Soft Edges:
The link between edge and soft modes

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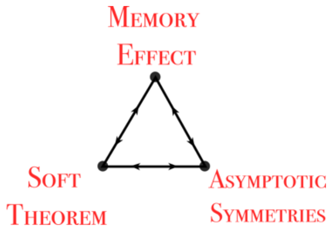
Based on:

2412.14548 w/ G. Araujo-Regado, P.A. Höhn and B. Tomova
2506.23459 w/ G. Araujo-Regado and P.A. Höhn

Soft Modes, Corners, and Subsystems

- Asymptotic regime (soft, IR)
  - IR dressing required in amplitudes
  - IR triangle: quantum and (semi)classical
  - Soft modes & asymptotic symmetry are physical

Ball, Chen, He, Mitra, Pasterski, Pate, Raclariu, Strominger, Myers, Zhiboedov, …



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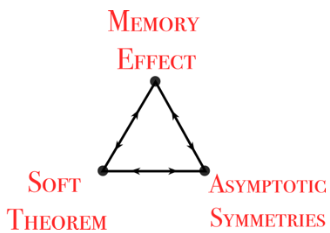

- Finite region's edge modes (UV)
  - Subsystems do not factorize
  - Local observables?
  - Emergence of corner symmetries
  - Observers and entropy

Carrozza, Donnelly, Freidel, Geiller, Höhn, Moosavian, Speranza, Pranzetti, Wall, …

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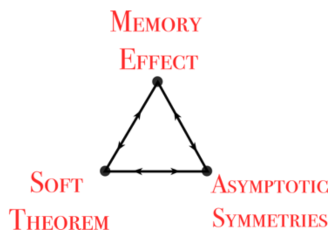

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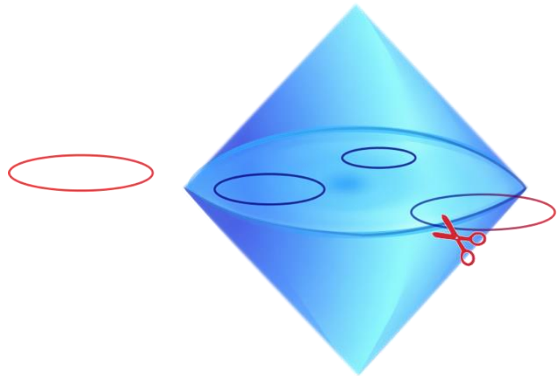
- A relational approach enhances clarity, establishes a link between the two and unravels rich structure in symmetries at finite distance
- **Symmetries as change in relational data**

Subsystems in gravity and gauge theories

"What is a local (quantum) subsystem in gauge theories and gravity?"

- Constraints imply non-factorisation of Hilbert space
- Gauge invariant observables (e.g Wilson loops) are non-local
- Subsystems are central for:
  - thermodynamic, entropy etc..,
  - isolated building blocks of spacetime?



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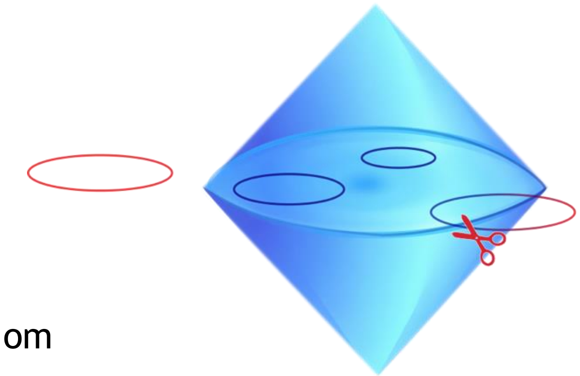

Gauge invariance ties bulk and boundary

- Defining local observables requires additional degrees of freedom
- Algebraically we get algebras with centre

(Donnelly '12; Casini, Huerta, Rosabal '13; Sonak, Trivedi '15; van Acoleyen et al. '15; Delcamp, Dittrich, Riello '16; …)

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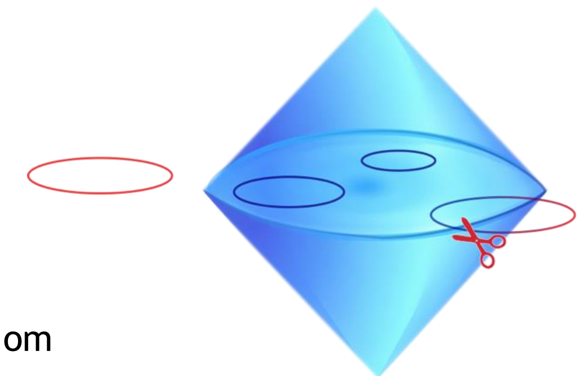

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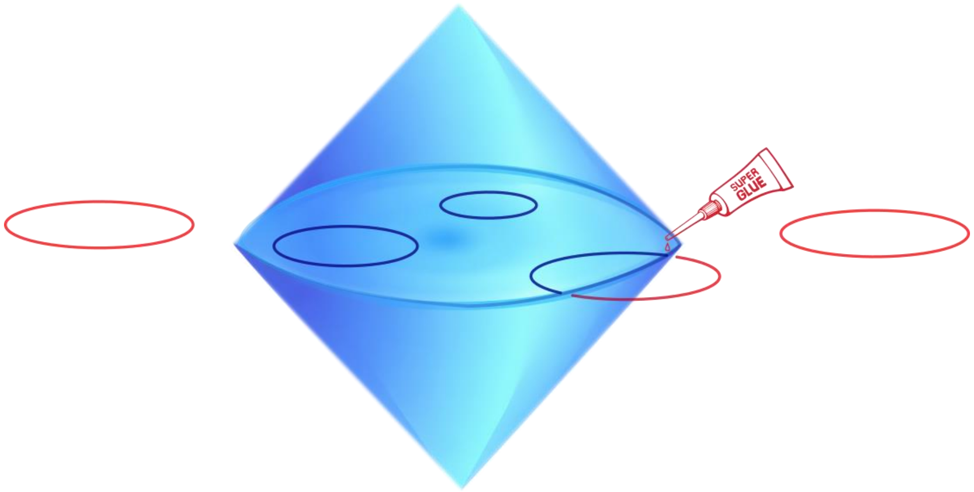
But:

- *Extension of phase space by hand*
- *Mismatch between entanglement and Von Neumann entropy*

Relational subsystems

Idea: *complete* the subregion with some information outside

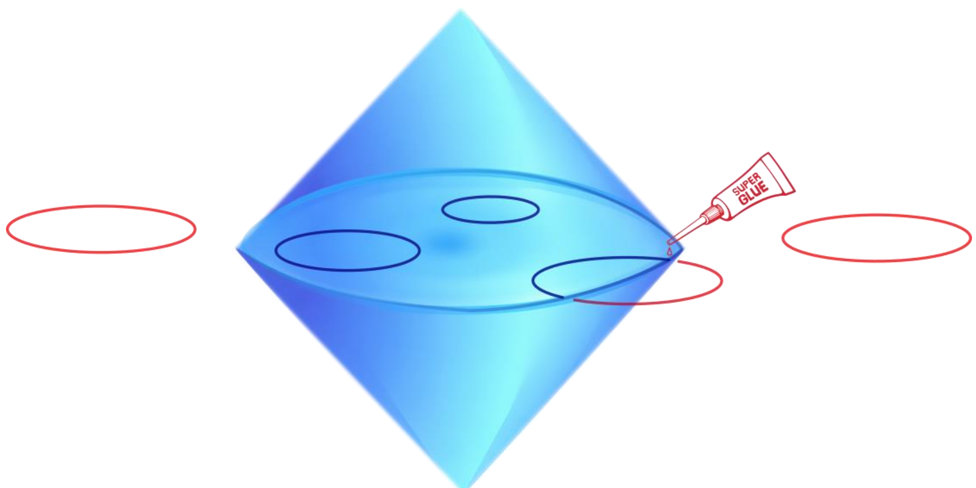


Relational subsystems

dress (Q)RF

Idea: ~~*complete*~~ the subregion with ~~some information outside~~

- Frames allow the construction of relationally local observables.
- Explicit connection between quasi-local symmetries and asymptotic physics *(soft edges)*



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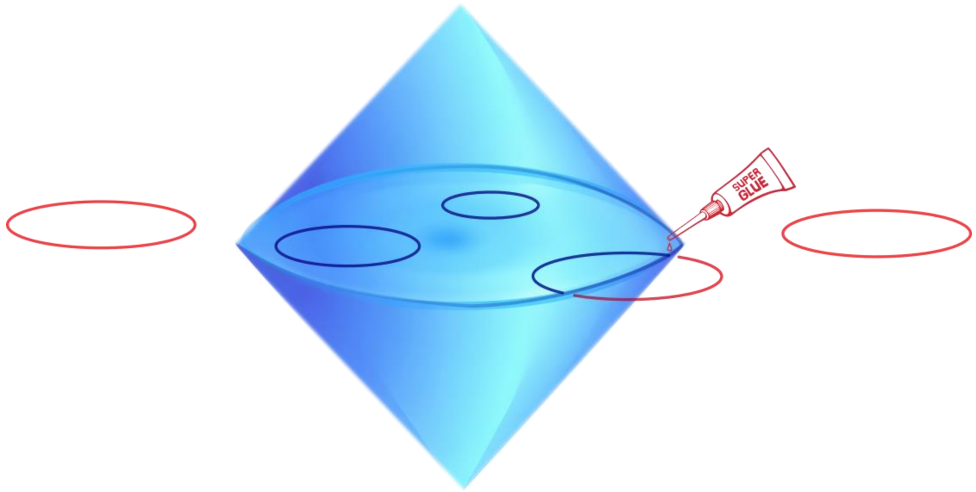
- Similarly to gravity, where QRF regularizes entropy (Crossed product De Vuyst, Eccles, Höhn, Kirklin ['24] Chandrasekaran, Longo, Penington, Witten ['23]), here the QRF perspective (Page-Wootters reduction) induces a factorization on $\mathcal{H}_{\text{phys}}$ [Philipp's talk]



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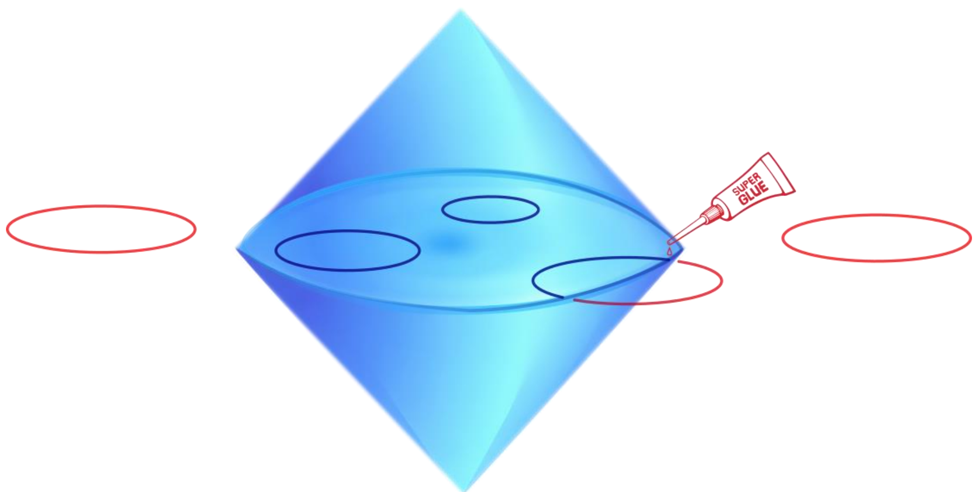
- **Cost:** Frame dependence of subsystems and entropy (**Subsystem relativity**)



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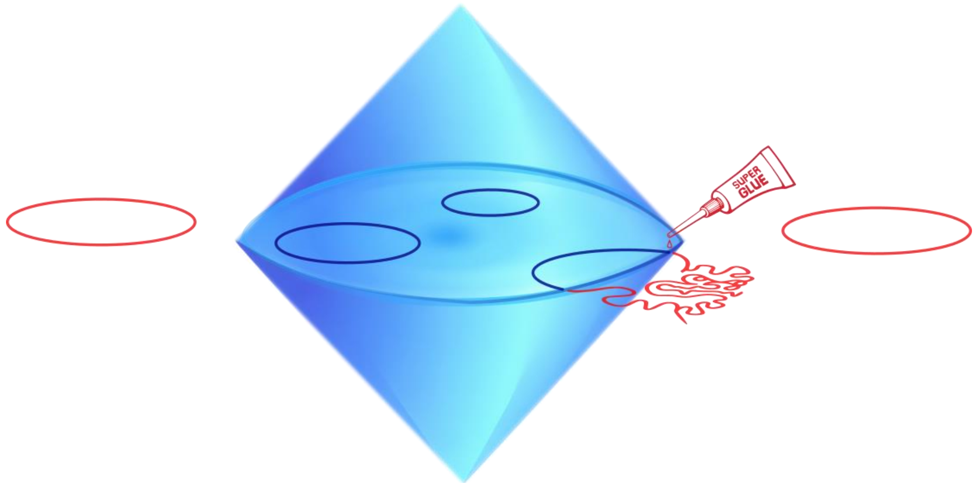

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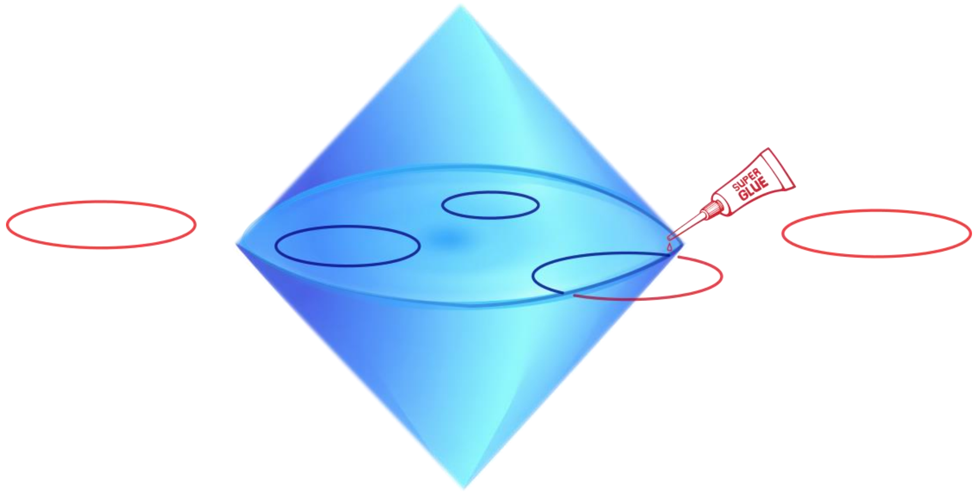
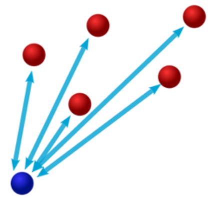
- **Cost:** Frame dependence of subsystems and entropy (**Subsystem relativity**)
- There is no **gauge-invariant** notion of subsystem decomposition without a frame



Edge modes as Reference frames (classical)



Analogous to the position of particles relative to a reference one

Dynamical reference frames to define **gauge invariant** observables

They are similar to coordinates, but on the gauge orbits

Carrozza, Höhn, Eccles [‘21, ’22],
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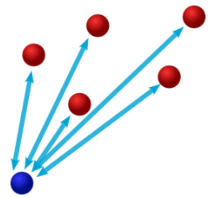

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Maxwell theory:

$$L = \int F \wedge \star F \,, \quad F = \mathrm{d}A$$

$$\Omega = \int_\Sigma \delta A \wedge \delta \star F$$

- Non-local functional: $\Phi[A]$

U(1) connection

Frame

- Gauge covariant:

$$A \to A + \mathrm{d}\lambda$$

$$\Phi \to \Phi + \lambda$$

Gauge transformation

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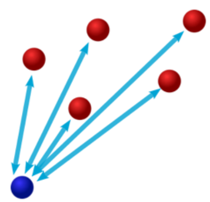

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Frame

- Gauge covariant:

$$A \to A + \mathrm{d}\lambda$$
$$\Phi \to \Phi + \lambda$$

Gauge transformation

- Dressed observables:

$$A^{\mathrm{dr}} := A - \mathrm{d}\Phi$$

Gauge invariant!

- Relational:

$$A^{\mathrm{dr}}\big|_{\Phi=0} = A$$

- Frame reorientations:

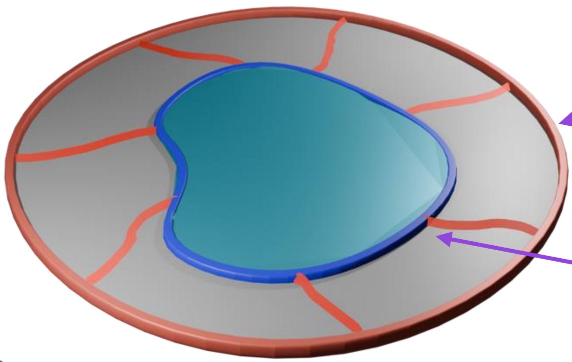
$$A \to A$$
$$\Phi \to \Phi - \rho$$

Extrinsic vs Intrinsic frames

Araujo-Regado, Höhn, FS, Tomova ['24]

Extrinsic frames: anchored in the complement or at infinity

- Wilson lines from the subregion to the global boundary



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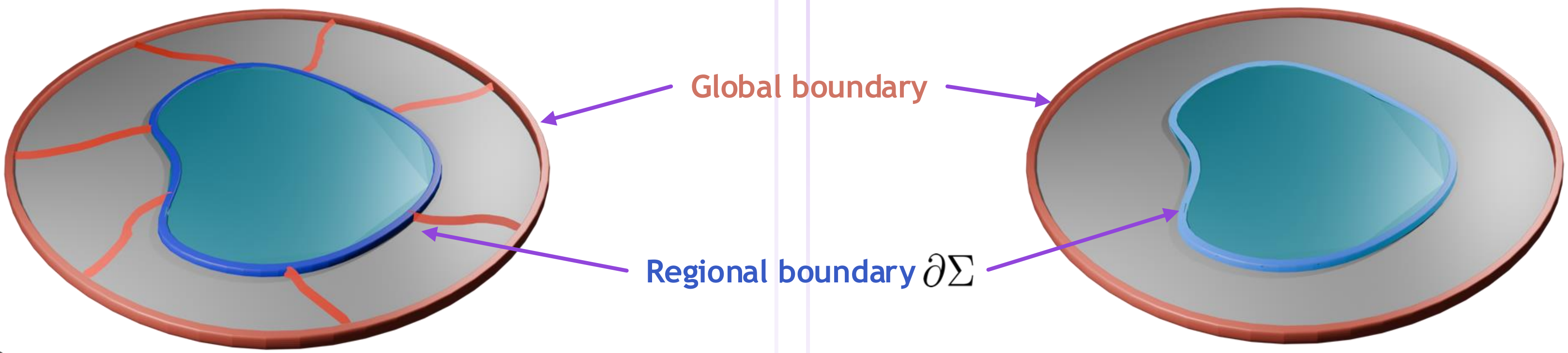
- Hodge decomposition **on** the subregion boundary



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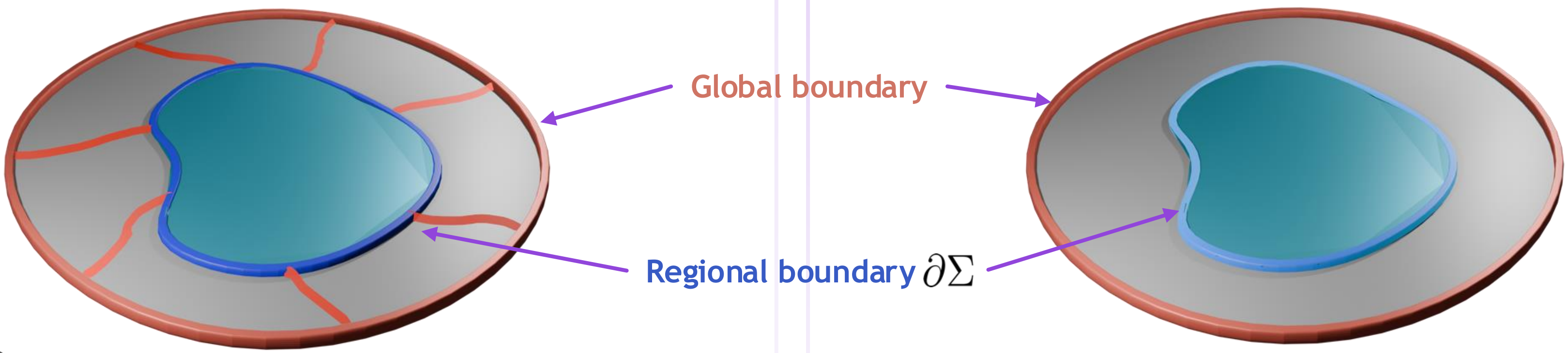

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Goldstone mode (GM): $\mathrm{d}\varphi = \mathrm{d}\tilde{\Phi} - \mathrm{d}\Phi = A^{\mathrm{dr}} - \tilde{A}^{\mathrm{dr}}$ Gauge invariant relational observable!

Extrinsic = Intrinsic x GM

Extended phase space: $\Omega = \int_\Sigma \delta A \wedge \delta \star F - \int_{\partial\Sigma} \delta\Phi\, \delta \star F = \int_\Sigma \delta\tilde{A}^{\mathrm{dr}} \wedge \delta \star F + \int_{\partial\Sigma} \delta\varphi\, \delta \star F$

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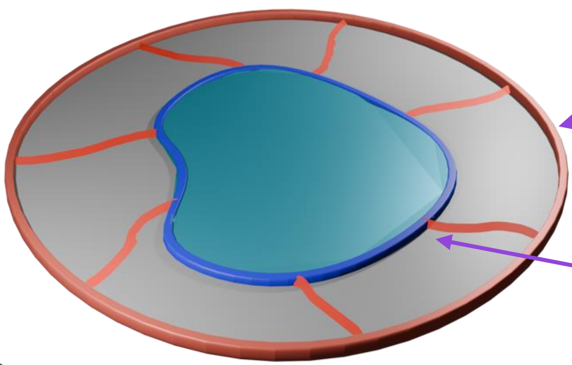

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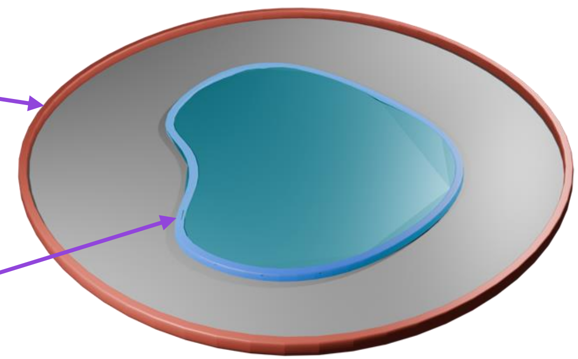

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Corner charge aspect

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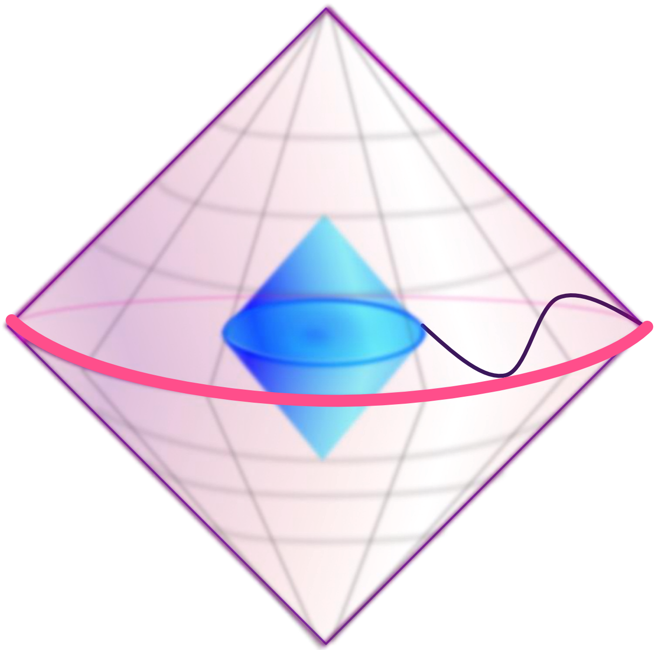

Operators in global flat space theory

- Large gauge transformations

  gauge transformations acting non trivially at the global corner

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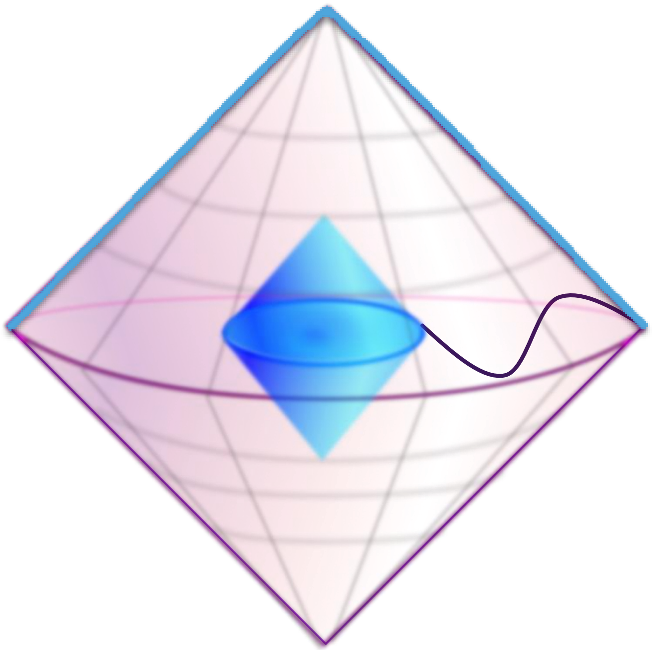

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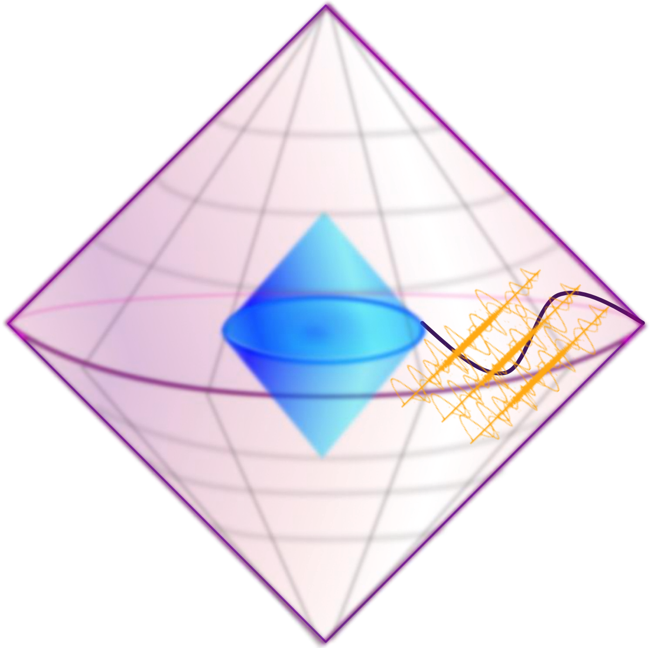

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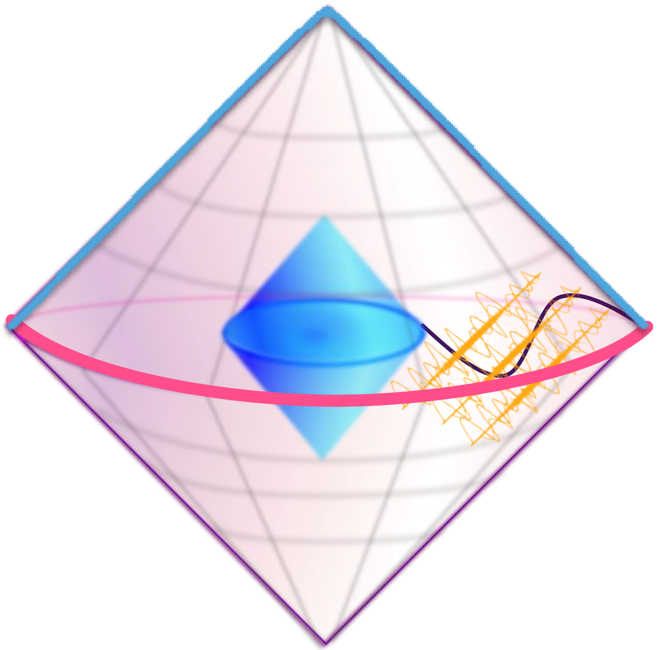

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- Add hard radiation
  Change electric field in causally disconnected region

Multiple ways of realizing subregion symmetries!

- However the region only knows about frame reorientations, not their origin in the complement
- Imprint of global vacuum on local superselection sectors -> Local memory effect
  Araujo-Regado, Höhn, FS [‘25]

Summary and outlook

- **Relational approach:**
  - Physical symmetries = frame reorientations
  - No gauge is broken

- **Explicit link (soft edges):**
  - Finite distance Goldstone = difference between frames
  - Only extrinsic frames give corner symmetries
  - Imprint of asymptotia (IR) into finite distance (UV?)
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- **Quantization and entanglement:**
  - QRF induces factorization: local subsystems
  - Relational entanglement entropy
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