Quantum Null Ray: Localized gauge invariant observables and Covariant quantization

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KEK
03-2026

Based on 2309.03932, 2404.11132 with Luca Ciambelli and Rob Leigh

2510.26589 & To appear with ***Josh Kirklin***

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Quasi-Local gauge invariant observable

- A central theme of my work is to develop a bottom-up approach to Quantum Gravity (QG) that focus on understanding universal features of QG and its phenomenology
- This includes the study of the nature of gravitational entanglement, of background independence, it requires a deeper dive into the constraints, and its holographic properties and focuses on the understanding of gravitational fluctuations and is also connected to the infrared issues.
- All these questions necessitate understanding the nature of gravitational subsystems and of the construction of quasi-local gauge invariant observables

Page-Wooters, Rovelli, Dittrich, Giddings, Marolf

- One of the main challenge we face in connecting QFt to QG is the challenge of background independence
- What became clear from many sources is that it requires a deeper understanding relational observables and QFT in the presence of Quantum Reference Frame (QRF)

CLPW,
Fewster, Janssen, Rezjner
Loveridge, Glowacki,...

Quasi-Local gauge invariant observable

The construction of localized gravitational observables requires three ingredients:

- An understanding of edge modes and dressing fields necessary to localize the gauge algebra

[LF Donnelly '16]
[Ciambelli, Leigh, Hoen, Carrozza]
LF, Kirklin 25

- The necessity to understand Dressing fields and their quantization
  Field theoretical QRF: Kahler reference frame

Strominger, Pate, Raclariu, LF, Kirklin 25

- An deeper understanding of null ray Physics: The importance of Raychaudhuri eq

[Wall '11. Bousso, Holland-Wald]

The presence of anomalies requires a deformation of the classical effective description and implies that [Quantization, Reduction] $\neq 0$

Null geometry

A universal and foundational element of classical and quantum geometry are null rays.

We have discoverered that null rays described by a universal structure that enables quantization. Quantum Null rays (called embadons) therefore constitute an fundamental building block of any effective description of quantum gravity and support proofs of the GSL along horizons

The key elements of Quantum Geometry: Corner symmetries, Quantum references frames, Dressing fields and localization are realized in the study of null Horizons where the localization program can be carried out explicitly.

In this context the quantum reference frame activated is the null time, so null geometry also allow to address the nature of time at the quantum level

Geometry of Null Hypersurfaces

[Levy-Leblond ’64, Ashtekar ’78 -’24, Henneaux ’81, Dautcourt ’97, Duval-Gibbons-Horvarthy ’14, …]

$\mathcal{N}$ 3d Null Manifold

Carrollian Structure (ℓ^a, q_{ab}) with $\ell^a q_{ab} = 0$ $\qquad \ell^a \partial_a = \partial_v$

Ruling k_a with $\ell^a k_a = 1$

Traceless

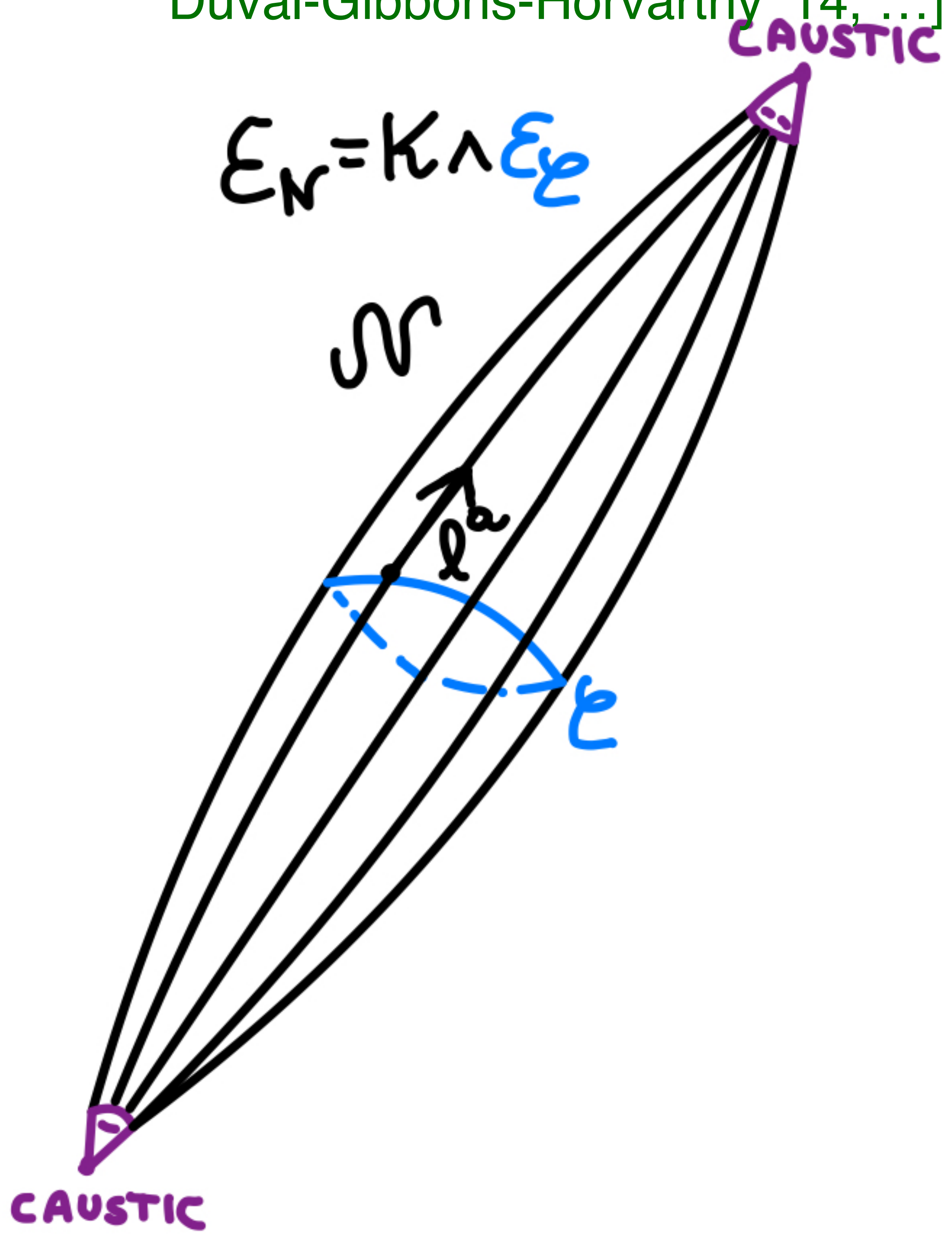
Expansion Tensor $\theta_{ab} = \frac{1}{2}\mathcal{L}_\ell q_{ab}$ such that $\theta_a{}^b = \frac{\theta}{2} q_a{}^b + \sigma_a{}^b$

Carrollian Connection: ‘preserve’ the metric as

$$D_a q_{bc} = -(k_b \theta_{ac} + k_c \theta_{ab})$$

It defines a boost connection $D_a \varepsilon_{\mathcal{N}} = -\omega_a \varepsilon_{\mathcal{N}}$ With $\omega_a = \kappa k_a + \pi_a$

Inaffinity



Dynamics on Null Hypersurfaces

[Ciambelli-Petropoulos'19
Donnay-Marteau '19,
Chandrasekaran-Flanagan-
Shehzad-Speranza '21]
[Freidel-Jai-akson '22]

Null Brown York Tensor $T_a{}^b := D_a \ell^b - \delta_a^b D_c \ell^c$

Einstein Gravity projected to $\mathcal{N}$: $D_b T_a{}^b = 8\pi G T^{\mathrm{mat}}_{a\ell}$

Projected along ℓ^a : $(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b \sigma_b{}^a - 8\pi G T^{\mathsf{mat}}_{\ell\ell}$ Null Raychaudhuri Equation

Projected along $q_c{}^a$: $(\mathcal{L}_\ell + \theta)\pi_a = D_a \mu - D_b \sigma_a{}^b + 8\pi G T^{\mathsf{mat}}_{a\ell}$ Damour Equation

$\mu = \kappa + \frac{\theta}{2}$ Surface Tension

Fluid-Gravity perspective, Membrane paradigm [Price-Thorne '88]

Symplectica

Starting from the gravitational action $\delta L_{\mathsf{EH}} = \mathrm{d}\Theta_{\mathsf{EH}} + G^{ab}\delta g_{ab}$

$$\Theta_{\mathsf{EH}} = \frac{1}{2} D_a \left(\sqrt{g}\delta\ell^a\right) - \delta\left(\sqrt{g} D_a \ell^a\right) + \Theta_{\mathsf{can}}$$

We extract after a canonical transformation a canonical Carrollian-fluid potential

Intrinsic to $\mathcal{N}$

Symplectic potential $\quad \Omega_{\mathsf{EH}} = \delta\Theta_{\mathsf{EH}}$

[Hayward '93, Ashtekar '00, Lewandowski '04, Lehner-Myers-Poisson-Sorkin '16, Parattu-Chakraborty-Padmanabhan '16, LF-Hopfmueller'16, DePaoli-Speziale '17, Adami-Grumiller-Sheikh-Jabbari-Taghiloo-Yavartanoo-Zwikel '21, Chandrasekaran-Speranza '21, Chandrasekaran-Flanagan-Shehzad-Speranza '21, LF-Jai-akson '22]

Kinematical Poisson Brackets

[Ciambelli, LF,Leigh'23]

$$\Omega^{\text{can}} = \frac{1}{8\pi G}\int_{\mathcal{N}} \varepsilon^{(0)}_{\mathcal{N}} \left(\delta\left(\tfrac{1}{2}\Omega\sigma^{ab}\right)\wedge\delta q_{ab} - \delta\mu\wedge\delta\Omega\right)$$

Spin2 : q_{ab} and $\Omega\sigma^{ab}$ are conjugate variable : Radiation modes

Spin0 : Ω and μ are conjugate variable : Newtonian modes

Raychaudhuri constraint

$$C = \partial_v^2\Omega - \mu\partial_v\Omega + \Omega\left(\sigma^2 + 8\pi G T^{\text{mat}}_{vv}\right) = 0$$

Time Reparameterization Charge:

$$\mathcal{E}_f = \frac{1}{8\pi G}\int_{\mathcal{N}} \varepsilon^{(0)}_{\mathcal{N}} fC + \frac{1}{8\pi G}\int_{\mathcal{C}} \varepsilon^{(0)}_{\mathcal{C}} (\Omega\partial_v f - f\partial_v\Omega)$$

area element

= local boosts generator

Decoupling regime

[LF, Kirklin]

Linearized regime for spin 2 $\quad q_{ab} = \Omega\left(q^{(0)}_{ab} + \varkappa h_{ab}\right)$

where $\quad \partial_v q^{(0)}_{ab} = l^a q^{(0)}_{ab} = 0 \qquad \det q^{(0)} = 1$

Chose matter to be a collection of massless scalar

At this order $\quad \partial_v h_a{}^b \partial_v h_b{}^a + 8\pi G T^{\text{mat}}_{vv} = 8\pi G \sum_{i=1}^{M} (\partial_v \phi_i)^2$

Radiative dof
Includes gravitons + matter fields

$$\boxed{C = \partial_v^2 \Omega - \mu\Omega + 8\pi G\Omega \left(\sum_i (\partial_v \phi_i)^2\right) = 0}$$

Coupling matter dof with spin 0 : cubic coupling + gravitational back-reaction

Spin 2 behaves like matter = radiative dof are the usual EFT degrees of freedom

Remarkably, we can achieve decoupling and create a QRF + clock system

Decoupling regime

One defines

$$\varphi_i = \sqrt{\Omega}\phi_i, \qquad \beta = \overset{\kappa+\theta/2}{\overset{\downarrow}{\mu}} + 4\pi G_{\mathrm{N}} \frac{1}{\sqrt{\Omega}} \partial_v\Big(\sqrt{\Omega}\sum_i \phi_i^2\Big),$$

In term of which the constraint becomes

$$C = \underbrace{\frac{1}{8\pi G}(\partial_v^2\Omega - \beta\partial_v\Omega)}_{=\tau} + \underbrace{\sum_{i=1}^{M}(\partial_v\varphi_i)^2}_{T_{\text{rad}}}$$

This achieve decoupling: The gravitational back-reaction is entirely encoded in spin 0 = Physical clock for the radiative degrees of freedom

$$C = H_{\text{Clock}} + H_{\text{System}}$$

This decoupling allows one to use the QRF technics to quantize C and impose $C = 0$

Ultra-Locality

Different null rays are out of causal contact, they are dynamically decoupled and can be decoupled through the principle of ultra-locality. One can perform quantization and constraint imposition ray-by-ray and we now focus on a single ray An embadon

$$\langle \partial_{v_1}\phi \partial_{v_2}\phi \rangle = \langle \partial_{v_1}\phi \rangle \langle \partial_{v_2}\phi \rangle$$

[Kay-Wald'91]

In term of which the constraint becomes

$$C = \partial_v^2 \Omega - \beta \partial_v \Omega + 8\pi G_N \sum_i (\partial_i \varphi_i)^2 .$$

And the null ray symplectic potential reads

$$\boldsymbol{\Omega} = \int_{\mathbb{R}} \mathrm{d}v \left(\frac{1}{8\pi G_{\mathrm{N}}} \delta\Omega \wedge \delta\beta + \sum_i \partial_v (\delta\varphi_i) \wedge \delta\varphi_i \right),$$

Cross-product QRF

The construction of the Quantum Gauge invariant observables
Requires the choice of a quantum dressing map

One starts with the kinematical Hilbert space $\mathcal{H}_{\text{Kin}} = \mathcal{H}_S \otimes L^2(G)$

And construct the set of gauge invariant observables

$$\mathcal{A}_{\text{S}} = \mathcal{B}(\mathcal{H}_S)$$

$$\mathcal{A}_{\text{QRF}} = \mathcal{B}(L^2(G)) = \mathbb{C}[\pi, g = e^{i\hat{T}}]$$

$$\boxed{\mathcal{A}_{\text{Phys}} = (\mathcal{A}_S \times \mathcal{A}_{\text{QRF}})^G}$$

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The group action is generated by the action of the constraint $C_a = \pi_a^{\mathsf{QRF}} + H_a^{\mathsf{S}} = 0$

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Thm: This algebra is equal to the cross-product algebra $\mathcal{A}_{\text{Phys}} = \hat{\mathcal{A}}_S \rtimes G$

Dressed observables

Reorientations

$D(\pi^{\text{QRF}})_a$

$$\hat{\mathcal{A}}_S = \{D(a) = e^{i\hat{T}^a H_a^{\text{S}}} a e^{-i\hat{T}^a H_a^{\text{S}}} | a \in \mathcal{A}_{\text{S}}\}$$

QRF

Chandrasekaran, Longo, Penington, Witten, Jensen, Sorce, Speranza, De Vuyst, Eccles, Höhn

Cross-product Challenges

$$\mathcal{A}_{\mathsf{Phys}} = \hat{\mathcal{A}}_S \rtimes G$$

Dressed observables

Reorientations

- The group $G = \mathrm{Diff}(\mathbb{R})$ is not locally compact $L^2(G)$ doesn't exist

- The construction of $\mathcal{H}_S$ relies on a choice of vacuum which breaks G

  In QFT given a diffeomorphism F $F \triangleright \mathcal{H}_{\mathsf{S}} \neq \mathcal{H}_{\mathsf{S}}$

- The symmetry becomes anomalous

- We develop a notion of Kahler QRF that allows the construction of Field theoretical QRF = Goldstones and allows the resolution of anomalies !

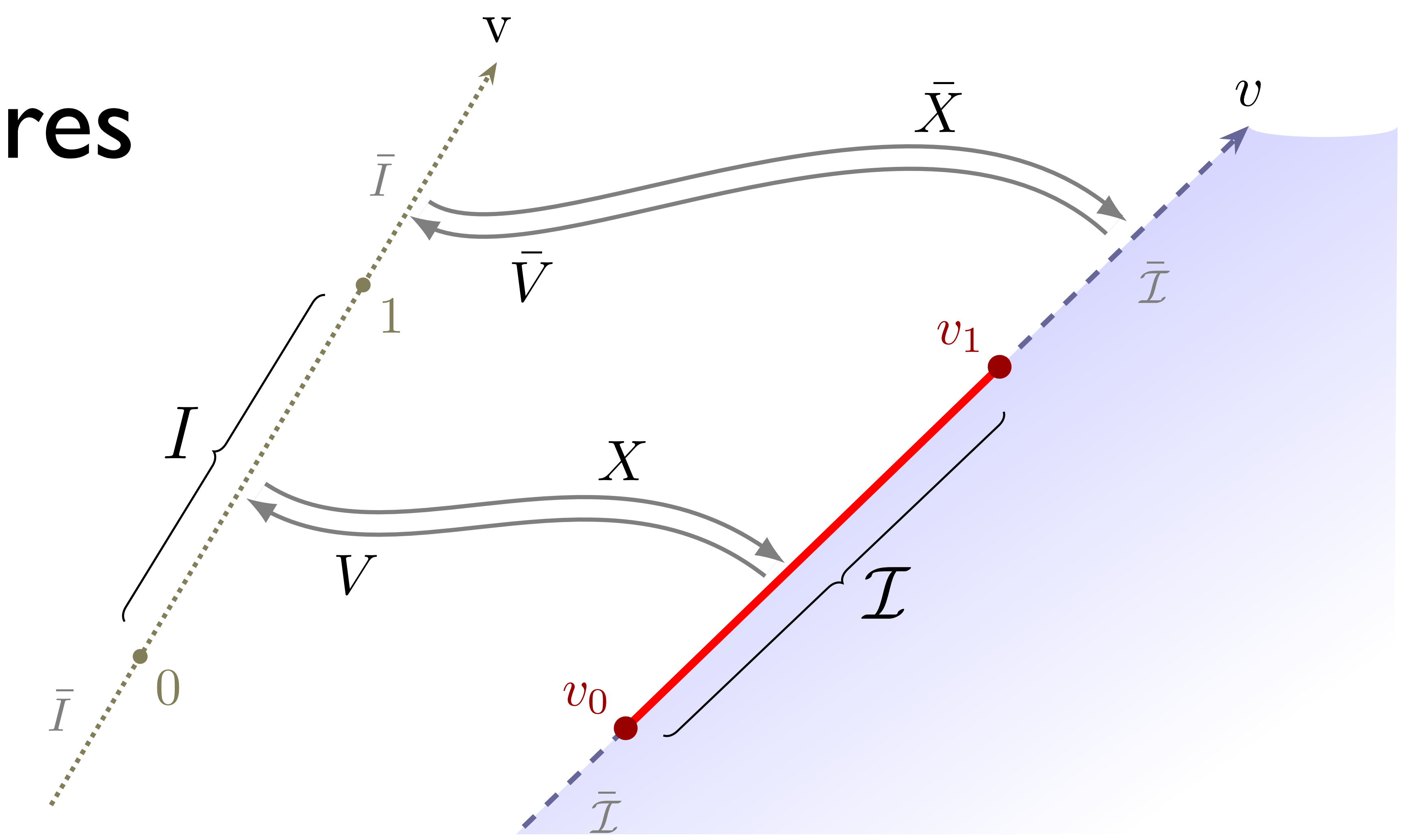
Dressing time I

[Ciambelli-Leigh-LF, Wieland]

The construction of GI observables inside $\mathcal{I}$ requires internal dressing

The dressing time $V : \mathcal{I} \to I = [0,1]$

is such that $$\boxed{\frac{\partial_v^2 V}{\partial_v V} = \beta}$$



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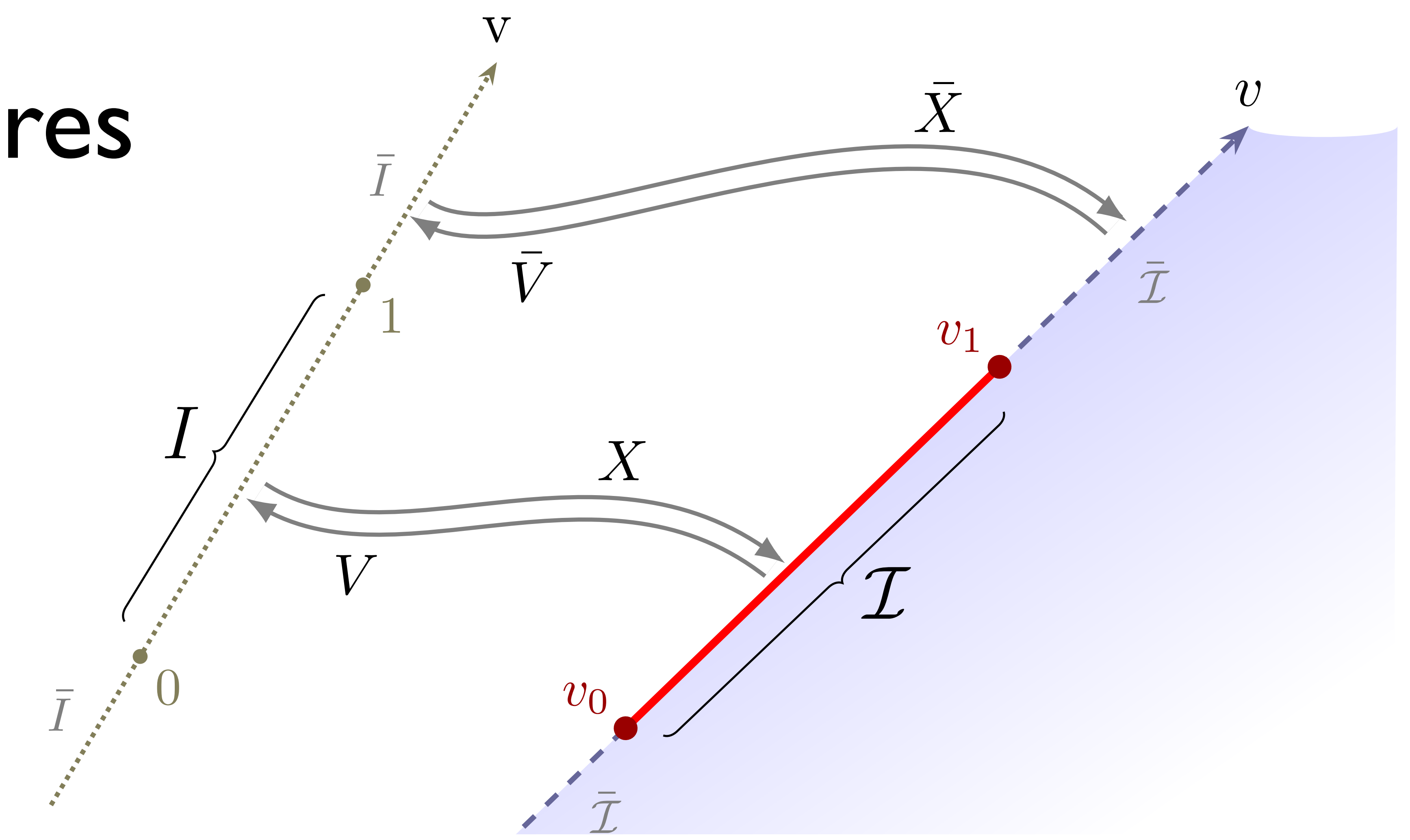

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β is not enough to determine V: Ambiguity $V \to AV + B$

We need edge modes

$\beta +$ boundary conditions $V(v_a) = a$ determine V

From the internal perspective we see that the edge modes represents the dof that allows to fix the difference between gauge fixing and frame fixing

1-Given V we can tread canonical variables for the spin 0 sector

$$(\Omega, \beta) \to (\tau, V)$$

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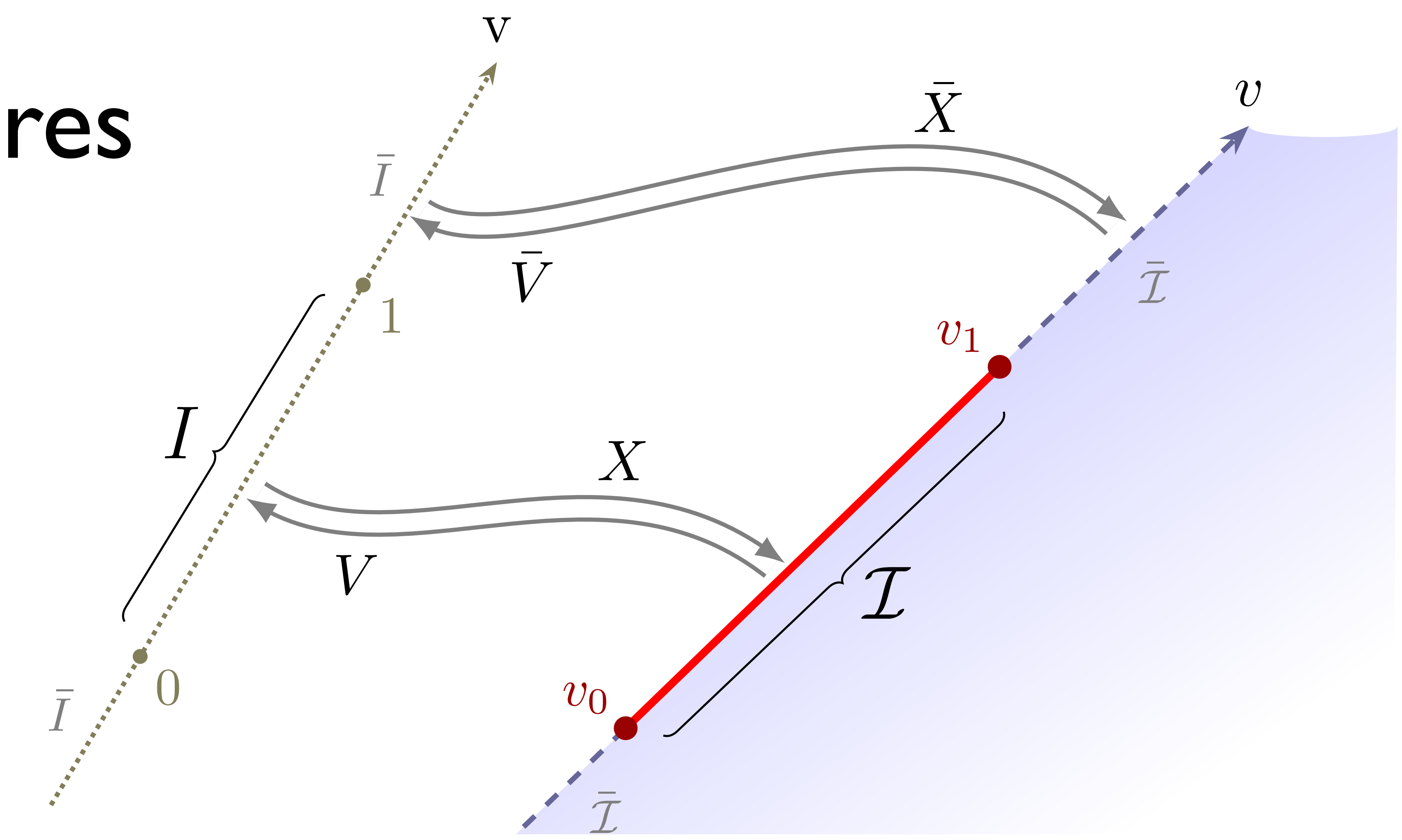

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$$(\Omega, \beta) \to (\tau, V)$$

$$\boxed{\tau = \underbrace{\frac{\partial_v(\partial_v \Omega - \mu\Omega)}{8\pi G}}_{\text{spin } 0 \text{ stress}}}$$

$$\Theta = \int_{\mathbb{R}} \Omega \delta\beta = \int_{\mathbb{R}} \tau \frac{\delta V}{\partial_v V}$$

$$C = \tau + \sum_i (\partial_v \varphi_i)^2 = 0$$

Dressing time II

The dressing time transform as a Goldstone mode under reparametrisation

$$\boxed{\frac{\partial_v^2 V}{\partial_v V} = \beta}$$

$$\delta_f V = f\partial_v V, \quad \delta_f \varphi_i = f\partial_v \varphi_i$$

$$X = V^{-1}$$

Hence we can construct Gauge invariant observable by dressing

$$\tilde{\varphi}_i = \varphi_i \circ X$$

While $\tilde{\tau} = (\partial_{\mathrm{v}} X)^2 (\tau \circ X),$ is the canonical generator of reorientations

It defines the area operator $\tilde{\tau} = \partial_v^2 \tilde{\Omega}$

Its action change of the dressing frame values $V \to F(V) \quad F \in \mathrm{Diff}(\mathbb{R})$

And leaves the field invariant $\varphi_i \to \varphi_i$

Gauge-invariant observables are $\tilde{O}[\tau, V, \varphi] = O[\tilde{\tau}, \tilde{\varphi}_i]$

Canonical Quantization

The dressing time commute with itself $[V(v), V(v')] = 0$

The canonical conjugate to V is $\Pi(v) = \dfrac{\tau}{\partial_v V}$

We can straightforwardly quantize the fields to obey

$$[\Pi(v), V(v')] = -i\hbar\delta(v, v') \qquad [\partial_v \varphi_i(v), \varphi_j(v')] = -\delta_{ij}\frac{i\hbar}{2}\partial_v \delta(v, v')$$

With a vacuum state annihilated by positive frequency modes

$$P_+\Pi|0\rangle = P_+V|0\rangle = P_+\varphi_i|0\rangle = 0$$

Beware that this kinematical quantization possesses negative norm states !

The challenge is to construct the gauge invariant Hilbert space and show that no negative norm state arises

Constraint Quantization

The quantization of the constraint require the choice of a normal ordering

$$C \to \hat{C} :=: \Pi \partial_v V : + \sum_i : (\partial_v \varphi_i)^2 :$$

[Ciambelli-LF-Leigh'24]

Which introduces a central charge. It is infinite in QFT

UV regularization due to QG molecularization makes it finite

$$C_f = \int_{\mathbb{R}} fC$$

$$\frac{1}{i\hbar}[C_f, C_g] = C_{[f,g]_{\mathrm{W}}} + \frac{\hbar c}{48\pi} \int_{\mathbb{R}} (\partial_v f \partial_v^2 g - \partial_v g \partial_v^2 f)$$

$$c_T = 2 + M$$

One needs to impose gauge invariance under this anomalous symmetry:

Solution: Find a nice (positive definite) representation of gauge invariant operators

Covariant Quantization

At the quantum level the generator of a gauge transformation associated with a diffeomorphism $F = \exp(f\partial_v)$ is

$$U[F] = \exp\left(\frac{i}{\hbar}C_f\right) = U_0[F] \otimes U_{\text{rad}}[F]$$

Gauge invariant operators satisfy $U[F]OU^{-1}[F] = O$ Unchanged by central charge

Linear operators $(\tilde{\tau},\, \tilde{\varphi}_i)$ are gauge invariant.

However, composite operators $O =: O[\tilde{\tau}, \tilde{\varphi}_i]:$ are not!

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Unchanged by central charge

Linear operators $(\tilde{\tau},\, \tilde{\varphi}_i)$ are gauge invariant.

However, composite operators $O =: O[\tilde{\tau}, \tilde{\varphi}_i]:$ are not!

Its because normal ordering (NO) is not invariant under diffeomorphism

It is a prescription for quantizing classical observables $O[\Phi]$ that tells us to put positive frequency components $P_+\Phi$ to the right, negative frequency components $P_-\Phi$ to the left. It depends on the choice of time hence

$$U[F] :O[\Phi]: U^{-1}[F] \neq : F \triangleright O[\Phi]:$$

This is an obstruction to simple quantization of gauge-invariant observables.

Covariant Normal ordering

C

Note that normal ordering is covariant in a background-dependent sense.

For each background time coordinate $F(v)$ there is a different notion of normal ordering defined using positive negative frequencies $P^F_\pm$ with respect to $F(v)$ (c.f. Minkowski time vs Rindler time, leads to Unruh effect)

We have the covariance property of NO $U[F] :O[\Phi]: U^{-1}[F] =: F \triangleright O[\Phi] :_F$

$$U[F] :\tilde{O}[\Phi]: U^{-1}[F] =: \tilde{O}[\Phi] :_F \neq : \tilde{O}[\Phi] :$$

Covariant Normal ordering

Using the dressing time we can define a NO with respect to V as a correction of the naive normal ordering. This provides a covariant notion of NO ie a background-independent one

For radiative fields we define $$O^* = U_{\mathsf{rad}}[V] :O[\varphi_i]: U^{-1}_{\mathsf{rad}}[V]$$

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we get $$O^{\star} =: \exp\left(\int \mathrm{d}u\mathrm{d}v G_V(u,v)\frac{\delta}{\delta\varphi_i(u)}\frac{\delta}{\delta\varphi_i(v)}\right)O[\varphi_i]:$$

Where $$G_V(u,v) := \ln\left(\frac{V(u)-V(v)}{u-v}\right)$$ Is a smooth operator.

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Where $$G_V(u,v) := \ln\left(\frac{V(u)-V(v)}{u-v}\right)$$ Is a smooth operator.

This construction which generalizes to spin zero operator is such that

$$\boxed{T^\star_{\text{rad}} = T_{\text{rad}} + \frac{\hbar M}{24\pi}\{V,v\}}$$ $\longleftarrow$ Schwarzian derivative

And $U[F]\tilde{O}^\star U^{-1}[F] = \tilde{O}^\star$ when $\tilde{O}[\Phi]$ is GI

Covariant Normal ordering

The covariant normal order provides a way to dress Kinematical observable.

Given a classical kinematical observable $O_K[V, \Pi, \varphi_i]$

We can construct a gauge-fixed observable $O[\Pi, \varphi_i] = O_K[v, \Pi, \varphi_i]$

And a gauge invariant observable $\tilde{O}[\Phi] = O[\tilde{\tau}, \tilde{\varphi}_i]$

At the quantum level we can construct two quantum observables

$O = :O[\Pi, \varphi_i]:$ and $\tilde{O}^* = {}^{\star}_{\star}\tilde{O}[\tilde{\tau}, \tilde{\varphi}_i]^{\star}_{\star} = D(O)$ ⟵ Dressed operators

If O is purely radiative then the dressing map is an algebra morphism

$$D(O_{\text{rad}})D(O'_{\text{rad}}) = D(O_{\text{rad}}O'_{\text{rad}})$$

Covariant Normal ordering

The covariant normal order provides a way to dress Kinematical observable.

Given a classical observable $O_K[V, \Pi, \varphi_i]$

We can construct a gauge-fixed observable $O[\Pi, \varphi_i] = O_K[v, \Pi, \varphi_i]$

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At the quantum level we can construct two quantum observables

$O =: O[\Pi, \varphi_i]:$ and $\tilde{O}^* = {}^\star_\star \tilde{O}[\tilde{\tau}, \tilde{\varphi}_i]^\star_\star = D(O)$ $\longleftarrow$ Dressed operators

In general we have a cross product algebra

$$[\tilde{\tau}(u), D(O_{\text{rad}})] = -D([T_{\text{rad}}(u), O_{\text{rad}}]$$

Three Diff(R) representations

Symmetry	Classical generator	Action
Reparametrization	C	$\delta_f V = f\partial_v V, \qquad \delta_f \varphi_i = f\partial_v \varphi_i$
Reorientations	$\tilde{\mathcal{T}}$	
Dressed reparametrization	$\tilde{C}$	

Three Diff(R) representations

Symmetry/Gauge	Classical generator	Action
Reparametrization	C	$\delta_f V = f\partial_v V, \qquad \delta_f \varphi_i = f\partial_v \varphi_i$
Reorientations	$\tilde{\mathcal{T}}$	$\delta_g V = g \circ V, \qquad \delta_g \varphi_i = 0$
Dressed reparametrization	$\tilde{C}$	

Three Diff(R) representations

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Reparametrization	C	$\delta_f V = f\partial_v V,$	$\delta_f \varphi_i = f\partial_v \varphi_i$
Reorientations	$\tilde{\mathcal{T}}$	$\delta_g V = g \circ V,$	$\delta_g \varphi_i = 0$
Dressed reparametrization	$\tilde{C}$	$\delta_{\tilde{f}} V = \tilde{f} \circ V$	$\delta_{\tilde{f}} \varphi_i = -\dfrac{(\tilde{f} \circ V)}{\partial_v V}\partial_v \varphi_i$

$$\tilde{C} = \tilde{\tau} + \sum_i (\partial_{\mathrm{v}} \tilde{\varphi}_i)^2$$

Three Diff(R) representations

Symmetry/Gauge	Classical generator	Quantum generator	Central charge
Reparametrization	C	$C^\star$	$c = 2 + M$
Reorientations	$\tilde{\mathcal{T}}$	$\tilde{\mathcal{T}}^\star$	$c_{\mathcal{T}} = 24$
Dressed reparametrization	$\tilde{C}$	$\tilde{C}^\star$	$\tilde{c} = 24 - M$

$$\tilde{\tau}^\star = \tilde{\tau} + \frac{\hbar}{\pi}\{V, v\}$$

Three Diff(R) representations

Symmetry	Classical generator	Quantum generator	Central charge
Reparametrization	C	$C^\star$	$c = 2 + M + c_{\text{cl}}$
Reorientations	$\tilde{\mathcal{T}}$	$\tilde{\mathcal{T}}^\star$	$c_{\mathcal{T}} = 24 - c_{\text{cl}}$
Dressed reparametrization	$\tilde{C}$	$\tilde{C}^\star$	$\tilde{c} = 24 - M - c_{\text{cl}}$

$$\tilde{C} = \tilde{\tau} + \sum_i (\partial_{\text{v}} \tilde{\varphi}_i)^2$$

We can shift the central charges by the addition of a classical counterterm.

And impose no quantum anomalies at the price of having a classical one

LF-Kirklin '25

$$c_{\text{cl}} = M - 24$$

Physical Hilbert space

$c_{\mathsf{cl}} = M - 24$ Implies no quantum anomalies

$$\boxed{\tilde{c} = 0,} \quad c_\mathcal{T} = M, \quad c = 26 + M$$

We can then construct a nice representation of the gauge-invariant operators. The physical Hilbert space when $\tilde{c} = 0$ is just a Fock space for the covariant radiative fields time the edge mode Hilbert space

$$\mathcal{H}_{\mathsf{phys}} = \mathcal{H}_{\mathsf{rad}} \otimes \mathcal{H}_{\mathsf{edge}}$$

$$\mathcal{H}_{\mathsf{edge}} = L^2(\mathbb{R}^2)$$

The gauge invariant algebra is a Cross-product

$$\mathcal{A}^G = \mathsf{Vir}_M \ltimes \mathcal{A}_{\mathsf{rad}}$$

Conclusions

We have described Quantum Gauge invariant observables and their localization, both requires the choice of a quantum dressing map. Here the dressing time. The proper quantization of GI observables requires a notion of covariant normal ordering.

The dressing time is preferred from entropic consideration. Its the only time for which the boost charge $S := \Omega - V\partial_V\Omega$ satisfy the second law $\partial_V S \geq 0$

LF-Leigh-Ciambelli '23

Still it would be interesting to understand the role of different choice of time: affine, conformal, areal and in what precise way the dressing time is optimal, ie does it minimizes fluctuations?

$F = \exp(f\partial_v)$

The presence of the central charge for frame reorientations creates new fluctuations between different time framed vacua proportional to 1/c $\langle F, F' \rangle = e^{-cS(F,F')}$

Are these extra vacuum fluctuations observables in principle in interferometry noise experiments.

Carney et al. '25, LF-Oberfrank '26

Questions

The dressing time is preferred from entropic consideration. Its the only time for which the boost charge $S := \Omega - V\partial_V \Omega$ satisfy the second law $\partial_V S \geq 0$

LF-Leigh-Ciambelli '23

Still it would be interesting to understand the role of different choice of time: affine, conformal, areal and in what precise way the dressing time is optimal, ie does it minimizes fluctuations?

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How does the QRF construction given here compares with the usual BRS approach of dealing with constraints

Porratti-Grassi'25