A Quantum Mechanical Model of Black Hole

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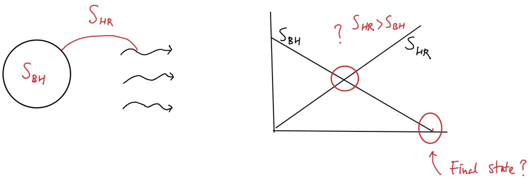
Fundamental problems of quantum black hole

Black hole provides an ideal (only!?) window to quantum gravity!

- Bekenstein-Hawking entropy:

$$\text{Microscopic origin of} \quad S_{\mathrm{B-H}} = \frac{A}{4G\hbar}?$$

- Information problem:

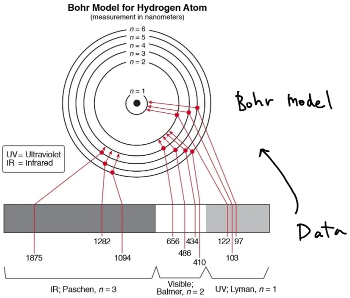


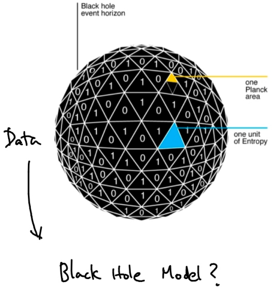
- BH chaos: most efficient quantum information scrambler (Susskind, Maldacena-Shenker-Stanford, Shenker etal)

$$\tau_s \sim T^{-1} \log S$$

Resolution of these problems require understanding of the quantum physics of black hole.

- my understanding: Horizon and its infinite redshift is the origin to ALL of the black hole problems.
- We want to construct a quantum mechanical model of horizon and learn about quantum black hole.
  This is like Bohr model (data: spectral line. learnt: nucleus, eigenstates, ...)





Outline

Quantum mechanical model of black hole

- We consider the large N quantum mechanics of 3d quantum space

$$L = \text{tr}\left[\frac{1}{2a_0^2 M_P}\dot{X}^{a2} + \frac{M_P}{N^2}([X^a, X^b]^2 + 4X^{a2}) + i\dot{\psi}^\dagger\psi - a_2\frac{M_P}{N^2}\psi^\dagger\sigma^a X^a\psi\right]$$

where $a = 1, 2, 3$, $X^a_{mn}, \psi_{mn}, \psi^\dagger_{mn}$ $N \times N$ traceless matrices.

- Remarks:
  1. Like BFSS but no SUSY.
  2. tachyonic mass term instead of Chern-Simons mass term: important for capturing the instability associated with the infinite red shift of horizon.
  3. a_0, a_2 coefficients to be fixed.

Quantum Schwarzschild as Fuzzy Sphere

- The EOM admits static solution given by $K \times K$ matrices obeying

$$[X^a, X^b] = i\epsilon^{abc} X^c, \quad \sum_a X^{a2} = \frac{K^2 - 1}{4}\mathbf{1},$$

- Introduce dimensional coordinates $Y^a = 2l_P X^a$, the fuzzy sphere solution becomes

$$[Y^a, Y^b] = \frac{2iR}{\sqrt{K^2 - 1}}\epsilon_{abc} Y^c, \quad \sum_a Y^{a2} = R^2\mathbf{1},$$

where $R = Kl_P$ is the radius of the fuzzy sphere for K large.

Fermionic states

- Quantizing the fermions over the fuzzy sphere background, we obtain $2N^2$ fermionic oscillators

$$H_F = \frac{a_2 M_P}{2N} \sum_{p,k=1}^{N} (\xi_k^{p\dagger} \xi_k^p + \chi_k^{p\dagger} \chi_k^p).$$

- For a half filled Fermi sea, we reproduce precisely the Schwarzschild radius

$$R = \frac{M}{2G}$$

for static black hole.

Microstate counting

- Let us consider the microstates counting. For the half-filled Fermi sea, there is a degeneracy of

$$\Omega_0 = \binom{2N^2}{N^2} = 2^{2N^2}$$

in the leading large N limit. These microstates give rises to the entropy $S = \log_2 \Omega_0$:

$$S = 2N^2 = \frac{A}{4G},$$

which is precisely the Bekenstein-Hawking entropy.

Kerr black hole

- A similar analysis can be performed for rotating black hole

$$ds^2 = -\left(1 - \frac{2M}{\rho^2}\right) dt^2 - \frac{4Mar\sin^2\theta}{\rho^2} dt d\phi + \frac{\Sigma}{\rho^2}\sin^2\theta d\phi^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2,$$

$$\rho^2 := r^2 + a^2\cos^2\theta, \Delta := r^2 - 2Mr + a^2, \Sigma := (r^2+a^2)^2 - a^2\Delta\sin^2\theta.$$

- The Kerr BH has a horizon at $r_+ = M + \sqrt{M^2 - a^2}$. Viewed in the Cartesian coordinates, the horizon is an elliptical surface

$$\frac{x^2+y^2}{r_+^2+a^2} + \frac{z^2}{r_+^2} = 1.$$

- Bekenstein-Hawking entropy, mass and angular momentum

$$S = \frac{A}{4G} = \frac{\pi(r_+^2+a^2)}{G}, \quad M = \frac{r_+^2+a^2}{2Gr_+}, \quad J = a\frac{r_+^2+a^2}{2Gr_+}.$$

- We showed that the rotating fuzzy sphere solution of the QM reproduces precisely all of these properties of Kerr black hole.
- In particular, matching of the angular momentum requires

$$a_0 = \frac{\pi}{3}.$$

This will be important later.

Outline

- Semi-classical QFT computation of Hawking radiation gives the decay rate of BH

$$\Gamma = -\frac{1}{M}\frac{dM}{dt} = -\frac{Q}{G^2M^3}.$$

- It gives also the Hawking temperature

$$T_H = \frac{1}{8\pi GM}.$$

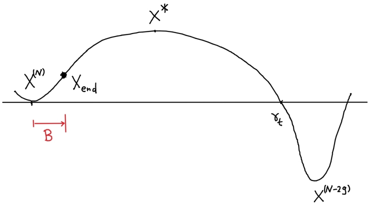
Our model of BH provides a natural explanation of this process in terms of QM tunneling of fuzzy spheres. Both results are reproduced.

Monopole channel and decay rate

- We have a collection of fuzzy sphere vacua

$$E = \frac{bNM_P}{2}$$

of different sizes (rank N). Decay can occur by tunneling.



- However since there is an excess of Fermi states in the Fermi sea of $X^{(N)}$, it cannot decay to the small fuzzy sphere with a smaller Fermi sea unless the decay path has the right number of fermi zero modes to soak up the excess fermi states.

- The criteria is so strong that the tunneling path is uniquely determined to be
$$X_a = X_a^{(N)} - \gamma(X_a^{(N)} - X_a^{(N')}), \quad 0 \leq \gamma \leq 1.$$
- The difference of two fuzzy spheres
$$A_a^{(n)} := X_a^{(N)} - X_a^{(N-n)},$$
is a gauge field which has the property that the Dirac equation on S_N^2 has exactly n zero modes,
- Moreover in the large N limit,
$$A = A_a dx^a = \frac{n}{2}(1 - \cos\theta)d\varphi$$
i.e. precisely the Dirac monopole connection on S^2 with charge $g = n/2$.

- For a quantum mechanics

$$L = \frac{1}{2} m\dot{x}^2 - V(x),$$

the tunneling rate is given by (Coleman)

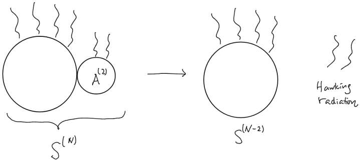
$$\Gamma = K e^{-S_b}, \quad K = \frac{1}{2}\sqrt{\frac{S[x_b] - S[x_0]}{2\pi\hbar m}}\sqrt{\frac{\mathrm{Det}\, S''[x_0]}{\mathrm{Det}'\, S''[x_b]}}.$$

- computing the path integral, we obtain the tunneling rate

$$\Gamma \sim \frac{1}{N^3} \sim -\frac{1}{G^2 M^3}.$$

Up to a numerical constant, this reproduces the semi-classical decay rate of black hole.

Hawking temperature



- Physically as the monopole is nucleated and leave $X^{(N)}$, it leaves behind the smaller fuzzy sphere. At the same time, the released fermi states appear as the Hawking radiation.
- We find the emission probability for a Hawking quanta with energy ω to be

$$P(\omega) = e^{-\omega/T_H},$$

  This is a thermal distribution with the Hawking temperature T_H!
- With a real time formulation of the tunneling process, one can go beyond the probabilistic description by determining the full wave function of the multi-patite Hawking radiation.

Interesting applications:

- other properties of Hawking radiation: Page curve? information?
- final state? PBH? remanants?
- memory burden?
- spacetime uncertainty principle from matrix model?
- black hole singularity?
- chaos?
- metric? forces between black hole ?
- holography?

Thank you for listening!