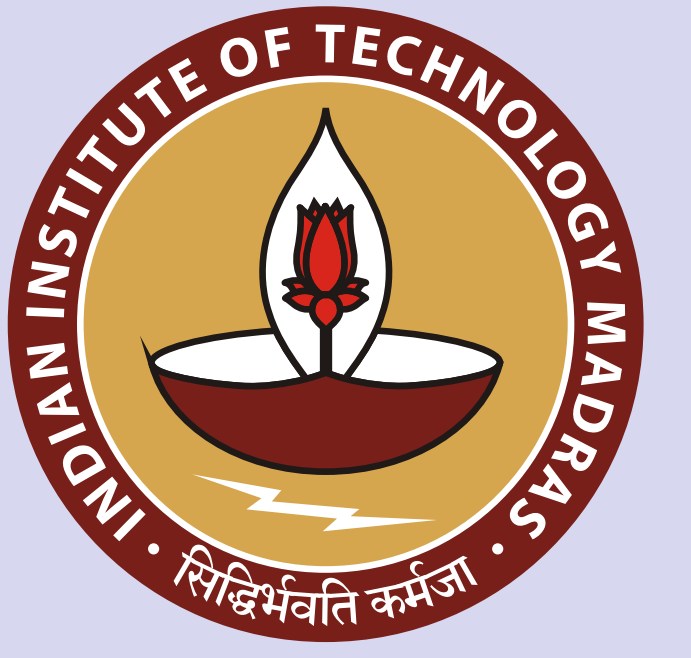




# Quantum cosmology with a reference fluid: Ambiguities and their imprints

Harkirat Singh Sahota | Indian Institute of Technology Madras, India



## Why Quantum Gravity

General Relativity successful *but predicts singularities*  
Signalling the breakdown of classical description of spacetime

Historically, quantization resolves the classical pathologies  
A quantum description of gravity is essential to probe singularities  
Canonical approach  $\rightarrow$  3+1 (ADM) decomposition

Quantization: Wheeler-DeWitt equation  
Intractable in full generality  
Symmetry reduction  $\rightarrow$  Quantum Cosmology

**Within Wheeler-DeWitt quantum cosmology, we address:**

- (i) Semiclassicality of quantum states
- (ii) Effects of quantization ambiguities on cosmic evolution

## Quantizing Cosmological Ansatz

- Maximal symmetry reduction  $\Rightarrow$  a mechanical model with finite d.o.f.  
 $ds^2 = -N(t)^2 dt^2 + a^2(t) d\mathbf{x}^2$

- Phase space  $\{a(t), p_a(t)\}$ , where  $p_a \sim$  Hubble constant.

- First Friedmann equation: Hamiltonian constraint

$$\mathcal{H} \equiv -a^{-1} p_a^2 + a^{-3} p_\phi^2 + V(a, \phi) = G^{\alpha\beta} p_\alpha p_\beta + V(a, \phi) \approx 0$$

- Time reparameterization invariance: free particle  $p^2 + m^2 = 0$

- Quantization: Like Klein-Gordon eq., no extrinsic time in  $\hat{\mathcal{H}}\Psi(a, \phi) = 0$

- Time rep. classically  $\Rightarrow$  Problem of time in quantum picture

- Ordering ambiguity: Operatorize  $\mathcal{H}$  uniquely? Covariance in  $(a - \phi)^{(1)}$

$$G^{\alpha\beta} p_\alpha p_\beta \rightarrow (|G|)^{-1/2} \partial_\alpha (\sqrt{|G|} G^{\alpha\beta} \partial_\beta)$$

## Dynamical Reference Frames

- If time rep. is a gauge redundancy, why not fix gauge?
- At what stage, trinity of relational dynamics<sup>(2)</sup>  
Here, quantization of the reduced system.
- Fixing gauge  $\Rightarrow$  preferred reference frame  $\rightarrow$  coordinate conditions.
- Kuchař-Isham: Coordinate conditions at action level  $\sim$  GR + non-canonical field<sup>(3)</sup>. Fields describe *dynamical reference frames*.
- Examples: Brown-Kuchař dust, Unimodular time
- Perfect fluid as reference frame:  $N(t) = a(t)^\gamma \Rightarrow \omega = \gamma/3$   
 $\mathcal{H} = \mathcal{H}_g + P_\tau$
- Here we ask, is  $\hat{a}$  alone enough to describe quantum universe?
- Observables like Hubble parameter and Ricci scalar depend on  $\hat{p}_a$ .

## Quantum Cosmology with a Reference Fluid

- With  $N = a^{3\omega}$ :  $\dot{a}^2 = \rho a^{2(1+3\omega)} \Rightarrow a(t) \propto t^{2/3(1-\omega)}$

- Hamiltonian constraint:  $-a^{3\omega-1} p_a^2 + P_\tau \approx 0$ , where  $P_\tau = \rho a^{3(1+\omega)}$

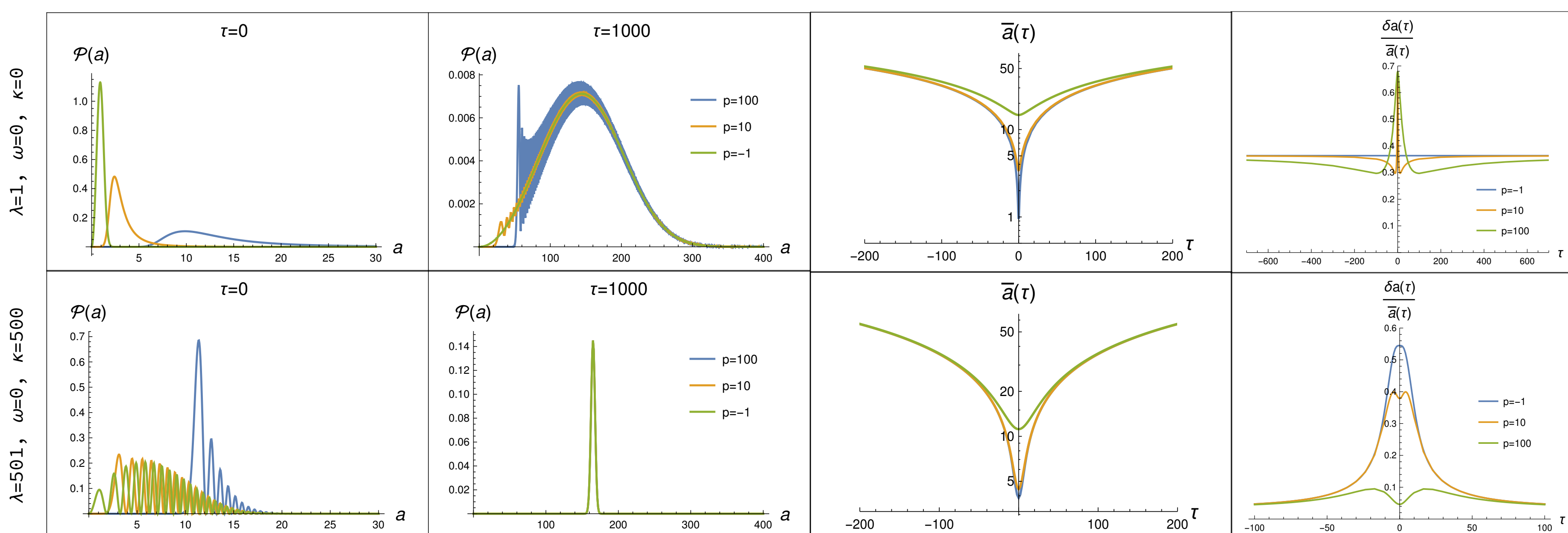
- Wheeler-DeWitt equation:  $\hat{\mathcal{H}}\Psi = 0 \Rightarrow \hat{\mathcal{H}}_g \Psi(a, \tau) = i\hbar \frac{d\Psi(a, \tau)}{d\tau}$

Self-adjoint  $\hat{\mathcal{H}}_g \Rightarrow$  Unitary evolution in  $\tau$

- Ordering ambiguity:  $\hat{\mathcal{H}}_g = \frac{1}{2} a^{3\omega-1+p+q} \frac{d}{da} a^{-p} \frac{d}{da} a^{-q}$ , on  $L^2(\mathbb{R}^+, a^{1-3\omega-p-2q})$
- Different  $p, q$  corresponds to inequivalent quantum theories
- Construct wave packet using distribution  $A(P_\tau)$  for  $P_\tau > 0$  eigenstates.
- This state represents ensemble of universes described by  $A(P_\tau)$

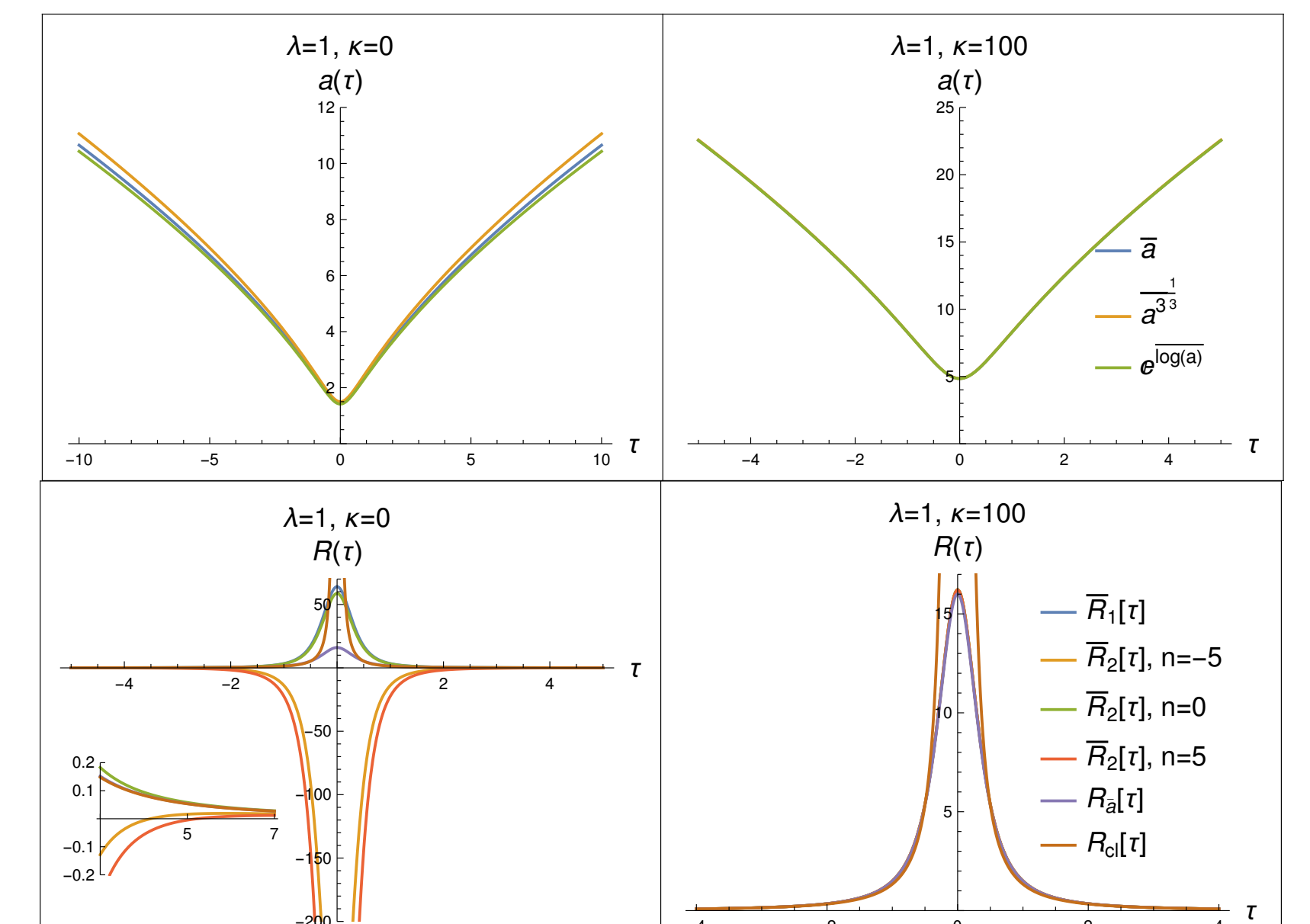
## Imprints of Ordering Ambiguity<sup>(4)</sup>

- Keep  $\langle P_\tau \rangle$  constant, change  $\Delta P_\tau / \langle P_\tau \rangle$ .  $\mathcal{P}(a) = a^{1-3\omega-p-2q} |\psi(a, \tau)|^2$



## Semiclassicality of Wave Packets<sup>(5)</sup>

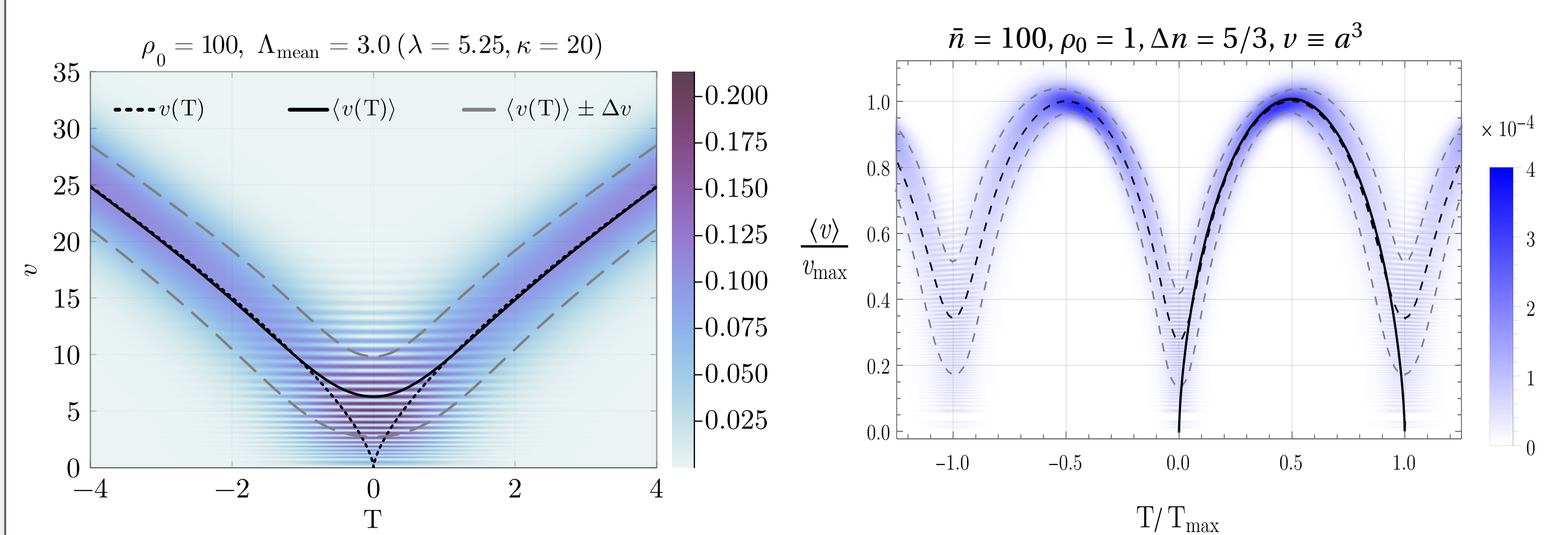
- Quantum observables vs observables of quantum universe  $\bar{a}(\tau)$
- We find  $\bar{O}(a, p_a) = \langle O_{cl} \rangle + O(\tau^{-1})$  and  $\bar{O}(a, p_a) = O(\bar{a}, \bar{a}) + O(\kappa^{-1})$



## Quantum cosmology with dust and $\Lambda$

- Unimodular framework:  $-a^{-4} p_a^2 + \rho + \Lambda \approx 0$
- Quantum cosmology of FLRW+dust+ $\Lambda \leftrightarrow$  radial Hydrogen atom with<sup>(6)</sup>  
 $a^3 \sim r, \Lambda \sim E, \rho \sim V_e, p \sim \ell$ .
- Spectrum of  $\Lambda > 0$  is  $\mathbb{R}^+$ , for AdS  $\Lambda_n = -\frac{\rho^2}{3n^2}, n > \frac{p}{2} + 1$
- Construct wave packets using  $A(\Lambda)$  for  $\Lambda > 0$  while  $C_n$  for  $\Lambda < 0$   
 $\psi(a, T) = \int_0^\infty A(\Lambda) e^{i\Lambda T} \Psi_\Lambda(a) d\Lambda \quad \psi(a, T) = \sum_n C_n \psi_n(a) e^{i\Lambda_n T}$
- These states represent ensemble of universes with different  $\Lambda$

## Bounce for $\Lambda > 0$ and Cyclic Evolution for $\Lambda < 0$



## Results and discussion

- Ordering ambiguity as conformal rescaling and covariance of minisuperspace arguments fails for reference fluid system with one-dimensional minisuperspace<sup>(1)</sup>.
- Dynamics of sharply peaked states is sensitive to ordering choice only near bounce.
- For broadly peaked states, the ordering imprints survive through fluctuations of scale factor far away from the bounce.
- Notion of quantum corrected spacetime through expectation value scale factor is unambiguous for sharply peaked states.
- FLRW + dust +  $\Lambda$  maps to Hydrogen atom where unimodular clock emerge naturally.
- Negative  $\Lambda$  is discretized with hydrogenic spectrum. The AdS-dust universe is cyclic.
- Models the interior of AdS black hole<sup>(6)</sup>, implications for black hole entropy.

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