String wave packets and termination of Hawking radiation

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Based on works in progress and on,
K. Hashimoto, Y. Matsuo, T. Yoda, Transient chaos analysis of string scattering, JHEP11(2022)147, arXiv:2208.08380

It is known that string amplitudes are exponentially suppressed at trans-Planckian energies, which leads to the generalized uncertainty principle and the spacetime uncertainty principle. Recently, based on this property of strings, it has been proposed that Hawking radiation terminates at the scrambling time. This proposal offers a new perspective in which string corrections appear in the **time dependence of the intensity of Hawking radiation**. However, it is desirable to support this prediction more directly from string theory. In this poster, we present our attempts to support the termination of Hawking radiation using **string scattering amplitudes** and **string wave packets**.

1. String amplitudes and Hawking radiation

Interactions are suppressed at trans-Planckian energies

$$\mathcal{A}_{n\text{-loop}} \sim e^{-\frac{1}{n+1} s \ln s} \ll \mathcal{A}_{\text{QFT}} \sim s^{-\#}$$

String amplitudes are UV-safe

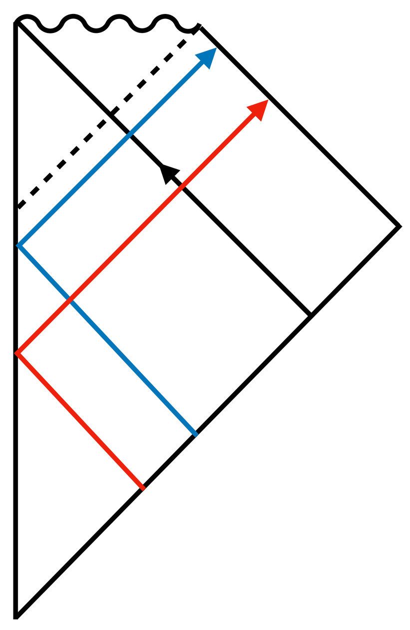
There exists an upper limit to the resolution of probing spacetime
The limit motivates

$$\Delta x \Delta p \geq \frac{1}{2}\left[1 + \ell^2 (\Delta p)^2\right]$$ Generalized uncertainty principle

$$\Delta X \Delta T \geq \ell^2$$ Spacetime uncertainty principle

Termination of Hawking radiation under GUP

Common belief : Small correction to the Hawking spectrum
New perspective: Large correction to time dependence of the intensity



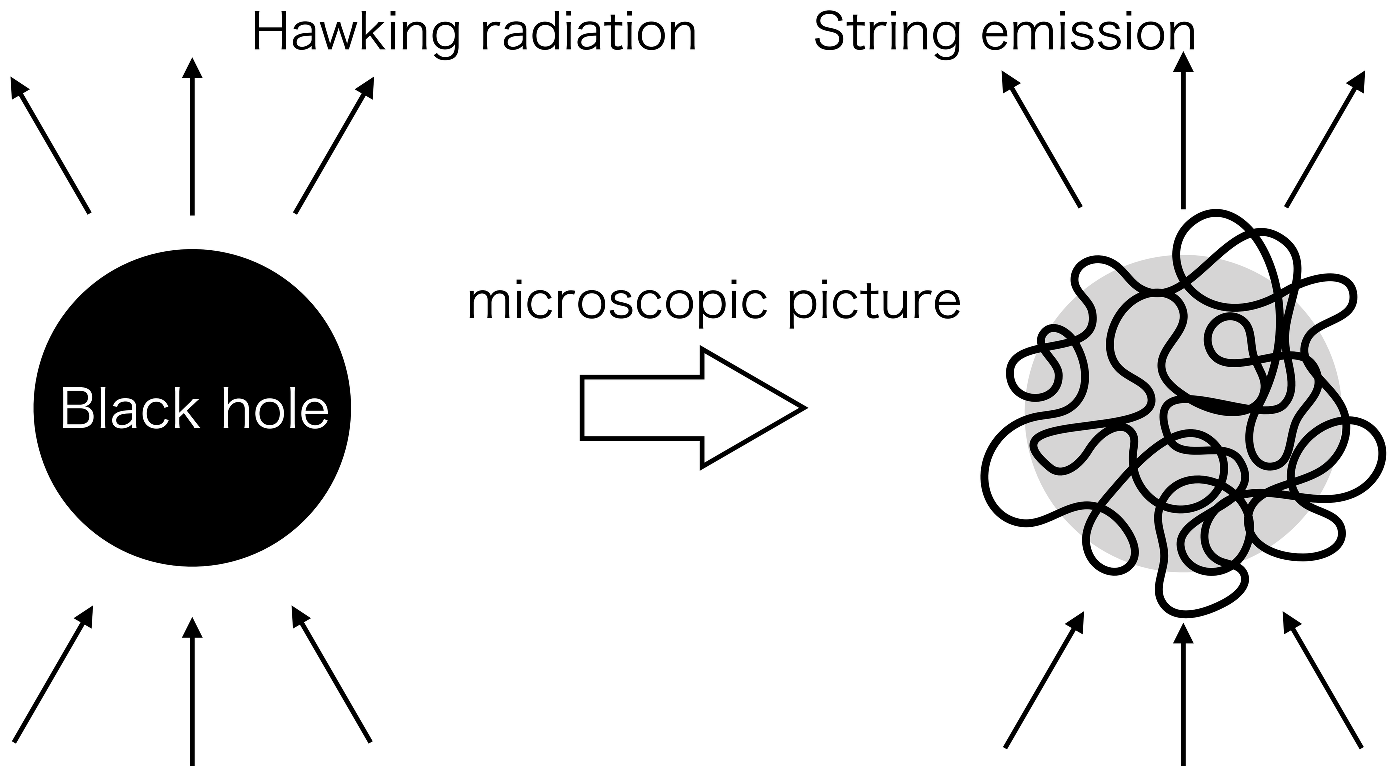
Standard derivation of Hawking radiation but with GUP

$$\hat{x} \equiv i\left(1 + \ell^2 p^2\right)\partial_p\,, \qquad \hat{p} = p$$

Short time: no correction

Late time: particles were trans-Planckian near horizon, thus, gravitational interactions are suppressed due to GUP

Problem: Can we derive termination of Hawking radiation from string amplitudes?



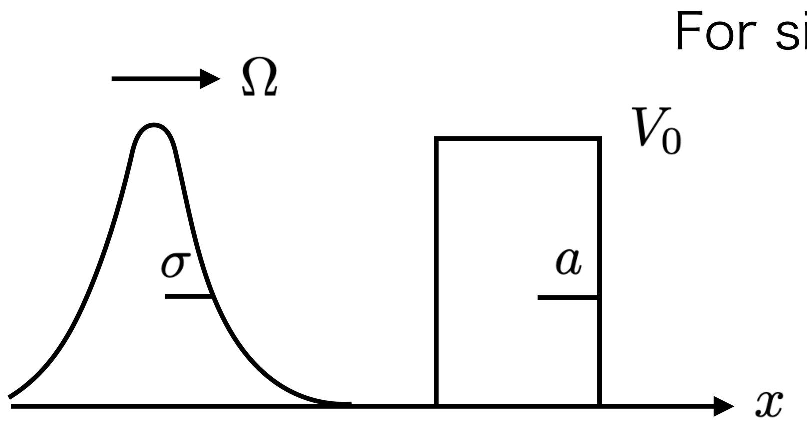
By evaluating the probability of string wave packets with some energy being emitted at fixed time, we will be able to follow the time development of radiation intensity

But pose here:

While scattering processes occur at string scale, observed particles are at the standard model scale
It is necessary to evaluate timedelay of low energy packets appropriately

2. Timedelay of low energy wave packets

Recall the Wigner timedelay



For simplicity, 1+1dim massless Klein-Gordon

$$\Phi_{\Omega,U}(x) = \frac{1}{\mathcal{N}}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\, f_{\Omega,U}(\omega)\phi_\omega(x)$$

$$f_{\Omega,U}(\omega) = e^{-\sigma^2(\omega-\Omega)^2/4 + i\omega U}$$

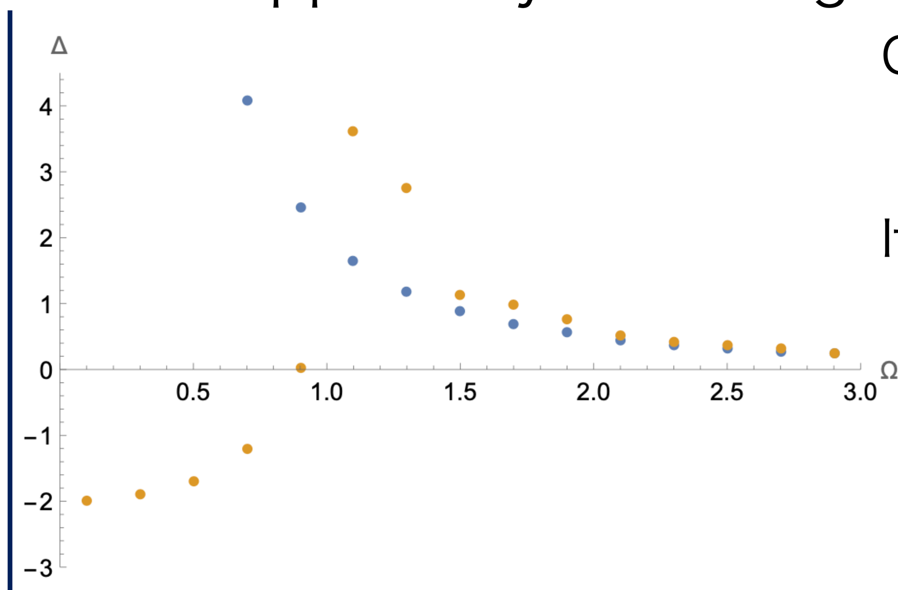
$$\Delta := \underset{\Delta}{\operatorname{argmax}}\left|\langle \Phi_{\Omega'=\Omega,U'=U+\Delta}, \Psi_{\Omega,U}\rangle_{t=+T}\right|$$

Possible approximations to evaluate the overlap integral:

$$\langle \Phi_{\Omega',U'}, \Psi_{\Omega,U}\rangle_{t=+T} \simeq \frac{1}{|\mathcal{N}|^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}(2\omega)\,\bar{f}_{\Omega',U'}(\omega')\cdot T_\omega \cdot f_{\Omega,U}(\omega)$$

Gauss approx. or non-Gauss
Perturbative or non-perturbative

Gauss approx. by assuming large wave packets $\sigma \to \infty$



One obtains the Wigner timedelay

$$\Delta \simeq \delta'_\Omega \qquad T_\omega = |T_\omega| e^{i\delta_\omega}$$

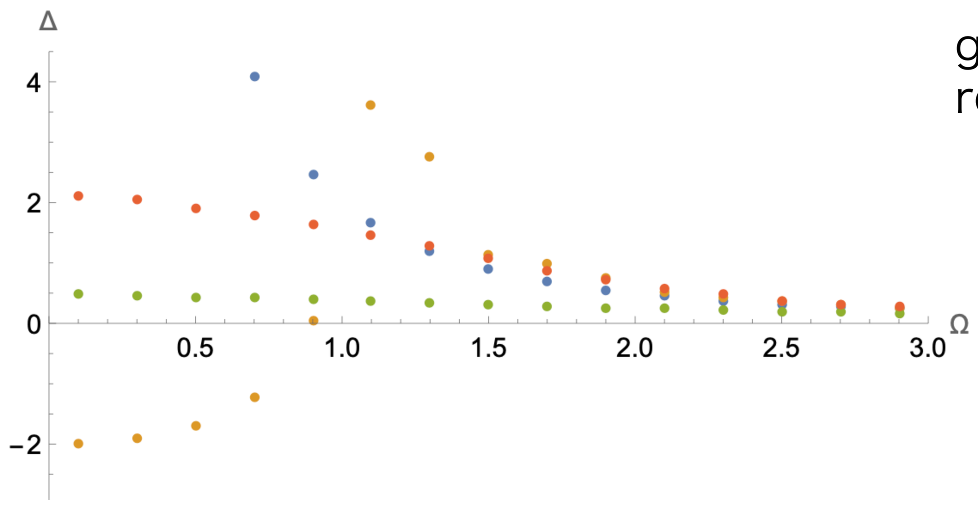
Its low energy behavior is undesirable

blue: Perturbative result diverges at low energies

orange: Even non-perturbative result becomes negative at low energies (apparently it exceeds the speed of light)

Solution

Use sufficiently small wave packets and give up the Gauss approx.

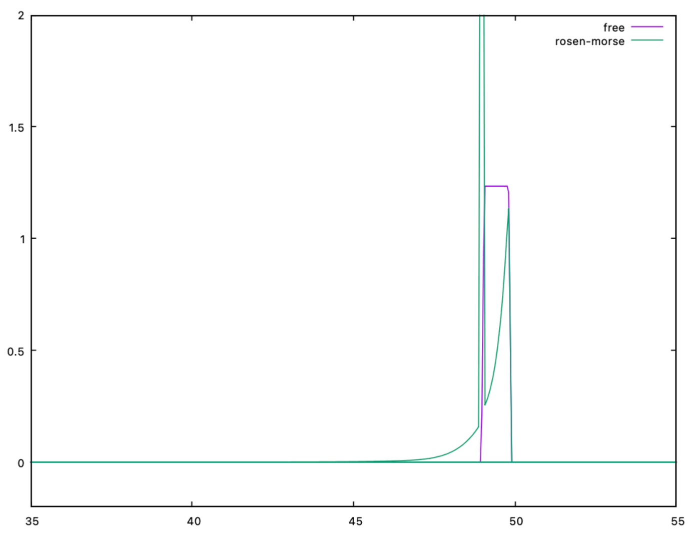


green: perturbative
red: non-perturbative

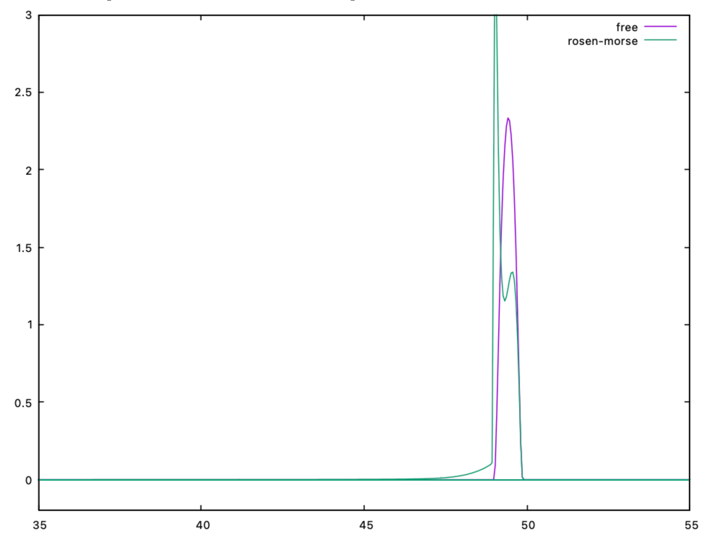
Both are safe at low energies
Perturbative result is qualitatively the same as the non-perturbative result

Use large but compactly supported wave packets

Rectangular shape:



Bump func. shape



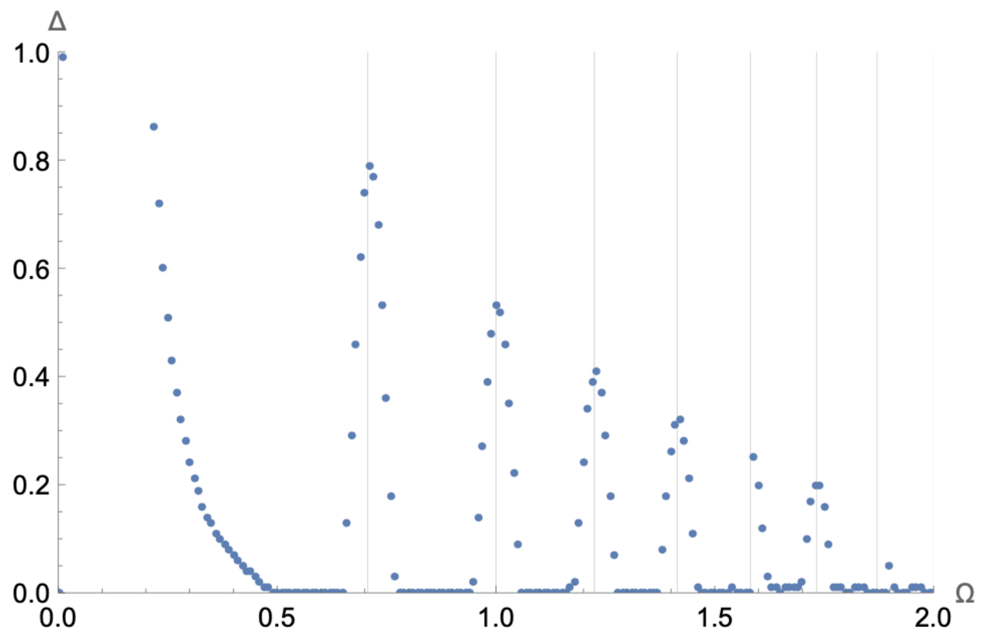
The wave fronts never exceed the speed of light

3. Timedelay of string wave packets

String timedelay probed by small wave packets

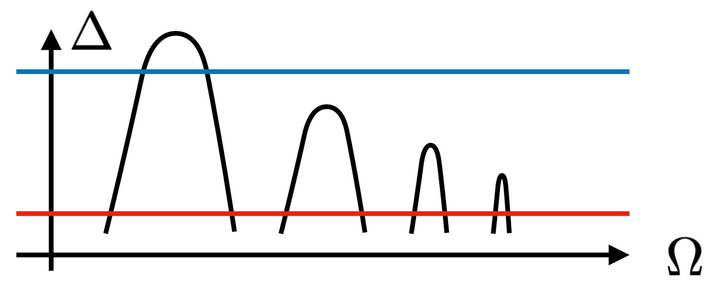
e.g. photon amplitudes but polarizations are ignored

$$\mathcal{A}_{\text{stringy}} = \frac{\Gamma(1+\alpha(s))\Gamma(1+\alpha(t))}{\Gamma(3+\alpha(s)+\alpha(t))} \sim \sum_{n=0}^{\infty}\frac{g_n}{s-n}$$



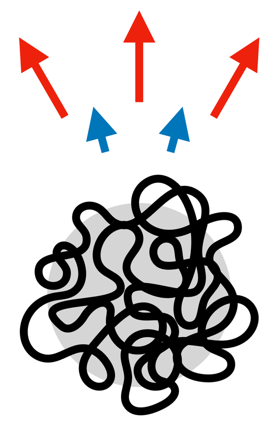
Timedelay is finite at all energies, goes to zero at various energies

Implications



Short time: strings with various energies are emitted

Late time: strings only with small energies are emitted



If similar phenomena occur in highly excited strings, it implies that radiation spectrum can be thermal only within short time, after that, only low energy strings are emitted

References

T.L. Chau, P.M. Ho, H. Kawai, W.H. Shao, C.T. Wang, Hawking Radiation Under Generalized Uncertainty Principle, Eur.Phys.J.C 83 (2023) 12, 1118
D. Amati and J. G. Russo, "Fundamental strings as black bodies," Phys. Lett. B 454 (1999) 207–212
G. Veneziano, "Transplanckian string collisions: An update," in 12th Marcel Grossmann Meeting on General Relativity, pp. 95–107. 7, 2009
G. Veneziano, "High-energy collisions of particles, strings, and branes," Les Houches Lect. Notes 97 (2015) 397–415.