Quantum fluctuations of geodesic deviation in Oppenheim's relativistic semi-classical gravity model

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1. Abstract

Gravity: does it have an inherently **quantum nature**, or is a **classical description** sufficient? This is one of the fundamental questions in modern physics.

We study the following three models:

1. The original Oppenheim model [1]
2. An environment-induced Oppenheim model — a model proposed by us (see Section 2)
3. A perturbative quantum gravity model

For these models, we calculate the fluctuations of geodesic deviation and compare the behavior of their spectra. We also discuss the possibility of detecting these effects with gravitational-wave interferometers and related experiments.

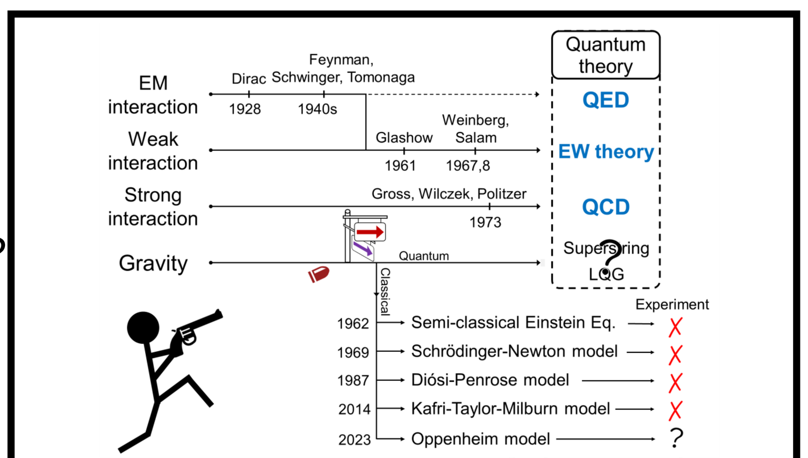
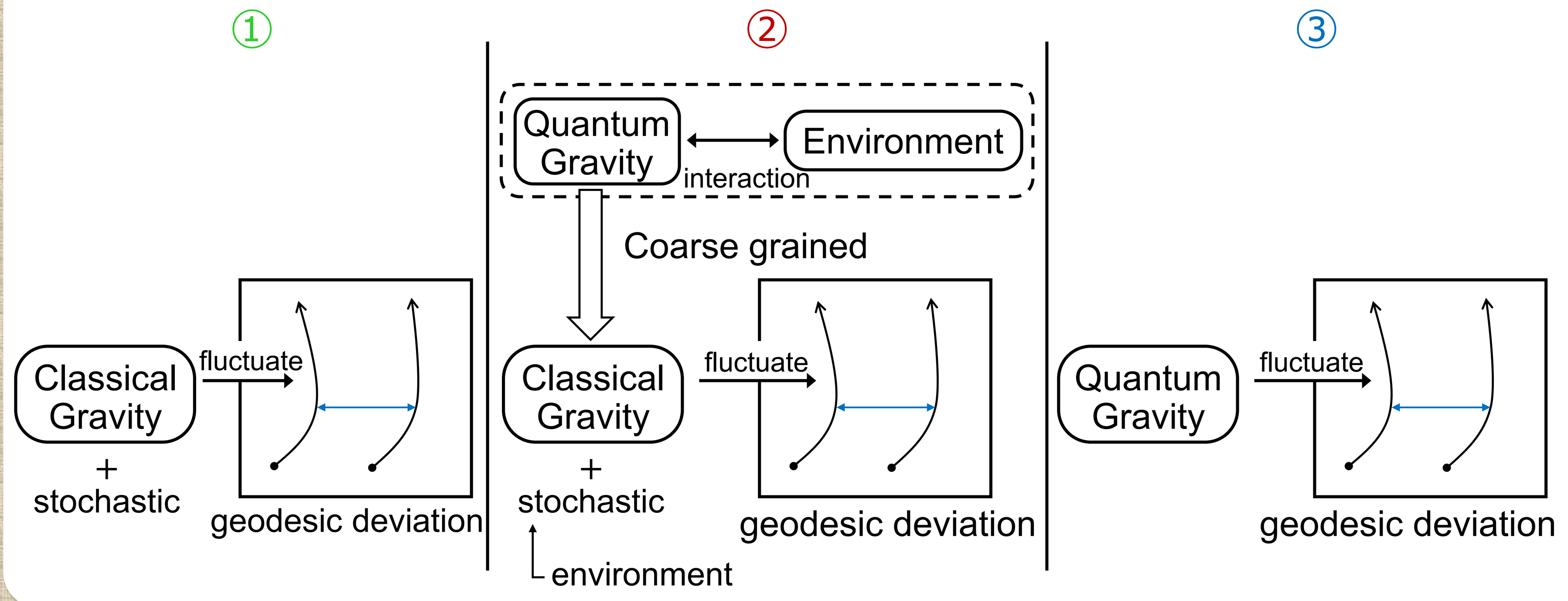


Figure 1: Overview of this study



2. Semi-classical gravity model

Schrödinger-Newton (SN) model

・It was proposed by adding a self-gravitational potential term to the standard Schrödinger equation [2]

$$i\hbar\partial_t\psi(\boldsymbol{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) + V_{SN}(\boldsymbol{r})\right)\psi(\boldsymbol{r}). \quad (1)$$

However, the SN equation can violate causality. Moreover, it has been **ruled out experimentally**.

Diósi-Penrose (DP) model

・To avoid the violation of causality mentioned above, this DP model introduces stochasticity into gravity [3].

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}\left[\hat{H}_0, \hat{\rho}\right] - \frac{G}{2\hbar}\iint \frac{d^3r\, d^3r'}{|\boldsymbol{r}-\boldsymbol{r}'|}\left[\hat{f}(\boldsymbol{r}), \left[\hat{f}(\boldsymbol{r}'), \hat{\rho}(t)\right]\right], \quad (2)$$

However, this model is also increasingly **disfavored by experiments** [5].

Oppenheim model

・Previously, relativistic semiclassical gravity models such as *the semiclassical Einstein equation*

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu}\rangle \quad (3)$$

had been proposed. The differences from those earlier models are as follows:

i) incorporates backreaction

ii) applies under large fluctuations

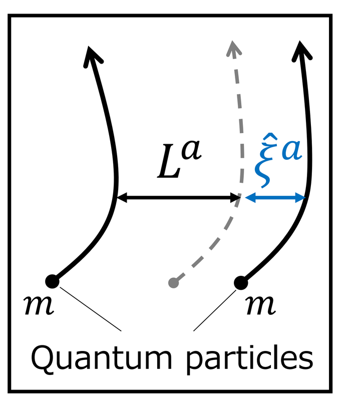
iii) gives gravity intrinsic **stochasticity**

iv) imposes a **trade-off relation** between decoherence and diffusion

Environment-induced Oppenheim model

・In this model, gravity is assumed to be fundamentally quantum. Through interactions between the quantum gravitational field and environmental quantum fields, it becomes coarse-grained and appears as classical gravity.

3. Setup



L^a : Initial geodesic deviation
$\hat{\xi}^a$: Deviation from the initial length
m : Mass of the particles
$\rho[x, \underline{x}, t] = \langle x|\hat{\rho}(t)|\underline{x}\rangle$
$T^{\mu\nu} = 2m\int_{-\infty}^{\infty} dt\, E^{\mu\nu}_{0a0b}\delta^4(x - X(t))L^a\xi^b(t)$

・Using the action for geodesic deviation

$$S_{eff} = \frac{m}{2}\int dt\left\{\delta_{ab}\frac{d\xi^a}{dt}\frac{d\xi^b}{dt} - 2R^{(1)}_{0a0b}L^a\xi^b\right\},$$

the dynamics of the system can be written in terms of a path integral as follows:

$$\rho\left[\xi^a_f, \underline{\xi}^a_f, h_{\mu\nu,f}, t_f\right] = \int^f D\xi^a D\underline{\xi}^a Dh_{\mu\nu}\,\delta\left[\partial^\mu(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h)\right] e^{I_{CQ}\left[\xi,\underline{\xi},h_{\mu\nu}\right]}\rho\left[\xi^a_i, \underline{\xi}^a_i, h_{\mu\nu,i}, t_i\right],$$

where

$$I_{CQ}\left[\xi,\underline{\xi},h_{\mu\nu}\right] = i\left(S_{eff}[\xi,h_{\mu\nu}] - S_{eff}\left[\underline{\xi},h_{\mu\nu}\right]\right)$$
$$-\frac{1}{2}\int_{t_i}^{t_f} d^4x d^4y\, \boldsymbol{D_{\mu\nu\rho\sigma}(x-y)}\left[T^{\mu\nu}(x) - \underline{T}^{\mu\nu}(x)\right]\left[T^{\rho\sigma}(y) - \underline{T}^{\rho\sigma}(y)\right]$$
$$-\frac{1}{2}\int_{t_i}^{t_f} d^4x d^4y\, \boldsymbol{N^{-1}_{\mu\nu\rho\sigma}(x-y)}\left[G^{(1)\mu\nu}(x) - 4\pi G_N(T^{\mu\nu}(x) + \underline{T}^{\mu\nu}(x))\right]$$
$$\times\left[G^{(1)\rho\sigma}(y) - 4\pi G_N(T^{\rho\sigma}(y) + \underline{T}^{\rho\sigma}(y))\right].$$

From this, we obtain the Langevin equation

$$\boldsymbol{m\frac{d^2\xi^a}{dt^2} - \zeta^a(t) = 0},$$

where

$$\langle\zeta_a(t)\rangle = 0, \qquad \langle\zeta_a(t)\zeta_b(t')\rangle = \left\{\boldsymbol{\Delta^D_{cadb}(t-t')} + \boldsymbol{\Delta^N_{cadb}(t-t')}\right\}L^cL^d,$$

$$\boldsymbol{\Delta^D_{cadb}(t-t')} = 4m^2 E^x_{0a0b}{}^{\mu\nu}E^y_{0c0d}{}^{\rho\sigma}\boldsymbol{D_{\mu\nu\rho\sigma}(x-y)}|_{x=X(t),y=X(t')},$$

$$\boldsymbol{\Delta^N_{cadb}(t-t')} = 16m^2\int_{t_i}^{t} d^4z\int_{t_i}^{t'} d^4w\, E^x_{0a0b}{}^{\mu\nu}G_R(x-z)E^y_{0c0d}{}^{\rho\sigma}G_R(y-w)\Big|_{x=X(t),y=X(t')}$$
$$\times\left(\delta^\alpha_\mu\delta^\beta_\nu - \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}\right)\left(\delta^\lambda_\rho\delta^\kappa_\sigma - \frac{1}{2}\eta_{\rho\sigma}\eta^{\lambda\kappa}\right)\boldsymbol{N_{\alpha\beta\lambda\kappa}(z-w)}.$$

Here, we define the power spectral density $(S^h_x)^2$ corresponding to the strain as

$$\left(S^h_x\right)^2 \equiv \frac{1}{m^2L^2\omega^4}\int dt\, e^{i\omega t}\langle\zeta_x(t)\zeta_x(0)\rangle.$$

① Oppenheim original

→ the gravitational field has white noise.

$$D_{\mu\nu\rho\sigma}(x-y) = \frac{D_0}{8}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - 2\beta\eta_{\mu\nu}\eta_{\rho\sigma})\delta^4(x-y),$$
$$N^{-1}_{\mu\nu\rho\sigma}(x-y) = \frac{D_0}{128\pi^2G_N^2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - 2\beta\eta_{\mu\nu}\eta_{\rho\sigma})\delta^4(x-y)$$

② Environment-induced model

→ environment-induced noise

$$D_{\mu\nu\rho\sigma}(x) = D_0\int\frac{d^4p}{(2\pi)^4}e^{ipx}\theta(-p^2 - 4\mu^2)\mathcal{P}_{\mu\nu\rho\sigma},$$
$$N_{\mu\nu\rho\sigma}(x) = \frac{(4\pi G_N)^2}{D_0}\int\frac{d^4p}{(2\pi)^4}e^{ipx}\theta(-p^2 - 4\mu^2)\mathcal{P}_{\mu\nu\rho\sigma}$$
$$\mathcal{P}_{\mu\nu\rho\sigma} = \mathcal{P}_{\mu\rho}\mathcal{P}_{\sigma\nu} + \mathcal{P}_{\mu\sigma}\mathcal{P}_{\rho\nu} - \frac{2}{3}\mathcal{P}_{\mu\nu}\mathcal{P}_{\rho\sigma}, \quad \mathcal{P}_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}.$$

③ Perturbative quantum gravity model

→ the gravitational field is plotted for the vacuum state, based on [6].

4. Result

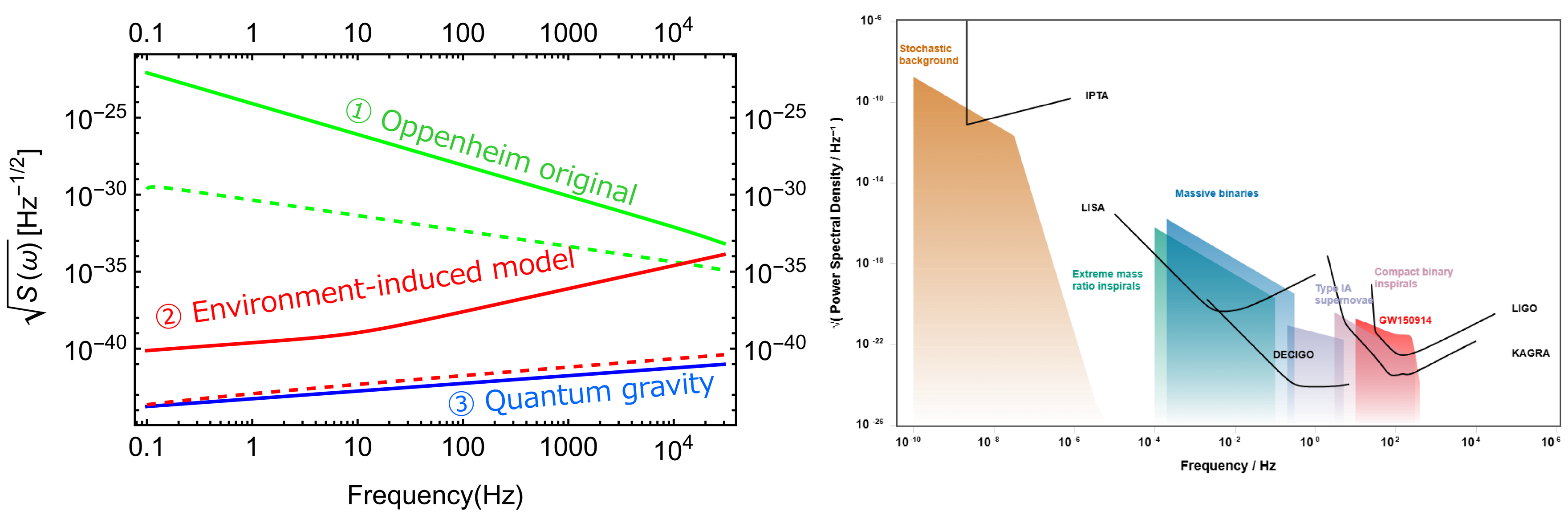


Figure 2: Strain spectra of each model and the sensitivity of gravitational-wave interferometers.

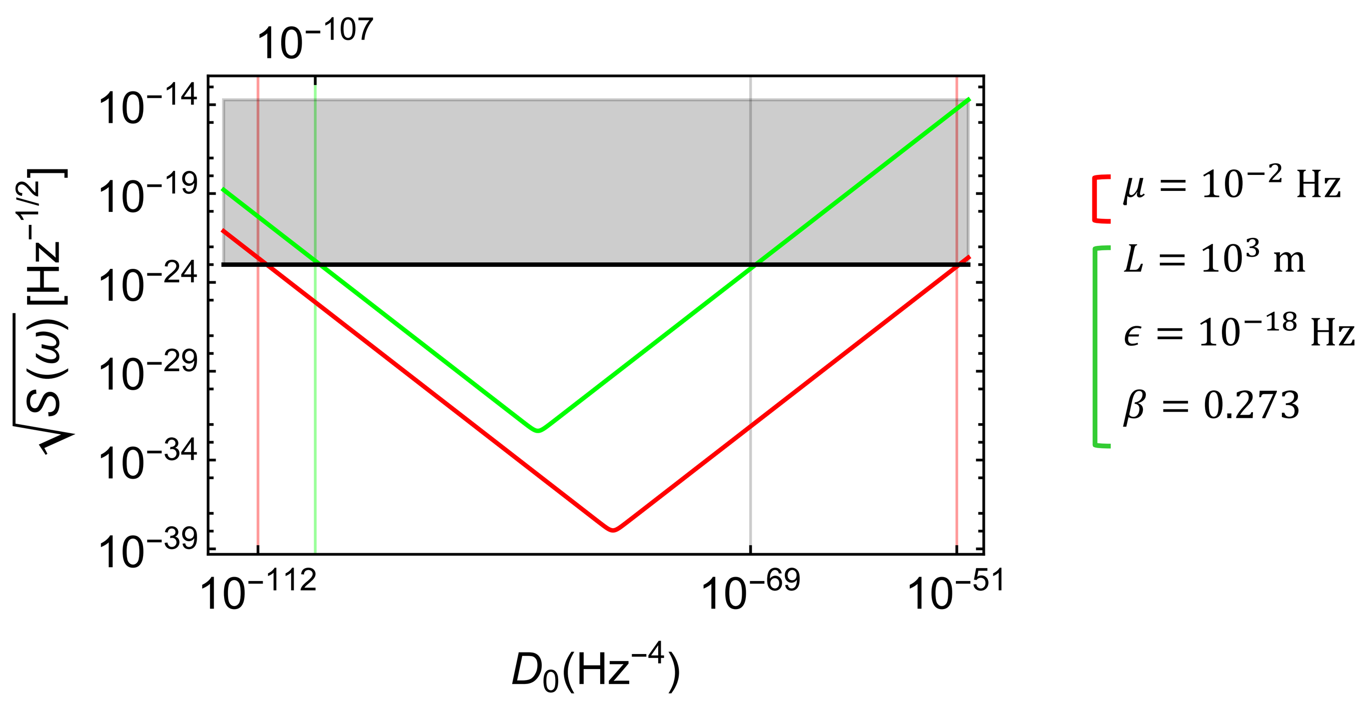
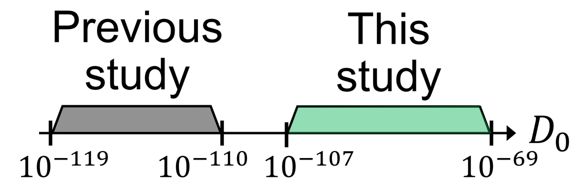


Figure 3: The constraint from LIGO

$\rightarrow 10^{-107} \leq D_0 \leq 10^{-69}$

10^{-119} [7] $\leq D_0 \leq 10^{-110}$ [8]



5. Conclusion

☑ This study is the first to derive fluctuations of geodesic deviation using the relativistic semiclassical gravity model proposed by Jonathan Oppenheim et al.
☑ In model ①, ②, the gravitational field is assumed to be classical, but there always exists a nonzero minimum fluctuation..
☑ The trade-off behavior in models ① and ② works in opposite ways.
☑ Model ① can be tested using the current gravitational-wave interferometer LIGO.
☐ Using model ②, we will investigate whether classical gravity can generate quantum entanglement.

Reference

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