

# Emergence of (3+1)-dimensional expanding spacetime from the Lorentzian type IIB matrix model

Jun Nishimura (KEK, SOKENDAI)

talk at the KEK-iTHEMS joint workshop  
“Concepts of Quantum and Spacetime”  
KEK, Tsukuba, Japan, March 9-12, 2026,

Ref.) Anagnostopoulos, Azuma, Hirasawa, Karydis, J.N., Tsuchiya, Yamamori,  
work in progress

# Plan of the talk

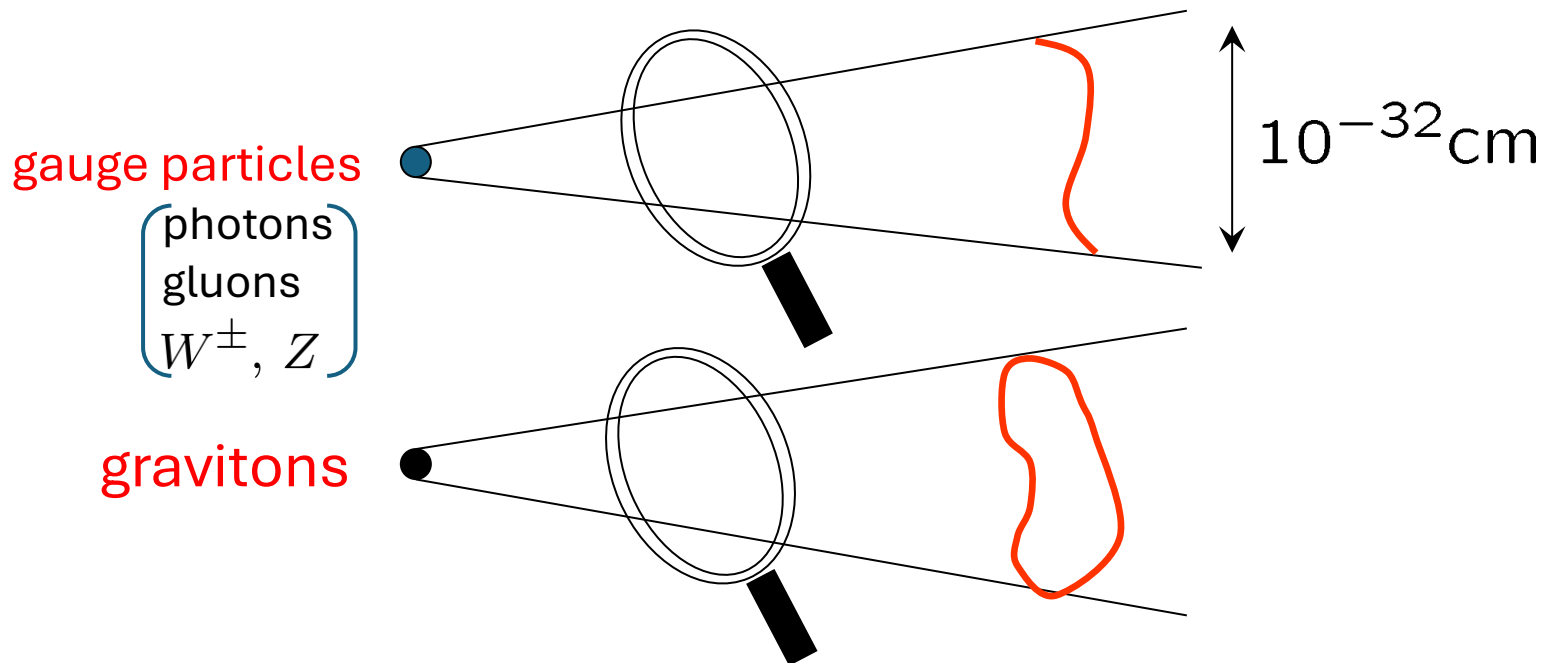
1. Brief review of string theory
2. Results of our simulation
3. Summary and discussions

# 1. Brief review of string theory

# Why strings?

1974 Sherk-Schwarz, Yoneya  
1984 Green-Schwarz

Various vibration modes  $\leftrightarrow$  various particles



- All the 4 fundamental interactions including gravity are described in a unified manner.
- No UV divergences. (No nonrenormalizability issues.)
- Spacetime dimensionality is fixed to (9+1) for consistency.

# Perturbative v.s. Nonperturbative

**perturbative**

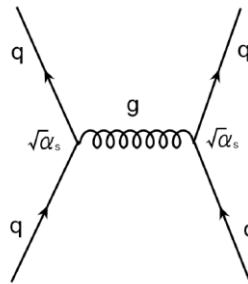
Effective in describing high-energy scattering

**nonperturbative**

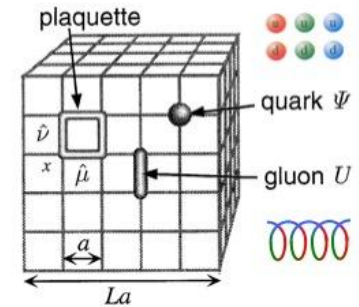
Indispensable in studying the vacuum and the excited states

**QCD**

QFT describing a many-body system of quarks & gluons



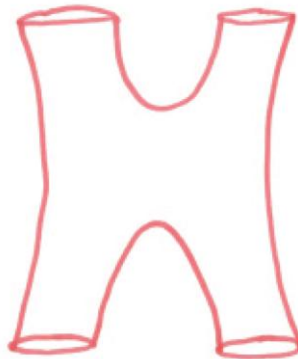
(Feynman diagram)



(lattice gauge theory)

**String theory**

Quantum theory describing matters, force & spacetime

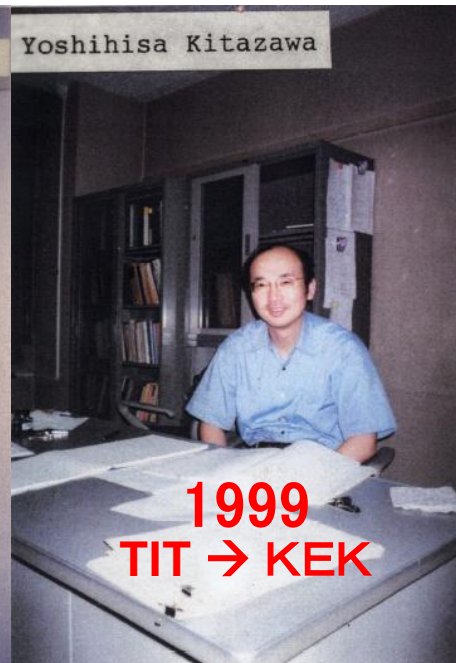


# A nonperturbative formulation of string theory

## IKKT matrix model (or type IIB matrix model)

Nucl.Phys.B 498 (1997) 467-491, e-Print: [hep-th/9612115](https://arxiv.org/abs/hep-th/9612115) [hep-th]

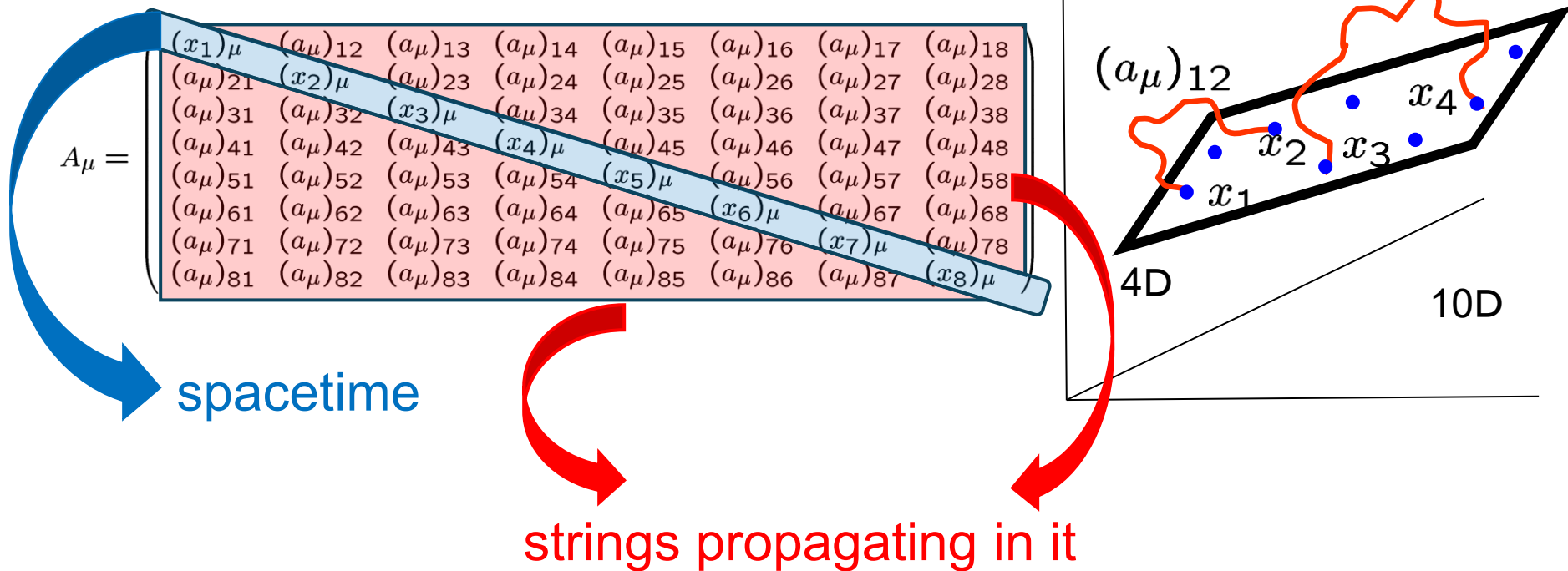
The fundamental d.o.f. are  $(9+1) N \times N$  Hermitian matrices.  
String picture appears in the large- $N$  limit.



# Spacetime picture in the IKKT matrix model

10  $N \times N$  matrices  $A_\mu$  ( $\mu = 0, 1, \dots, 9$ )

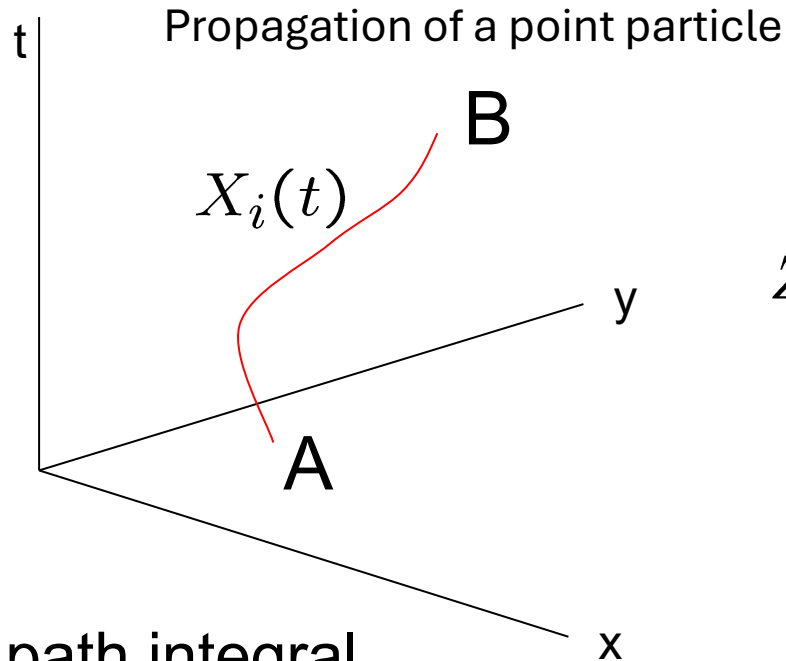
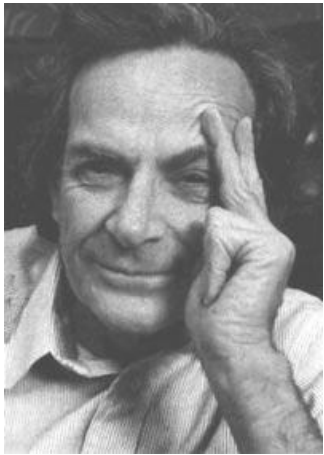
Example of  $N = 8$



Does four-dimensional spacetime appear dynamically from this model?

# Quantum theory based on summation over histories

c.f.) Fay Dowker's talk on 3/11



$$Z = \int \mathcal{D}X_i(t) e^{iS[X]}$$

## Feynman's path integral

Quantum time evolution can be described by summing over all the histories.  
(integral) (paths)

In this case, equivalent to Schrödinger eq., but in general, path integral is considered more fundamental than Schrödinger eq.

# matrix integral as a summation over histories

$$Z = \int dA e^{iS[A]}$$
$$S[A] = -\frac{1}{4g^2} \text{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) + \dots$$

$\mu, \nu$  : Lorentz indices

integration over all the elements of (9+1) Hermitian matrices

$$A_\mu \quad (\mu = 0, 1, \dots, 9)$$

- Standard Monte Carlo sampling does not work due to “**sign problem**”.  
→ **complex Langevin method** Parisi ('83), Klauder ('83)
- Use **supercomputers** to generate  
(general complex) **matrix configurations**
- Expectation values of **holomorphic observables**  
can be calculated by taking **an ensemble average**.

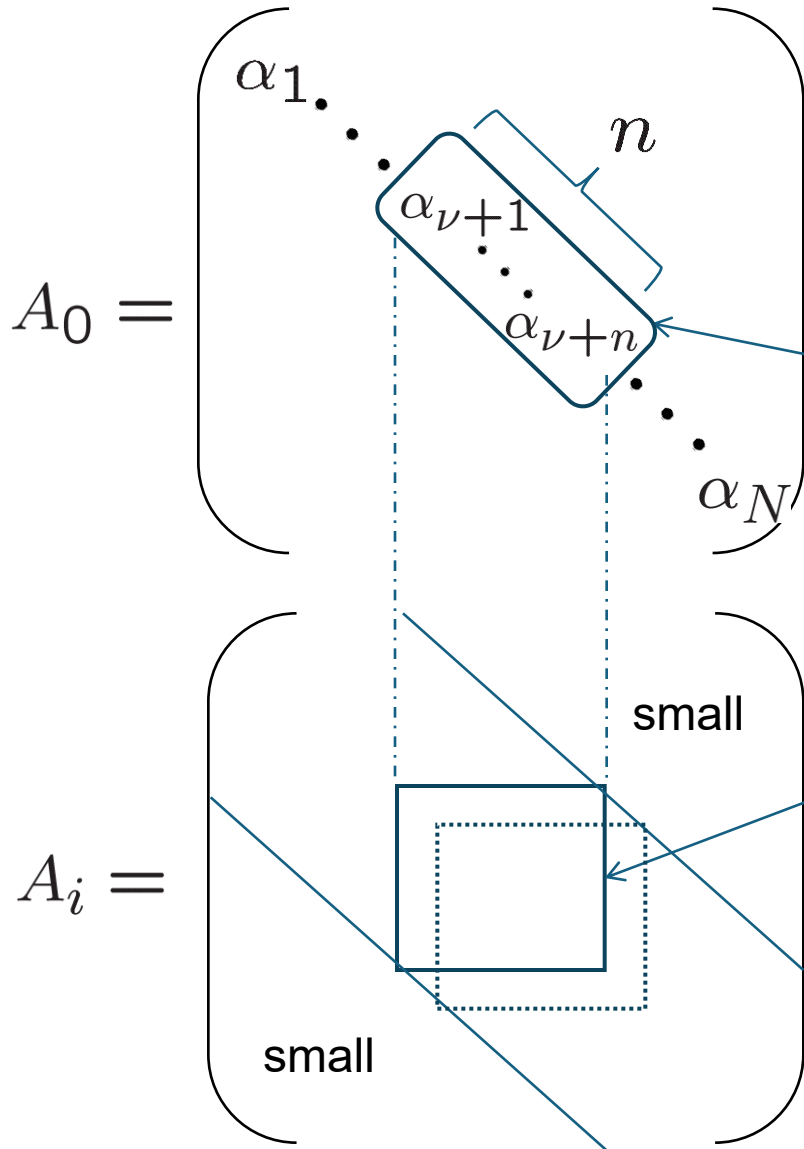


# extracting time-evolution from the Lorentzian model

Using  $A_\mu \rightarrow UA_\mu U^\dagger$ ,  $U \in SU(N)$

Kim-J.N.-Tsuchiya

PRL 108 (2012) 011601 [arXiv:1108.1540]



$$\alpha_1 < \dots < \alpha_N$$

definition of "time"

$$t_\nu = \frac{1}{n} \sum_{i=1}^n \alpha_{\nu+i}$$

The state of the universe  $\bar{A}_i(t_\nu)$  at time  $t_\nu$

$A_i$  has a band diagonal structure

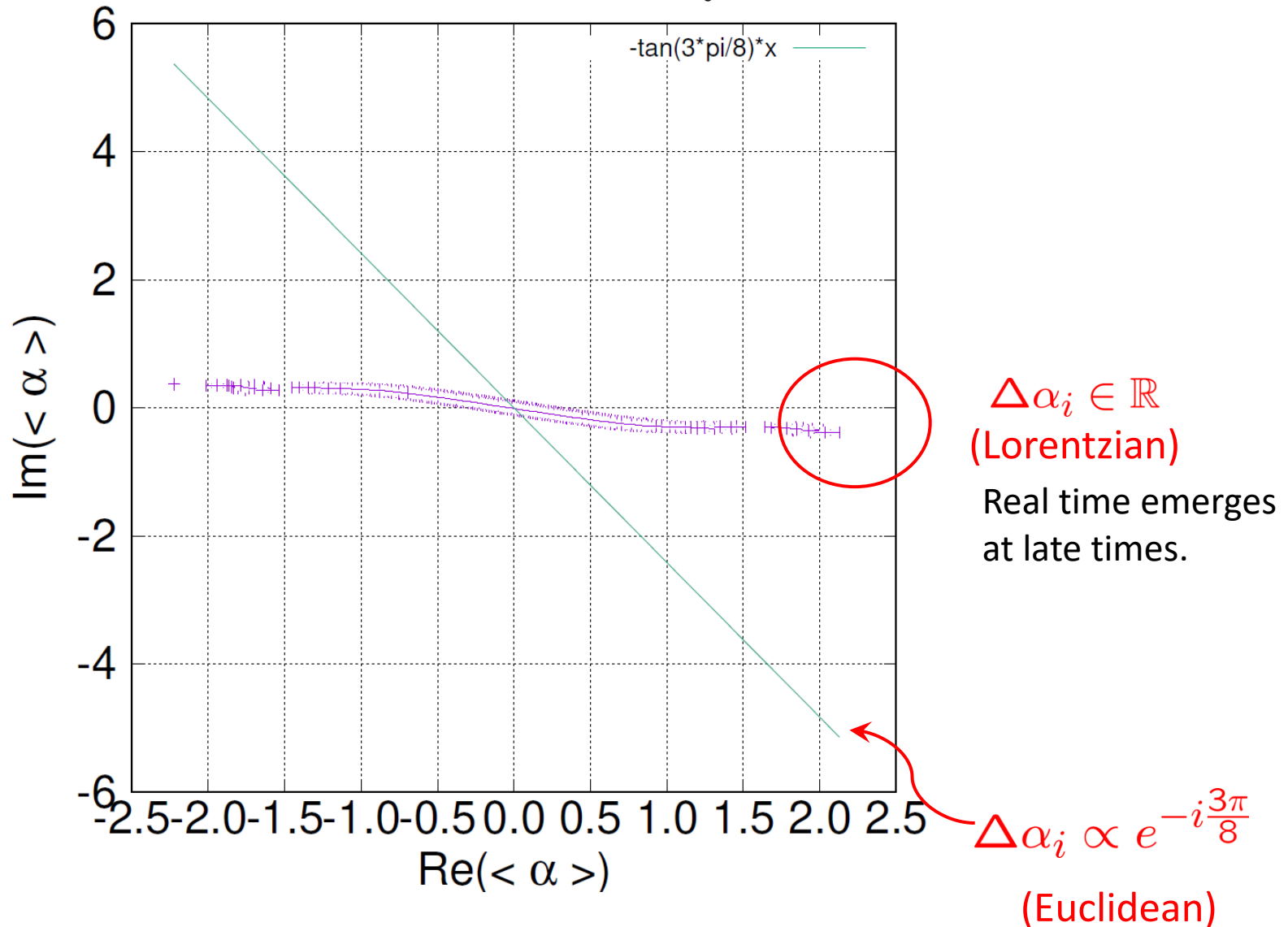
non-trivial dynamical property

## 2. Results of our simulations

Anagnostopoulos, Azuma, Hirasawa, Karydis, J.N., Tsuchiya, Yamamori, work in progress

# eigenvalues of $A_0$

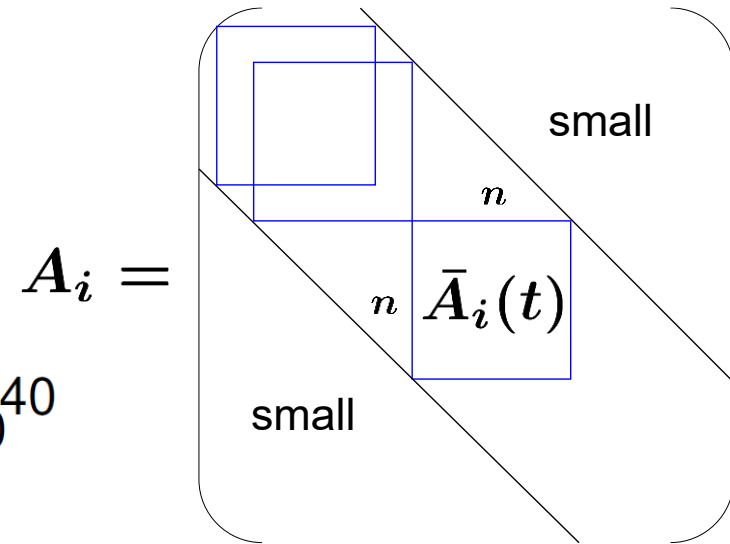
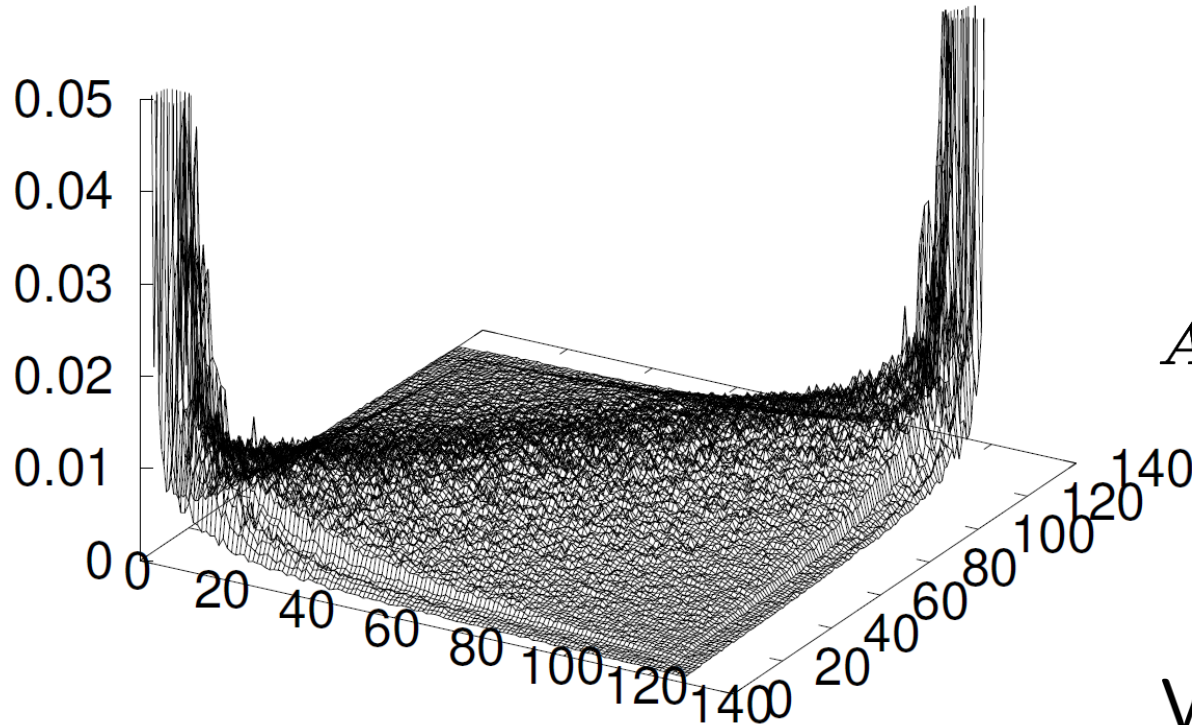
$$N = 128, \gamma = 4, m_f = 5, \xi = 10, \tilde{d} = 5$$



# band-diagonal structure

$$A_{pq} = \frac{1}{9} \sum_{i=1}^9 |(A_i)_{pq}|^2$$

$$N = 128, \gamma = 4, m_f = 5, \xi = 10, \tilde{d} = 5$$



We choose  $n = 6$ .

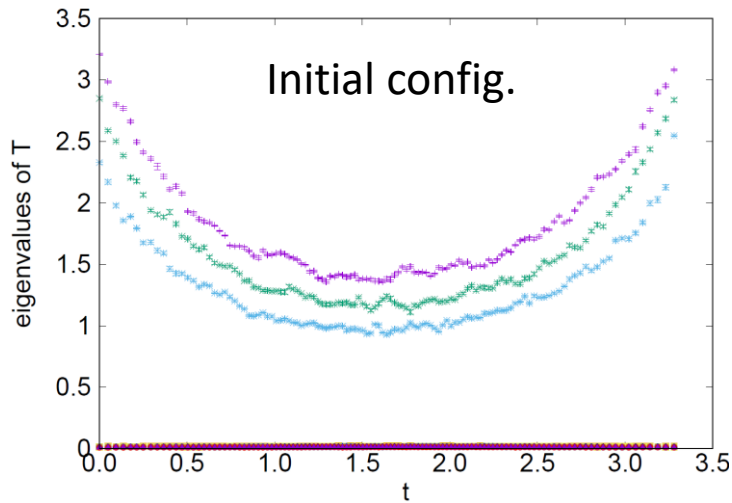
emergence of band-diagonal structure



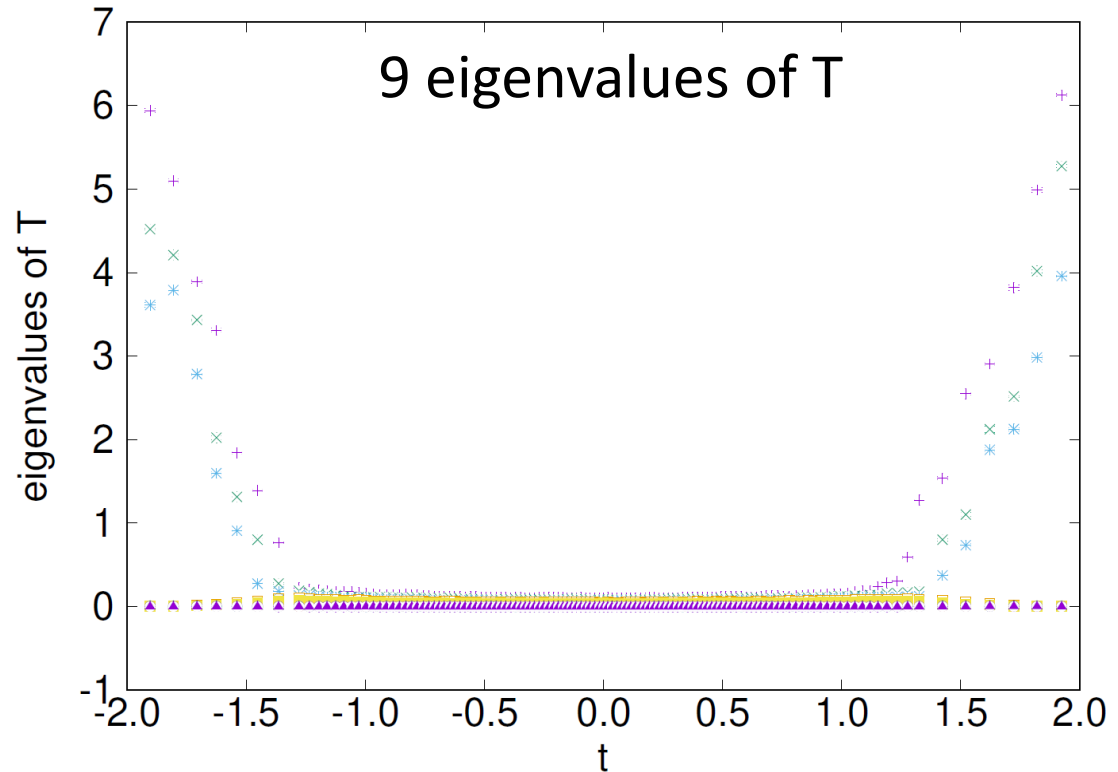
important for extracting the time evolution from matrix configs.

# emergence of 3d expanding space-time

$$T_{ij}(t) = \frac{1}{n} \text{tr} \left( \bar{A}_i(t) \bar{A}_j(t) \right)$$



$$N = 128, \gamma = 4, m_f = 5, \xi = 10, \tilde{d} = 5$$



3 out of 9 directions start to expand at some point in time.  
(SSB of rotational symmetry)

### 3. Summary and discussions

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c.f.) Opening address  
by Yuki Yokokura



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2) How does the **classical spacetime picture** appear from quantum spacetime?

3) Can we study the quantum dynamics of spacetime from the viewpoint of **many-body quantum systems**?

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Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob,  
*PTEP* 2020 (2020) 4, 043B10 , e-Print: 1911.08132 [hep-th]

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Numerical simulations of the Lorentzian IKKT matrix model.

# Future prospects

- To be more precise, we added **quadratic terms** in the IKKT action in order to make the **complex Langevin method** work.

Need to understand their effects.

Can we remove them by new simulation algorithms?

**Lefschetz thimble method**

Chou- JN-Tripathi

Phys.Rev.Lett. 134 (2025) 21, 211601, e-Print: 2501.17798 [hep-th]

- Can we obtain **the Standard Model** from the IKKT matrix model?

**Intersecting D-branes** in the extra dimensions typically give rise to **vector-like** gauge theories. How can we get **chiral** gauge theories?

Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob,

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