Concepts of Quantum and Spacetime

Gravity sourced by quantum matter : a subjective overview

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March 10th 2026

Outline

◆ Introduction

◆ Quantum scenario

◆ Semiclassical scenario

◆ Summary

Introduction

Does gravity follow quantum mechanics?

Can gravitational field be in a quantum superposition?

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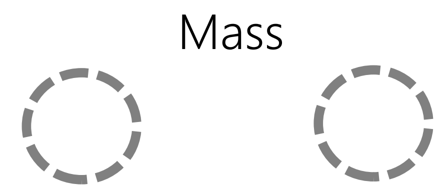
Quantum

Quantum superposition



$|\mathrm{L}\rangle$ $|\mathrm{R}\rangle$

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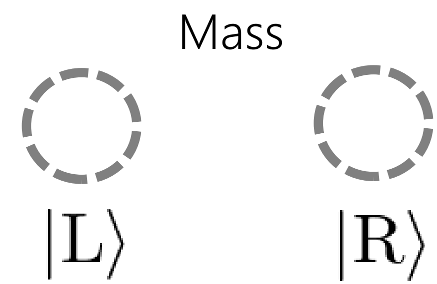
Can gravitational field be in a quantum superposition?

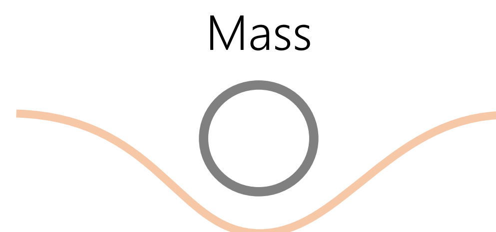
Quantum

Quantum superposition



Gravity

Mass generates gravity =
spacetime curvature

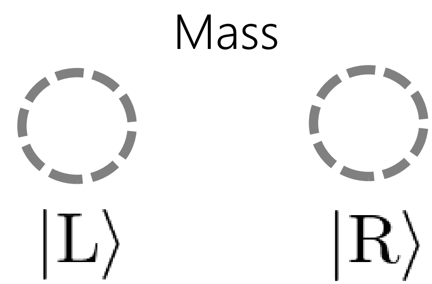


Gravitational field = Spacetime curvature

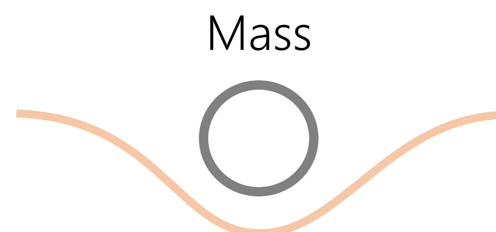
Does gravity follow quantum mechanics?

Can gravitational field be in a quantum superposition?

Quantum — Quantum superposition



Gravity — Mass generates gravity = spacetime curvature



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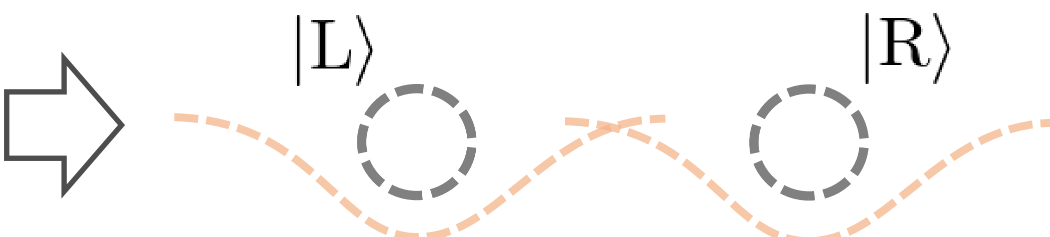
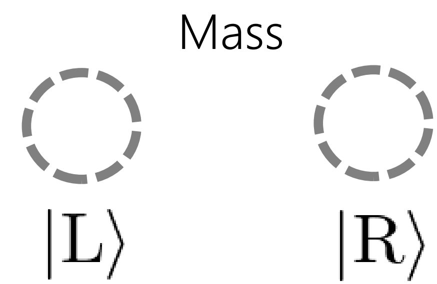


Quantum superposition of grav. field

Does gravity follow quantum mechanics?

Can gravitational field be in a quantum superposition?

Quantum — Quantum superposition



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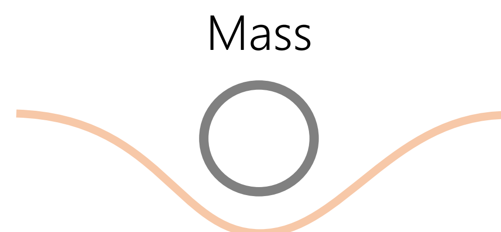

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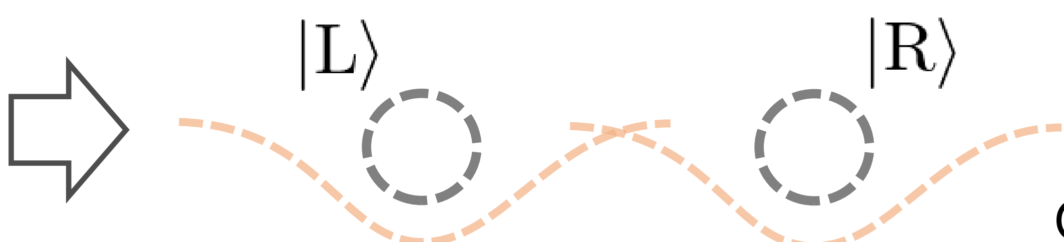


Quantum superposition of grav. field

This is yet to be tested in very low energy and non-relativistic regime.

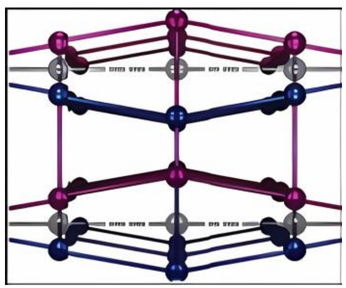
Status of experiment

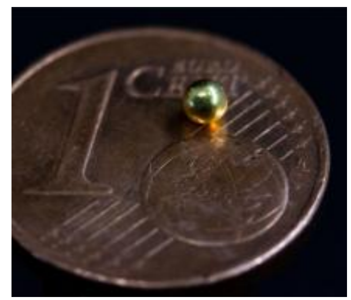
Quantum experiment and gravity experiment

$\sim 10^{33}\mathrm{eV/cm^3}$

Quantum | Future crossover region | Gravity

$\sim 10^{-8}\mathrm{g}$ $\sim 10^{-5}\mathrm{g}$ $10^{-1}\mathrm{g} \sim$ Mass





Gravity of small object is measured

Westphal+ Nature (2021)

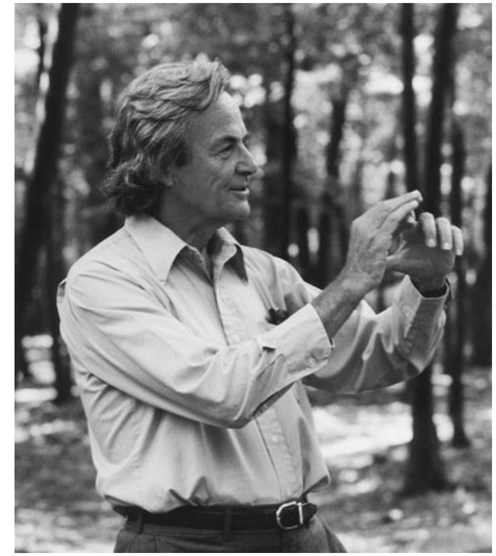
Oscillation follows quantum mechanics

Cripe+ Nature (2019) Bild+ Science (2023)

The gap between the two regions will be filled ⇨ Testing the gravity of quantum matter

Quantum scenario

A benchmark signal



What happens if grav. field is in a quantum superposition?

Feynman’s thought experiment

A benchmark signal

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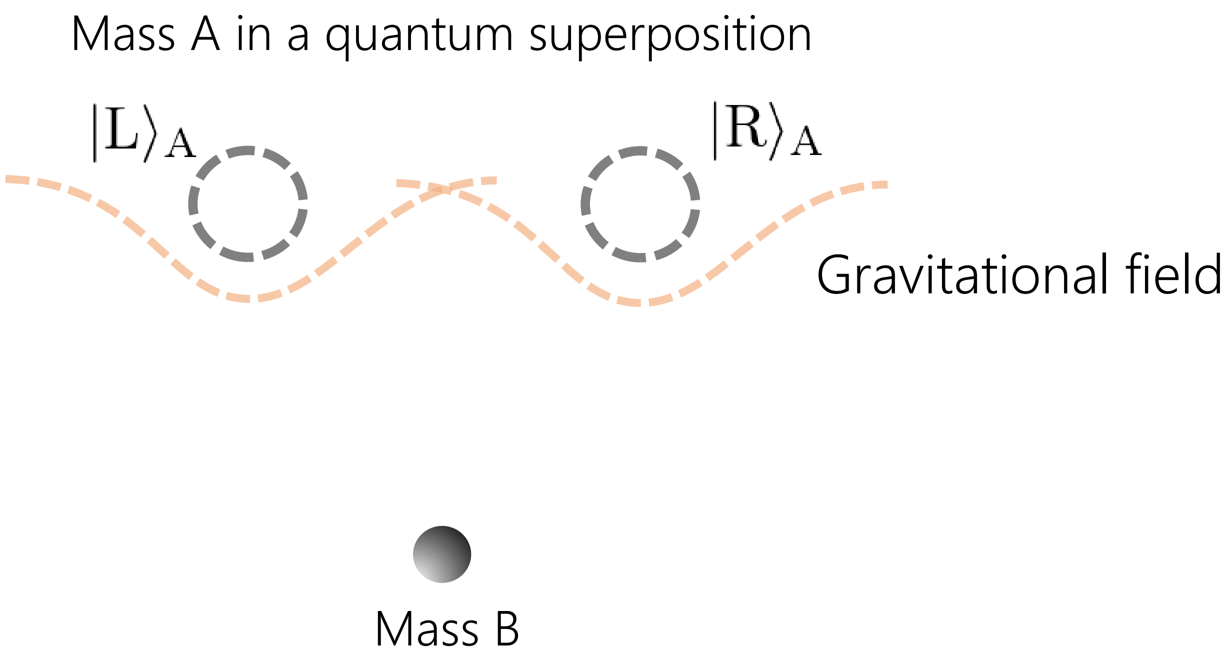

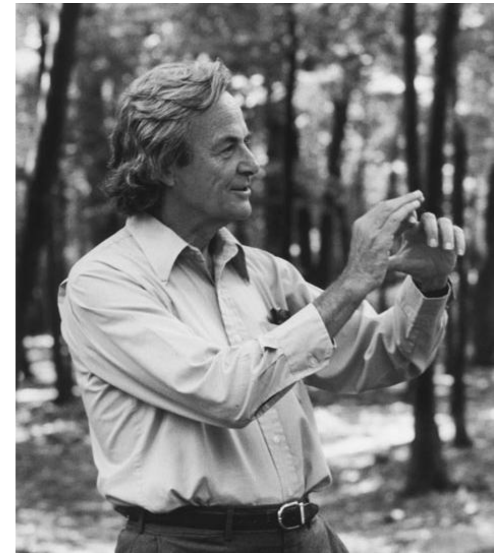

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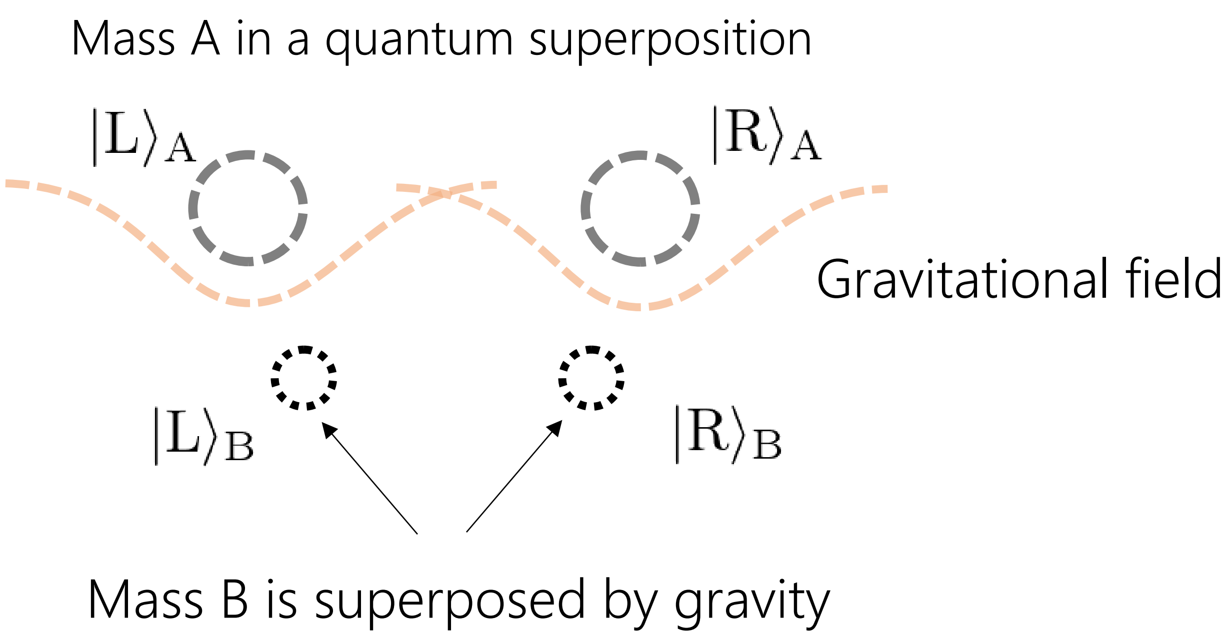

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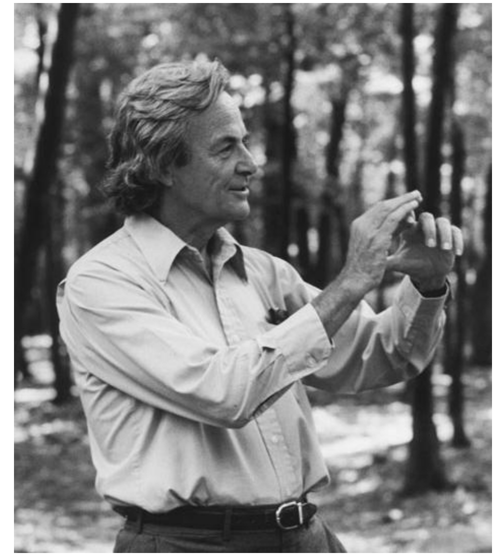

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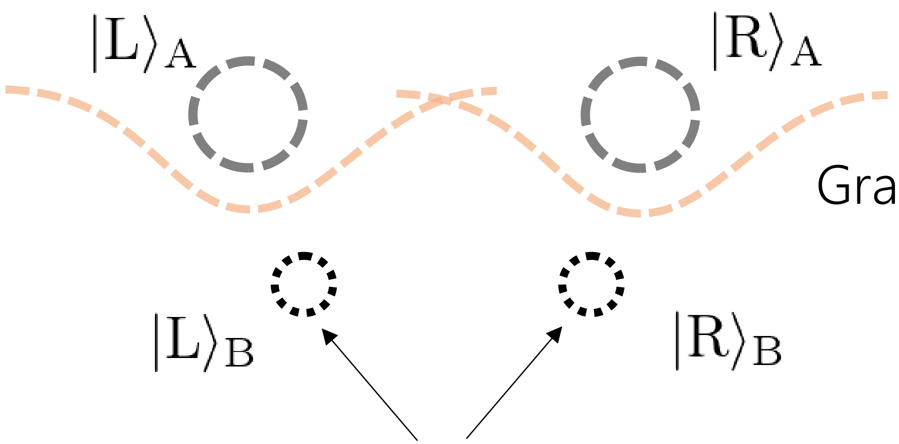
Feynman's thought experiment

Mass A in a quantum superposition



Gravitational field

Mass B is superposed by gravity

$$|\mathrm{L}\rangle_\mathrm{A}|\mathrm{L}\rangle_\mathrm{B} + |\mathrm{R}\rangle_\mathrm{A}|\mathrm{R}\rangle_\mathrm{B}$$

Gravity induces quantum entanglement

This is considered as a benchmark for proving the quantum superposition of gravitational field, but this is yet to be tested.

What theory predicts gravity-induced entanglement?

Quantum model: Perturbative quantum gravity

Donoghue, arXiv:1209.3511

$$S = M_{\mathrm{pl}}^2 \int d^4x \sqrt{-g} \left(R + \mathcal{L}_{\mathrm{matter}} + \frac{c_1}{M_{\mathrm{pl}}^2} R^2 + \frac{c_2}{M_{\mathrm{pl}}^2} R_{\mu\nu} R^{\mu\nu} + \cdots \right) \qquad \hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$$

In a weak-field and non-relativistic regime,

$$\nabla^2 \hat{\Phi} = 4\pi G \hat{\mu}(\boldsymbol{x}) \qquad \hat{V} = \frac{1}{2} \int d^3x \, \hat{\mu}(\boldsymbol{x}) \hat{\Phi}(\boldsymbol{x}) \qquad \hat{\mu}(\boldsymbol{x}) = \sum_i m_i \delta^3(\boldsymbol{x} - \hat{\boldsymbol{x}}_i)$$

Grav. potential is a q-number and quantum mechanically superposed

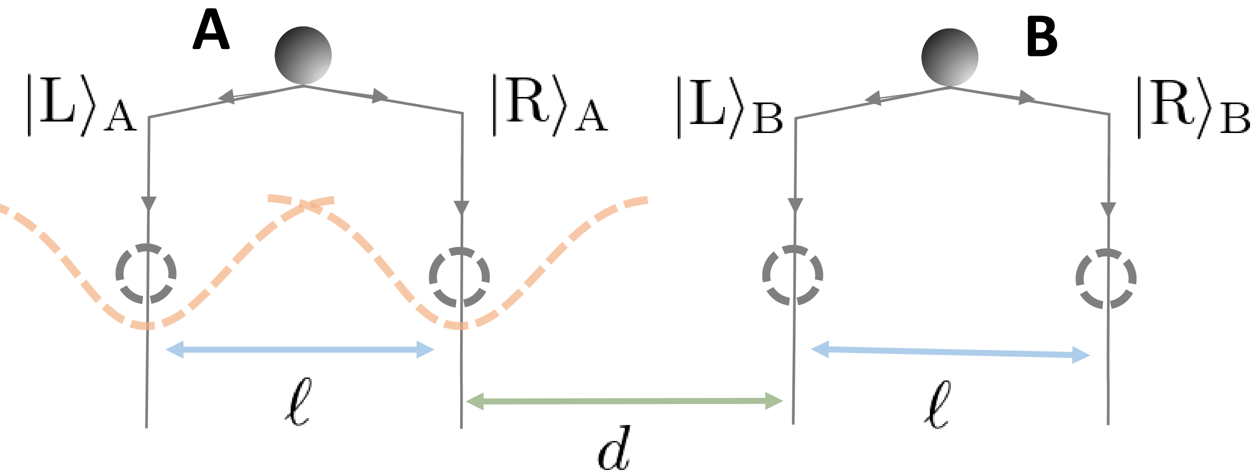
Quantum masses follow

$$i\hbar \frac{d}{dt} |\psi\rangle = \hat{H} |\psi\rangle \qquad \hat{H} = \sum_i \frac{\hat{\boldsymbol{p}}_i^2}{2m_i} - \sum_{i \neq j} \frac{G m_i m_j}{|\hat{\boldsymbol{x}}_i - \hat{\boldsymbol{x}}_j|}$$

The quantized potential induces the entanglement of masses

Matter-wave interferometer setting

Bose+ PRL(2017), Marletto and Vedral, PRL (2017)



Preparing each mass being in a superposition state of two locations, for example, by Stern-Gerlach effect

They feel the gravitational potential

$$\hat{V} = -\frac{G m_A m_B}{|\hat{x}_A - \hat{x}_B|}$$

$(|L\rangle_A + |R\rangle_A)(|L\rangle_B + |R\rangle_B)$ No entangled

$$\xrightarrow{e^{-i\hat{V}t/\hbar}} |L\rangle_A(e^{i\phi_{LL}}|L\rangle_B + e^{i\phi_{LR}}|R\rangle_B) + |R\rangle_A(e^{i\phi_{RL}}|L\rangle_B + e^{i\phi_{RR}}|R\rangle_B)$$ Entangled

$$\phi_{LL} = \phi_{RR} = \frac{t}{\hbar}\frac{G m_A m_B}{d+\ell} \quad \phi_{LR} = \frac{t}{\hbar}\frac{G m_A m_B}{d+2\ell} \quad \phi_{RL} = \frac{t}{\hbar}\frac{G m_A m_B}{d}$$

The entanglement characterized by $d \gg \ell$ $m_A \sim m_B \sim m$

$$(\phi_{RL} - \phi_{RR}) - (\phi_{LL} - \phi_{LR}) \sim \frac{t}{\hbar}\frac{G m^2 \ell^2}{d^3} \sim 1.0\left(\frac{t}{1.5\mathrm{s}}\right)\left(\frac{m}{10^{-13}\mathrm{kg}}\right)^2\left(\frac{\ell}{10\mu\mathrm{m}}\right)^2\left(\frac{100\mu\mathrm{m}}{d}\right)^3$$

large mass, large superposition, long coherence time

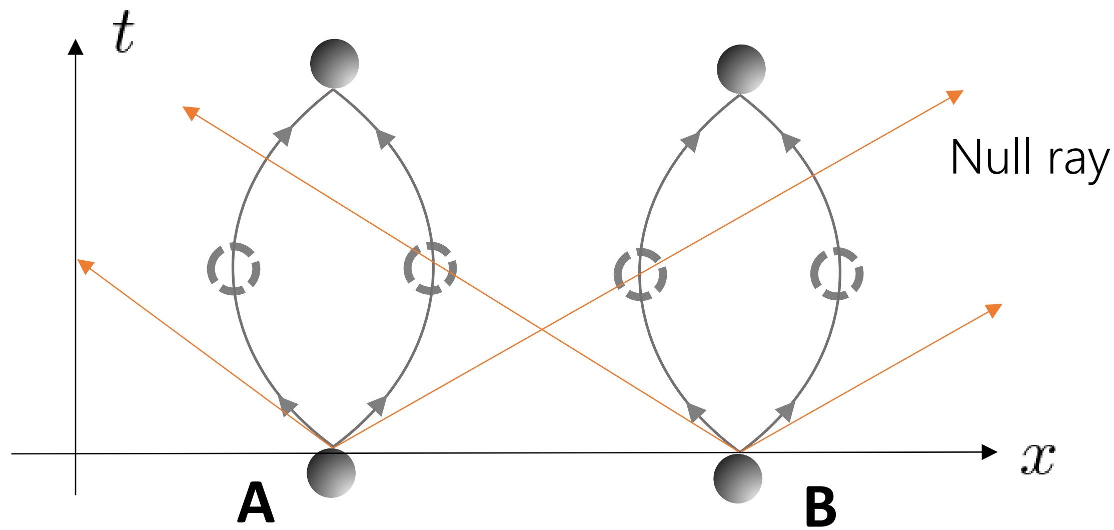
First principle calculations

Entanglement generation is calculated taking field quantization into account.

QED $\hat{J}^{\mu}\hat{A}_{\mu}$ PQG $\hat{T}^{\mu\nu}\hat{h}_{\mu\nu}$

The interaction between A and B causally occurs

Decoherence due to photon/graviton emission



Coulomb/Gravity-induced entanglement can be reproduced.

Christodoulou+ PRL(2023)

Entanglement occurs when they are causally connected.

Entanglement does not occur when they are spacelike separated or only have one-way information.

AM PRD (2021) Sugiyama, **AM**, Yamamoto PRD (2022)

Relation to the quantization of gravitational field

Paradox between causality (no-faster-than-light information) and complementarity (trade-off of coherence and which-path information) in a thought experiment involved with gravity-induced entanglement **is resolved by the entangling radiation of gravitons and the vacuum fluctuations of gravitons.**

Belenchia+ PRD (2018)

Consistency between causality and complementarity is guaranteed by the Robertson inequality of quantized gravitational field (the uncertainty relation of gravitational field).

Sugiyama, **AM**, Yamamoto PRD (2022) See also Mitrakos+ arXiv:2601.03214

Newtonian gravitational entanglement + Lorentz invariance + unitarity + micro causality lead to the graviton-like degrees of freedom.

Carney PRD (2022)

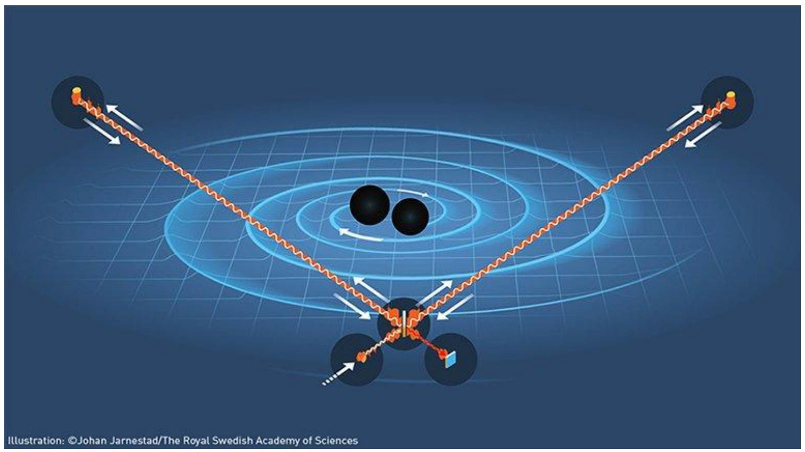
There is no clear distinction between entanglement mediated by the Newtonian gravitational field of a body and entanglement mediated by on-shell gravitons emitted by the body.

Danielson+ PRD (2022)

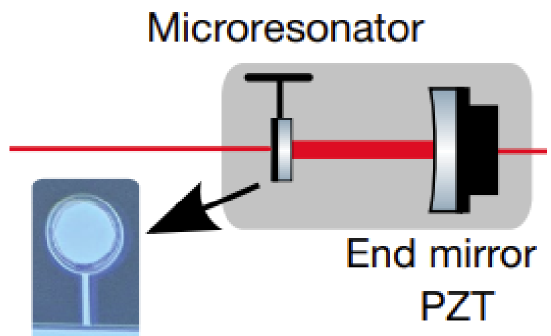
Gravity-induced entanglement is predicted from perturbative quantum gravity, but further assumptions are necessary to say about gravitons and their quantum nature.

Optomehcanical setting

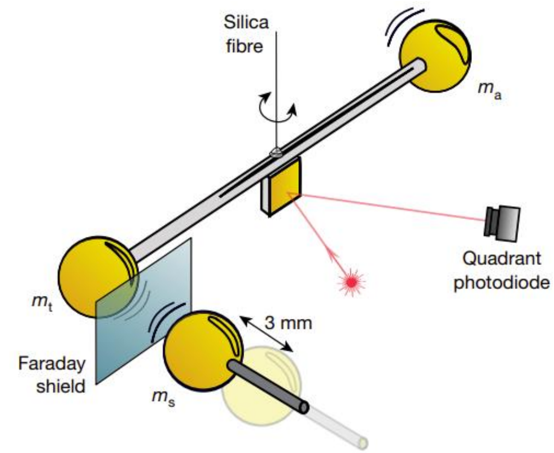
An optomechanical system is the coupled system of light and mechanical motions, which is a platform of gravitational wave experiments, realizing macroscopic quantum states, and measuring gravitational forces.



LIGO experiment



Cripe+ Nature (2019)

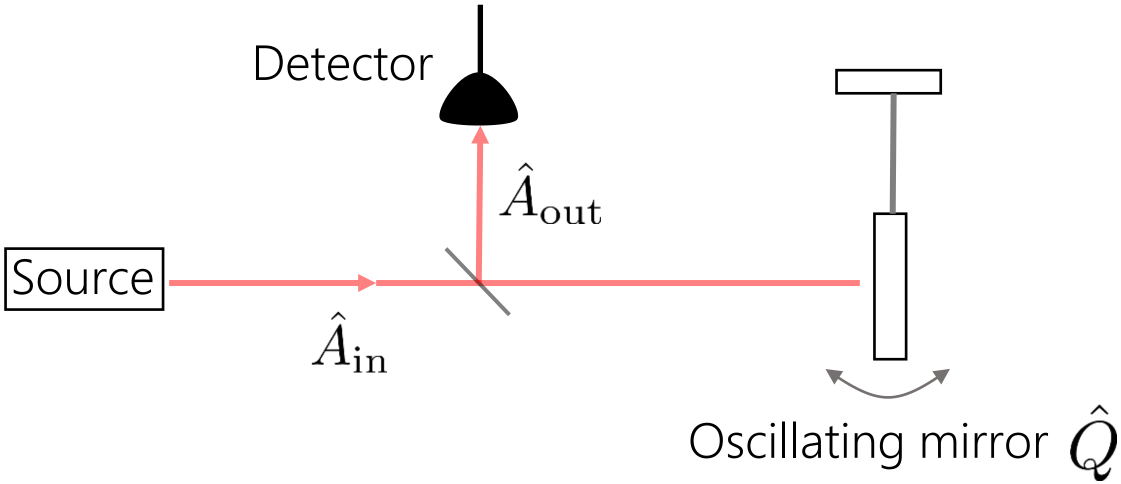


Westphal+ Nature (2021)

Using optomechanical systems is considered as a promising way for proving gravity-induced entanglement.

e.g. Balushi+ PRA (2018) Miao+ PRA (2020)
AM, Yamamoto PRD (2020)
Kaku, Fujita, **AM** PRD (2023)

Gravity-induced entanglement in optomechanical systems



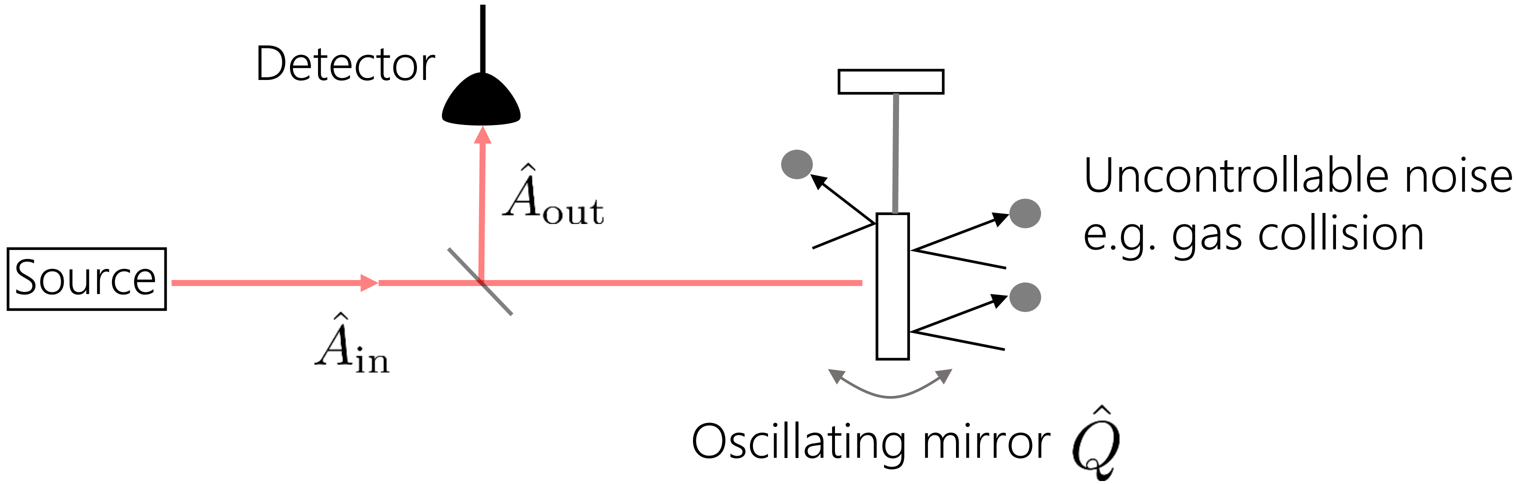
EoM of the oscillating mirror

$$m\frac{d^2\hat{Q}}{dt^2} = -m\Omega^2\hat{Q} + 2\hbar k\hat{A}_{\rm in}^{\dagger}\hat{A}_{\rm in}$$

The readout optical field

$$\hat{A}_{\rm out} = \hat{A}_{\rm in}e^{2ik\hat{Q}}$$

Gravity-induced entanglement in optomechanical systems

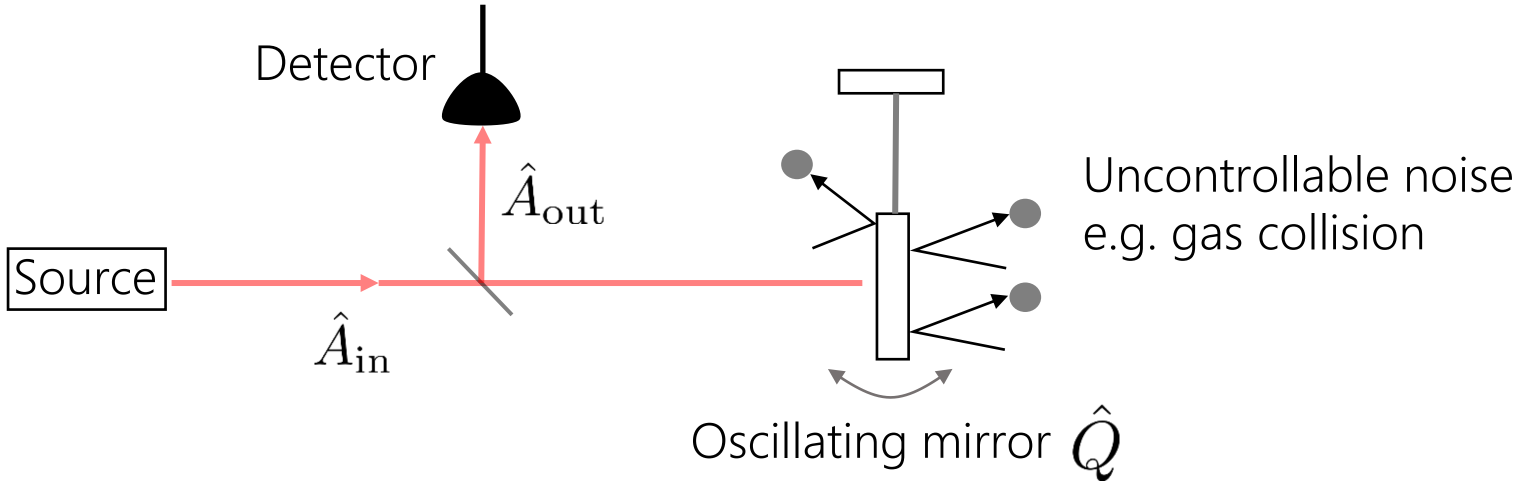


EoM of the oscillating mirror
$$m\frac{d^2\hat{Q}}{dt^2} = -m\Omega^2\hat{Q} + 2\hbar k\hat{A}^\dagger_{\rm in}\hat{A}_{\rm in} - m\Gamma\frac{d\hat{Q}}{dt} + F_{\rm n}$$

The readout optical field
$$\hat{A}_{\rm out} = \hat{A}_{\rm in}e^{2ik\hat{Q}}$$

$$\langle F_n(t)F_n(t')\rangle = 2m\Gamma k_{\rm B}T\delta(t-t')$$

Gravity-induced entanglement in optomechanical systems



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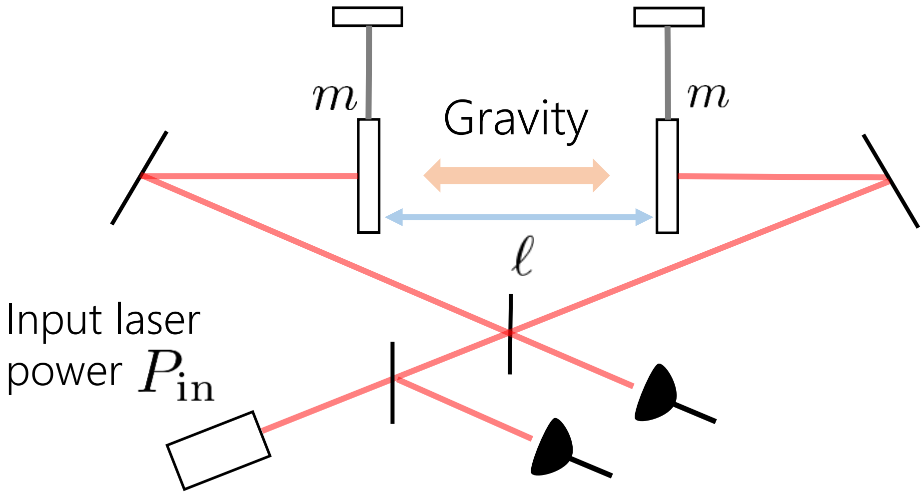
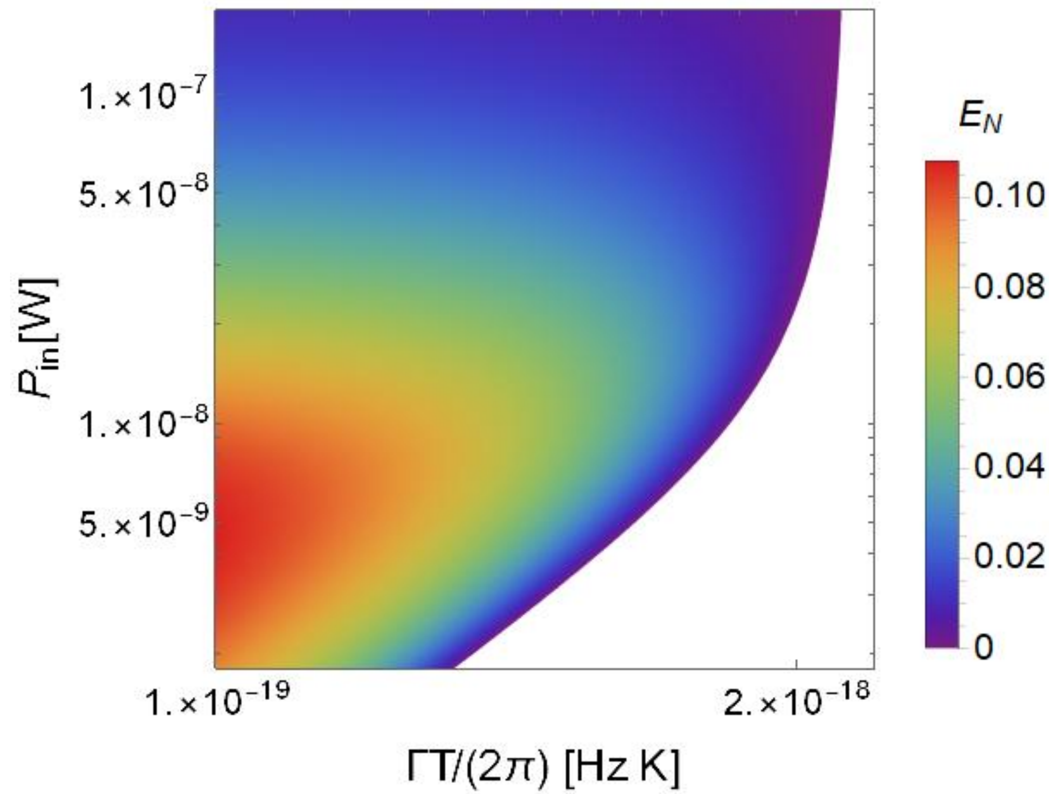
Measurement and feedback of oscillating mirror realize the quantum state of mechanical motion

Following the setting and focusing on small quantum fluctuations of lights and oscillating mirrors, we can get the steady state of the mirrors and can demonstrate the entanglement between them as





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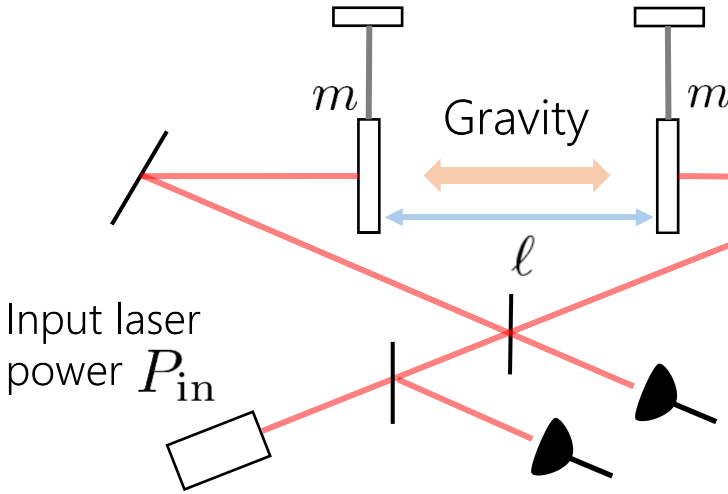
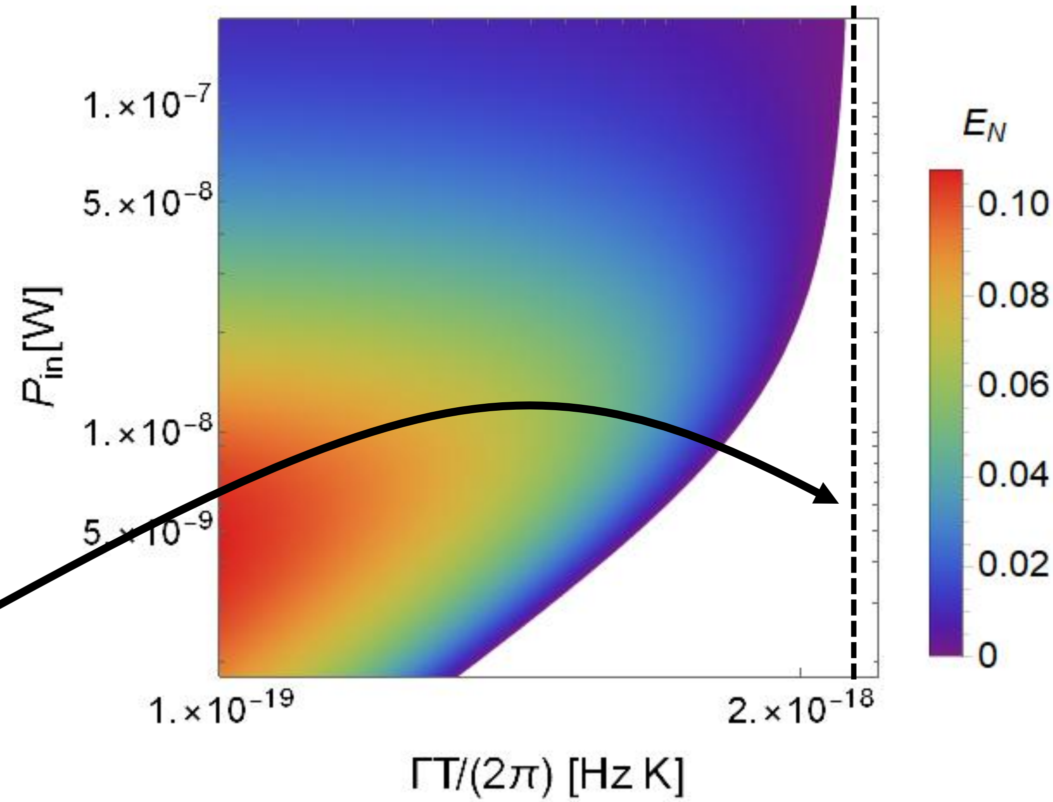




Entangling condition $\frac{\hbar G m}{\sqrt{2}\ell^3} > k_{\rm B} T \Gamma$

Miao+ PRA (2020) Miki, **AM**, Yamamoto PRD (2024)

Matsumoto, Sakai, Izumi, Miki, Iso, Hatakeyama, **AM,** Yamamoto arXiv:2507.12899

Super small temperature and dissipation are required

$$\left(\frac{\Gamma T/2\pi}{10^{-18}\ \mathrm{Hz}\cdot\mathrm{K}}\right)\left(\frac{20\ \mathrm{g/cm^3}}{m/\ell^3}\right) < O(1)$$

c.f. gas pressure requirement is 10^(-15) Pa at 1 K << pressure of interstellar

Enhancement technics

There are many approaches to give large quantum superposition of mass and large gravity-induced entanglement.

- Releasing mass (free mass) Krisnanda+ npj Quantum Inf. (2020)
- Inverted oscillator Fujita, Kaku, **AM**, Michimura CQG(2025)
- Delocalizing levitated mass Weiss+ PRL(2021)
- Enhancing through the other force by massive mediator Pedernales+ PRL(2022)
- Optomechanical position anti-squeezing Fukuzumi+ arXiv:2508.14337

and so on

We are looking for the best experimental setting to test gravity-induced entanglement and to prove the quantum superposition of gravitational field.

Semiclassical scenario

Quantum superposition of gravitational field should be tested

Since it is the basic assumption of quantum gravity. Also, we have many theoretical proposals, in particular, semiclassical models: mass is quantum, but gravity is classical.

- High energy such as the Planck scale (energy density) $\sim 10^{123}\mathrm{eV}/\mathrm{cm}^3$

  String theory, Loop quantum gravity, . . .

- Low energy scale such as a tabletop scale $\sim 10^{33}\mathrm{eV}/\mathrm{cm}^3$

Quantum

Perturbative quantum gravity

Donoghue, arXiv:1209.3511

Semiclassical

Post-quantum classical gravity

Oppenheim, PRX (2023)

Diósi-Penrose model

Diósi, PLA (1987) Penrose, GRG (1996)

Schrödinger-Newton model

Bahrami+ New J. Phys. (2014, review)

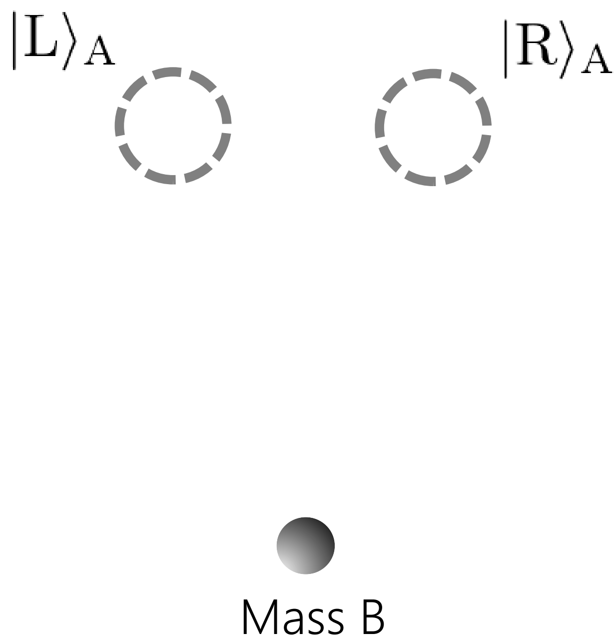
and so on

Well-known semiclassical models

- **Schrödinger-Newton model** Bahrami+ New J. Phys. (2014, review)

$$\nabla^2 \Phi = 4\pi G \langle \hat{\mu} \rangle$$

Gravitational potential is given by the quantum expectation value of mass density, a c-number and has only one realization



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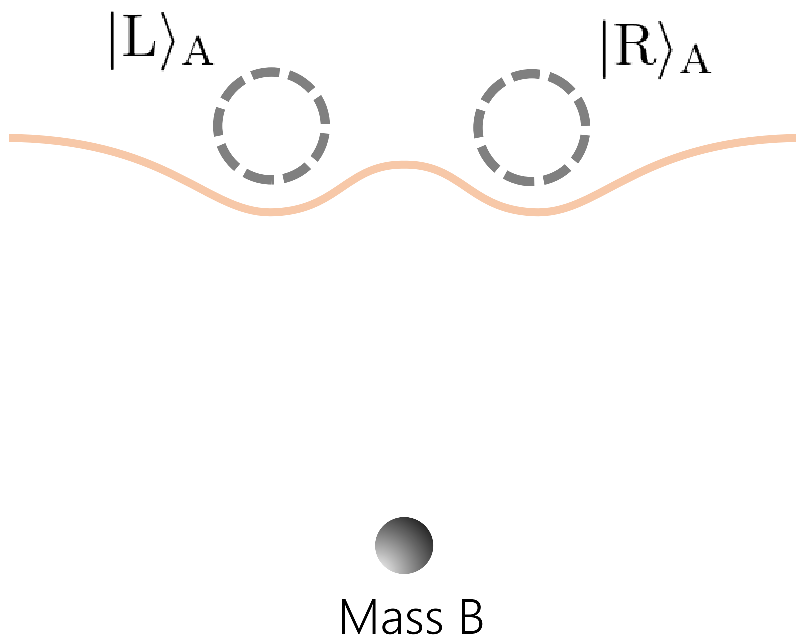

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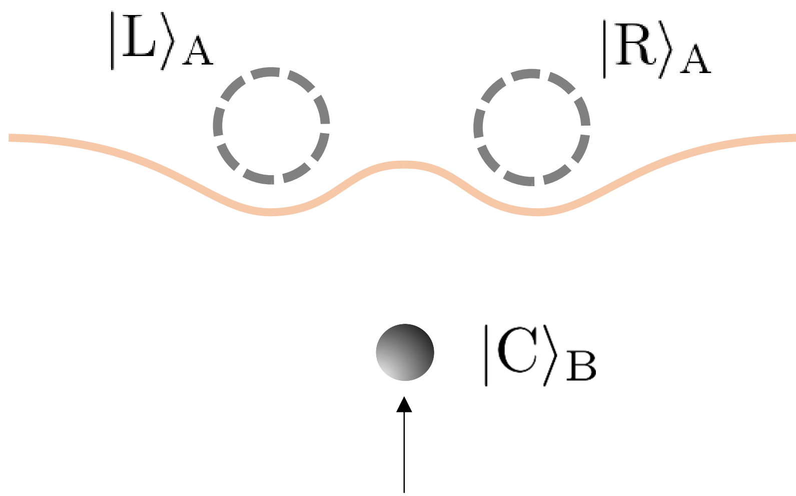

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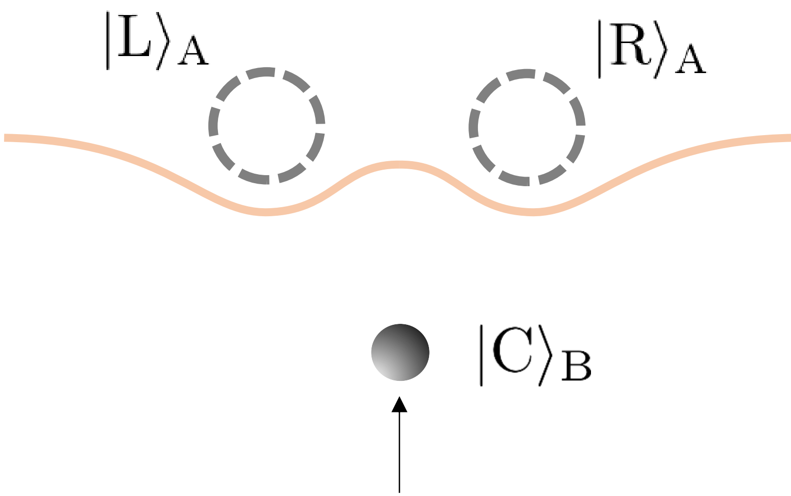


Mass B is not superposed and moves to a definite position

$$(|L\rangle_A + |R\rangle_A)|C\rangle_B$$

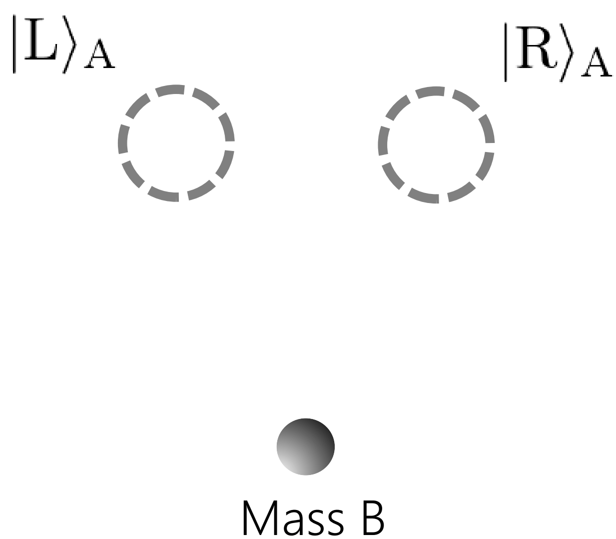
no entanglement

Well-known semiclassical models

- **Diósi-Penrose model** Diósi, PLA (1987) Penrose, GRG (1996)

$$\frac{d}{dt}\hat{\rho}_{\mathrm{M}} = -\frac{i}{\hbar}[\hat{H},\hat{\rho}_{\mathrm{M}}] - \frac{G}{2\hbar}\int \frac{d^3x d^3y}{|\boldsymbol{x}-\boldsymbol{y}|}[\hat{\mu}(\boldsymbol{x}),[\hat{\mu}(\boldsymbol{y}),\hat{\rho}_{\mathrm{M}}]] \quad \hat{\mu}(\boldsymbol{x}) = \frac{m}{(2\pi)^{3/2}R_0^3}e^{-\frac{(x-\hat{x})^2}{2R_0^2}}$$

When R0 goes to zero, even if you try to prepare a superposed mass, **the gravitational field of mass is realized around the mass's position,** and then **the mass is decohered.**



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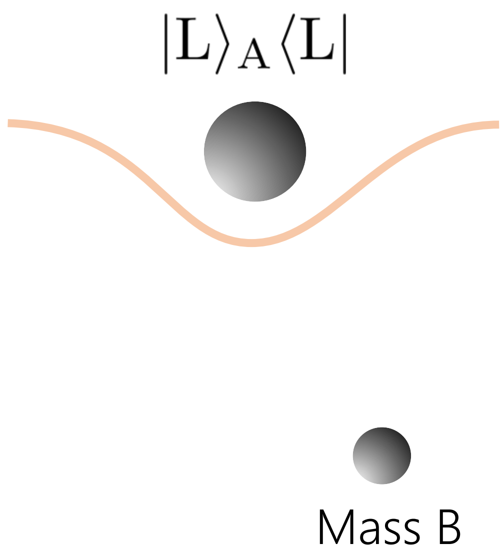

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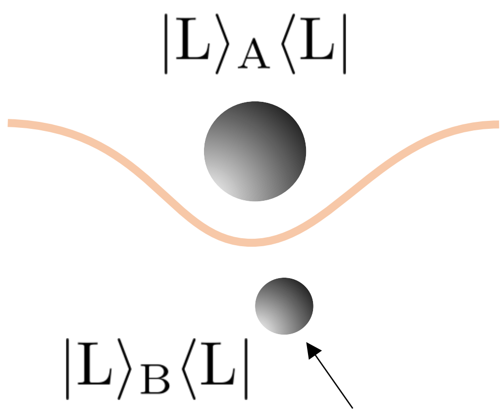

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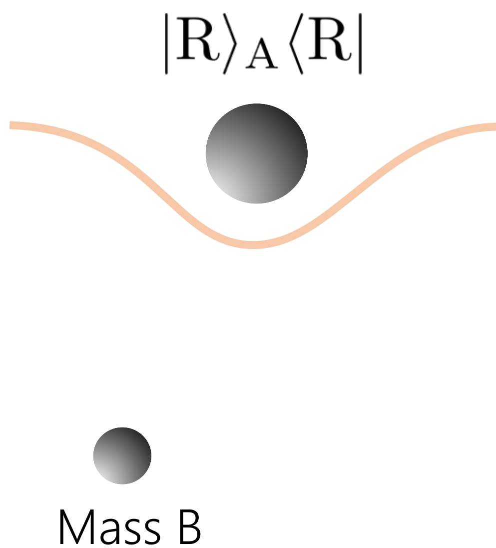
When R0 goes to zero, even if you try to prepare a superposed mass, **the gravitational field of mass is realized around the mass's position,** and then **the mass is decohered.**



Well-known semiclassical models

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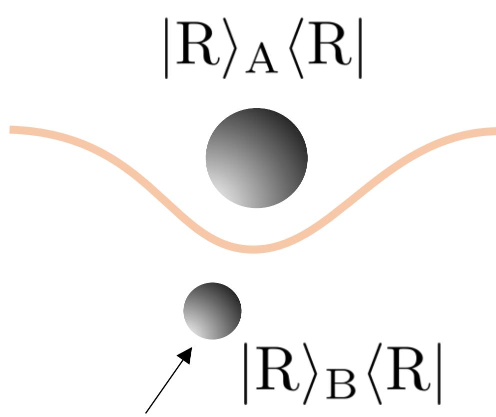

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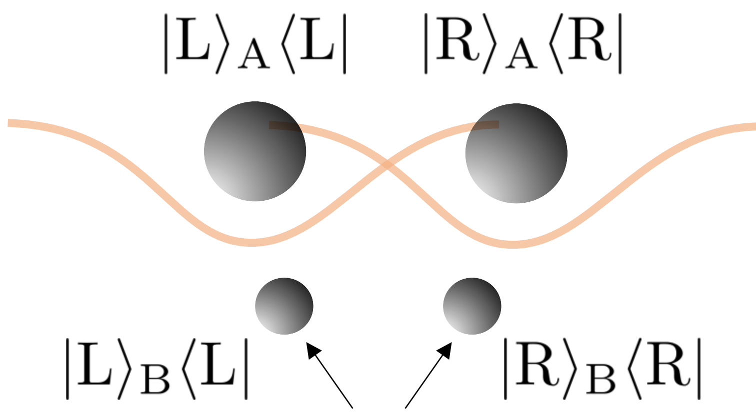


Mass B is not superposed and moves to either of the two locations

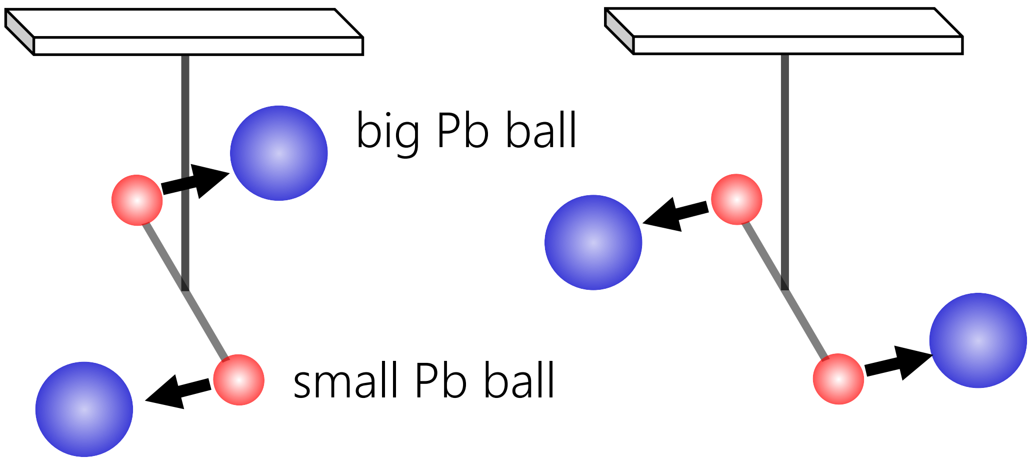
$$|\mathrm{L}\rangle_{\mathrm{A}}\langle\mathrm{L}| \otimes |\mathrm{L}\rangle_{\mathrm{B}}\langle\mathrm{L}| + |\mathrm{R}\rangle_{\mathrm{A}}\langle\mathrm{R}| \otimes |\mathrm{R}\rangle_{\mathrm{B}}\langle\mathrm{R}|$$

They are correlated but not entangled.

Testing them

Test of Schrödinger-Newton (SN) model $\nabla^2\Phi = 4\pi G\langle\hat{\mu}\rangle$ Page+, PRL (1981)

Torsion pendulum



Configuration A

Configuration B

Either of the configurations AB is selected by observing a quantum mechanical phenomenon, e.g. gamma-ray from Co-60

SN model predicts **no correlation between the torsion angle and the choice AB**

In the experiment, the correlation is observed, and **the standard version of SN model is negative.**

Various versions of SN models Miki+, PRD (2025)

Experiments and proposals for testing it

Yan+, PRD (2025) Liu+, PRD (2026) Kent, PRD (2021) Yang+, PRL (2013) etc.

Testing them

Test of Diósi-Penrose model

Donadi+, Nat. Phys. (2021)

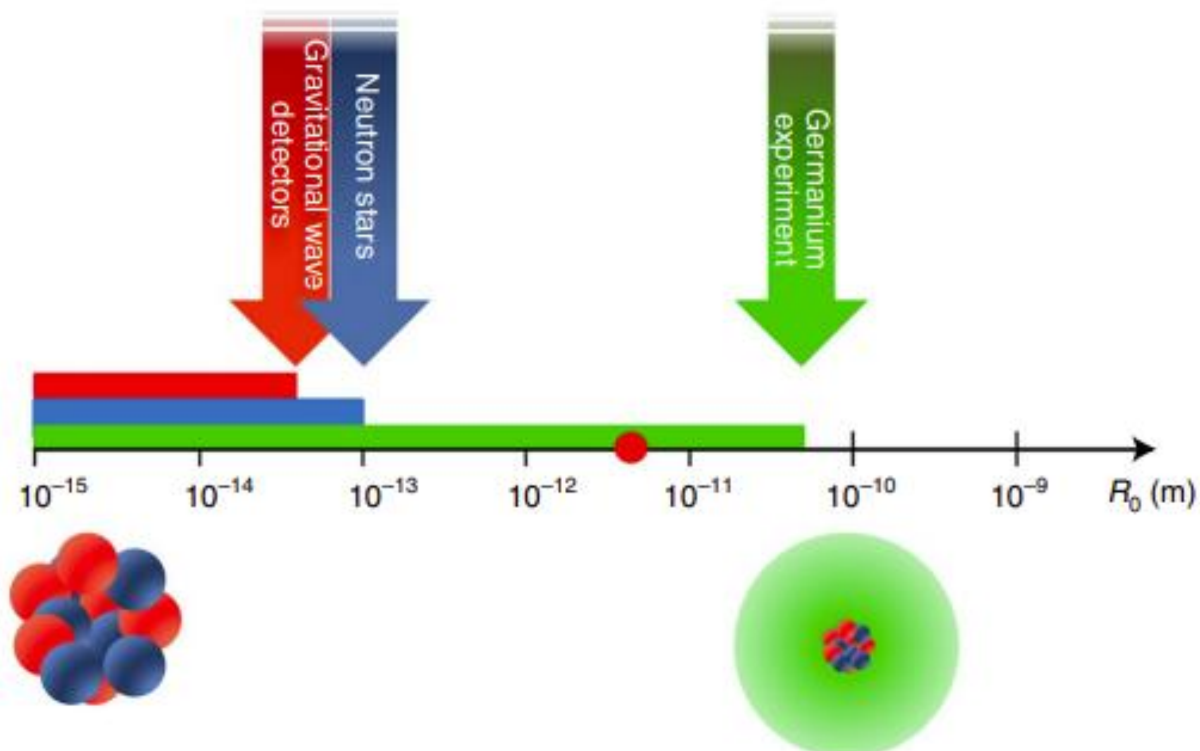
$$\frac{d}{dt}\hat{\rho}_{\mathrm{M}} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{\mathrm{M}}] - \frac{G}{2\hbar}\int \frac{d^3x d^3y}{|\boldsymbol{x}-\boldsymbol{y}|}[\hat{\mu}(\boldsymbol{x}), [\hat{\mu}(\boldsymbol{y}), \hat{\rho}_{\mathrm{M}}]] \quad \hat{\mu}(\boldsymbol{x}) = \frac{m}{(2\pi)^{3/2}R_0^3}e^{-\frac{(x-\hat{x})^2}{2R_0^2}}$$

The decoherence term leads to the Brownian motion of particle

If a charged particle follows the Brownian motion, the radiation of light is emitted.

No observations of the radiation give the constraint of R0 (Green)

We find that R0 should be larger than 10^(-11) m, which is much larger than a nuclear size



Classical-Quantum gravity

Constraining and ruling out semiclassical models, such as SN and DP, are a route to clarify the gravity of quantum masses.

A novel relativistic semiclassical model was proposed : **classical-quantum gravity**

Oppenheim, PRX (2023) Oppenheim+, arXiv2302.07283 Oppenheim+, Nat. Comm. (2023)

This has been attracting attentions. However, it is not clear what prediction is obtained.

As illustrative observables, I have focused on

- Scattering process Carney, **AM** CQG (2025)

  O(1) anomalous feature compared with QFT prediction

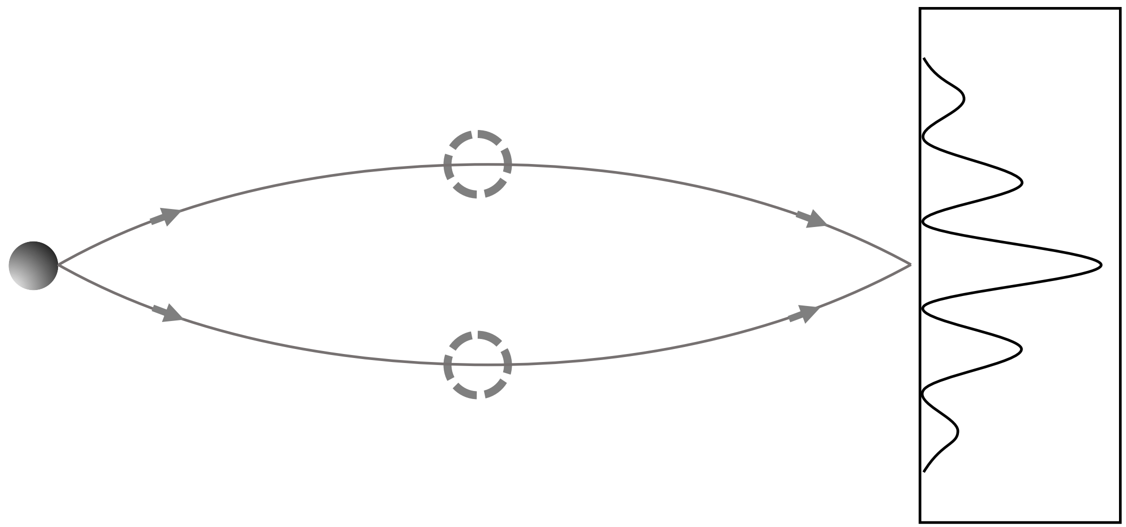
- **Geodesic deviation** with Hirotani on going

  Towards the constraints from, for example, the LIGO experiment

A schematical view

In the classical-quantum framework, classical system follows a **stochastic** evolution.

To understand why we adopt stochasticity, let us consider an interference exp.



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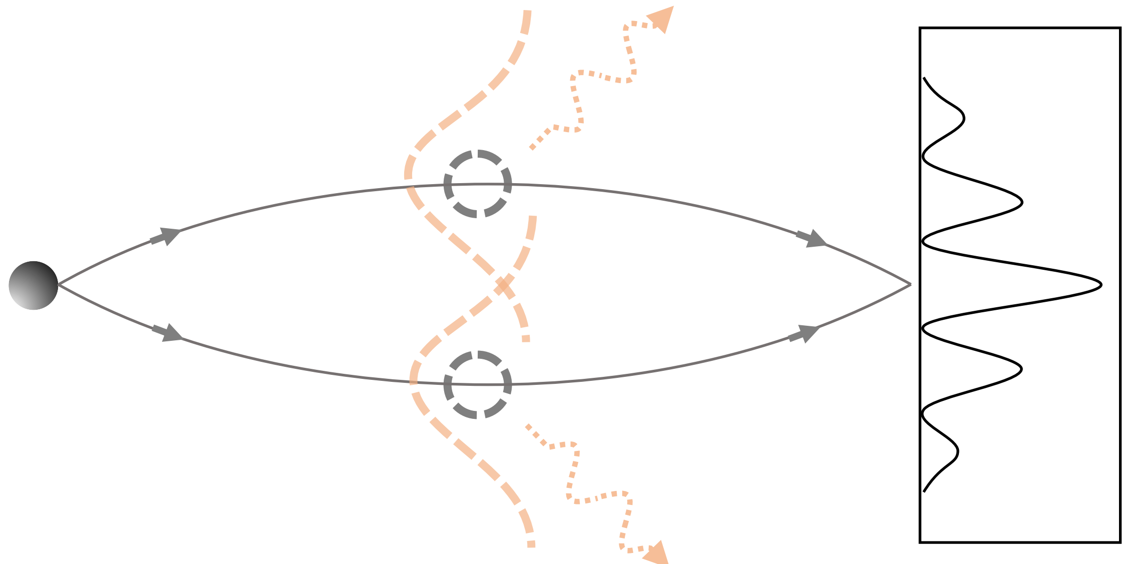

If gravitational field is quantum, the gravitational field is in a quantum superposition.

The particle can be entangled with the gravitational field (graviton), which leads to decoherence but it is very small if the particle has a small mass and velocity.

$$\text{Visibility} \sim \exp\left[-\frac{m^2 v^4}{m_{\rm pl}^2}\right] \sim \exp\left[-5\times 10^{-59}\left(\frac{m}{7\times 10^3\times 10^{-27}\,\mathrm{kg}}\right)^2\left(\frac{v/c}{10^{-7}}\right)^4\right]$$

(vacuum state)

S. Gerlich+. Nat Commun 2, 263 (2011).

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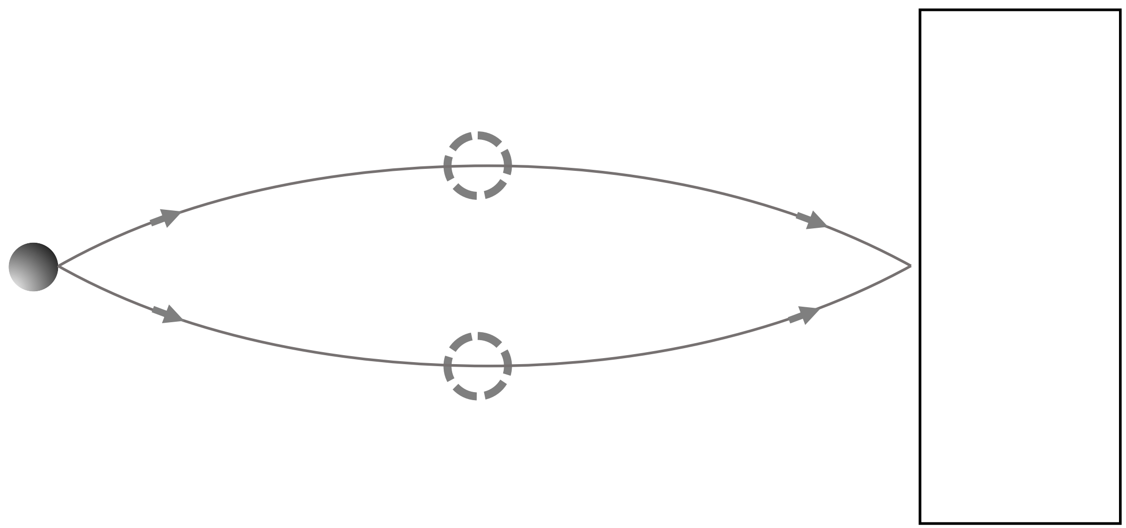

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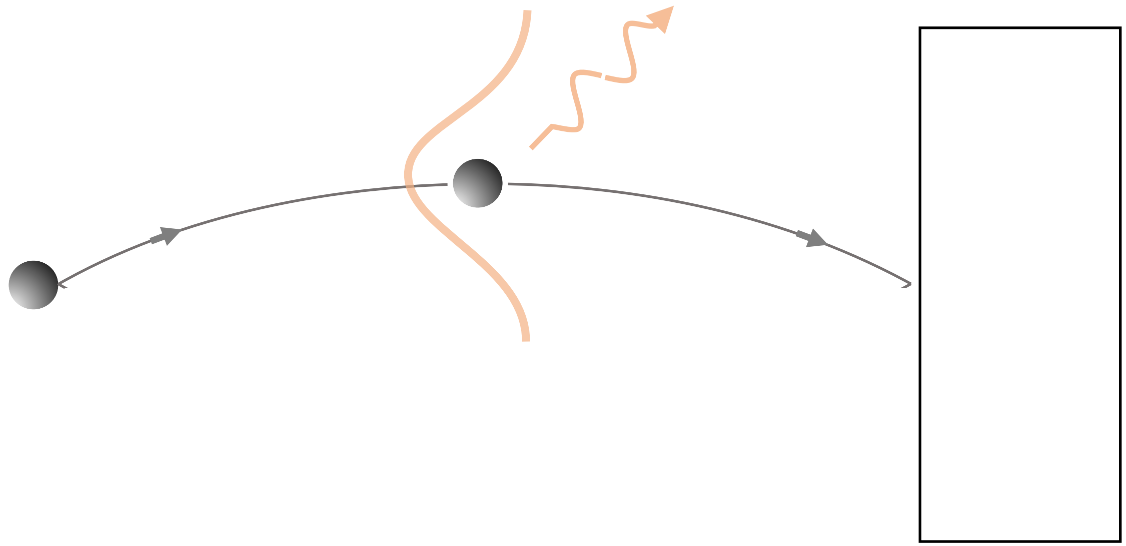

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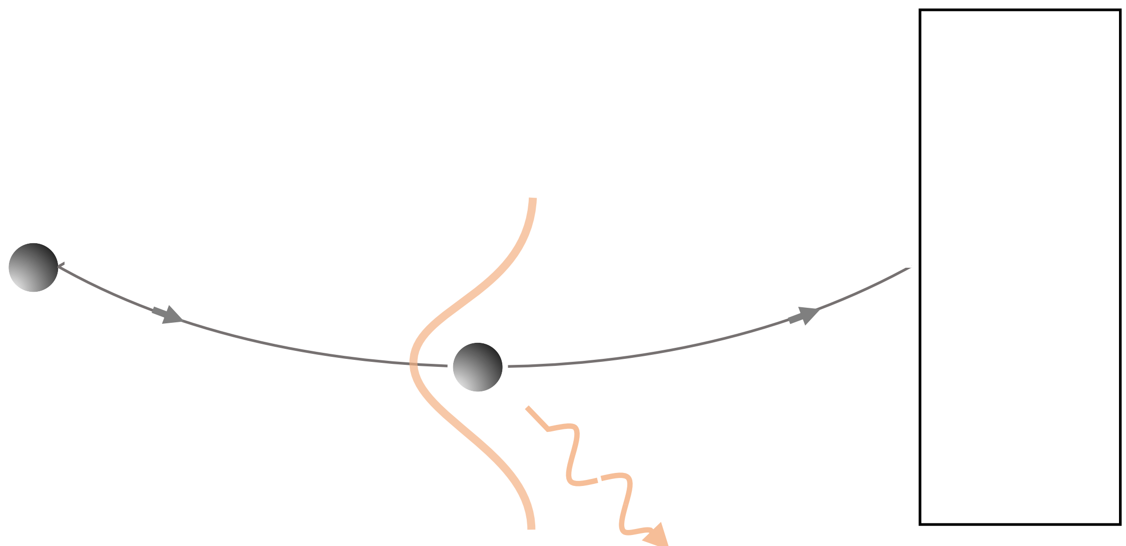

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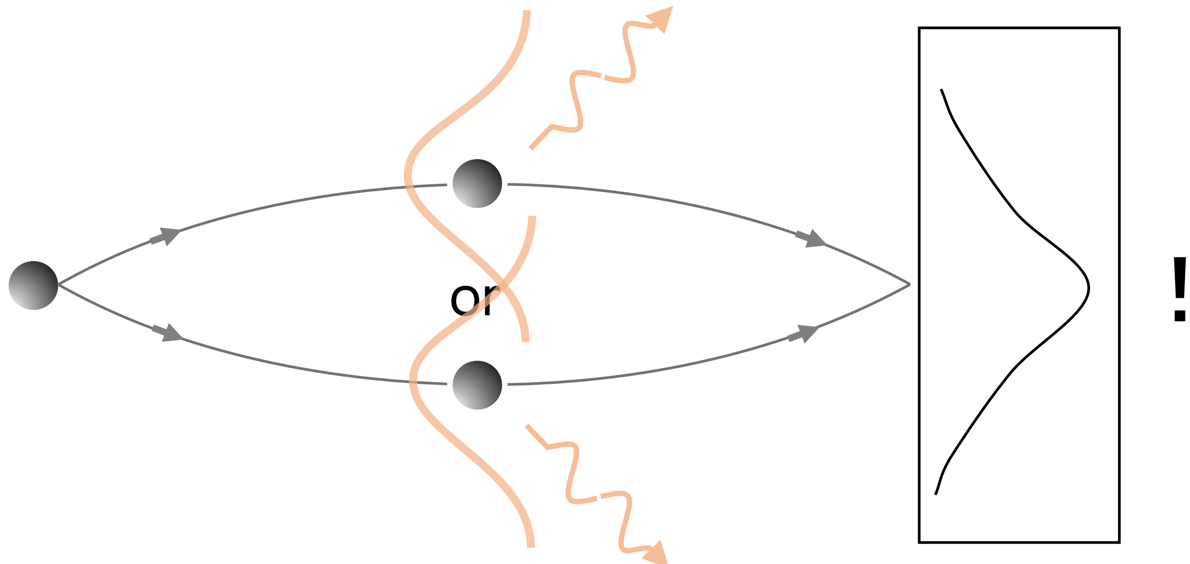

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➡ The gravitational field leads to decoherence

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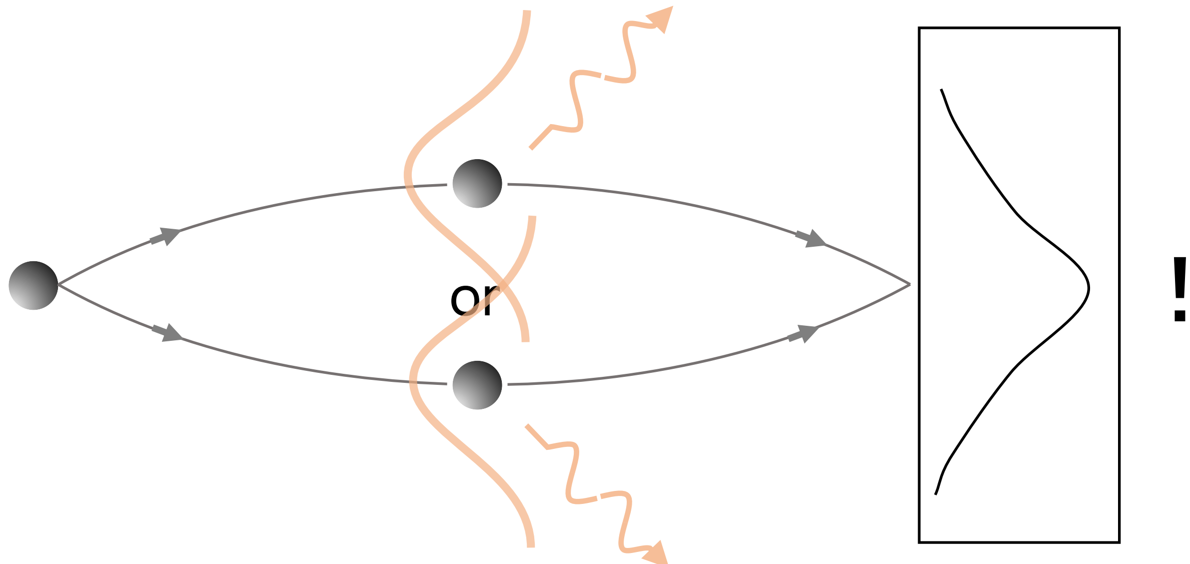

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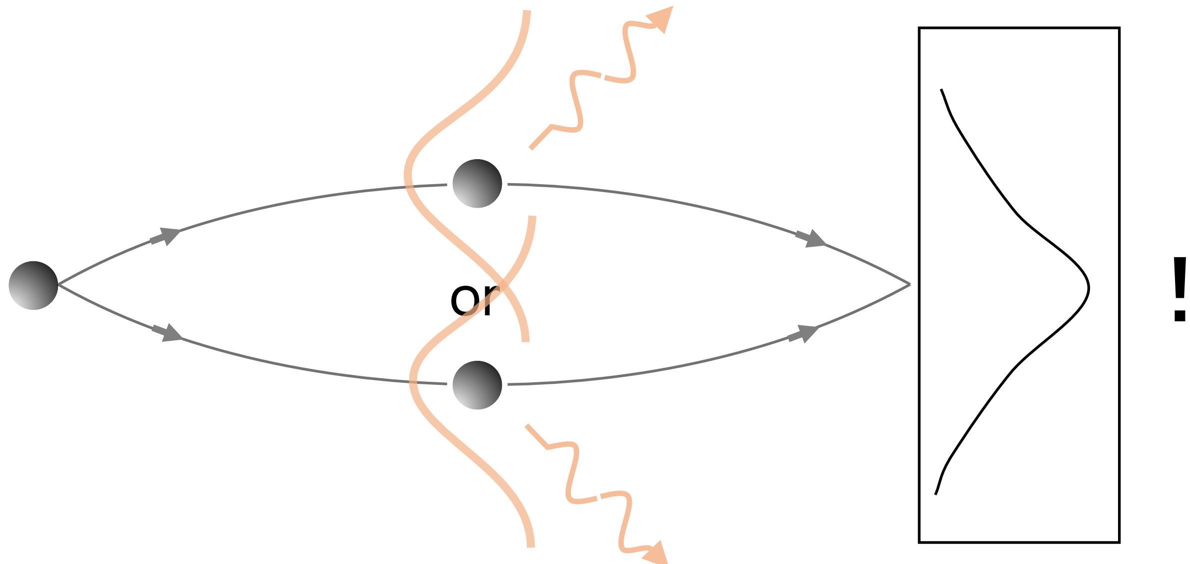

We consider one of the stories that grav. field is classical: it realizes at either of particle positions every time we throw the particle.

One can get **which-path info of particle by measuring the grav. field with high precision and without disturbing the field,** and then **one never gets interference patterns.**

The measurement of gravity is not essential since the state of classical grav. field is determined independently of measurement.

The stochasticity

The thought experiment suggests the grav. field is quantum and can be disturbed by measuring.

Related to Eppley and Hannah, Found. Phys. (1975)

But if **the classical grav. field has a stochastic noise**, the interference fringes may be observed.

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This limits the which-path info of particle obtained from the measurement of grav. field because the stochasticity reduces measurement precision.

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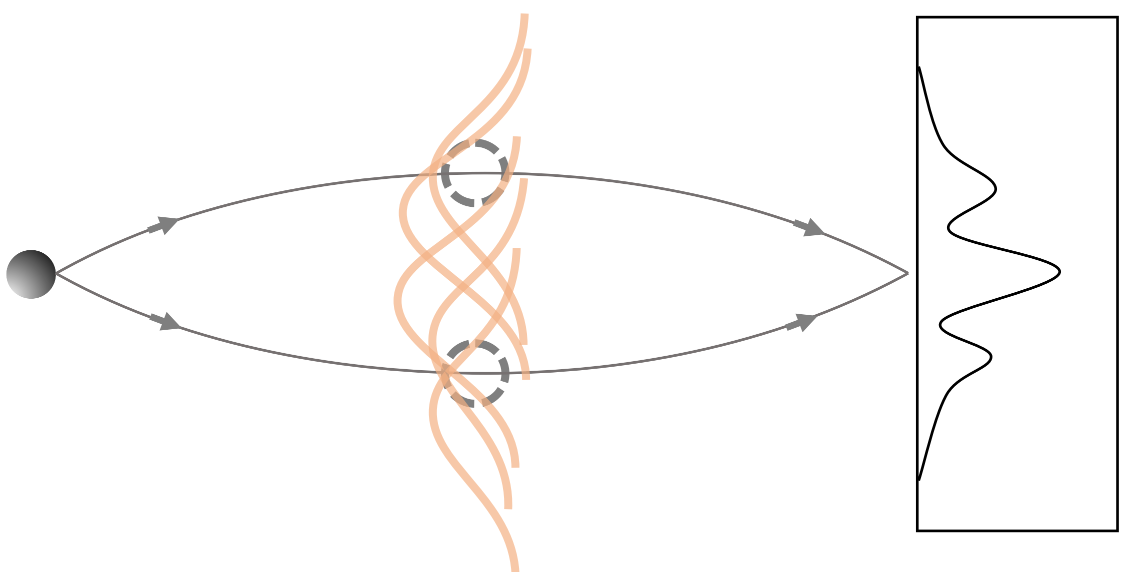
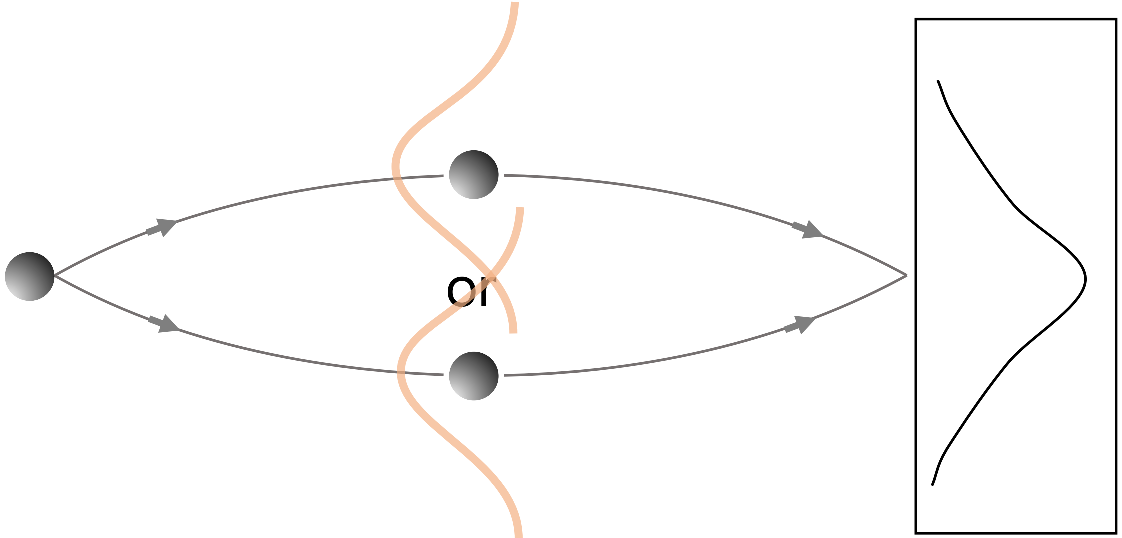
Schematically,

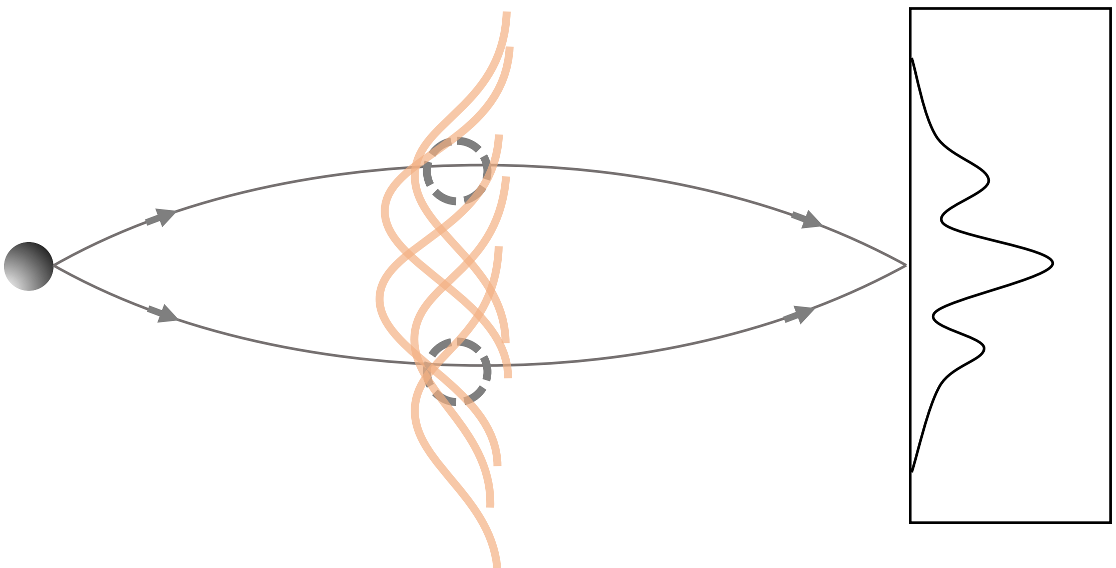


We cannot get which-path info and the interference pattern consistently appears.

The trade-off relation



Small noise and large decoherence



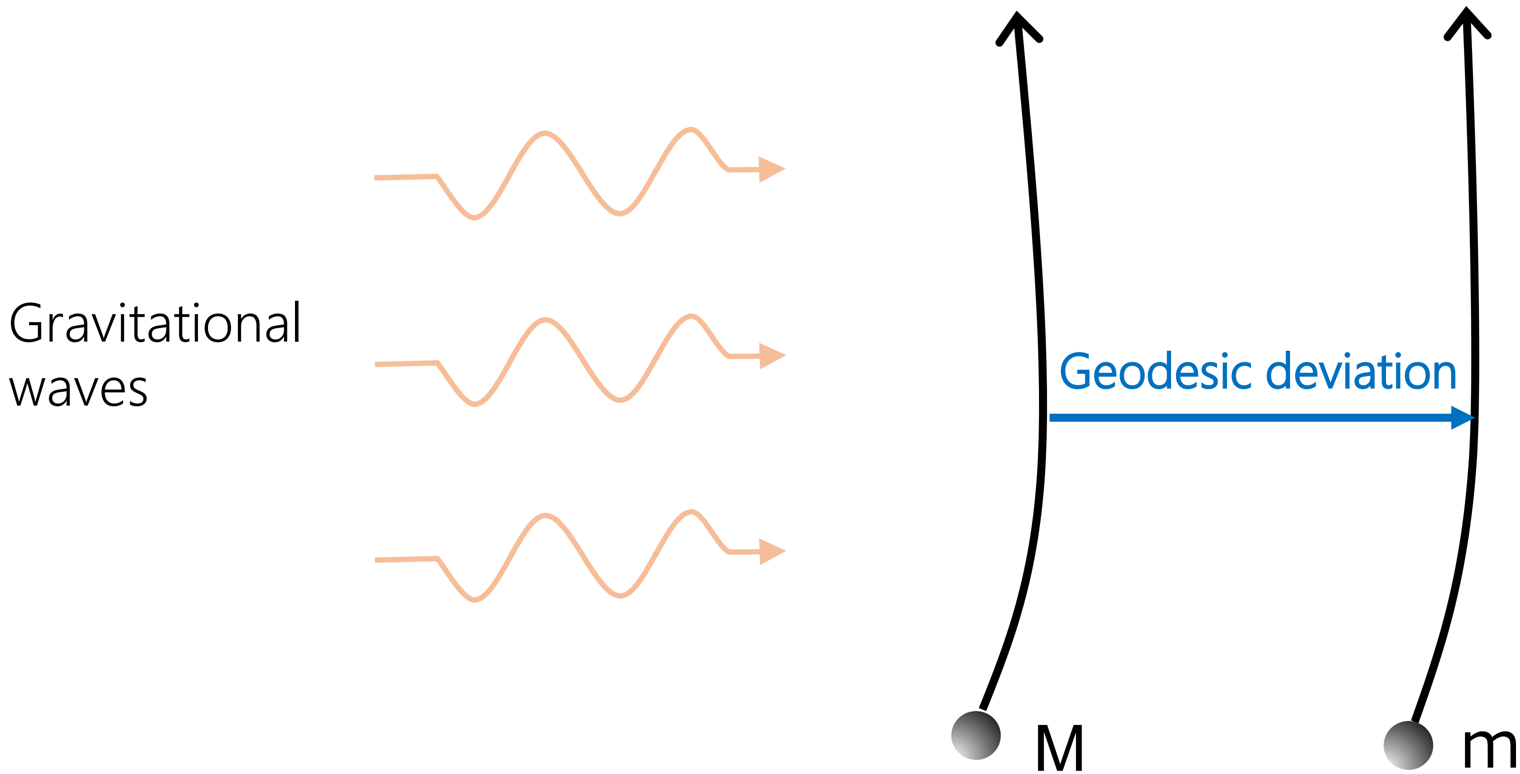
Large noise and small decoherence

Trade-off relation

But, note that too large noise leads to decoherence again

Geodesic deviation

The geodesic deviation by gravity is an illustrative observable in general relativity



What happens in the classical-quantum gravity?

Can we get some constraints on the model from experiments?

The action of geodesic deviation

$$S = -M\int d\tau - m\int d\tau\sqrt{-g_{\mu\nu}\dot{y}^{\mu}\dot{y}^{\nu}} + S_{\mathrm{EH}}$$

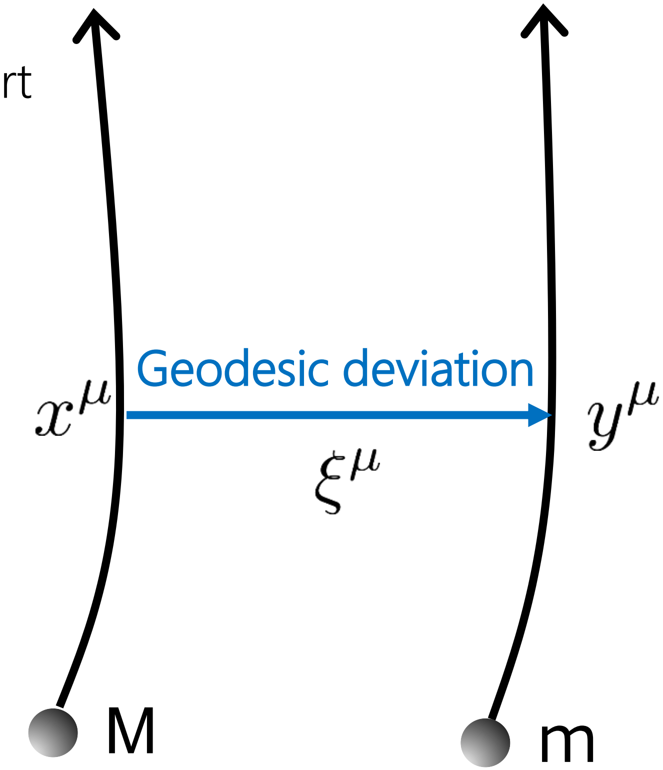
Einstein-Hilbert

x^{μ} follows geodesic equations

$$y^{\mu} = x^{\mu} + \xi^{\mu}$$

$$u^{\mu} = \frac{dx^{\mu}}{d\tau}$$

in the Fermi normal coordinate



$$S \simeq \int d\tau\left[\delta_{ab}\frac{d\xi^{a}}{d\tau}\frac{d\xi^{b}}{d\tau} - R_{\mu a\nu b}u^{\mu}u^{\nu}\xi^{a}\xi^{b}\right] + S_{\mathrm{EH}}$$

$a, b = 1, 2, 3$

$R_{\mu\nu\rho\sigma}$ Riemann curvature

Maulik+, Int. J. Mod. Phys. (2020)

Kanno+, PRD(2020)

The classical-quantum formalism for perturbations

The perturbation of the geodesic deviation around the averaged separation L

$$\xi^a = L^a + q^a \quad |q^a| \ll |L^a|$$

The perturbation of gravitational field around the flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$

Up to the second order,

O(h) Riemann curvature

$$S_{\text{eff}}[q,h] = \int dt[(\dot{q}^a)^2 - 2\overset{(1)}{R}_{0a0b}L^a q^b] + \underline{S_{\text{EH}}[h]}$$

$$(\dot{q}^a)^2 = \delta_{ab}\frac{dq^a}{dt}\frac{dq^b}{dt}$$

O(h^2) Einstein-Hibert action

Classical-quantum state and its evolution

Oppenheim+, arXiv2302.07283

$$\rho_{\rm cq}[q^a, \underline{q}^a, h_{\mu\nu}, \dot{h}_{\mu\nu}, t_{\rm f}]$$

← The density matrix of q and the probability distribution of h

$$= \int DqD\underline{q}Dh\,\delta\left[\partial_\mu h^\mu_\nu - \frac{1}{2}\partial_\nu h^\mu_\mu\right] e^{I_{\rm cq}[q,\underline{q},h]}\rho_{\rm cq}[q^a_{\rm i}, \underline{q}^a_{\rm i}, h^{\rm i}_{\mu\nu}, \dot{h}^{\rm i}_{\mu\nu}, t_{\rm i}]$$

Gauge condition

Classical-Quantum action

Classical-Quantum action

$$I_{\rm cq} = iS_\Delta - \int d^4x d^4y N^{-1}_{\mu\nu\rho\sigma}(x,y)\frac{\delta S_{\rm c}}{\delta h_{\mu\nu}(x)}\frac{\delta S_{\rm c}}{\delta h_{\mu\nu}(y)}$$
$$- \int d^4x d^4y D_{\mu\nu\rho\sigma}(x,y)\frac{\delta S_\Delta}{\delta h_{\mu\nu}(x)}\frac{\delta S_\Delta}{\delta h_{\mu\nu}(y)}$$

$$S_\Delta = S_{\rm eff}[q,h] - S_{\rm eff}[\underline{q},h]$$
$$S_{\rm c} = S_{\rm eff}[q,h] + S_{\rm eff}[\underline{q},h]$$

$N_{\mu\nu\rho\sigma}(x,y)$: Noise kernel for the classical gravitational field h

$D_{\mu\nu\rho\sigma}(x,y)$: Decoherence kernel for the quantized geodesic deviation q

These kernels satisfy the trade-off relation $ND \succeq 1 \quad N \succeq 0 \quad D \succeq 0$

How to make the classical-quantum action

Explicitly

O(h) Einstein tensor

$$I_{\rm cq} = i\int dt\Big[(\dot{q}^a)^2 - 2\overset{(1)}{R}_{0a0b}L^a q^b - (\dot{\underline{q}}^a)^2 + 2\overset{(1)}{R}_{0a0b}L^a \underline{q}^b\Big]$$

$$-\int d^4x\, d^4y\, N^{-1}_{\mu\nu\rho\sigma}(x,y)\Big\{\frac{\overset{(1)}{G}{}^{\mu\nu}(x)}{8\pi G_N} - \frac{1}{2}\big[T^{\mu\nu}(x) + \underline{T}^{\mu\nu}(x)\big]\Big\}\Big\{\frac{\overset{(1)}{G}{}^{\rho\sigma}(y)}{8\pi G_N} - \frac{1}{2}\big[T^{\rho\sigma}(y) + \underline{T}^{\rho\sigma}(y)\big]\Big\}$$

$$-\frac{1}{4}\int d^4x d^4y D_{\mu\nu\rho\sigma}(x,y)[T^{\mu\nu}(x) - \underline{T}^{\mu\nu}(x)][T^{\rho\sigma}(y) - \underline{T}^{\rho\sigma}(y)]$$

$$T^{\mu\nu} = -2\frac{\delta S_{\rm eff}}{\delta h_{\mu\nu}}$$

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$$T^{\mu\nu} = -2\frac{\delta S_{\rm eff}}{\delta h_{\mu\nu}}$$

- Gravitational field h is stochastic

$$\overset{(1)}{G}{}^{\mu\nu} = 8\pi G_{\rm N}\Big[\langle T^{\mu\nu}\rangle_{\rm q} + \xi^{\mu\nu}\Big] \qquad \langle \xi_{\mu\nu}(x)\rangle = 0 \qquad \langle \xi_{\mu\nu}(x)\xi_{\rho\sigma}(y)\rangle = N_{\mu\nu\rho\sigma}(x,y)$$

- Decoherence in the energy-momentum basis

Effective Langevin

After integrating out the gravitational field, we meet the following equations

Langevin eqs. of the geodesic deviation

$$m\frac{d^2q^a}{dt^2} = F^a(t)$$

$$\langle F_a(t) \rangle = 0$$

$$\langle F_a(t)F_b(t') \rangle = [\Delta^N_{cadb}(t,t') + \Delta^D_{cadb}(t,t')]L^c L^d$$

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$$\Delta^N_{abcd}(t,t') = 16(8\pi G_N m)^2 \int d^4z \int d^4w E^{x,\mu\nu}_{0a0b} G_R(x-z) E^{y,\rho\sigma}_{0c0d} G_R(y-w)|_{x=X(t),y=X(t')}$$
$$\times (\delta^\alpha_\mu \delta^\beta_\nu - \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta})(\delta^\lambda_\rho \delta^\kappa_\sigma - \frac{1}{2}\eta_{\rho\sigma}\eta^{\lambda\kappa}) N_{\alpha\beta\lambda\kappa}(z,w)$$

The retarded Green function

$$\Delta^D_{abcd}(t,t') = 2m^2 E^{x,\mu\nu}_{0a0b} E^{y,\rho\sigma}_{0c0d} D_{\mu\nu\rho\sigma}(x,y)|_{x=X(t),y=X(t')} \qquad X^\mu(t) = [t,\mathbf{0}]^{\mathrm{T}}$$

$$E^{\mu\nu}_{\rho\alpha\sigma\beta} = \frac{1}{2}[\partial_\alpha\partial_\sigma\delta^{(\mu}_\rho\delta^{\nu)}_\beta - \partial_\alpha\partial_\beta\delta^{(\mu}_\rho\delta^{\nu)}_\sigma - \partial_\rho\partial_\sigma\delta^{(\mu}_\alpha\delta^{\nu)}_\beta + \partial_\beta\partial_\rho\delta^{(\mu}_\alpha\delta^{\nu)}_\sigma]$$

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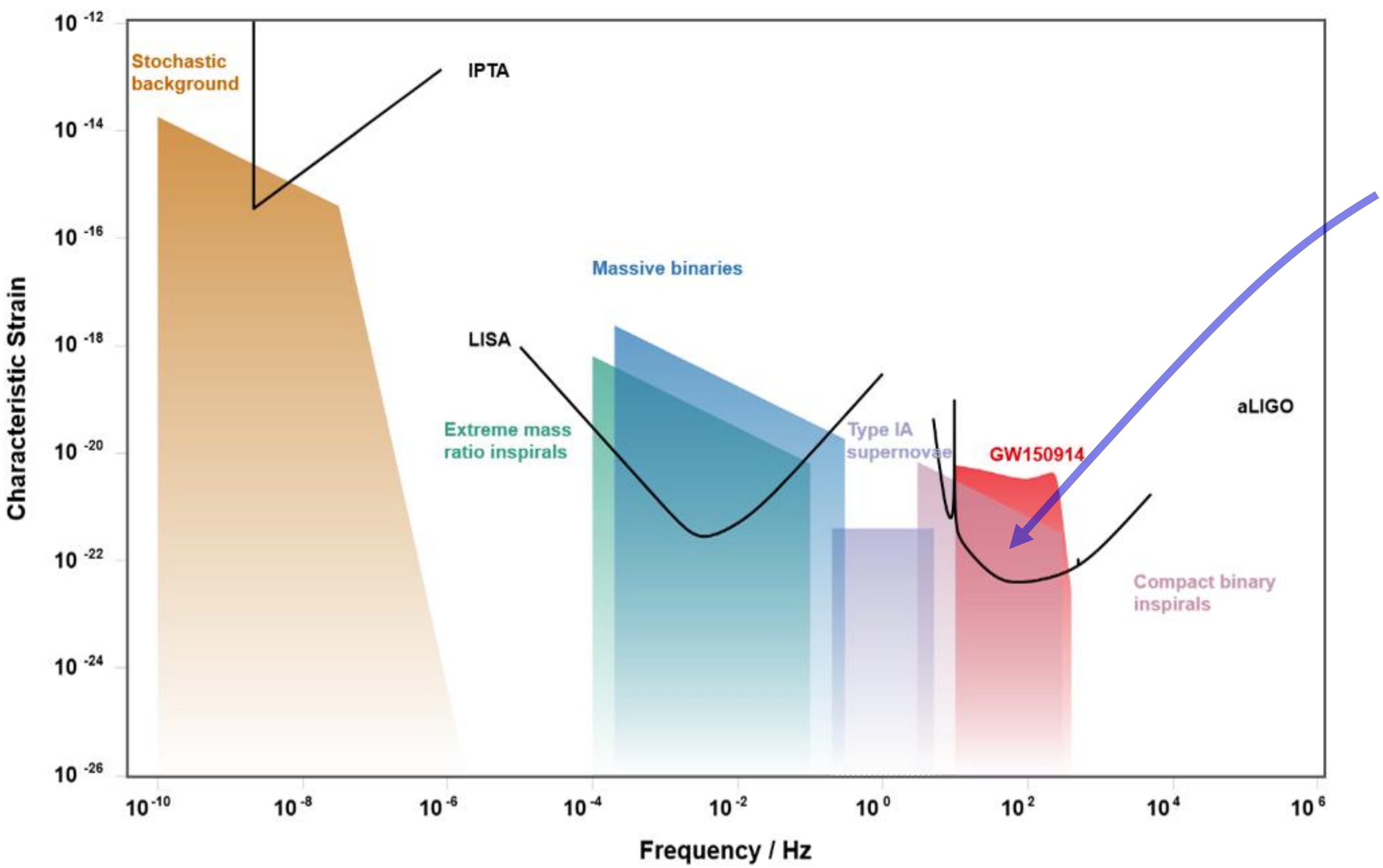
$$X^\mu(t) = [t, \mathbf{0}]^{\mathrm{T}}$$

- The stochastic noise of gravitational field propagates and perturbs the deviation
- The decoherence in the energy-momentum basis leads to the fluctuation of the deviation due to the uncertainty relation

Strain

Two-time correlation $\langle q(\tau)q(0)\rangle = \int d\omega S_{qq}(\omega)e^{i\omega\tau}$ $\quad q = q^a L_a / L$

Strain spectrum $\sqrt{S_{hh}}$ $\quad S_{hh} = S_{qq}/L^2 \quad q = Lh$



LIGO sensitivity is roughly 10^(-23)/$\sqrt{\mathrm{Hz}}$ at 100 Hz.

The strain spectrum predicted from CQ gravity should be smaller than the value.

A demonstration

$$N_{\mu\nu\rho\sigma}(x,y) = \frac{1}{D}\left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} + \frac{2\beta}{1-4\beta}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\delta^4(x-y)$$

J. Oppenheim+ arXiv:2302.07283
A. Grudka+ arXiv:2402.17844

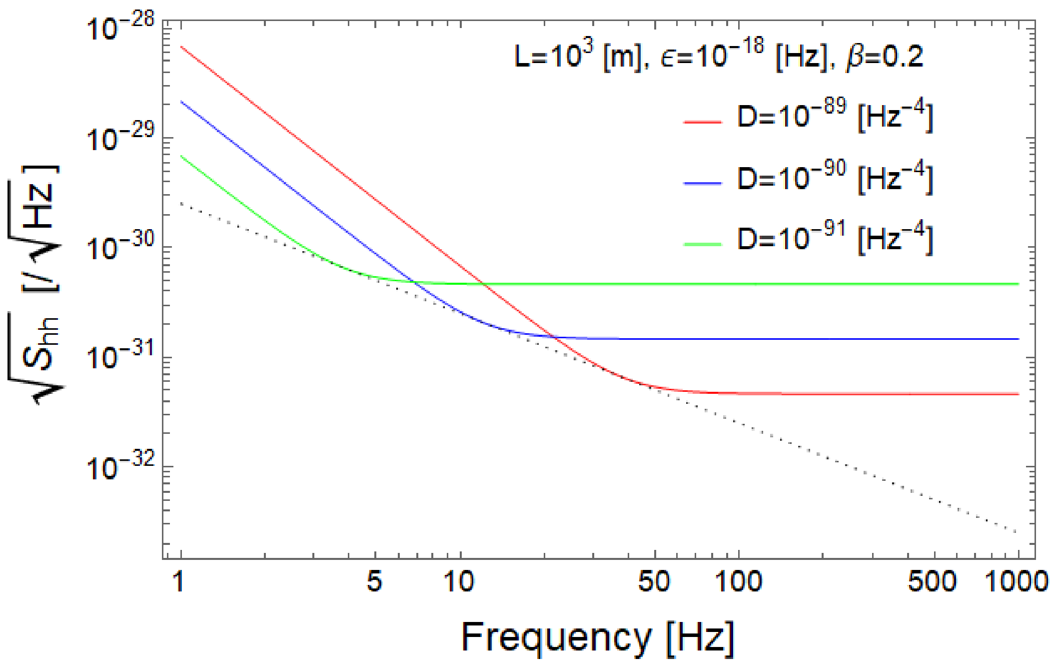
$$D_{\mu\nu\rho\sigma}(x,y) = D(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - 2\beta\eta_{\mu\nu}\eta_{\rho\sigma})\delta^4(x-y)$$

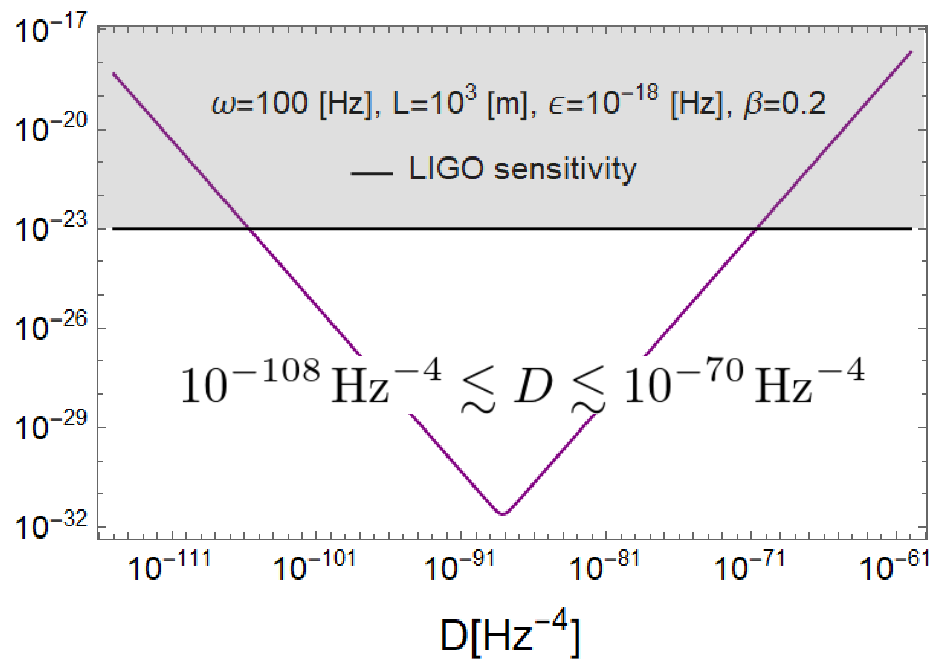
See also T. Hirotani's poster

$$S_{hh} \sim \frac{512\pi(1-3\beta)G_N^2}{15(1-4\beta)D\epsilon} + \frac{2D(1-\beta)}{35\pi^2 L^7\omega^4}$$

L : UV cutoff ~ the size of the initial separation

ϵ : IR cutoff ~ the size of the present universe





Further discussion and problems

On the consistency with Einstein equations

Several semiclassical models including classical-quantum gravity require the stochastic noise of gravitational field.

Is it consistent with the diffeomorphism invariance or the Bianchi identity in general relativity?

c.f. Hu+, Living Rev. Relativity (2008)

On gravity-induced entanglement

Entanglement has been considered as a benchmark for ruling out semiclassical models, however, it was reported that, even if gravity is not fundamentally quantum, it can produce entanglement.

Trillo+, PRD (2025) Carney+, PRX (2025) Aziz+, Nature (2025)

Does the classical-quantum gravity predict the entanglement? What is the meaning of the classicality? Is there a better signature than entanglement?

Summary

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- Verifying the quantum superposition of gravitational field is a milestone to clarify the crossover regime of quantum and gravitational physics.

- Testing gravity-induced entanglement is so hard and a long-term task, and it supports perturbative quantum gravity approach.

- Testing semiclassical models is relatively easy and a short-term task, and it gives an interplay research of quantum and gravitational physics that is linked to near-future experiments.

- There remains room for discussions on the implications of the gravity-induced entanglement and the validity of semiclassical models.

Thank you for your attention.