

What is the Gravitational Path Integral?

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Plan

- The path integral is the basis of a framework for quantum theory, alternative to the canonical framework.
- Events!
- A fork in the road.
- With Gravity?
- Issues and questions take particular forms: e.g. topology change, the cosmological constant problem, the discrete-continuum dichotomy.
- Path integral for causal set quantum gravity

What is the Path Integral For?

- A propagator $K(x', t'; x, t) = \int [d\gamma] e^{iS[\gamma]}$
- A partition function: quantum mechanics (in imaginary time) = classical statistical mechanics in one higher spatial dimension
- A generating function for QFT correlators
- **The basis for an alternative framework for quantum theory** (Hartle, Sorkin)

The path integral quantum framework is based on a **double** path integral. Why? Essentially because a probability in quantum mechanics is the **square** of a sum of amplitudes.

The textbook calculation for the probability for the **outcomes of a sequence of position measurements** between $t = 0$ and T gives

$$\text{Prob}(\Delta) = \int_{\gamma \in \Delta} [d\gamma] \int_{\gamma' \in \Delta} [d\gamma'] D(\gamma, \gamma')$$

$$D(\gamma, \gamma') = \text{Amp}(\gamma)^* \text{Amp}(\gamma') \delta(\text{final endpoints}) \quad \text{and}$$

$$\text{Amp}(\gamma) = e^{i\frac{S[\gamma]}{\hbar}} \Psi(\gamma(0))$$

Copenhagen redone, for positions

- The double path integral (DPI)

$$\text{Prob}(\Delta) = \int_{\gamma \in \Delta} [d\gamma] \int_{\gamma' \in \Delta} [d\gamma'] D(\gamma, \gamma')$$

can be generalized to N particles: now a history γ is a set of N spacetime trajectories.

- The double path integral gives everything the Born rule & collapse postulate give when Δ is a sequence of position measurement outcomes.
- If “every measurement is a position measurement” then the DPI recovers Copenhagen.
- The DPI also gives *more*:
 - ▶ Δ can be **any** set of histories¹ such as the famous Feynman-event “passes through spacetime region R ”.
 - ▶ The path integral *decoherence functional* (Hartle, Sorkin):

$$D(A, B) = \int_{\gamma \in A} [d\gamma] \int_{\gamma' \in B} [d\gamma'] D(\gamma, \gamma')$$

¹up to the question of which sets are ‘measurable’ –c.f. Wiener process

Diagram from "The Quantum Mechanics of Cosmology" Hartle

Illustration of the histories summed over in $D(A, B)$

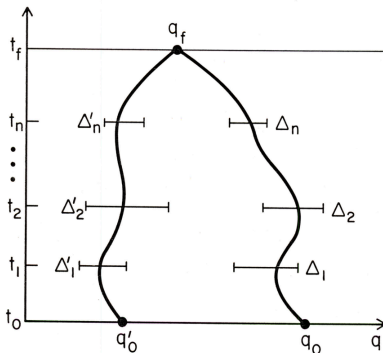


Fig. 2: *The sum-over-histories construction of the decoherence functional.*

Double path integral for a **closed** system: Copenhagen outdone

Now suppose an N particle history, γ , describes the microsystem, the instruments, and the environment together. *Nothing is external.*

- Each possible sequence of outcomes registered by the instruments corresponds to an **event** \equiv a set X of histories of the closed system.
- Claim: For X an instrument event, then For All Practical Purposes (FAPP)

$$D(X, X) = \text{Copenhagen Prob}(\text{sequence of outcomes } X)$$

Arguments for the claim are heuristic and based on the assumption the instruments work as they should and on the efficiency of environmental decoherence. Arguments are widely accepted and are tantamount to similar claims about “von Neumann measurements”.

Copenhagen has been outdone because the **same predictions** have been made—with same assumptions—but the whole world has been treated quantum mechanically.

In the path integral framework, each instrument event, X , **definitely either happens or doesn't happen**, in the physical world, in spacetime.

Axioms for Path Integral Quantum Theory for closed system

- A set of histories Ω
- A Boolean algebra \mathfrak{A} of **events** (aka subsets of Ω)
- A decoherence functional $D(X, Y)$ satisfying
 1. **Hermiticity:** $D(X, Y) = D(Y, X)^*$
 2. **Linearity:** $D(X \cup Y, W) = D(X, W) + D(Y, W)$ when $X \cap Y = \emptyset$
 3. **Positivity:** $D(X, X)$ is positive semidefinite
 4. **Normalization:** $D(\Omega, \Omega) = 1$
 5. $\mu(X) \equiv D(X, X)$ is the *quantum measure* of X .

This structure is a **unifying** principle. A history is as detailed a description of the world as is possible in ones full theory. Everything is in there.

One decides, in any given physical situation, what is treated as background and what is treated as dynamical and is summed over.

All predictions are about events.

We can recover our beloved Hilbert space using the GNS (Gelfand-Naimark-Segal) construction² in which every event $A \rightarrow |A\rangle$ s.t.

$$\langle A | B \rangle = D(A, B).$$

²Martin, O'Connor, Sorkin, gr-qc/0403085

Fork in the Road for Interpretation: Generalised Quantum Mechanics (Hartle) or Quantum Measure Theory (Sorkin)

Hartle's GQM (also known as “Decoherent Histories”, a path integral version)

- Decide one needs probabilities to make predictions.
- Consider subalgebras of \mathfrak{A} generated by *decoherent partitions* of Ω .
In each such partition, $\{A_1, A_2, \dots, A_n\}$, the events have measures $\mu(A_i) = D(A_i, A_i)$ that add up to 1.
- There are many decoherent partitions (Dowker&Kent; Kent)
- Postpone the problem by declaring that the “quasiclassical” partition into “instrument events” is good enough FAPP.

Sorkin's QMT

- **Every** event in \mathfrak{A} either happens or doesn't happen, not just the instrument events.
- μ is not additive on \mathfrak{A} , not a probability distribution.
- Adopt the concept of **precluded** events: events of measure zero (Geroch).
- Seek a scheme that recovers Copenhagen.

Have a Quick and Dirty interpretation: predict whatever dominates the path integral.

Path integral for gravity?

From QFT and GR: the set of histories Ω is a set of spacetimes.

Questions and issues take certain forms in this framework, e.g.:

- Topology change: can or should or must Ω include topology changing spacetimes?
- Atomic vs Continuous: for the physical system of a finite volume of spacetime, is the number of “building blocks” of the spacetime finite or infinite?
- Cosmological Constant: how does our large, old universe of low curvature emerge from a path integral in which most histories are not like that?

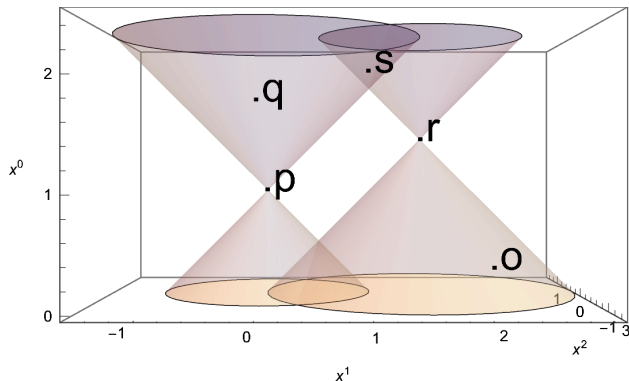
For rest of talk, take Causal Set Quantum Gravity as an example. Its 3 pillars are:

- the spacetime causal order from General Relativity
- the path integral
- atomicity of spacetime (answering 2nd question above: finite)

NOTE

The continuum is recovered as an **approximation** and not as a **limit**. The continuum is an approximation to the discrete and not the other way around.

Spacetime causal order



- The so-called spacetime *causal order* is a precedence order: before and after
- $o < r$, $r < s$, $o < s$, $p < q$, $p < s$. No physical order between p and r .
- The spacetime causal order is a **partial order** on spacetime points
- In GR this order relation is central to the physics e.g. the physics of black holes.

Causal order marries discreteness

- Spacetime is a **discrete order**
= transitive directed acyclic **graph**
= causal set
- Physically, elements = spacetime atoms = smallest, indivisible **events**
- $\approx 10^{240}$ spacetime atoms make up the observable universe

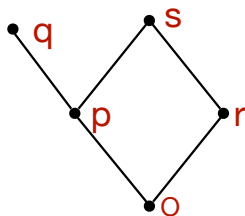


Figure: Hasse diagram

- Consequence: The Path Integral for Quantum Gravity is a SUM over causal sets:

$$“ Z(V) = \sum_{M \in \mathcal{M}} \int_{g \in \mathcal{G}} [dg] e^{iS[g]} \rightarrow Z(N) = \sum_{\text{causal sets with } N \text{ elements}} e^{iS(C)} ”$$

- Quick and dirty prediction: there are no closed timelike curves in nature.
- Topology change is naturally included
- Cosmological constant: some classes of nonmanifoldlike causal sets are suppressed in the sum by a nonlocal action (Carlip, Carlip & Surya).
- Constructing a D is a work in progress.

Summary

- The (double) path integral offers an alternative to the canonical framework for quantum theory
- Physics is about **events**, events in spacetime or (for gravity) events of spacetime
- Hilbert space is recovered but is not fundamental
- There are two interpretational directions as well as the Q&D interpretation.
- In causal set quantum gravity, finding a physically motivated D is work in progress. Many challenges, e.g. how to implement the “final delta function”
- In other approaches, how do issues/questions look in the path integral context? Can one define a D ? Even in the continuum, can one write down a formal expression for a D that satisfies the axioms?

Some Background References

- Hilbert Spaces from Path Integrals: arXiv:1002.0589
- Quantum Mechanics as Quantum Measure Theory: arXiv:gr-qc/9401003