

Based on 2511.10738v1, with Jonathan Oppenheim

CQ gravitational waves background

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“Concepts of Quantum and Spacetime”, KEK

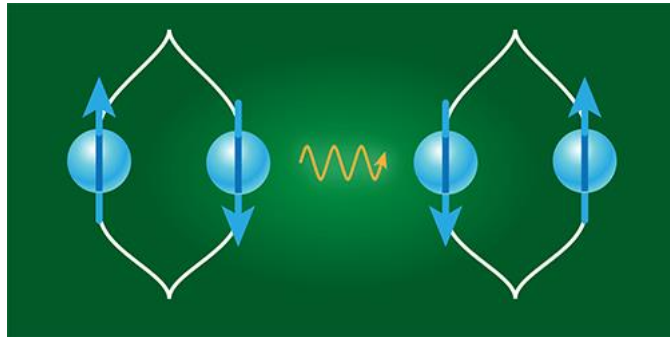
Outline

1. Classical-quantum (CQ) systems
 - ▶ The decoherence-diffusion trade-off
2. CQ gravity: stochastic gravitational waves background

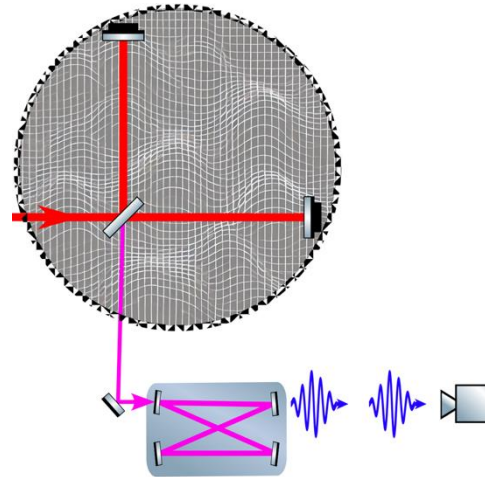
Classical-quantum systems

Can gravity be non-quantum?

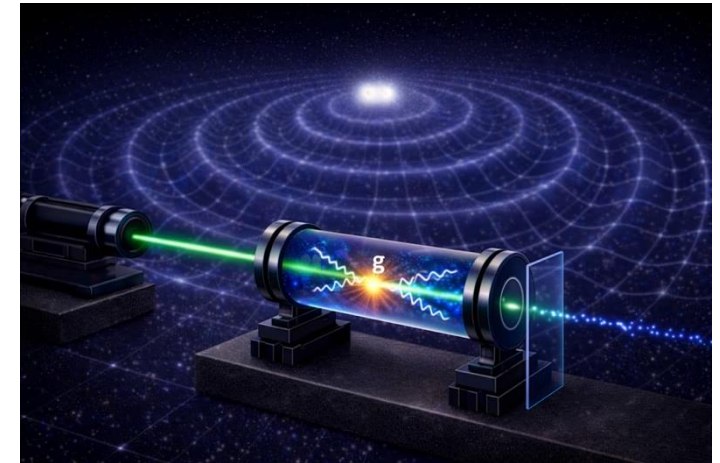
- ▶ Over the last few years, many (technically challenging) experimental proposals to test the quantum nature of spacetime have been put forward:



Entanglement experiments
(eg BMV [Bose '17, Marletto and Vedral '17])



Interferometric measurements
(eg G-Quest @ Caltech)



Single graviton detection
(eg [Tobar et al '24])

- ▶ But what's the point? Doesn't the quantisation of matter imply that spacetime cannot be classical, for consistency?

Classical-quantum dynamics

Not only classical-quantum (CQ) consistent dynamics exists [Diosi '95], but it is unique if one requires [Oppenheim '23]:

- ▶ Linearity in classical-quantum state $\rho(z, t) = E[|\psi\rangle\langle\psi|_t \delta(Z_t - z)]$
- ▶ Complete positivity and trace preservation
- ▶ Continuous trajectories in phase space and markovianity

The most general CQ dynamics can be written in terms of trajectories as [Layton et al '22]

$$\dot{z} = \{z, H_C(z)\} + \underbrace{\langle\{z, H_I(z)\}\rangle}_{\text{backreaction}} + \underbrace{\sigma(z)\xi}_{\text{noise}}$$

$$|\dot{\psi}\rangle = \left[-i \left(H_Q + \underbrace{H_I(z)}_{\text{feedback}} \right) + \frac{1}{2\sigma} \left(\{z, H_I(z)\} - \langle\{z, H_I(z)\}\rangle \right) \xi - \frac{1}{8\sigma^2} \left(\{z, H_I(z)\} - \langle\{z, H_I(z)\}\rangle \right)^2 \right] |\psi\rangle$$

Testing quantum gravity *ab absurdo*

- ▶ Measuring directly quantum gravity effects is hard
- ▶ CQ models offer an alternative: assuming classicality and looking for an experimental falsification
- ▶ If gravity is fundamentally classical (+ linearity + positivity), the decoherence-diffusion trade-off needs to be satisfied
 - ▶ The gravitational field has a stochastic component (controlled by D_2)
 - ▶ Macroscopic superpositions decohere (controlled by $1/D_2$)

These effects cannot be both made small.

- ▶ Measuring violations of the trade-off by:
 - ▶ Pushing coherence times of spatial superpositions in the lab
 - ▶ Bounding the size of the stochasticity in the gravitational field



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Stochastic gravitational waves

Stochastic gravity

- ▶ Solving for the combined classical-quantum system is extremely complicated:
 - ▶ Imposing covariance in a stochastic theory is an open problem (choice of foliation needs to be irrelevant)
 - ▶ Numerics are often unavoidable for non-linear partial stochastic differential equations
- ▶ Our approach today:
 - ▶ Work in vacuum
 - ▶ Add noise to Einstein's equations (in CQ the noise tensor needs to be *independent* of the quantum state of matter – it is always there!)
 - ▶ Linearise the gravitational perturbations
 - ▶ Project on gauge-invariant degrees of freedom (transverse-traceless modes)
 - ▶ Wait for a long time: diffusion has the age of the Universe to develop significant deviations from GR predictions!

Stochastic gravitational waves

- ▶ In standard GR, linear perturbations of the gravitational field on flat-space can be decomposed in two physical independent modes (polarisations) $h^{(\lambda)}$ that
 - ▶ Are transverse to the direction of propagation \hat{n}
 - ▶ Obey the massless wave equations

- ▶ Stochasticizing Einstein's equations and projecting on transverse-traceless modes leads to

$$(\partial_t^2 - \nabla^2)h^{(\lambda)} = \sqrt{D_2} \xi^{(\lambda)}, \quad E[\xi^{(\lambda)}(x)\xi^{(\lambda')}(x')] = \delta_{\lambda\lambda'}\delta(x-x')^* \quad \lambda = +, \times$$

- ▶ The amplitudes of the two modes follow independent *stochastic wave equations*
- ▶ Decomposing in Fourier space, every k mode is an independent 1D Ornstein-Uhlenbeck motion

$$\pi_k = \dot{h}_k \quad \dot{\pi}_k = -k^2 h_k + \sqrt{D_2} \xi_k$$

* Choosing the Wheeler-deWitt metric as noise tensor and projecting onto TT modes

Aside: spurious drift

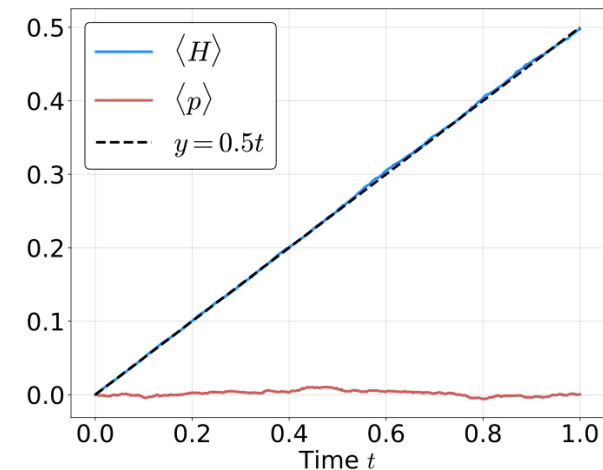
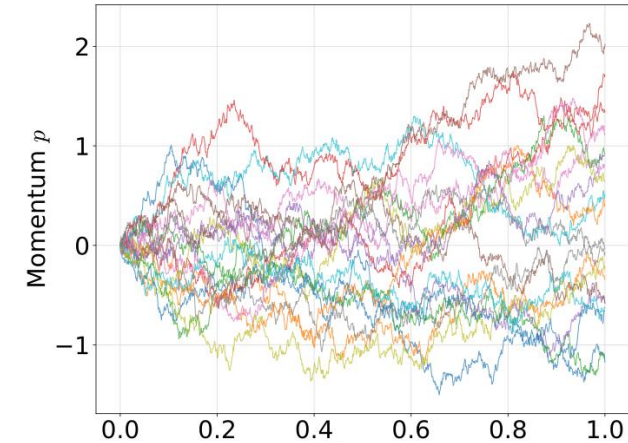
A function f of a stochastic variable z driven by Gaussian white noise with variance D obeys (Itô's lemma):

$$df = \frac{\partial f}{\partial z} dz + \frac{D}{2} \frac{\partial^2 f}{\partial z^2} dt$$

A collection of free particles with stochastic forcing from a white noise process with variance D :

$$\dot{p} = \sqrt{D}\xi$$

- ▶ Conserve the average momentum
- ▶ Do not conserve the average energy



Energy production in GW sector

- ▶ Using Itô's lemma mode-by-mode on the K-G Hamiltonian density, we find the energy per unit logarithmic frequency (ignoring redshift) of the stochastic background to be:

$$d\rho_{GW} = \frac{D_2}{G} \omega^3 T_{Hubble} d\ln(\omega)$$

- ▶ Gravitational waves detector measure energy density per logarithmic frequency $\Omega(\omega) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\ln(\omega)}$
 - ▶ LIGO and VIRGO bound stochastic signal at $\Omega(25 \text{ Hz}) < 10^{-9}$ [LIGO collab. '25]
 - ▶ Combined with decoherence-diffusion trade-off [Grudka '24], this implies

$$10^{-71} \lesssim D_2 \lesssim 10^{-68} *$$

- ▶ The allowed parameter space is tight!
 - ▶ Redshift and modelling of the interferometer response could (mildly) soften the upper bound

* Assuming decoherence rate of superposition of positions and of quadrupole moments are equal

Conclusions

- ▶ Classical gravity is in principle possible, but necessarily irreversible
- ▶ Diffusion in the classical sector is lower-bounded by decoherence rate in the quantum matter
 - ▶ Observing violations of the decoherence-diffusion trade-off proves non-classicality of gravity
- ▶ Randomness in the gravitational field produces a stochastic gravitational waves background
- ▶ Combining lower bounds coming from decoherence experiments with upper bounds from GW experiments squeezes CQ gravity from both sides

Challenges and natural directions:

- ▶ Understand constrained hybrid systems: covariance?
- ▶ Compute redshift corrections due to cosmological expansion

THANK YOU!



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Thank you!