

GRAVITY FROM ENTROPY

*Concepts of Quantum and Spacetime
9 March 2026, KEK, Tsukuba, Japan*

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[Bianconi, G., Gravity from entropy. Phys. Rev. D 111, 066001 (2025)]

GRAVITY FROM ENTROPY

Gravity is information

GRAVITY AND STATISTICAL MECHANICS

Since the discovery
that black-hole have an entropy
and emit Hawking radiation,
a big challenge
of theoretical physics
is to understand the deep relations between
gravity,
statistical mechanics
and
quantum information theory

GRAVITY AND STATISTICAL MECHANICS

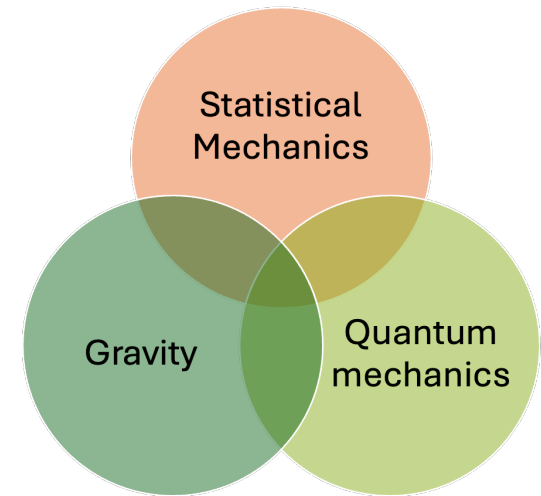
Since the discovery
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and emit Hawking radiation,
a big challenge
of theoretical physics
is to understand the deep relations between
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My belief is that
for building this theory we cannot start
from thermodynamic properties of black-holes and horizons
but we need a proper statistical mechanics approach

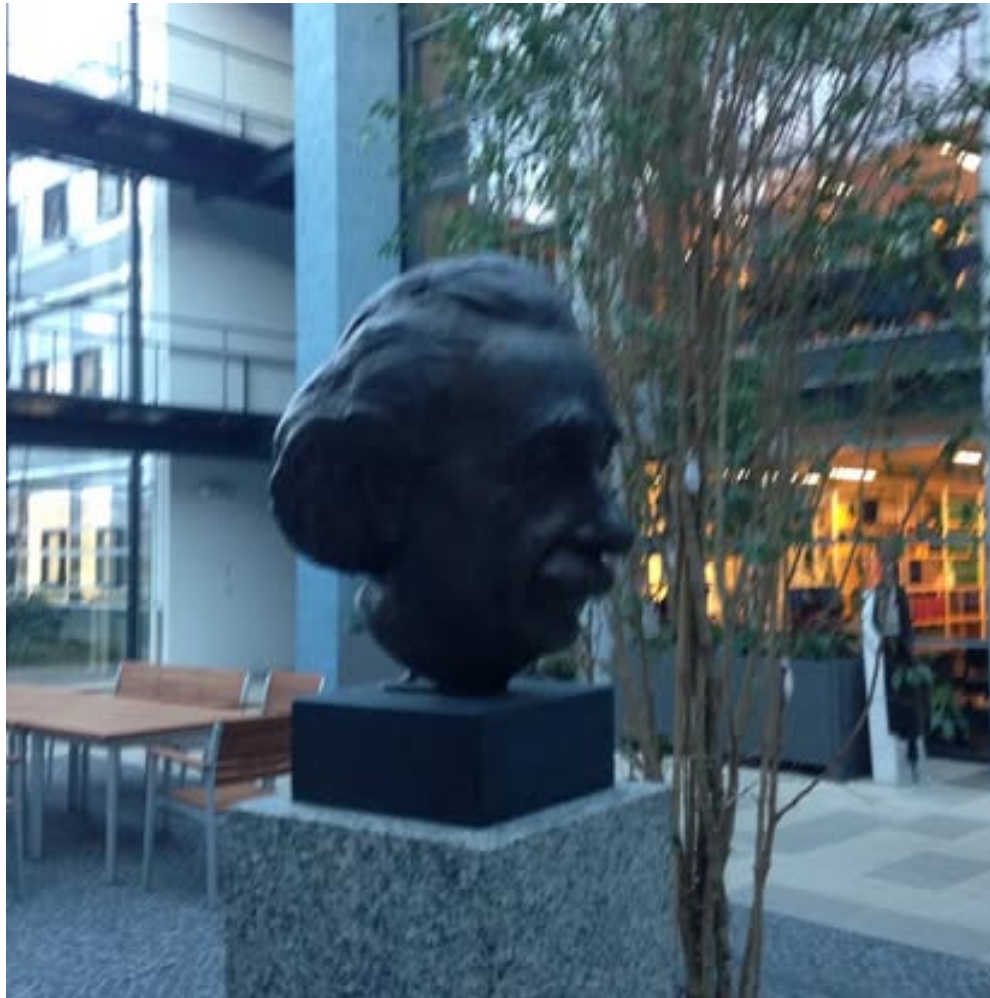
OUTLINE OF THE TALK

- Introduction of the GfE theory in a nutshell
- Discussion of the warm-up scenario
- General theory: Gravity from Entropy (GfE)
- The area law for the Schwarzschild metric
- Conclusions

GRAVITY FROM ENTROPY IN A NUTSHELL



EINSTEIN GENERAL RELATIVITY



GENERAL RELATIVITY

Einstein-Hilbert action

Gravity is the result of the curvature of space time

$$\mathcal{L}_G = \frac{1}{2\kappa} R$$

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GENERAL RELATIVITY

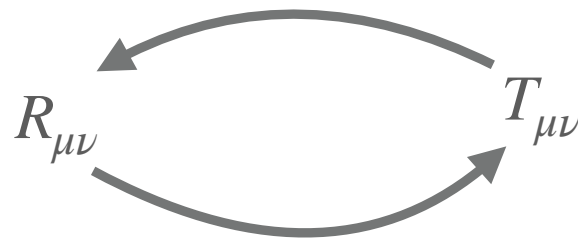
Einstein-Hilbert action

Gravity is the result of the curvature of space time

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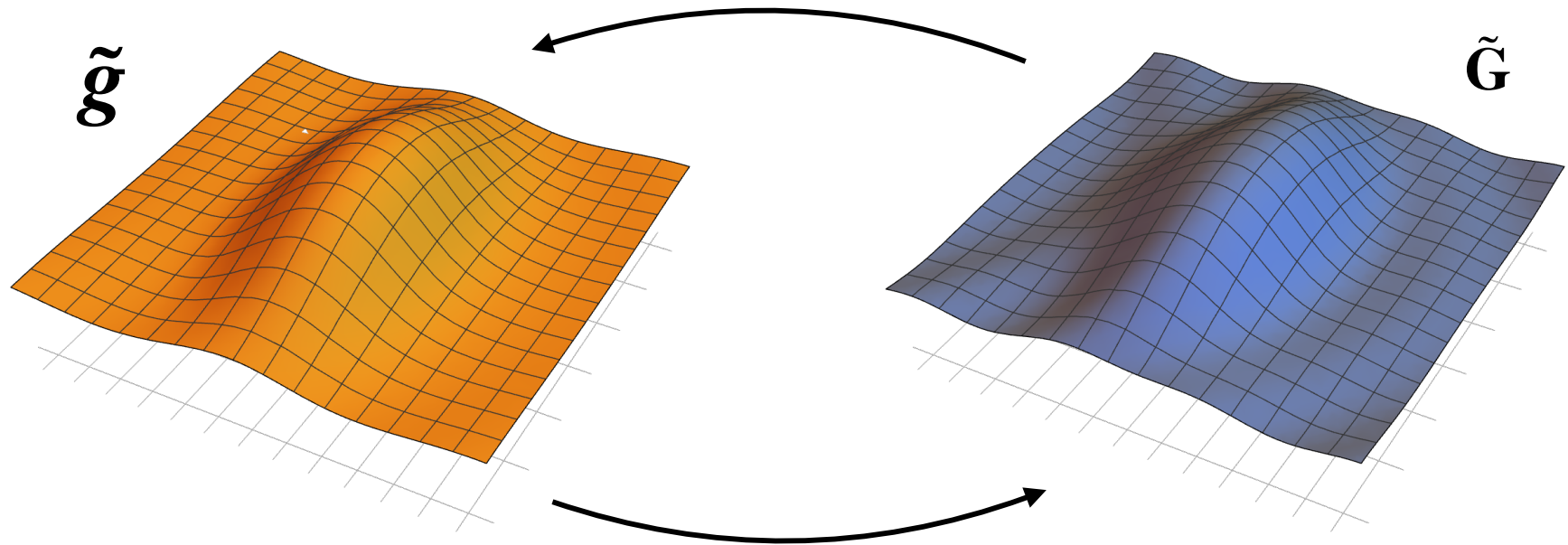
John Wheeler interpretations of Einstein equations

“Matter tells spacetime how to curve, and curved spacetime tells matter how to move”



Can we elevate the role that in gravity has
the interplay between matter and geometry
as the key physical insight
to be reflected in the action?

GRAVITY FROM ENTROPY



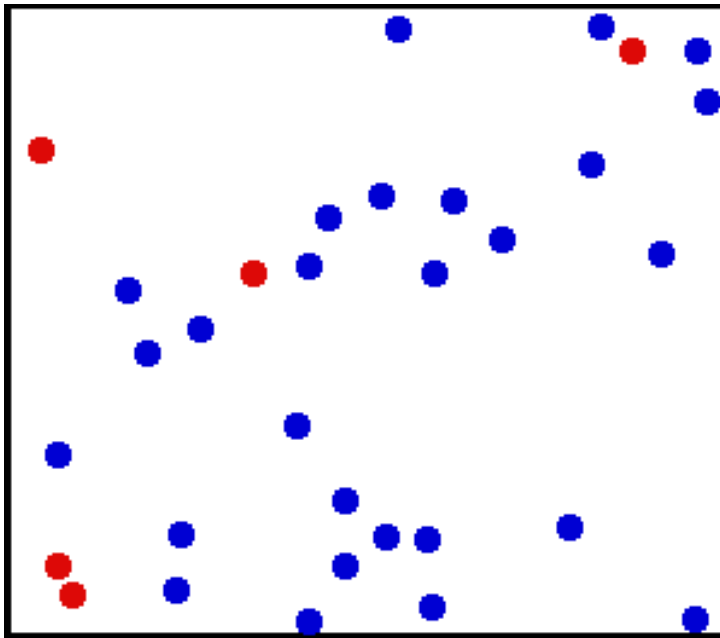
The Gravity from Entropy (GfE) Lagrangian is the Geometrical Quantum Relative Entropy (GQRE)

Gravity emerges

from the tension between the actual metric and the metric induced by matter and curvature

THE GEOMETRIC QUANTUM RELATIVE ENTROPY

ENTROPY



$$S = \ln \Omega$$

*The entropy is the logarithm
of the number Ω of microstate configurations
compatible with a macrostate of the system*

*According to the
2nd principle of thermodynamics
the entropy of a system never decreases*

QUANTUM – VON NEUMANN– ENTROPY

The Von Neumann entropy of a quantum system described by the density matrix ρ is given by

$$H(\rho) = -\text{Tr} \rho \ln \rho$$

which can also be expressed as

$$H = - \sum_n \lambda_n \ln \lambda_n$$

- The entropy quantifies the uncertainty in a quantum system
- Mixing increases the entropy (uncertainty)

QUANTUM RELATIVE ENTROPY

The quantum relative entropy

$$S(\rho||\sigma) = \text{Tr}\rho(\ln \rho - \ln \sigma)$$

is a measure of distinguishability between two quantum states.

If ρ, σ commute

$$S(\rho||\sigma) = \sum_n \lambda_n (\ln \lambda_n - \ln \tilde{\lambda}_n)$$

- **Quantum distinguishability never increases**
- (upon completely positive trace preserving maps)

STATISTICAL MECHANICS EMERGENCE AND INFORMATION IS PART OF NATURE

More is different

Paul W. Anderson

It from bit

John Wheeler

A theory

*of quantum cosmology cannot be logically consistent
if it does not describe a complex universe*

Lee Smolin, The Life of the Cosmos

STATISTICAL MECHANICS AS A FUNDAMENTAL THEORY

*Can statistical mechanics and information theory
be actually part of our
fundamental understanding of nature?*

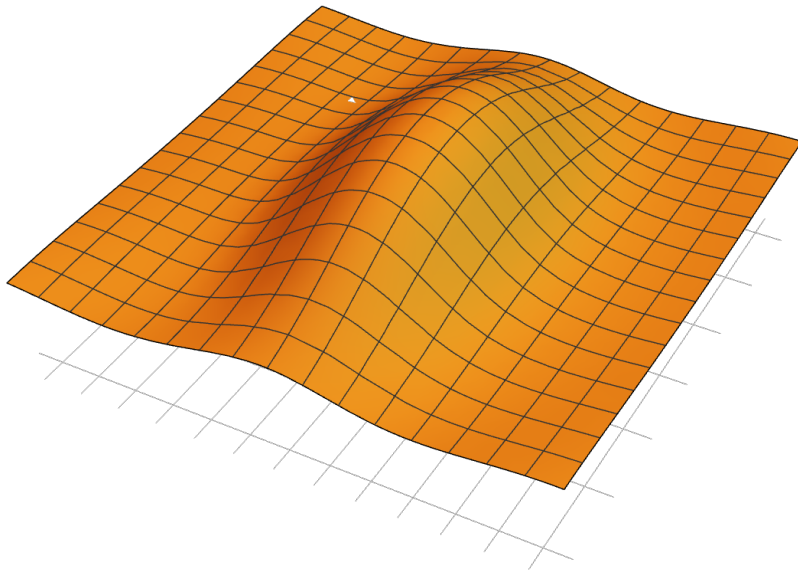
GRAVITY

AS A THE FUNDAMENTAL STATISTICAL MECHANICS THEORY
OF GEOMETRY

which allows interactions to occur in spacetime

Gravity from Entropy

GRAVITY FROM ENTROPY



Gravity from Entropy

*The metrics $g_{\mu\nu}$, $G_{\mu\nu}$
associated to the manifold
define quantum operators*

COMBINING GEOMETRY WITH QUANTUM STATISTICAL MECHANICS

The main differences of the metrics with standard density matrices are:

We require metrics to be non singular.

This will allow us to have a full Lorentz invariant theory.

For instance in this way we treat g and g^{-1} on the same footing.

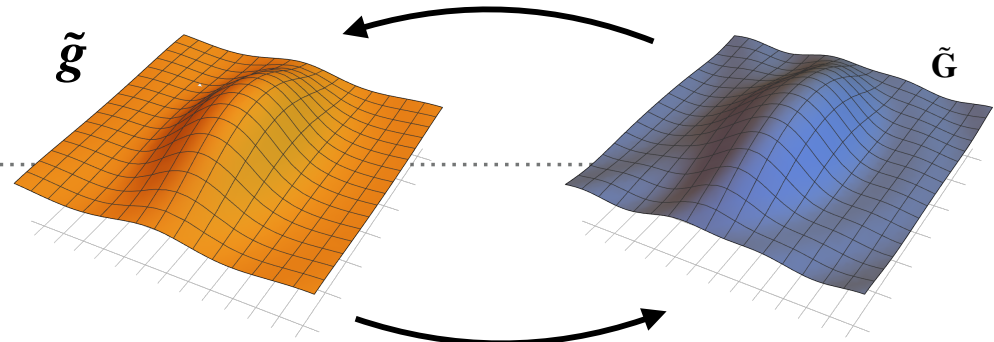
Horizons are allowed but true singularities are not.

We require metrics to have a trace at each point p of the manifold \mathcal{K} ,

but their trace will be in general different from one.

Also this generalisation can be traced back to the requirement of having a Lorentz invariant theory.

GRAVITY FROM ENTROPY ACTION

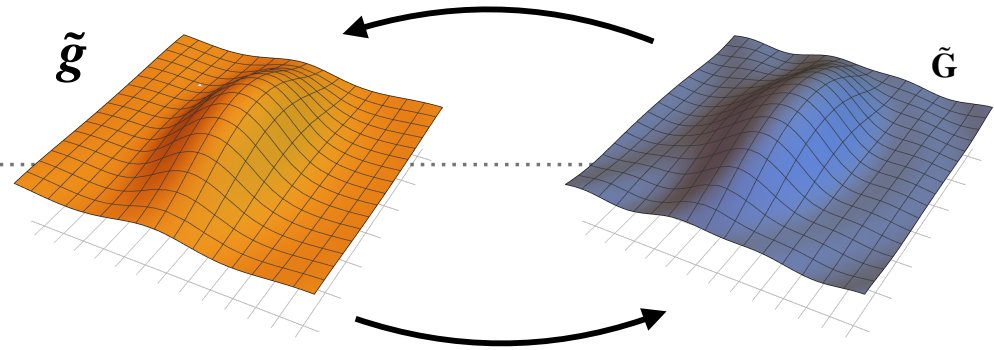


Gravity from entropy

$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{G}^{-1} = - \text{Tr}_F \ln \tilde{G} \tilde{g}^{-1}$$

The Gravity from Entropy Lagrangian
is the geometrical quantum relative entropy between
the metric of the manifold and the metric induced by the matter fields and curvature
[Bianconi, Gravity from entropy. Phys. Rev. D 111, 066001 (2025)]

THE GfE ACTION



The GfE action for gravity

$$\mathcal{S} = \frac{1}{\ell_P^d} \int \sqrt{|-g|} \mathcal{L} d^d \mathbf{x}$$

where

$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{G}^{-1} = - \text{Tr}_F \ln \tilde{G} \tilde{g}^{-1}$$

quantifies the information content

carried by the true metric that

that can be codified by the matter fields and the curvature

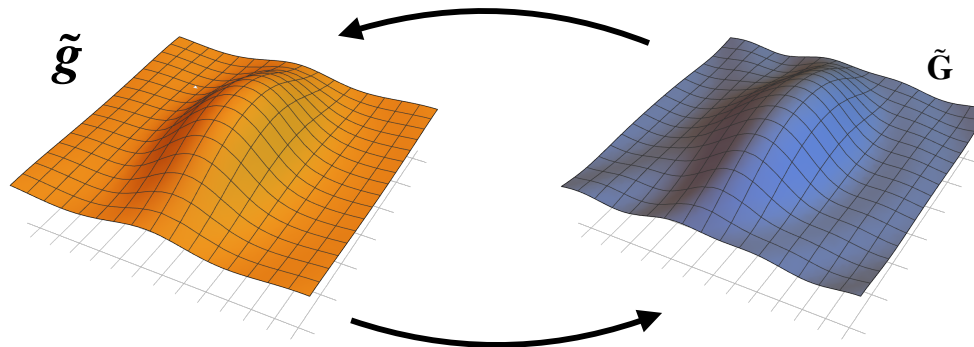
GRAVITY FROM ENTROPY AND ENTANGLEMENT

The GfE action for gravity

$$\mathcal{L} = \text{Tr}_{\tilde{g}} \ln \tilde{\mathbf{G}}^{-1} = - \text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

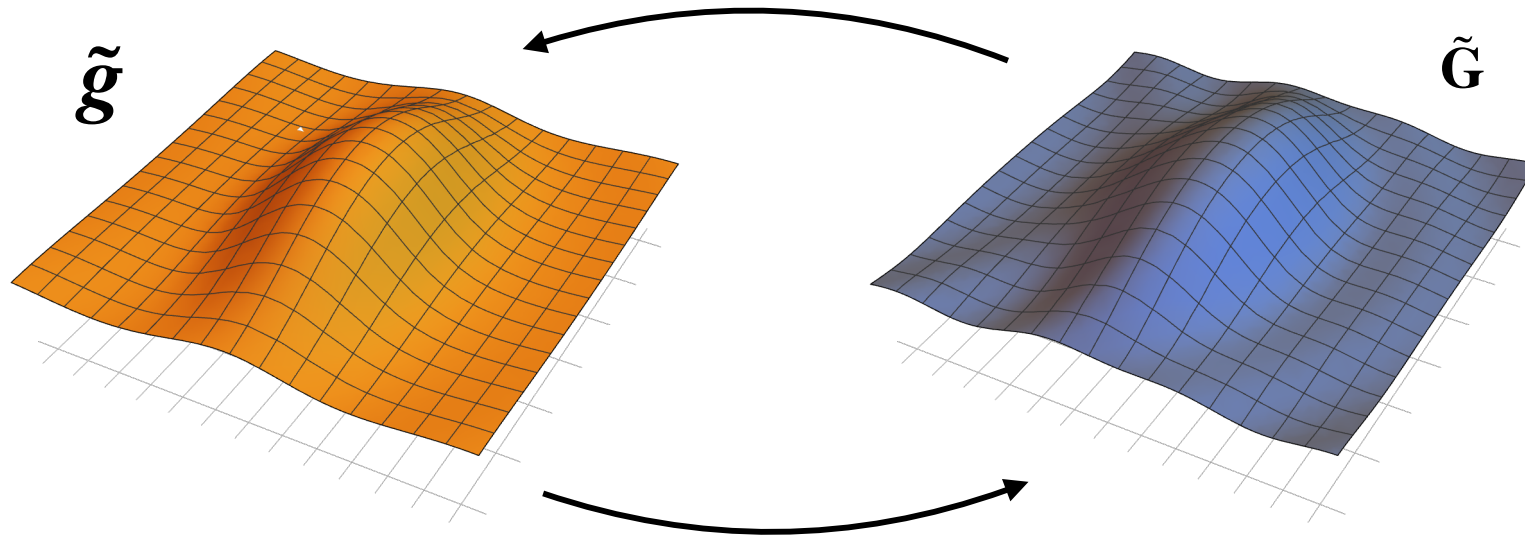
is the Geometric Quantum Relative Entropy (GQRE)

between the actual metric and the metric induced by the matter fields, and curvature



The proposed GQRE is a generalisation of the Araki entropy, notoriously quantifying quantum entanglement, to the topological Hilbert space under consideration.

THE INTERPLAY BETWEEN MATTER AND GEOMETRY



\tilde{g} encodes the actual metric
defines the scalar product at each point

\tilde{G} encodes both the matter and the curvature
Depends on the connection and the curvature
Prescribes how the metric changes from point to point

WARM-UP SCENARIO

OUTLINE OF THE WARM-UP SCENARIO DISCUSSION

- Motivation and warm-up scenario

- 2. Eigenvalues and entropy of the metrics
- 3. Metric induced by the matter field
- 4. GfE action and GQRE
- 5. Emergence of the massless Klein-Gordon equation
- 6. Achievements and limitations of the warm-up scenario

METRICS BETWEEN ONE FORMS

- We consider a torsion-free, d -dimensional manifold \mathcal{K} with metric $g_{\mu\nu}$ having Lorentz signature $(-1, 1, \dots, 1)$
- The metric g is assumed invertible and such that $g_{\mu\nu} V^\nu = V_\mu$, $g^{\mu\nu} V_\nu = V^\mu$ where at each point p of \mathcal{K} the metric $g_{\mu\nu}$ is real and symmetric
- On the manifold \mathcal{K} we consider a metric compatible Levi-Civita connection determining the covariant derivative ∇_μ
- In addition to the metric $g_{\mu\nu}$ we will consider alternative invertible metrics between vectors $G_{\mu\nu}$ which we might allow to be Hermitian

EIGENVALUES OF A METRIC TENSOR

Definition of eigenvalues

Given the metric tensor $G_{\mu\nu}$ at a given point p of the manifold \mathcal{K} , we define its eigenvalues λ in a Lorentz invariant way,

$$G_{\mu\nu} V^\nu = \lambda V_\mu$$

This eigenvalue problem can be reduced to the eigenvalue problem of the matrix $\mathbf{G}_{\mu\nu} \mathbf{g}^{\nu\alpha}$ as we have

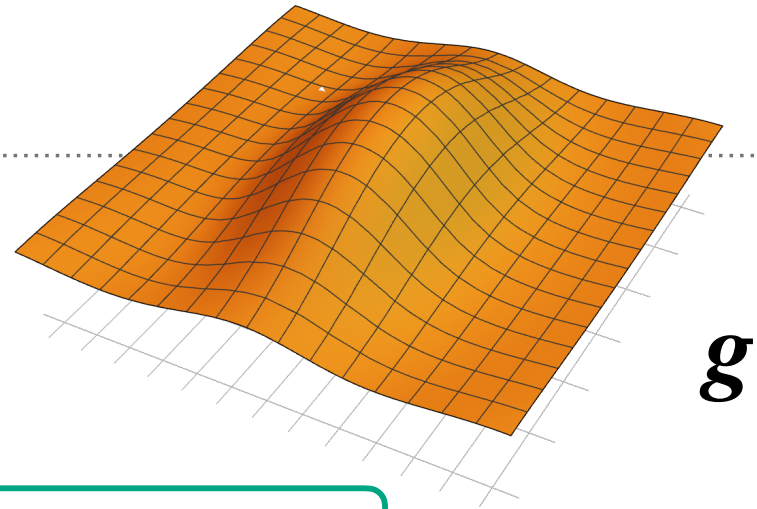
$$G_{\mu\nu} V^\nu = G_{\mu\nu} g^{\nu\alpha} V_\alpha = \lambda V_\mu$$

The logarithm of \mathbf{G} is given by

$$[\ln \mathbf{G}]_{\mu\nu} = \sum_{\lambda} (\ln \lambda) V_\mu^{(\lambda)} V_\nu^{(\lambda)} \text{ with } V_\mu^{(\lambda)} g^{\mu\nu} V_\nu^{(\lambda)} = 1$$

EIGENVALUES OF THE METRIC g

Eigenvalues of the metric g



► Any invertible real metric $g_{\mu\nu}$ has only eigenvalues $\lambda = 1$.

► Indeed we have that $g_{\mu\nu}V^\nu = V_\mu$ is an identity.

► Equivalently $g_{\mu\nu}g^{\nu\alpha} = \delta_\mu^\alpha$ is a matrix having only eigenvalues equal to one.

THE GEOMETRIC QUANTUM ENTROPY OF THE METRIC TENSORS

► The geometric quantum entropy of a general metric tensor $G_{\mu\nu}$ is given by

$$H(G) = \text{Tr } G \ln G^{-1} = - \sum_n \lambda_n \ln \lambda_n$$

Since the metric $g_{\mu\nu}$ has all the eigenvalues λ_n equal to one, $\lambda_n = 1$

it follows that

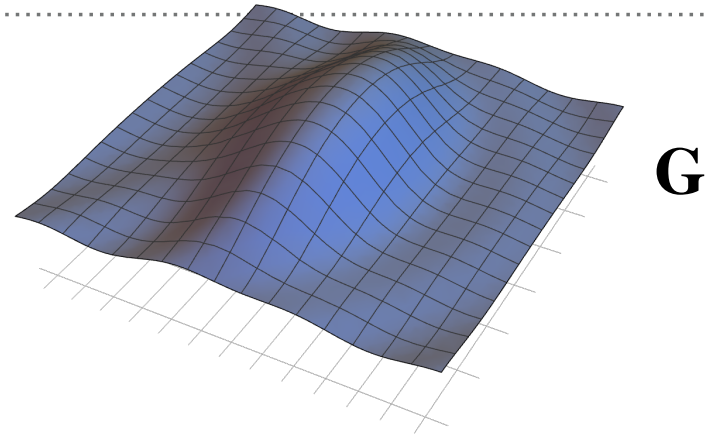
the entropy associated to the metric $g_{\mu\nu}$ is null,

$$\mathbf{H(g) = Tr g \ln g^{-1} = 0}$$

GAUSS LEGACY



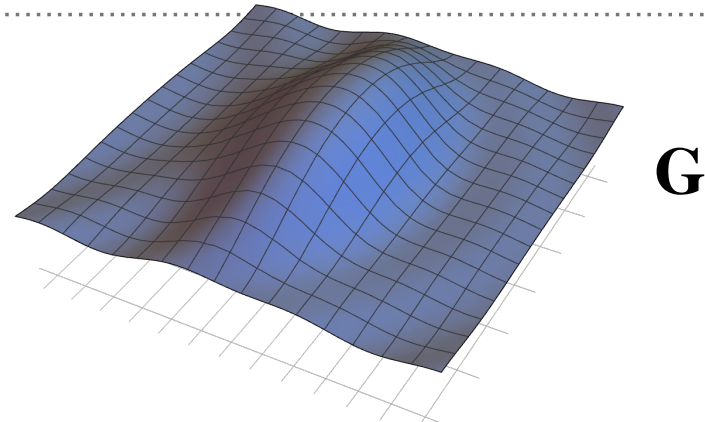
THE METRIC INDUCED BY THE MATTER FIELD



- Consider a boson matter field $\phi(\mathbf{x}) \in \mathbb{C}$, at any point p of \mathcal{K} we consider the manifold $\mathcal{K} \otimes \mathbb{C}$ of coordinates $(\mathbf{x}, \phi(\mathbf{x}))$.
- The metric induced by this manifold on \mathcal{K} is indicated by \mathbf{G} of elements

$$G_{\mu\nu} = g_{\mu\nu} + \alpha M_{\mu\nu} \text{ with } M_{\mu\nu} = (\nabla_{\mu} \bar{\phi})(\nabla_{\nu} \phi) \text{ where } \alpha = \alpha' \ell_P^d$$

THE METRIC INDUCED BY THE MATTER FIELD



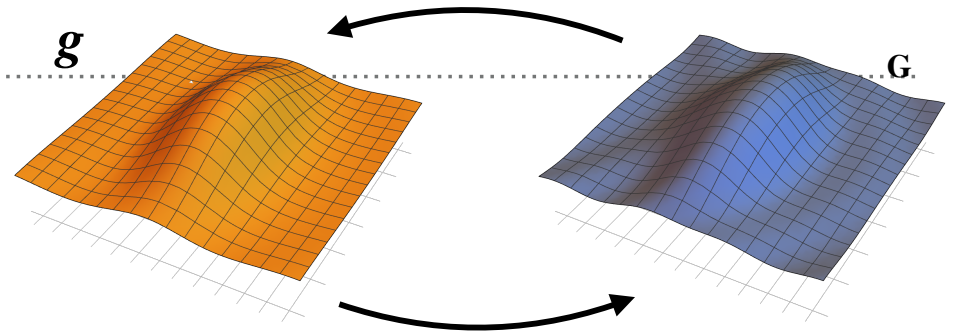
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- Thus if the metric is Euclidean and flat, i.e. $g_{\mu\nu} = \delta_{\mu\nu}$ and ϕ is real, the metric \mathbf{G} reduces to the first fundamental form of Gauss, i.e. in 2d

$$G = \begin{pmatrix} 1 + \alpha\phi_x^2 & \alpha\phi_x\phi_y \\ \alpha\phi_x\phi_y & 1 + \alpha\phi_y^2 \end{pmatrix}$$

THE GFE LAGRANGIAN



- ▶ The Geometrical Quantum Relative Entropy between g and \mathbf{G} , where \mathbf{G} is the metric induced by the matter fields assumed to be positively defined is given by

$$\mathcal{L} = -\text{Tr}g \ln g^{-1} + \text{Tr}g \ln \mathbf{G}^{-1}$$

- ▶ Since the entropy of g is null, the GfE Lagrangian is given by

$$\mathcal{L} = \text{Tr}g \ln \mathbf{G}^{-1} = -\text{Tr}_M \ln \mathbf{G}g^{-1}$$

SOME TECHNICAL DETAILS

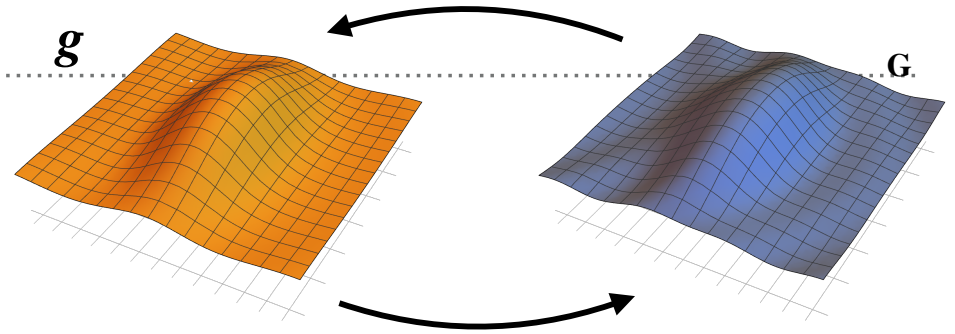
We want to discuss and prove that the adopted quantum relative entropy can be equivalently expressed as

$$\mathcal{L} = \text{Tr} g \ln \mathbf{G}^{-1} = - \text{Tr}_M \ln \mathbf{G} g^{-1}$$

- ▶ The eigenvalues λ_n of the metric $g_{\mu\nu}$ are equal to one.
- ▶ The eigenvalues $\tilde{\lambda}_n$ of the metric $G_{\mu\nu}$ induced by the matter field reduce to the eigenvalues of the matrix $G_{\mu\nu} g^{\nu\rho}$
- ▶ Thus we have

$$\mathcal{L} = \text{Tr} g \ln \mathbf{G}^{-1} = - \sum_{n=1}^d \lambda_n \ln \tilde{\lambda}_n = - \sum_{n=1}^d \ln \tilde{\lambda}_n = - \text{Tr}_M \ln \mathbf{G} g^{-1}$$

THE GFE THEORY ACTION



- ▶ The GfE Lagrangian is given by

$$\mathcal{L} = \text{Tr}_g \ln \mathbf{G}^{-1} = -\text{Tr}_M \ln \mathbf{G} g^{-1}$$

- ▶ The GFE action is

$$\mathcal{S} = \frac{1}{\ell_P^d} \int \sqrt{|-g|} \mathcal{L} d^d \mathbf{x}$$

THE MASSELESS KLEIN-GORDON EQUATION EMERGES FROM THE ENTROPIC ACTION

- The action can be proved to become simply

$$\mathcal{L} = -\ln(1 + \alpha |\nabla\phi|^2) \text{ with } |\nabla\phi|^2 = \nabla_\mu \bar{\phi} \nabla^\mu \phi$$

- The linearised Lagrangian $\mathcal{L} = -\alpha \nabla_\mu \bar{\phi} \nabla^\mu \phi$ is one of the massless Klein-Gordon field.

- The equation of motion is $\nabla_\mu h(|\nabla\phi|^2) \nabla^\mu \phi = 0$ with $h(w) = \frac{\alpha}{1 + \alpha w}$

THE ACHIEVEMENTS AND THE LIMITATIONS OF THIS WARM-UP SCENARIO

Achievements

A great achievement of this warm-up scenario is that we get the Klein-Gordon equation out of the GfE action

Limitations

- A limitation is that we only get the massless Klein-Gordon equation
- The other limitation is that the Lagrangian $\mathcal{L} = -\ln(1 + |\nabla\phi|^2)$ is null in absence of matter field, so the metric g is undetermined in absence of matter fields

In the following we are going to address both limitations of this warm-up scenario

FURTHER COMMENTS ON THE WARM-UP SCENARIO

RELATION TO ARAKI ENTROPY

The adopted GfE Lagrangian can be equivalently expressed as

$$\mathcal{L} = \text{Tr}_g \ln \mathbf{G}^{-1} = -\text{Tr}_M \ln \mathbf{G}g^{-1} = -\text{Tr}_M \ln \Delta_{\mathbf{G},g}^{1/2}$$

where $\Delta_{\mathbf{G},g}^{1/2} = \mathbf{G}g^{-1}$

► Here $\Delta_{\mathbf{G},g}$ plays the role of the modular operator and its square root is given by

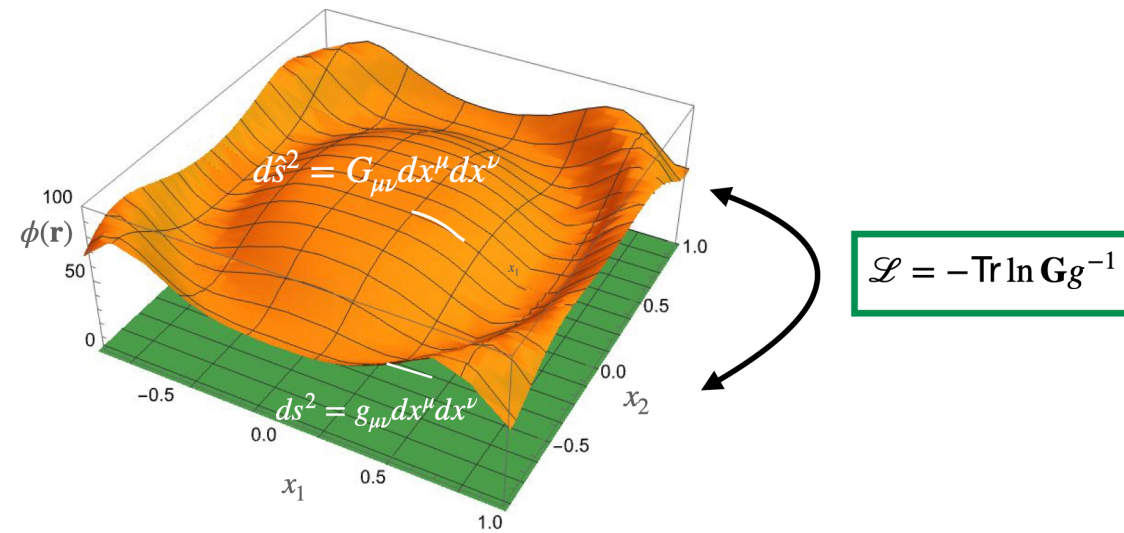
$$\Delta_{\mathbf{G},g}^{1/2} = \mathbf{G}g^{-1} = \sqrt{\mathbf{G}\mathbf{G}^*}$$

► where $\mathbf{G}^* = g^{-1}\mathbf{G}g^{-1}$ is the adjoint metric operator of \mathbf{G} .

► The Geometric Quantum Relative Entropy generalises the Araki entropy to metric tensors.

BEYOND HOLOGRAPHY: GRAVITY FROM ENTROPY ACTION AS A COMPUTING AI PARADIGM

The gradient flow that maximises the the Gravity from Entropy action, corresponding to this warm-up scenario, leads to the well known Perona-Malik algorithm of anisotropic diffusion in image processing.

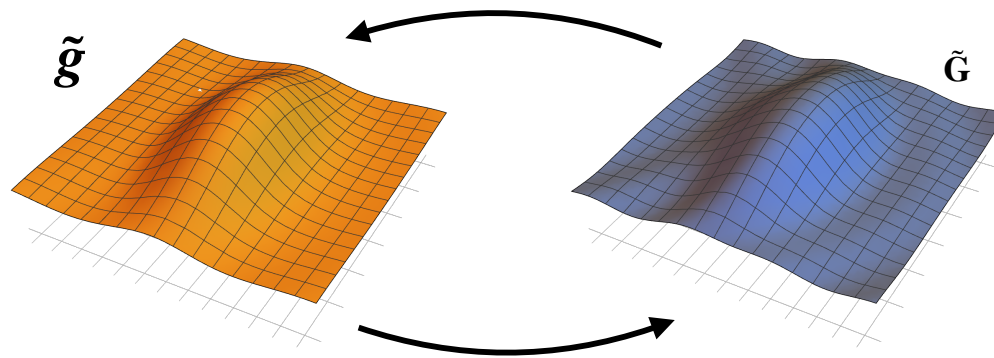


G. Bianconi PRE 112, L043301 (2025)

The maximisation of the entropy captured by the Gravity from Entropy action is compatible with the presentation of non-trivial structures

GENERAL THEORY

$$\mathcal{L} = -\text{Tr} \tilde{g} \ln \tilde{\mathbf{G}} = -\text{Tr}_M \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$



OUTLINE OF THE THE GENERAL THEORY DISCUSSION

- ▶ Discussion of the general entropic theory
 1. Topological Dirac-Kähler description of the matter fields
 2. Higher-order metric and curvature
 3. Metric induced by the matter field as a “density operator”
 4. GfE action
 5. Introduction of the G-field
 6. Modified gravity equations

TOPOLOGICAL MATTER FIELDS

The topological fields are vector of the Hilbert space

$$|\Phi\rangle = \phi \oplus \omega_\mu dx^\mu \oplus \frac{1}{2} \zeta_{\mu\nu} dx^\mu \wedge dx^\nu$$

and their adjoint is given by

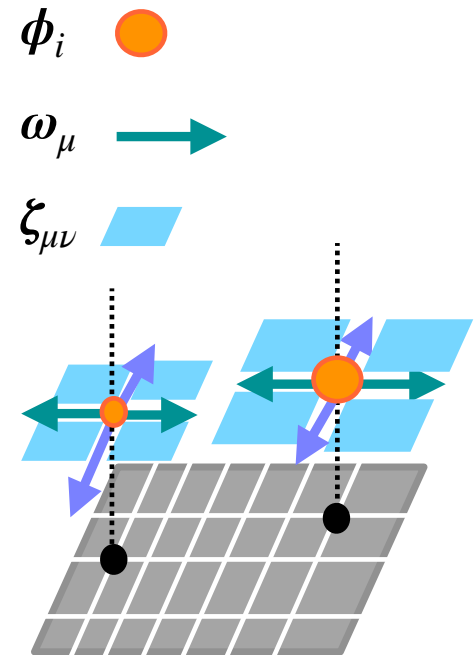
$$\langle\Phi| = \bar{\phi} \oplus \bar{\omega}_\mu dx^\mu \oplus \frac{1}{2} \bar{\zeta}_{\mu\nu} dx^\mu \wedge dx^\nu$$

The Hodge-Dirac operator $\mathbf{D} = d + \delta$ acts of the topological fields as

$$\mathbf{D} |\Phi\rangle = -\nabla^\mu \omega_\mu \oplus (\nabla_\mu \phi - \nabla^\rho \zeta_{\mu\rho}) dx^\mu \oplus \nabla_\mu \omega_\nu dx^\mu \wedge dx^\nu$$

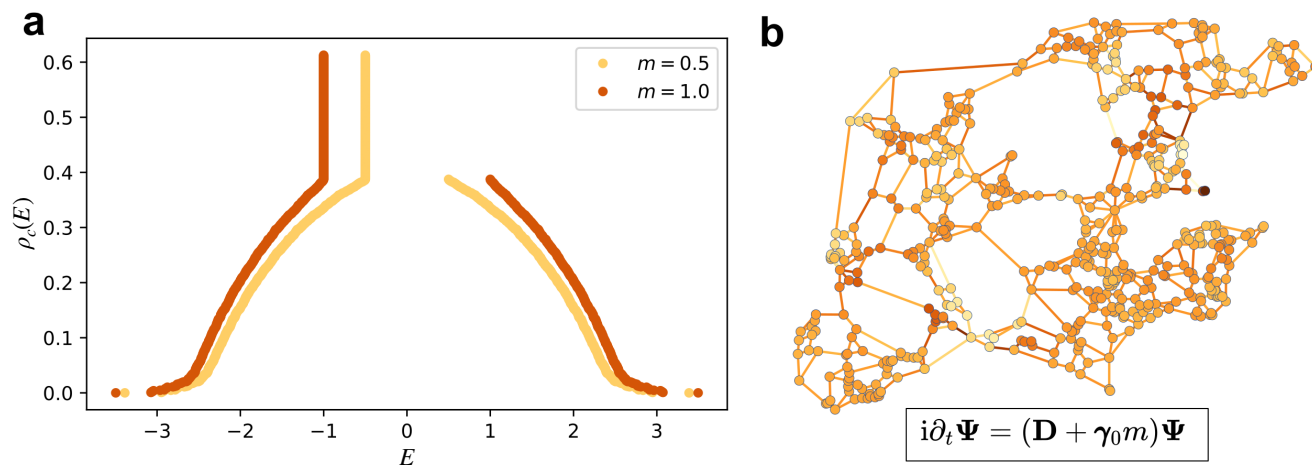
its adjoint is

$$\langle\Phi| \mathbf{D}$$



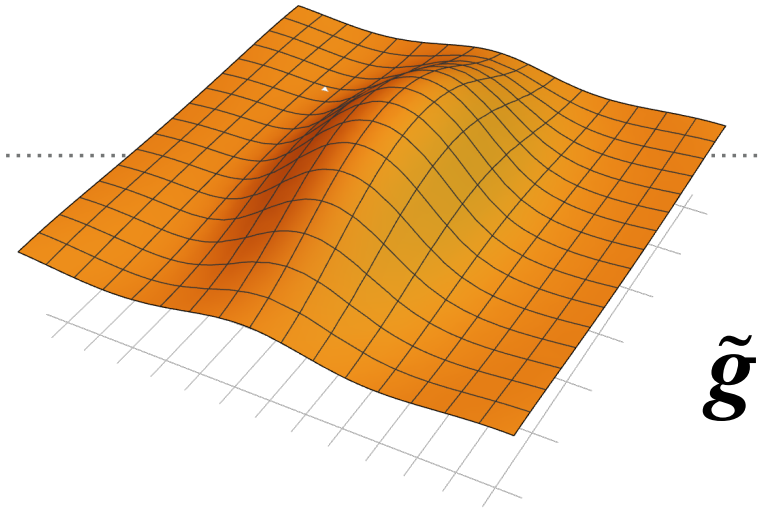
THE DIRAC OPERATOR IN THE DISCRETE

On the discrete the Dirac operators acts on the direct sum of a zero, a one and a two cochains, etc. defined on nodes, edges, triangles, etc.



See for instance Millan et. al. Nature Physics, 2025 and ICTP lectures that I gave in 2024 (available in YouTube).

TOPOLOGICAL METRIC OF THE MANIFOLD

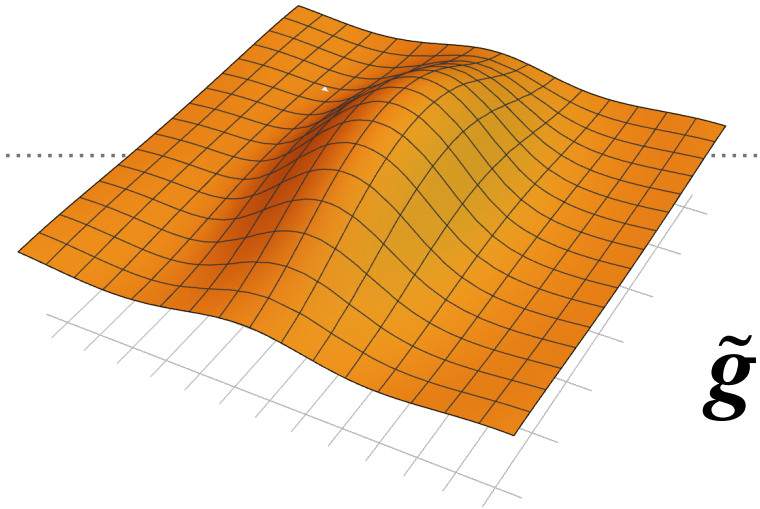


► We consider the topological metric \tilde{g} given by

$$\tilde{g} = 1 \oplus g_{\mu\nu} dx^\mu \otimes dx^\nu \oplus g^{(2)}_{\mu\nu\rho\sigma} (dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma)$$

$$\text{where } g^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})$$

EIGENVALUES OF THE METRIC \tilde{g}



Eigenvalues of the metric \tilde{g}

- ▶ Any invertible real metric $g_{\mu\nu}$ is associated to a metric $g^{(2)}$ which has only eigenvalues $\lambda = 1$.
- ▶ The entropy of \tilde{g} is identically zero.

HIGHER_ORDER METRIC INDUCED BY MATTER FIELDS

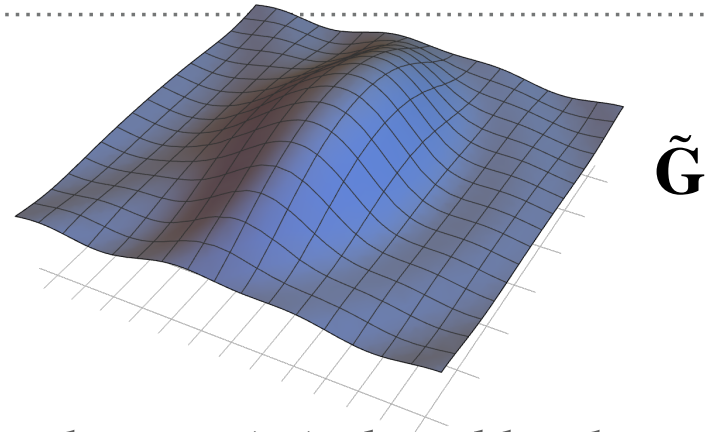
- The metrics induced by the matter fields and the geometry given in general by

$$\tilde{G} = G^{(0)} \oplus G_{\mu\nu}^{(1)} dx^\mu \otimes dx^\nu \oplus G_{\mu\nu\rho\sigma}^{(2)} (dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma)$$

Observations

- We can easily extend the notion of eigenvalues, entropy and quantum relative entropy to metrics between n-forms [see G. Bianconi 2025 for details].

THE METRIC INDUCED BY THE MATTER FIELD AND THE GEOMETRY



- In the warm-up scenario the metric induced by the matter field was given by \mathbf{G} of elements

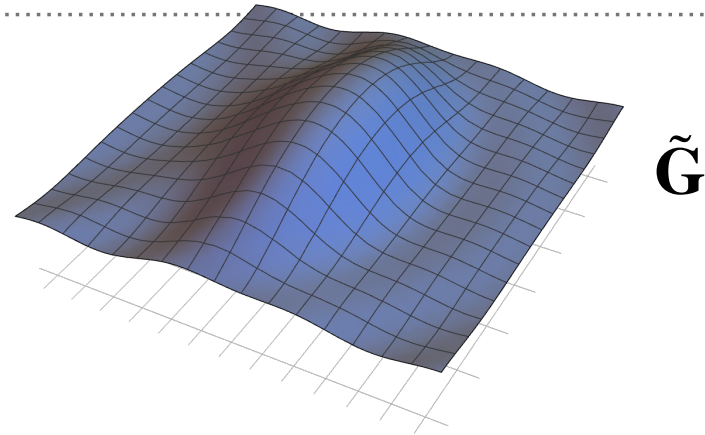
$$G_{\mu\nu} = g_{\mu\nu} + \alpha M_{\mu\nu} \text{ with } M_{\mu\nu} = (\nabla_{\mu} \bar{\phi})(\nabla_{\nu} \phi) \text{ where } \alpha = \alpha' \ell_P^d$$

- By considering topological matter fields it is natural to assume

$$\tilde{\mathbf{G}} = \tilde{\mathbf{g}} + \alpha \mathbf{D} |\Phi\rangle \langle \Phi| \mathbf{D}$$

where we substitute the covariant derivative with the Dirac operator \mathbf{D}

THE METRIC INDUCED BY THE MATTER FIELD AND THE GEOMETRY AS A DENSITY MATRICS



- If we interpret the metric \tilde{G} as a density matrix $\mathbf{D} |\Phi\rangle\langle\Phi| \mathbf{D}$ can be seen as a projector, in the spirit of Von Neumann algebras let us start adding relevant other terms...

$$\tilde{G} = \tilde{g} + \alpha \tilde{M}$$

where

$$\tilde{M} = \mathbf{D} |\Phi\rangle\langle\Phi| \mathbf{D} + m^2 |\Phi\rangle\langle\Phi|$$

HIGHER-ORDER CURVATURE

- The curvature of the manifold \mathcal{K} is described topologically as

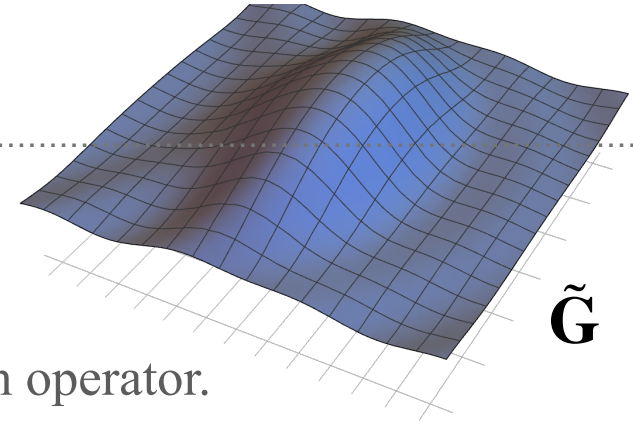
$$\tilde{\mathcal{R}} = R \oplus R_{\mu\nu} dx^\mu \otimes dx^\nu \oplus R_{\mu\nu\rho\sigma} (dx^\mu \wedge dx^\nu) \otimes (dx^\rho \wedge dx^\sigma)$$

where R is the Ricci scalar, $R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu\rho\sigma}$ is the Riemann tensor.

Observations

- We observe that $\tilde{\mathcal{R}}$ has the same structure as the general metric $\tilde{\mathbf{G}}$
- We also observe that $\tilde{\mathcal{R}}$ cannot be interpreted as a projector in general.

TOWARD A GEOMETRICAL QUANTUM MECHANICS



We interpret the metric $\tilde{\mathbf{G}}$ as a quantum operator.

Building on the first fundamental form of Gauss and the theory of von Neumann algebra [Witten RMP 2018],

$\tilde{\mathbf{G}}$ for boson matter field is given by

$$\tilde{\mathbf{G}} = \tilde{g} + \alpha\tilde{\mathbf{M}} - \beta\tilde{\mathcal{R}}$$

where

$$\tilde{\mathbf{M}} = \mathbf{D}|\Phi\rangle\langle\Phi| + m^2|\Phi\rangle\langle\Phi|$$

Here $\tilde{\mathcal{R}}$ includes Ricci scalar, Ricci tensor, and Riemann tensor as well

ABELIAN GAUGE FIELDS

The Abelian gauge fields A_μ can be included in this framework by postulating that the metric induced by the matter fields and the geometry is given by

$$\tilde{\mathbf{G}} = \tilde{g} + \alpha \left[\tilde{\mathbf{M}} - \frac{1}{4} \tilde{\mathbf{F}} \right] - \beta \mathcal{R}$$

where includes the electromagnetic field

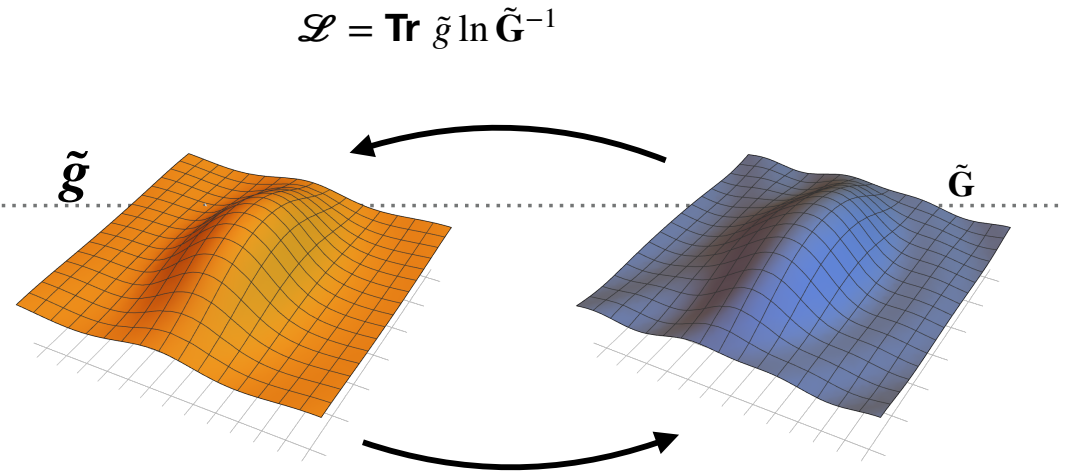
$$\tilde{\mathbf{F}} = 0 \oplus 0_{\mu\nu} dx^\mu \otimes dx^\nu \oplus F_{\mu\nu} F_{\rho\sigma} (dx^\mu \wedge dx^\nu \otimes dx^\rho \wedge dx^\sigma)$$

where here $F_{\mu\nu}$ is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

and where we consider the minimal substitution to the covariant derivative

$$\nabla_\mu \rightarrow \nabla_\mu - ieA_\mu$$

THE GfE ACTION



The GfE action reads

$$\mathcal{L} = -\text{Tr} \tilde{g} \ln \tilde{g}^{-1} + \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1}$$

Since the entropy of \tilde{g} is null, the Lagrangian is given by

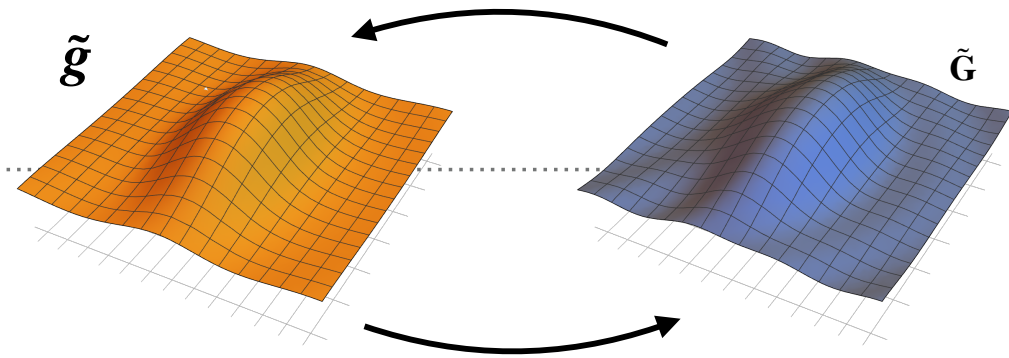
$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

GRAVITY FROM ENTROPY

Gravity is fundamental

Gravity is information

THE GfE THEORY ACTION



The GfE theory action for gravity

$$\mathcal{S} = \frac{1}{\ell_P^d} \int \sqrt{|-g|} \mathcal{L} d^d \mathbf{x}$$

where

$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = - \text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

quantifies the information content

carried by true metric

that can be codified by the matter fields and the curvature.

RESULTS

THE LOW ENERGY-SMALL CURVATURE LIMIT

**In the low-energy, small curvature limit
we recover
the Einstein-Hilbert action
with zero-cosmological constant**

$$\mathcal{L} = \hat{\mathcal{L}}_G + \hat{\mathcal{L}}_M$$

$$\hat{\mathcal{L}}_G = \frac{1}{2\kappa} R$$

LOW COUPLING REGIME: EINSTEIN-HILBERT ACTION WITH ZERO COSMOLOGICAL CONSTANT

In the low coupling regime the entropic action

$$\mathcal{L} - \text{Tr}_M \ln \tilde{\mathbf{G}} \tilde{g}^{-1} = - \text{Tr}_M \ln \left((\tilde{g} + \alpha \tilde{\mathbf{M}} - \beta \tilde{\mathcal{R}}) \tilde{g}^{-1} \right) = - \text{Tr}_M \ln \left(\tilde{\mathbf{I}} + (\alpha \tilde{\mathbf{M}} - \beta \tilde{\mathcal{R}}) \tilde{g}^{-1} \right)$$

can be linearised and

reduces to the Einstein-Hilbert Lagrangian with zero cosmological constant

plus the matter field Lagrangian

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M$$

where we have

$$\mathcal{L}_G = \beta \text{Tr} \tilde{\mathcal{R}} \tilde{g}^{-1} = 3\beta R, \quad \mathcal{L}_M = - \alpha \text{Tr} \tilde{\mathbf{M}} \tilde{g}^{-1}$$

- The general theory leads to Einstein equations in this limit,
- overcoming the warm-up scenario limitations

THE G-FIELD

The geometrical quantum entropy action
leads to a modified theory of gravity
expressed in terms of
the G-field $\tilde{\mathcal{G}}$

THE INTRODUCTION OF THE G FIELD

The considered Lagrangian

$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

is not linear but only dependent on

$$\tilde{\Theta} = \tilde{\mathbf{G}} \tilde{g}^{-1}$$

Imposing this set of constraints as Lagrangian multipliers using the auxiliary fields $\tilde{\mathcal{G}}, \tilde{\Theta}$ we obtain the Lagrangian

$$\tilde{\mathcal{L}} = -\text{Tr} \ln \tilde{\Theta} - \text{Tr} \tilde{\mathcal{G}} (\tilde{\mathbf{G}} \tilde{g}^{-1} - \tilde{\Theta})$$

which is now a Lagrangian linear in the curvature.

G-FIELD

The G-field,
introduced as a set of Lagrangian multipliers,
is here interpreted as
a physical and measurable field

Other examples of physical Lagrangian multipliers
are
the temperature and the chemical potential

THE ROLE OF THE G-FIELD

The effective metric is dressed by the G-field

$$\tilde{g}_{\mathcal{G}} = \tilde{g} \tilde{\mathcal{G}}^{-1}$$

The dressed Ricci scalar is given by the Ricci tensor
contracted with the dressed metric

$$R_{\mathcal{G}} = \mathbf{Tr}_F \tilde{g}_{\mathcal{G}}^{-1} \tilde{\mathcal{R}}$$

THE ACTION AS A FUNCTION OF THE G-FIELD

The Gravity from Entropy Lagrangian in terms of the G-field is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M$$

where

$$\mathcal{L}_G = \frac{1}{2\kappa}(\mathcal{R}_{\mathcal{G}} - 2\Lambda_{\mathcal{G}}) \quad \mathcal{L}_M = \mathbf{Tr}_F \tilde{g}_{\mathcal{G}}^{-1} \tilde{\mathbf{M}}$$

It has the same structure of the Einstein-Hilbert action + matter action
with the metric $\tilde{g}_{\mathcal{G}} = \tilde{g} \mathcal{G}^{-1}$ dressed by the G-field
and the emergent cosmological “constant” $\Lambda_{\mathcal{G}}$

EQUATIONS OF MOTION

The action depends now on
the metric, on the matter-fields and on the G-field.

By variational methods we get therefore three main equations of motion

1. Equation of motion for the matter fields, and the gauge fields
2. Modified gravity equations for the metric
3. Equations for the G-field (the Θ field is the inverse of the G-field)

THE EQUATION OF MOTION FOR THE MATTER FIELDS

The topological matter field $|\Phi\rangle$ obeys the equation of motion

$$\mathbf{D}\tilde{g}_{\mathcal{G}}^{-1}\mathbf{D}|\Phi\rangle + \tilde{g}_{\mathcal{G}}^{-1}m^2|\Phi\rangle = 0$$

Thus the topological matter fields obey
the full topological Klein-Gordon equation associated to
the dressed metric

$$\tilde{g}_{\mathcal{G}} = \tilde{\mathcal{G}}^{-1}\tilde{g}$$

MODIFIED GRAVITY: VARIATION WITH RESPECT TO $\tilde{\Theta}$

We now consider the variation of the Lagrangian

$$\tilde{\mathcal{L}} = -\text{Tr} \ln \tilde{\Theta} - \text{Tr} \tilde{\mathcal{G}} (\tilde{\mathbf{G}} \tilde{g}^{-1} - \tilde{\Theta})$$

with respect to $\tilde{\Theta}$ getting clearly

$$\tilde{\mathcal{G}} = \tilde{\Theta}^{-1}$$

Using $\tilde{\mathbf{G}} \tilde{g}^{-1} = \tilde{\mathbf{I}} + \alpha \tilde{\mathbf{M}} \tilde{g}^{-1} - \beta \tilde{\mathcal{R}} \tilde{g}^{-1} = \tilde{\Theta}$ we get

$$\tilde{\mathcal{G}} = \left[\tilde{\mathbf{I}} + \alpha \tilde{\mathbf{M}} \tilde{g}^{-1} - \beta \tilde{\mathcal{R}} \tilde{g}^{-1} \right]^{-1}$$

GRAVITY FROM ENTROPY EQUATIONS

► The Gravity from entropy equations are

$$R_{(\mu\nu)}^{\mathcal{E}} - \frac{1}{2}g_{\mu\nu}(\mathcal{R}_{\mathcal{E}} - 2\Lambda_{\mathcal{E}}) + \mathcal{D}_{(\mu\nu)} = \kappa\mathcal{T}_{(\mu\nu)},$$

Where

- $\mathcal{T}_{\mu\nu} = -\frac{1}{\sqrt{|-g|}}\frac{\delta\mathcal{S}_M}{dg_{\mu\nu}}$ is the **dressed** energy-momentum tensor,
- $R_{\mu\nu}^{\mathcal{E}}$ is the dressed Ricci tensor
- $\mathcal{D}_{\mu\nu}$ includes exclusively second order derivatives of the G-field

In Gravity from Entropy

$\Lambda_{\mathcal{E}}$, $\mathcal{D}_{\mu\nu}$ and $\mathcal{T}_{\mu\nu}$ might be interpreted as the dark energy and dark matter contributions

EMERGENT COSMOLOGICAL CONSTANT

Gravity is fundamental

The cosmological “constant” is emergent

THE EMERGENT COSMOLOGICAL “CONSTANT”

The emergent cosmological “constant” given by

$$\Lambda_{\mathcal{G}} = \frac{1}{\beta' \ell_P^2} \text{Tr}_F[\tilde{\mathcal{G}} - \tilde{\mathbf{I}} - \ln(\tilde{\mathcal{G}})]$$

is positive

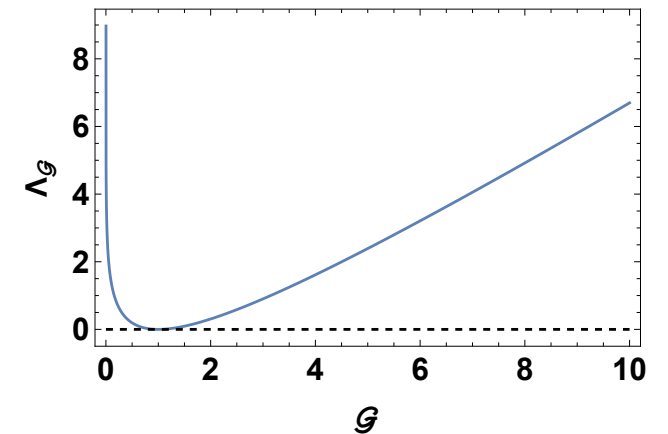
and only zero in the Minkowski vacuum

In the low energy small curvature limit

$$\tilde{\mathcal{G}} = \left[\tilde{\mathbf{I}} + \alpha' \ell_P^d \tilde{\mathbf{M}} \tilde{g}^{-1} - \beta' \ell_P^2 \tilde{\mathcal{R}} \tilde{g}^{-1} \right]^{-1} = \tilde{\mathbf{I}} + \tilde{\mathcal{E}}$$

the cosmological constant $\Lambda_{\mathcal{G}}$ is small

$$\Lambda_{\mathcal{G}} \simeq \frac{1}{2\beta' \ell_P^2} \text{Tr}_F \tilde{\mathcal{E}}^2$$



THE EMERGENT COSMOLOGICAL “CONSTANT”

The emergent cosmological “constant” given by

$$\Lambda_{\mathcal{G}} = \frac{1}{\beta' \ell_P^2} \text{Tr}_F[\tilde{\mathcal{G}} - \tilde{\mathbf{I}} - \ln(\tilde{\mathcal{G}})]$$

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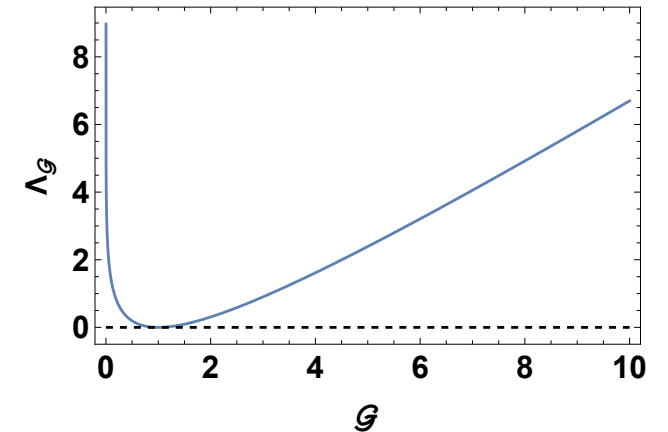
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In the low energy small curvature limit

$$\tilde{\mathcal{G}} = \left[\tilde{\mathbf{I}} + \alpha' \ell_P^d \tilde{\mathbf{M}} \tilde{g}^{-1} - \beta' \ell_P^2 \tilde{\mathcal{R}} \tilde{g}^{-1} \right]^{-1} = \tilde{\mathbf{I}} + \tilde{\epsilon}$$

the cosmological constant $\Lambda_{\mathcal{G}}$ is small

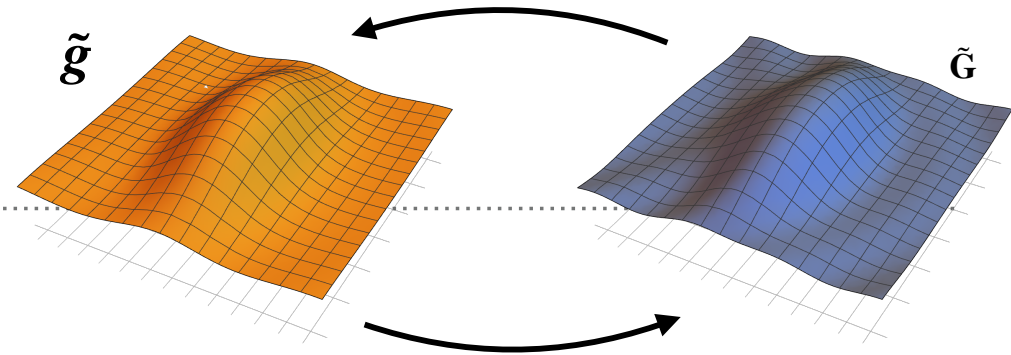
$$\Lambda_{\mathcal{G}} \simeq \frac{1}{2\beta' \ell_P^2} \text{Tr}_F \tilde{\epsilon}^2$$



*Note that $\Lambda_{\mathcal{G}}$ diverges
for high curvature
and high energies*

THE GfE THEORY

$$\mathcal{L} = -\text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{\mathbf{g}}^{-1}$$



Gravity

results

from a new **GfE** action

given by

Geometric Quantum relative Entropy

between the metric and the metric associated to the matter fields and curvature.

The matter fields are described by a Dirac-Kahler type of description.

DERIVATION OF THE AREA LAW

HOLOGRAPHIC ARGUMENT

We consider the Gibbs-entropy

$$\mathcal{S} = \ln \Omega$$

Assuming that each elementary volume in space-time is associated to the same number of degrees of freedom

(example 2 for spin $s = 1$ and spin $s = -1$)

$$\Omega = 2^V$$

The entropy is therefore extensive on the volume of spacetime

$$S = V \ln 2$$

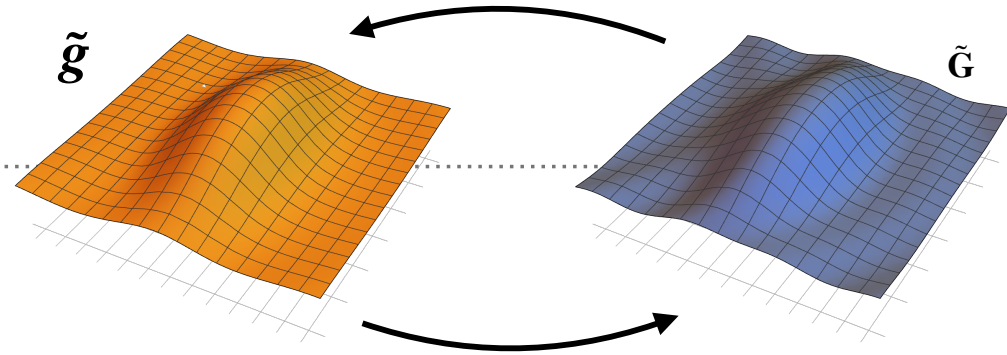
Holographic argument:

In order to obey the area-law $S = \frac{A}{4G}$

the degrees of freedom must be only associated to the horizon.

GEOMETRIC QUANTUM RELATIVE ENTROPY

The Gravity from Entropy Lagrangian given by



$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = -\text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

can be expressed also as

$$\mathcal{L} = \ln \Omega$$

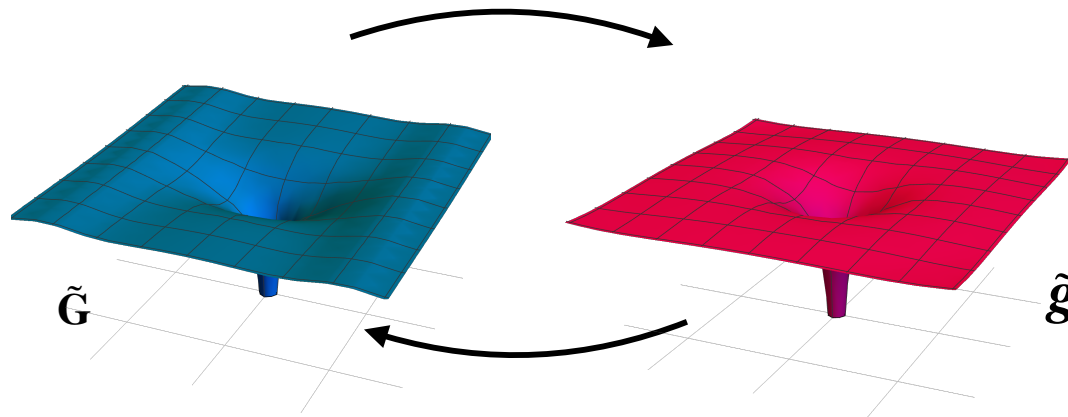
where

$$\Omega = G_{(0)}^{-1}(\det(\mathbf{G}_{(1)}^{-1}g))(\det(\mathbf{G}_{(2)}^{-1}g_{(2)}))$$

*Ω evaluates the number of degrees of freedom of the metric,
and is not uniform for a black-hole*

QUANTUM RELATIVE ENTROPY OF THE SCHWARZSCHILD METRIC

$$\mathcal{L} = \text{Tr} \tilde{g} \ln \tilde{\mathbf{G}}^{-1} = - \text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{g}^{-1}$$

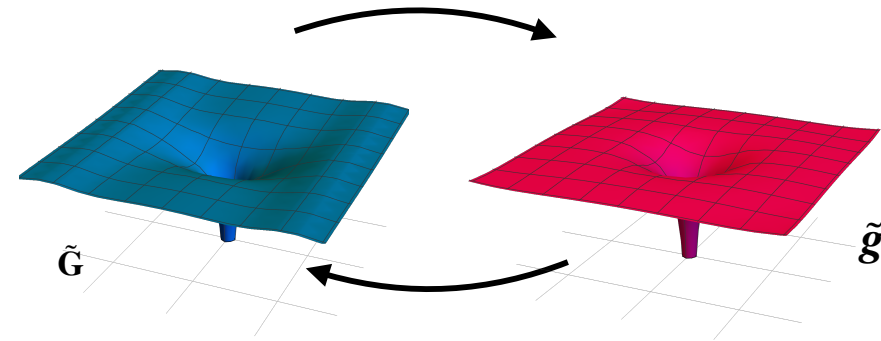


We can calculate the geometric quantum relative entropy of the Schwarzschild metric also if this metric is only the approximate solution of the equation of modified gravity

QUANTUM RELATIVE ENTROPY OF SCHWARZSCHILD METRIC

In absence of matter,
the metric induced by the curvature is

$$\tilde{\mathbf{G}} = \tilde{\mathbf{g}} - \beta \tilde{\mathcal{R}}$$



The Schwarzschild metric
is an approximate solution
of the modified gravity equations for a black hole
good for large Schwarzschild radius

[Bianconi, G., Entropy 27, 266 (2025)]

EINGENVALUES OF THE METRIC INDUCED BY THE CURVATURE

For the Schwarzschild metric
the metric induced by the curvature is

$$\tilde{\mathbf{G}} = \tilde{\mathbf{g}} - \beta \tilde{\mathcal{R}}$$

has non-trivial eigenvalues

$$1 - 2\beta \frac{R_s}{r^3}, \text{ with deg} = 2;$$

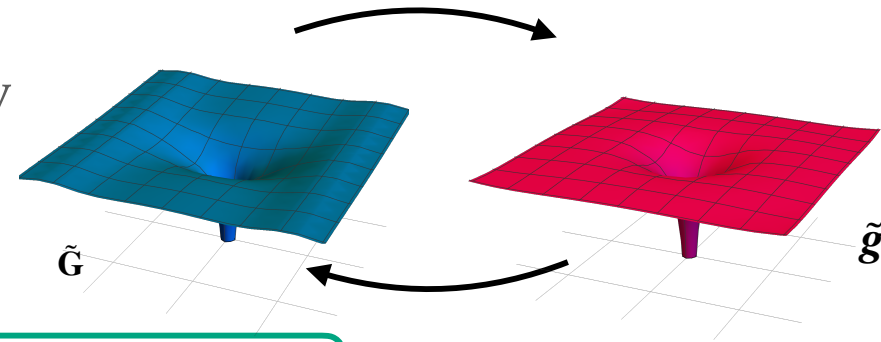
$$1 + \beta \frac{R_s}{r^3}, \text{ with deg} = 4$$

It is therefore positively defined only for

$$r > r_0 = (2\beta R_s)^{1/3}$$

GEOMETRICAL QUANTUM RELATIVE ENTROPY OF SCHWARZSCHILD METRIC

The geometrical quantum relative entropy of the Schwarzschild metric is



$$\mathcal{S} = \frac{1}{\ell_P^4} \int_0^{\kappa^{-1}\tau'} dt \int_{r_0}^{R_s} dr \sqrt{-|g|} \mathcal{L} \text{ with } \mathcal{L} = -\ln \left[\left(1 - 2\beta \frac{GR_s}{r^3} \right)^2 \left(1 + \beta \frac{GR_s}{r^3} \right)^4 \right]$$

only defined for $R_s > R_0 = \sqrt{2\beta G}$

[Bianconi, G., Entropy 27, 266 (2025)]

QUANTUM RELATIVE ENTROPY OF SCHWARZSCHILD METRIC

For large Schwarzschild R_s

when the Schwarzschild metric is a better approximation of the solution of the modified gravity equations

the geometric quantum relative entropy obeys the area law

$$\mathcal{S} \simeq \mathcal{S}_A = \mathcal{C} \frac{A}{4G}$$

where $\mathcal{C} \propto \beta' \tau'$ is adimensional

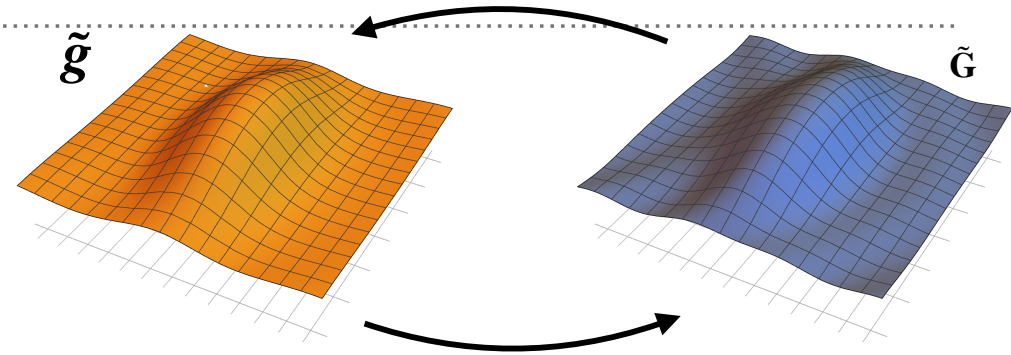
The temperature of the black hole is proportional to the Hawking temperature

$$T \simeq \frac{T_H}{\mathcal{C}}$$

[Bianconi, G., Entropy 27, 266 (2025).]

THE GfE THEORY

$$\mathcal{L} = -\text{Tr}_F \ln \tilde{\mathbf{G}} \tilde{\mathbf{g}}^{-1}$$



Gravity

results

from the GfE action

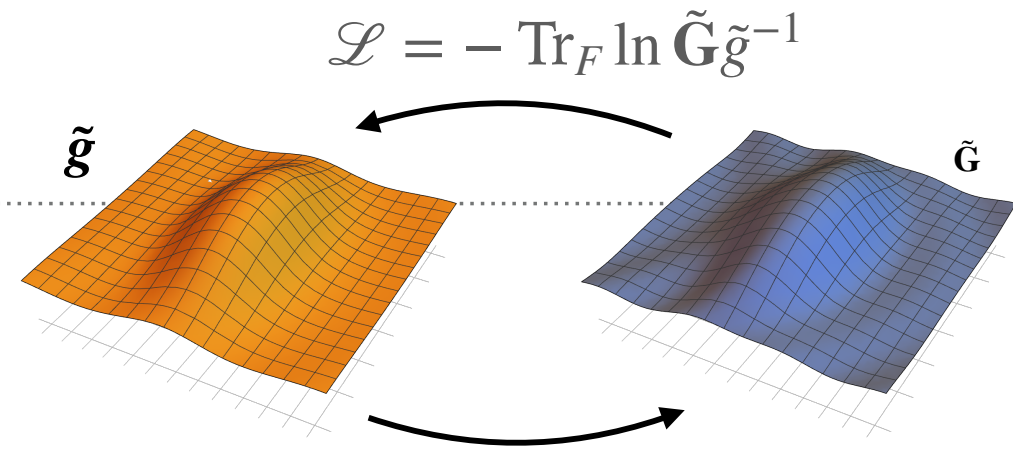
given by

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SUMMARY



- The Gravity from Entropy action is proposed
- In the low-energy, small curvature limit we recover the Einstein-Hilbert action with zero-cosmological constant
- The Gravity from Entropy action whose Lagrangian is given by the Geometrical Quantum Relative Entropy leads to a modified theory of gravity expressed in terms of the G -field, predicting a dynamical cosmological constant
- The area law for the Schwarzschild black-holes is derived from the Gravity from Entropy approach

FURTHER NEW RESULTS ON THE COSMOLOGICAL AND ASTROPHYSICAL IMPLICATIONS OF THE GFE THEORY

► Thermodynamics of GfE Cosmologies

Bianconi, G., 2025. The Thermodynamics of the Gravity from Entropy Theory. *arXiv preprint arXiv:2510.22545*.

► Inflation from Entropy

Thattarampilly, U. and Zheng, Y., 2025. Inflation from entropy. *The European Physical Journal C*, 85(12), p.1433.

► Weak-field corrections

Thattarampilly, U., Zheng, Y. and Kakkat, V., 2026. Spherically symmetric black holes in Gravity from Entropy and spontaneous emission. *arXiv preprint arXiv:2602.13694*.

BENEFITS OF THE GfE THEORY

- **Field theory formulation**
 - The GfE theory is the unique entropic gravity approach that stem from an action (reducing to Einstein-Hilbert action in the low-energy small-curvature limit)
- **GfE leads to modified gravity**
 - The GfE theory leads to modified equation of motion that can lead to testable predictions of the theory in cosmology and astrophysics.
- **The theory is local, within its Dirac-Kalher treatment of quantum dynamics**
 - Thus the theory can be potentially extended to all matter fields
- **GfE Lagrangian is given by GQRE related to the Araki relative entropy**
 - This connection provides a possible direction for the second quantisation of the GfE theory

OPEN QUESTIONS

➤ Statistical mechanics questions

- How to best reconcile gravity with the second principle of thermodynamics?
- Are actually quantum gravitational states thermal?

➤ Cosmological and Astrophysical questions

- Can the emergent cosmological constant of the GfE theory predict phenomena related to the increasing evidence for a dynamical cosmological constant?
- Do the GfE modified gravity solutions dynamically avoid singularities?

➤ Quantum questions

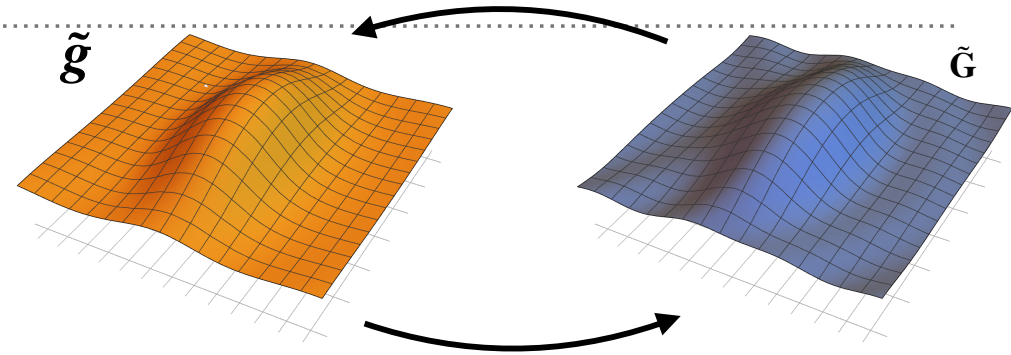
- To what extent is the GQRE an entanglement entropy?
- How to fully quantize the GfE action?

TO DO LIST

- Cosmological implications of the GfE theory
- Second quantisation of the GfE theory
- Relation to the theory of von Neumann algebras and the theory of entanglement
- Extension of this framework to non-Abelian gauge fields, and to Dirac fields
- Connection with dark energy and dark matter experimental evidence
- Discretisation of this geometrical quantum entropic action

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