Observables in Table Top Quantum Gravity

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Table top quantum gravity in a nutshell

Feynman: ***"We're in trouble if we believe in quantum mechanics but don't quantize gravitational theory […] One should think about designing an experiment which uses a gravitational link and at the same time shows quantum interference"***

The picture arising from GR is

matter sources a spacetime geometry

$$G_{\alpha\beta} = T_{\alpha\beta}$$

$$\downarrow$$

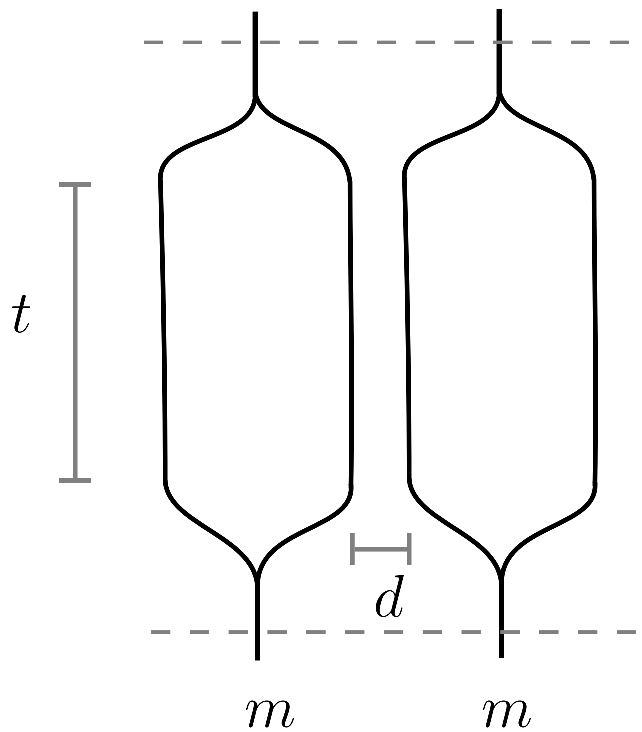
We expect that

***quantum* matter sources a *quantum* geometry**

$$\hat{G}_{\alpha\beta} = \hat{T}_{\alpha\beta}$$

The role of gravitation in physics: report from the 1957 Chapel Hill Conference
Max Planck Institute for the History of Science (2011)

Entanglement through gravity



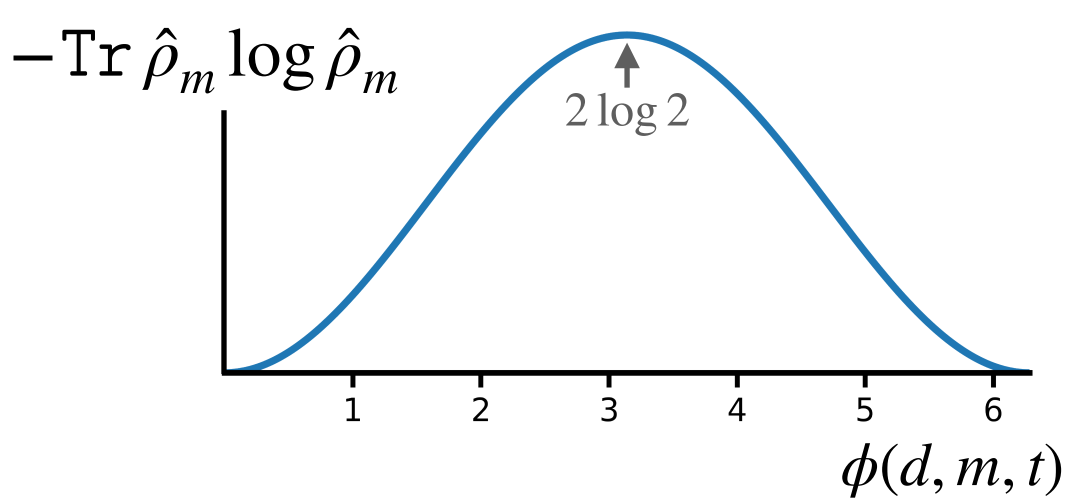
$$|\Psi_f\rangle \propto |LL\rangle + |RR\rangle + |LR\rangle + e^{i\phi(m,d,t)}|RL\rangle$$

Entangled state

$$\phi = \frac{Gm^2 d}{t\hbar}$$

Product state

$$|\Psi_i\rangle \propto |LL\rangle + |RR\rangle + |LR\rangle + |RL\rangle = \big(|L\rangle + |R\rangle\big) \otimes \big(|L\rangle + |R\rangle\big)$$

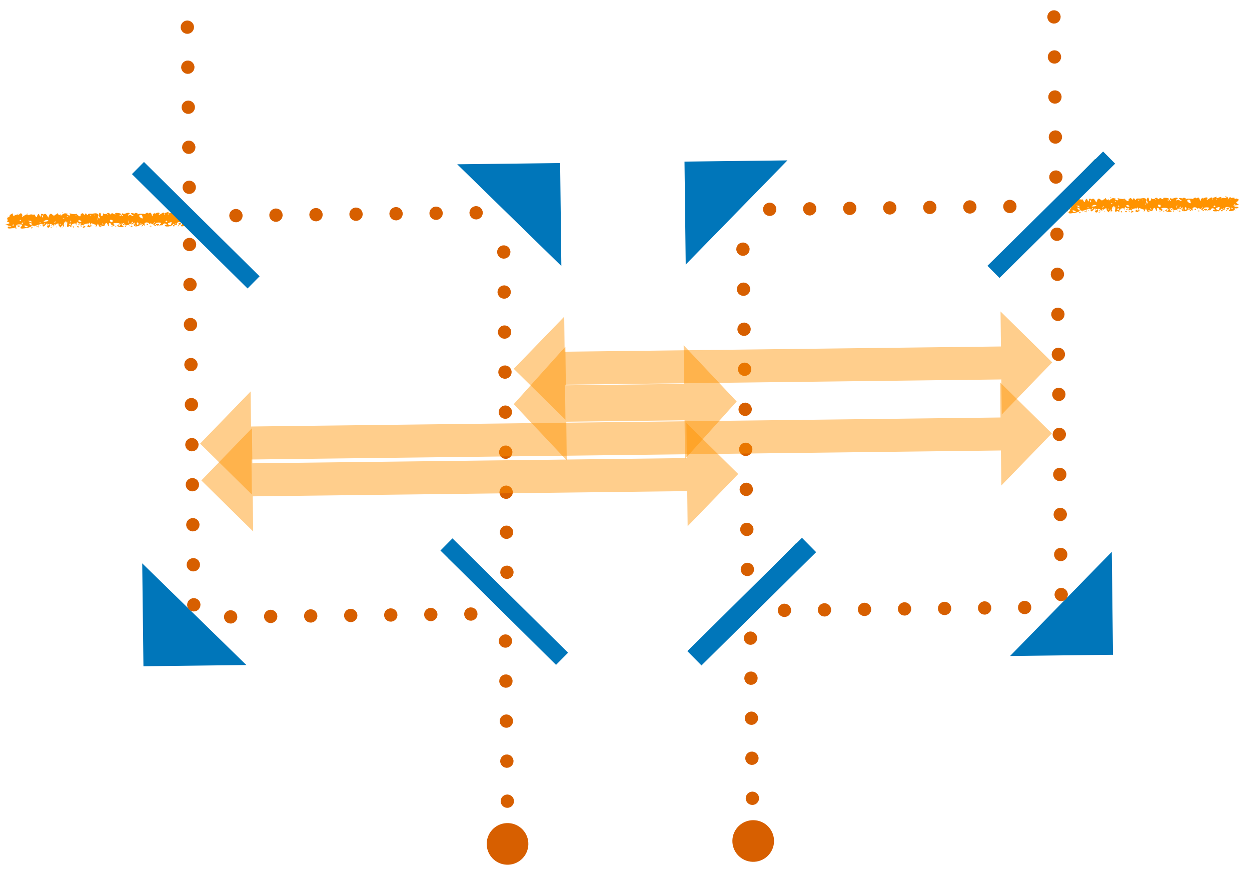


Bose et al. (1707.06050, 1707.06036) Krisnanda et al. (1906.08808)

Superposition of spacetimes in the laboratory

Quantum superposition of semiclassical states of spacetime, a gravitational 'cat state', implied by the superposition of gravitational potentials.

Geometrically, the effect is a superposition of proper times along each trajectory.



$$\phi = \frac{Gm^2 d}{t\hbar} = \frac{m\ \delta\tau}{m_P\ t_P}$$

On the possibility of laboratory evidence for quantum superposition of geometries
MC, C. Rovelli. *PLB* (2018)

Gravitationally mediated entanglement between oscillators as quantum superposition of geometries
O. Bengyat, A. Di Biagio, M. Aspelmeyer, MC. *PRD (2024)*

Why a theory independent argument is needed

Since 1915 we updated our understanding of spacetime from a background box to a dynamical entity.

An instantaneous potential is not satisfactory physical explanation: we have detected gravitational waves.

Fixing specific theories or models it is easy to 'see' that in order to create entanglement the gravitational field is in a quantum superposition. But, this assumes quantum to conclude quantum.

A different task is to arrive at a theory independent line of argument which excludes classical descriptions of gravity.

Bondi: ***"Does this mean, then, if there were no gravitational waves you would not feel that this experiment would prove the need for quantization?"***

Feynman: ***"Now listen we can analyze as we go along and cut the thing in the middle if we want to, and say that this produces a field and the field acts on the other one. That's one way of representing it. If we do it that way, then we have to have an amplitude for the field being here and an amplitude for the field being there. The gravitational field has to be quantized."***

The role of gravitation in physics: report from the 1957 Chapel Hill Conference
Max Planck Institute for the History of Science (2011)

Attempt at theory-independent argument

If A and B become entangled, then M cannot be a classical variable.

GR is a classical field theory.

Thus, GR would be falsified.

Bell's theorem in a nutshell

Theory independent arguments usually appear as no-go theorems. They exclude some theories from some space of theories, given an experimental outcome. Prototypical example is Bell's theorem.

Caricature of Bell no-go theorem(s):

All three cannot be true:

a) *Correlations in two parties have some property (violate an inequality)* ✓ *Experiment*

b) *Assume independent runs ('free choice')* ✓ ✗ ✗

c) *Assume local realism (local hidden variables)* ✗ ✓ ✗

Quantum mechanics 'goes through' the first option: it satisfies a) and b).

Classical mechanics is excluded, as it does not predict the violation of Bell's inequalities.

In particular, it satisfies b) and c) and the theorem says that no theory of this kind can explain a).

The search for a no-go theorem

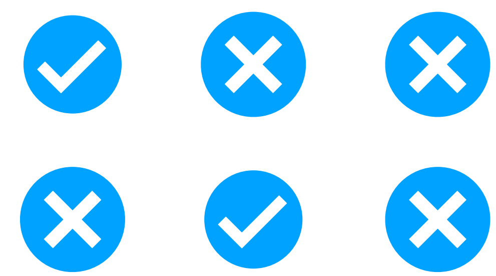
Caricature of sought after no-go theorem(s) for TTQG

All three cannot be true:

a) ***Measurements of correlations in two parties have some property*** ✓ Experiment

b) ***Assume locality of interactions***

c) ***Assume gravity is classical***



The idea is to force a dilemma: drop the locality of interactions, or drop classicality.

This would imply no theory 'like' General Relativity can explain a).

But, the problem is, the kind of locality assumed holds neither in classical nor in quantum field theories, it is different than relativistic locality.

This makes the theorem non-informative regarding classicality, as it is natural to drop the kind of locality presupposed as not fundamental.

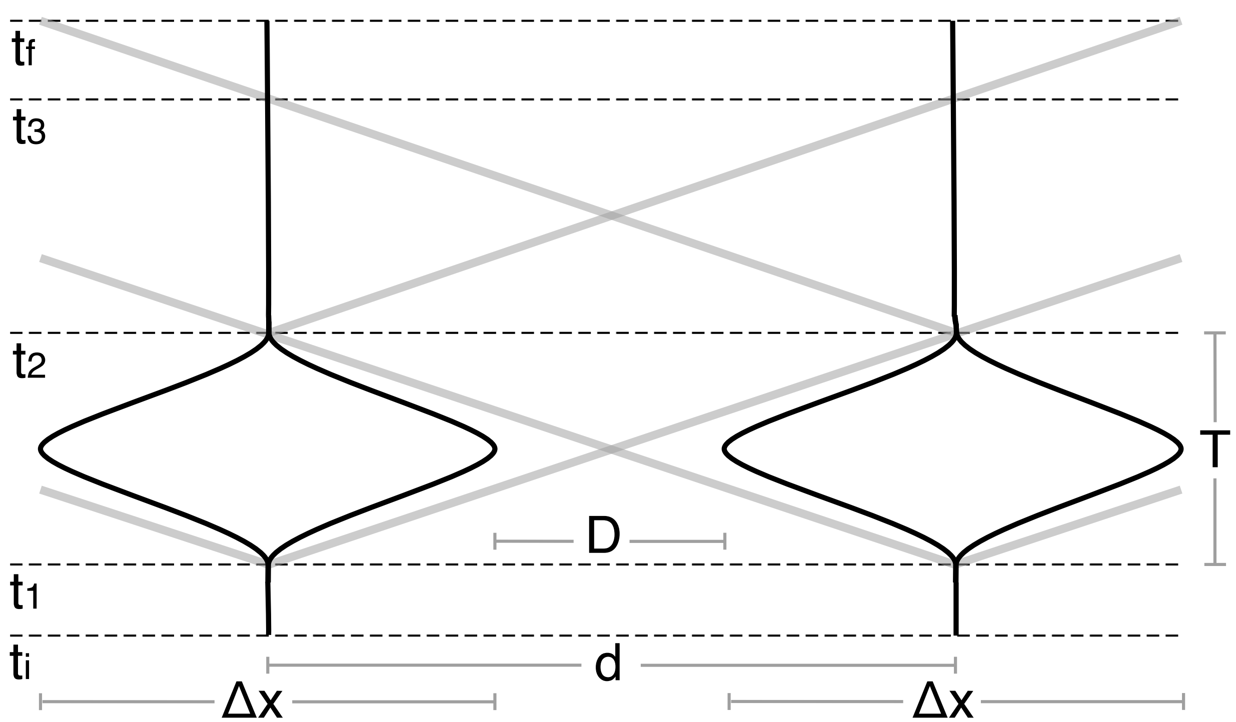
Galley et al. (2012.01141) Marletto et al. (2003.07974) Lami et al. (2302.03075)

Entanglement is generated locally in spacetime

Quantum Information language often relies on Hamiltonian formalism, which obscures Lorentz invariance.

In path integral formalism, can explicitly see that quantum phases do not build up instantaneously.

$$\phi_\sigma = \frac{S_F^\sigma\left[x_a^{s_a}, F[x_a^{s_a}]\right]}{\hbar}$$



$$U_{i\to f} = \sum_\sigma |\sigma\rangle\langle\sigma| \otimes U_{i\to f}^\sigma$$

$$U_{i\to f}^\sigma \propto \int_i^f \mathscr{D}x' \exp\left(\frac{i\,S[x_a', F[x_a']]}{\hbar}\right) |\psi^f\rangle\langle\psi^i|$$

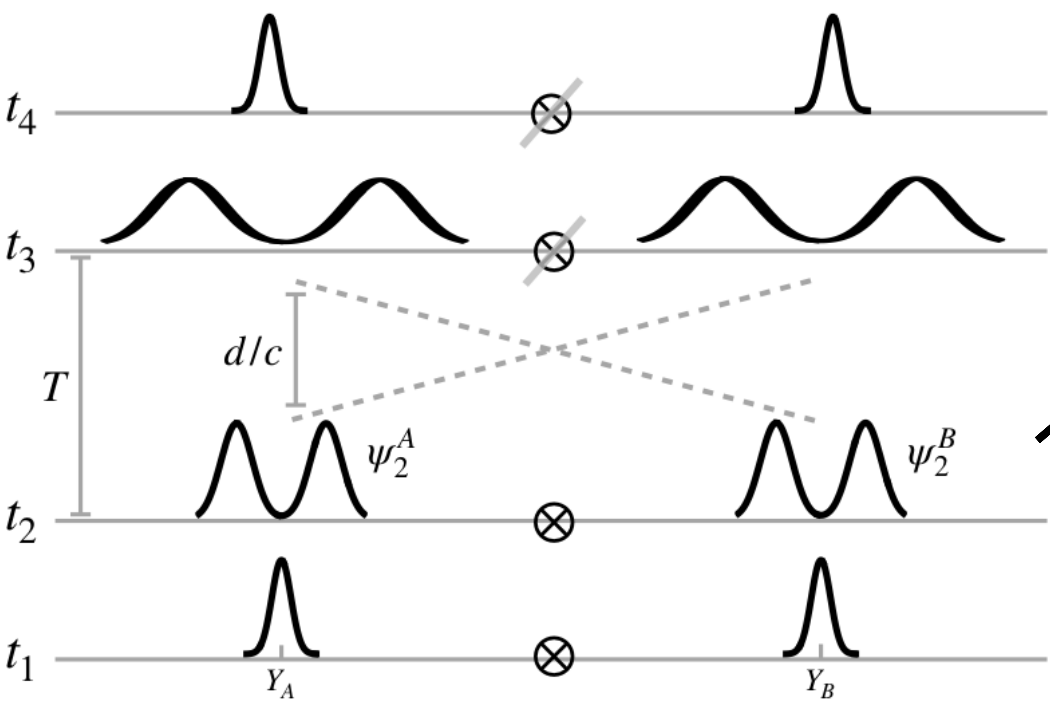
$$S_F = \frac{1}{4}\int d^4x\, h_{\mu\nu}T^{\mu\nu}$$

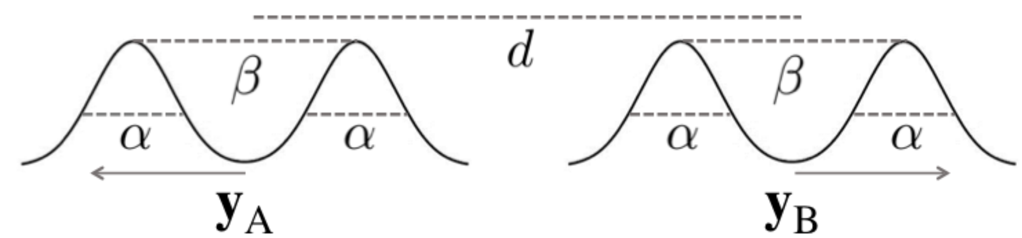
$$S_F = \frac{G}{c^4}\sum_{a,b}^{a\neq b}\int dt\, \frac{m_a m_b\, \bar{V}_a^{\mu\nu}(t_{ab})\, V_{b\mu\nu}(t)}{d_{ab}(t) - \mathbf{d}_{ab}(t)\cdot\mathbf{v}_a(t_{ab})/c}$$

$$|\Psi^f\rangle = U_{i\to f}|\Psi^i\rangle \propto |\psi^f\rangle \otimes \sum_\sigma A_\sigma e^{i\phi_\sigma}|\sigma\rangle$$

Locally mediated entanglement from linearised quantum gravity. MC et al. *PRL* (2023)

Continuous superposition of spacetimes in the laboratory





$$\mathscr{E} = \frac{2Gm^2 t}{\hbar d}\frac{1}{d^2}\left[\frac{1}{4}\beta^4 + \alpha^2\beta^2\left(1+\frac{1}{4}(\omega t)^2\right) + \alpha^4\left(f_0\left(\frac{\beta}{\alpha}\right) + f_2\left(\frac{\beta}{\alpha}\right)\frac{(\omega t)^2}{3} + f_4\left(\frac{\beta}{\alpha}\right)\frac{(\omega t)^4}{9}\right)\right]^{1/2}$$

$$\psi_4(y_4^a) = \int dy_1^a\, \psi_1^a(y_1^a) \int_{y_1^a,\varphi_1}^{y_4^a,\varphi_1} Dx_a'\, D\mathscr{G}'\, e^{iS[x_a',\mathscr{G}']}$$

$$\mathscr{E}_{\beta\gg\alpha} = \frac{Gm^2 T}{\hbar d}\left(\frac{\beta}{d}\right)^2$$

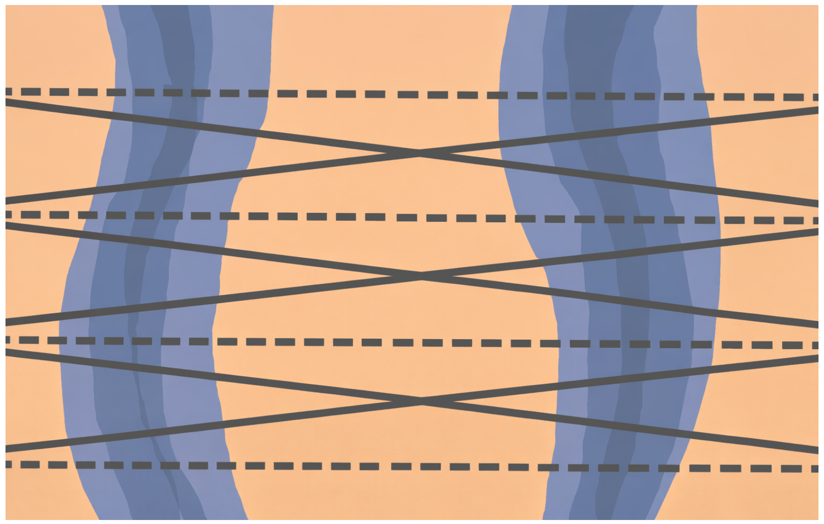
$$\mathscr{E}_{\beta\ll\alpha} = \frac{Gm^2 T}{\hbar d}\left(\frac{\sqrt{2}\alpha}{d}\right)^2\sqrt{1+\frac{(\omega T)^2}{3}+\frac{(\omega T)^4}{9}}$$

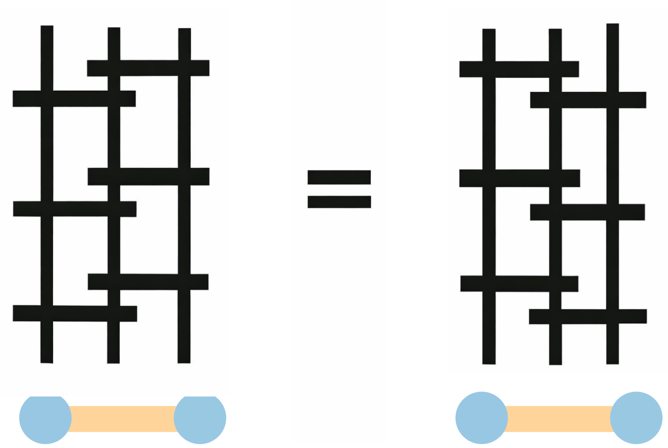
$$\Delta\mathscr{E}_{\beta\ll\alpha}^{\omega T\gg 1} = -\frac{3}{2}\left(\frac{d}{ct}\right)^2 \mathscr{E}_{\beta\ll\alpha}^{\omega T\gg 1}$$

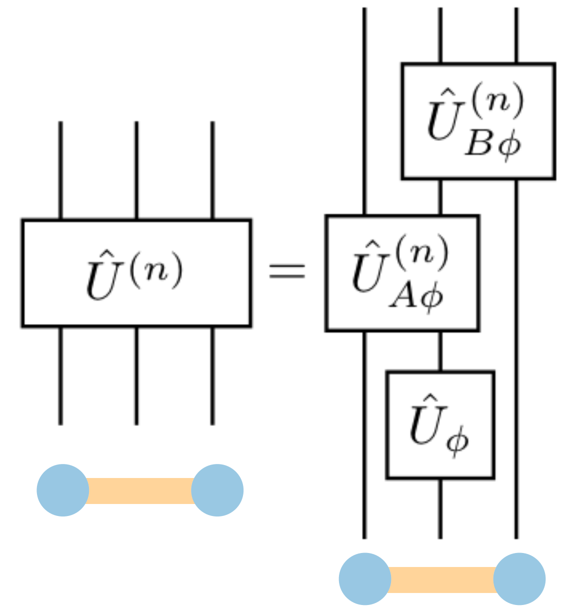
"…we may say that during the experiment, spacetime is in a superposition of a continuum of different geometries."

Gravitationally mediated entanglement between oscillators as quantum superposition of geometries
O. Bengyat, A. Di Biagio, M. Aspelmeyer, MC. *PRD (2024)*

Relativistic locality vs subystem locality







$$\hat{U} = \sum_{rs} |rs\rangle\langle rs| \otimes \hat{U}^s_B\,\hat{U}^r_A\,\hat{U}^{rs}_\phi$$

$$\hat{U}^{rs}_\phi = e^{\Omega^{rs}}\,\hat{U}^r_\phi \hat{U}^s_\phi\, e^{-i\hat{H}_0\,(t'-t)}$$

$$\hat{U}(t_i, t_f) = \prod_{n=1}^{N-1} \left(\hat{U}^{(n)}_{B\phi} \hat{U}^{(n)}_{A\phi} \hat{U}_\phi \right)$$

$$\Omega^{rs} = \int_{t_1}^{t_2} dt\, dt' \iint d^3x\, d^3x'\ \rho^r_A(t,x)\rho^s_B(t',x') \left[\hat{\phi}_I(t,x), \hat{\phi}_I(t',x')\right]$$
$$+ \int_{t_1}^{t_2} dt \int_{t_1}^{t} dt' \iint d^3x\, d^3x'\ \left(\rho^r_A(t,x)\rho^s_B(t',x') + \rho^r_A(t',x')\rho^s_B(t,x)\right) \left[\hat{\phi}_I(t,x), \hat{\phi}_I(t',x')\right]$$

$$\Delta s^2(X,Y) > 0 \Rightarrow [\hat{\phi}(X), \hat{\phi}(Y)] = 0$$

Approximate relation between the two notions of locality, with scalar field.

Gauge dependence makes things worse.

Circuit locality from relativistic locality in scalar field mediated entanglement
A. Di Biagio, R. Howl, C. Brukner, C. Rovelli, MC (2305.05645)

Transverse modes are not local

Difficult to identify degrees of freedom which are 'mediators' in the sense assumed in LOCC type theorems.

Root of the issue is that the transverse part (photons, gravitons) is not a spacetime local function of the source:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \partial_\mu F^{\mu\nu} = J^\nu$$

$$A_\mu = (A_0, A_i)$$

$$A_i = \psi_i + \partial_i C \quad \partial_i \psi_i = 0$$

$$\phi = A_0 - \partial_0 C$$

$$\phi = -\frac{1}{\nabla^2} J_0$$

$$\Box\, \psi_i = -J_i + \partial_i \frac{1}{\nabla^2} J_0$$

Although the transverse part satisfies a wave equation, the source is not a local spacetime function.

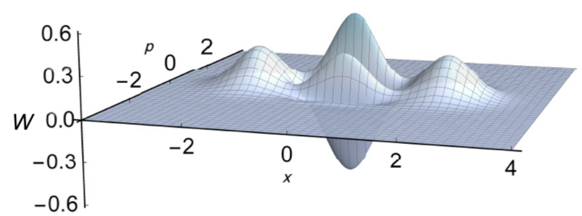
Gravity entanglement, quantum reference systems and degrees of freedom.
MC, A. Di Biagio, R. Howl, C. Rovelli. *CQG* (2023)

Single system tests

Production of non-Gaussianity in self-interactions of condensates implies quantum coupling

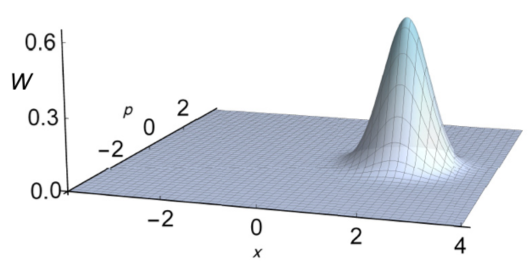
$$\hat{H}_{QG} = \frac{1}{2} m \int d^3 r \ : \hat{\Psi}^\dagger(r) \hat{\Psi}(r) \hat{\Phi}(r) :$$

$$\hat{\Phi}(r) = -Gm \int d^3 r' \ \frac{\hat{\Psi}^\dagger(r') \hat{\Psi}(r')}{|r - r'|}$$



Non-Gaussian

$$\hat{H}_{QG} = \frac{1}{2} \lambda_{QG} \, \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$$



Gaussian

If gravity is classical variable, non-gaussianity not produced

$$\Phi = \langle \hat{\Phi} \rangle$$

$$\hat{H}_{CG} = m \int d^3 r \ \hat{\Psi}^\dagger(r) \hat{\Psi}(r) \Phi[\Psi](t, r)$$

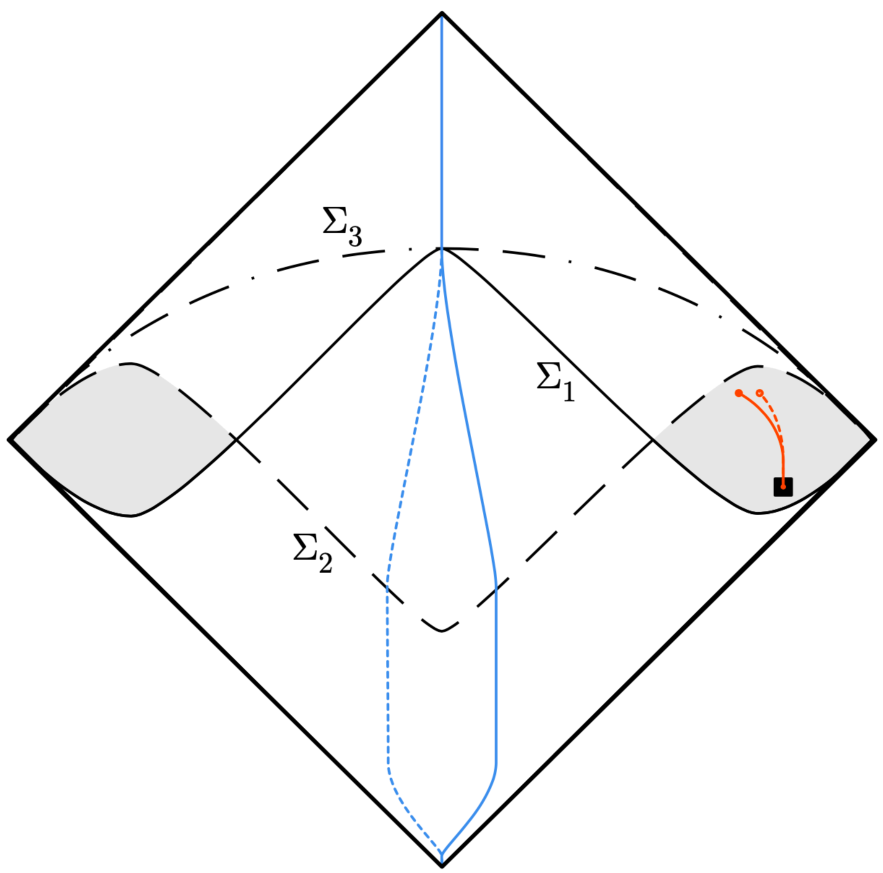
$$\hat{H}_{CG} = \lambda_{CG}[\Psi] \, \hat{a}^\dagger \hat{a}$$

Same mechanism produces QG corrections in heat capacity of a condensate

Non-Gaussianity as a Signature of a Quantum Theory of Gravity
Howl et al. *PRX Quantum 2021*

Evidencing quantum gravity with thermodynamical observables
T. Strasser, MC, R. Howl, C. Brukner *PRR 2025*

Additional motivating aspects that I will not go into



Mari et al (1509.02408) Belenchia et al (1807.07015) Danielson et al (2112.10798)

Hidaka, Iso, Shimada (2205.08403, 2211.09441, 2211.09441)

Sugiyama, Matsumura, Yamamoto (2308.03093, 2206.02506)

When does entanglement through gravity imply gravitons?
Mitrakos, Papageorgiou, Perche, MC (2601.03214)

Demanding complementarity and no-signaling implies gravitons, *if* retarded entanglement through gravity is detected.

But, this is by far *not* a theory independent argument.

Discreteness of geometry

On the possibility of experimental detection of the discreteness of time.
MC, Rovelli. *Front. in Phys.* 2020

An experiment to test the discreteness of time
MC, Di Biagio, Martin-Dussaud. *Quantum* 2022

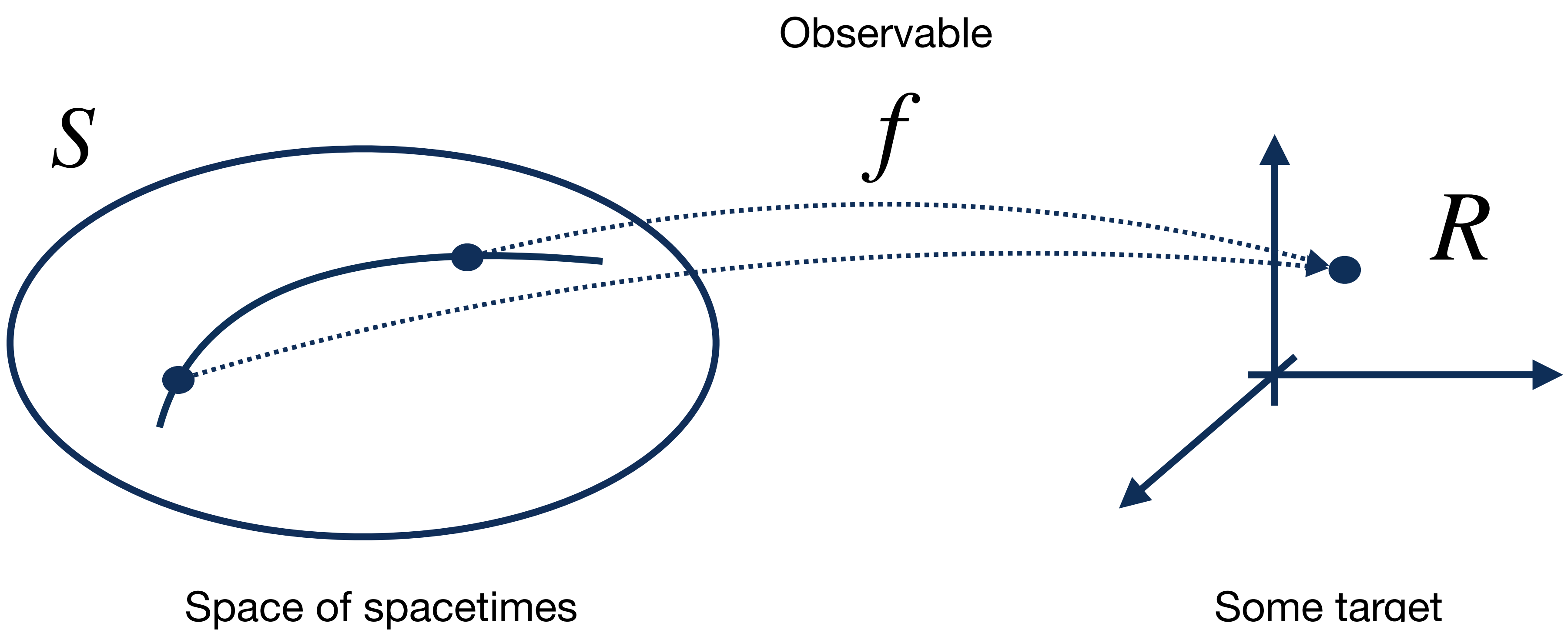
Detecting gravitationally interacting Dark Matter with Quantum Interference
MC, Perez, Rovelli *PRL 2024*

Quantum sensing of gravitational frame dragging with a superfulid Helium-4 gyrometer
Ellers, MC, Schwab, Whaley (2510.20772)

Observables

A reasonable input is to assume that 'the gravitational field is *something* in its own right'.

Instead of assuming local dofs, we might assume the existence of 'not purely global' observables.



We now run into the *Problem of Observables.*

The Problem of Observables

Take S to be the space of solutions of GR.

Observable $f : S \to R$ such that $g_{\mu\nu} \cong \tilde{g}_{\rho\sigma} \Rightarrow f(g_{\mu\nu}) = f(\tilde{g}_{\mu\nu})$

f is called concrete if R is some Polish space.

Complete Observable $f_1, \ldots, f_r : S \to R$ such that $g_{\mu\nu} \cong \tilde{g}_{\rho\sigma} \Leftrightarrow f_i(g_{\mu\nu}) = f_i(\tilde{g}_{\mu\nu})\ \forall i = 1,\ldots,r$

Theorem: *No concrete observable* $f : S \to R$ *is both complete and Borel definable*

Essentially, this says it is impossible to write down with mathematical formulas a complete set of observables for full general relativity.

Incompleteness of Observables in General Relativity
Panagiotopoulos, Sterling, MC. *PRL* 2023

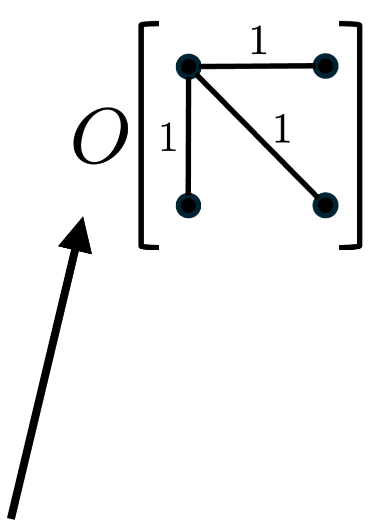
Encode local information globally

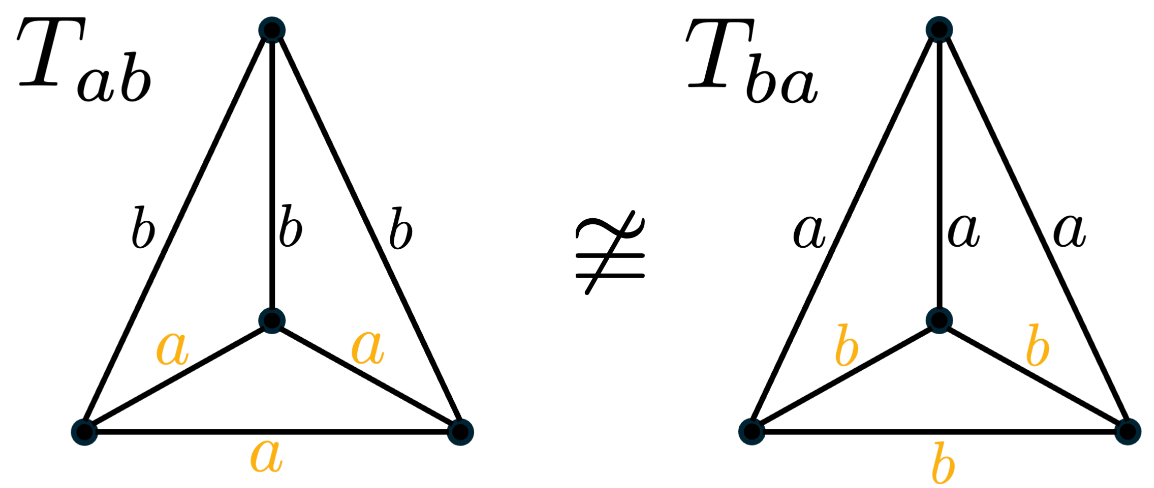
Another intriguing aspect of the observables of general relativity, is that they are global

How do global objects capture local information? Can they capture *all* local information?

Becomes clear when studying discrete geometry with the theory of invariants on weighted graphs

The analogue of diffeomorphism invariance is here permutation invariance





$$O(T_{ab}) = 3a^2b + b^3 \neq 3ab^2 + a^3 = O(T_{ba})$$

Example of background independent function (observable) that distinguishes configurations with same global information, but different local structure

Observables are glocal

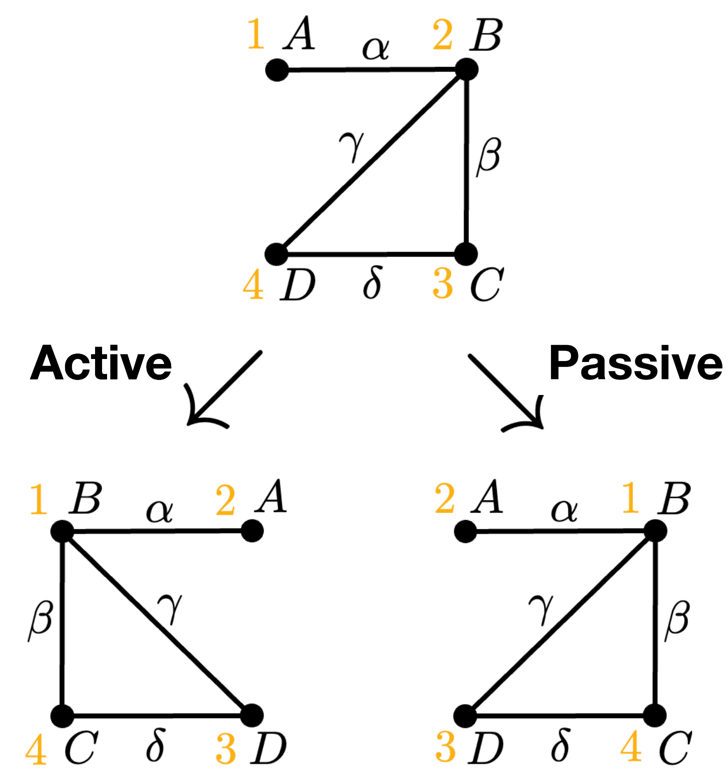
E. Broukal, A. Di-Biagio, E. Bianchi, MC (2508.02346)

Glocal observables

$$G = \sum_{i\leq j}^{N} g_{[i,j]} e_{[i,j]} \in \mathscr{G}_N, \quad \text{dim}(\mathscr{G}_N) = \frac{N}{2}(N+1)$$

$$\sigma \cdot G = \sum_{i\leq j}^{N} g_{[i,j]} e_{[\sigma(i),\sigma(j)]} \qquad \textbf{Passive}$$

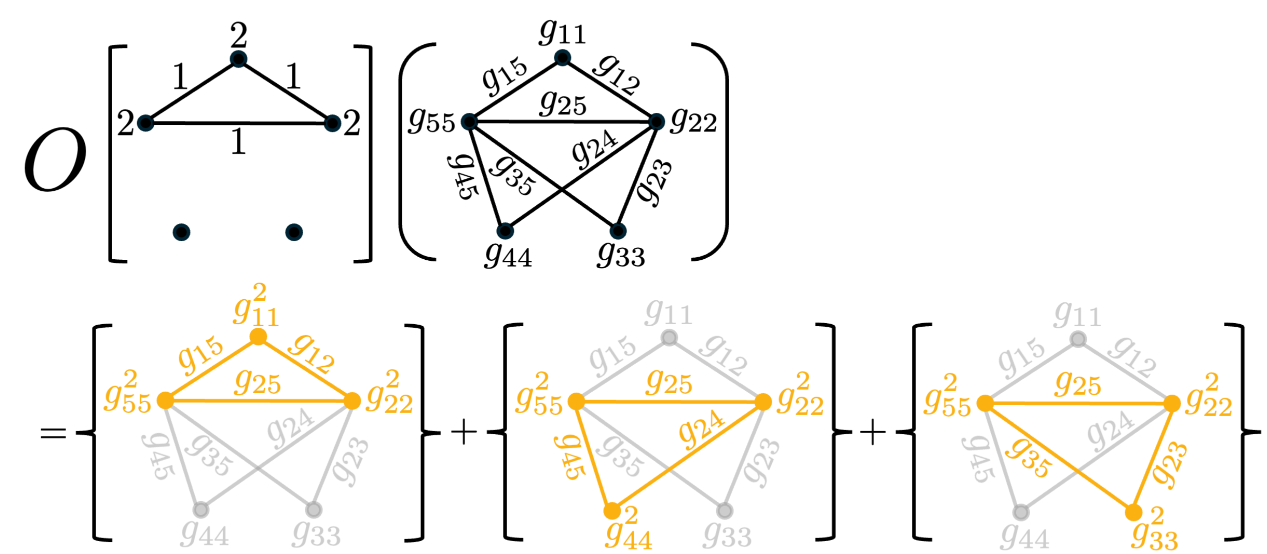
$$= \sum_{i\leq j}^{N} g_{[\sigma^{-1}(i),\sigma^{-1}(j)]} e_{[i,j]} \qquad \textbf{Active}$$



Local. **Correlation specified by connected subgraph**

Global. **Observable probes every instance of the local correlation across the whole graph**

Glocal = Local + Global. Captures local information in a way that is fully invariant under changes of labels



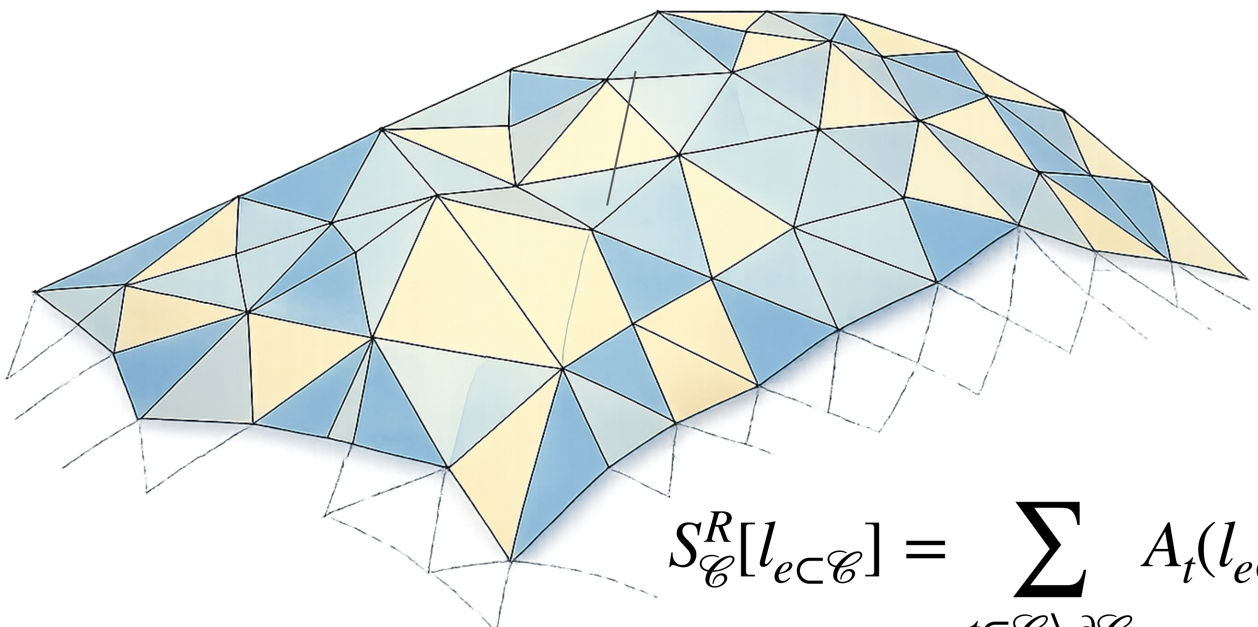
$$= g_{11}^2 g_{12} g_{22}^2 g_{25} g_{55}^2 g_{15} + g_{22}^2 g_{24} g_{44}^2 g_{45} g_{55}^2 g_{25} + g_{22}^2 g_{23} g_{33}^2 g_{35} g_{55}^2 g_{25}$$

Example of an observable that constructs an invariant from triangular correlations

Observables are glocal

E. Broukal, A. Di-Biagio, E. Bianchi, MC (2508.02346)

Complete characterisation of discrete geometries with invariants



Theorem: Algorithmic construction of finite sets of glocal observables that are complete on the space of weighted graphs up to N nodes.

$$S^{R}_{\mathscr{C}}[l_{e\subset\mathscr{C}}] = \sum_{t\subset\mathscr{C}\setminus\partial\mathscr{C}} A_t(l_{e\subset t})\epsilon_t(l_{e\subset t}) + \sum_{t\subset\partial\mathscr{C}} A_t(l_{e\subset t})\psi_t(l_{e\subset t}) + \Lambda \sum_{s\subset\mathscr{C}} V_s(l_{e\subset s})$$

Glocal observables encode types of local correlations.

Each is a global average of all appearances of some type of local subgraph structure.

Space of Regge configurations on finite graphs fully characterised with background independent objects.

Easy to promote the invariants to operators, yields complete characterisation with invariants under permutations for spin networks on finite graphs.

Observables are glocal

E. Broukal, A. Di-Biagio, E. Bianchi, MC (2508.02346)

Quantum observables as invariants under quantum coordinate transformations

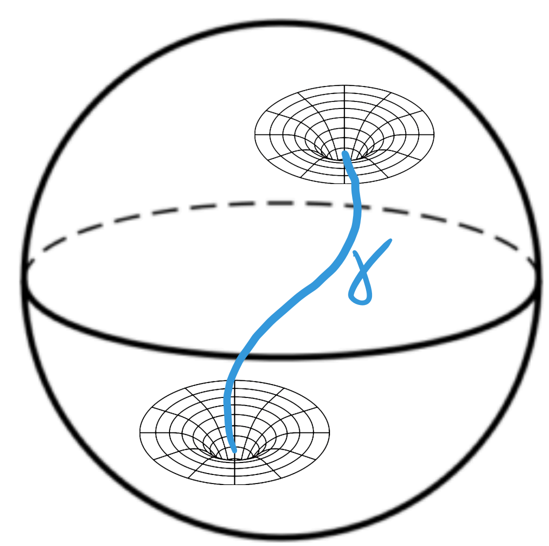
Back to entanglement through gravity: a simple 'quantum coordinate' transformation shows that distance must be quantised

$$d_{(i,j)}(t) = \int_0^1 \sqrt{g_{xx}^{(i,j)}(x,t)}\, dx \qquad |\psi(t)\rangle = |X=0\rangle \otimes |Y=1\rangle \otimes \sum_{(i,j)} |d_{(i,j)}(t)\rangle$$

Then, an observable that encodes the distance between the masses would need to be 'quantised'. For instance, the following observable:

$$\mathcal{O}(L) = \int dx^4 dy^4 \sqrt{-g(x)}\sqrt{-g(y)}\, R(x)R(y)\, \delta\left(\max_\gamma L_\gamma \,,\, L\right)$$

γ **any spacelike curve connecting** x **and** y **and** L_γ **its proper length**



Gravity entanglement, quantum reference systems, degrees of freedom
MC, Di Biagio, Howl, Rovelli. CQG 2022

Indefinite Causal Order and Quantum Coordinates
A. Hamette, V. Kabel, MC, C. Brukner. *PRL* 2025

Summary

- **Entanglement-through-gravity and other TTQG protocols may imply general relativity is falsified.**

- **Clear argument not at hand. Related difficulties are (i) different notions of locality considered fundamental in quantum information and quantum field theory (ii) effects sought after are near field, usual approximate notions of local degrees of freedom are impertinent.**

- **However, it seems that the observables of any reasonable theory of gravity would need to be quantised for entanglement to arise.**

- **Due to the lack of background, observables are global functions, which nevertheless encode the local information of the geometry.**

- **Next step, work in progress: formalise observables in a quantum setting, as invariants under beyond quantum controlled transformations.**

Quantum permutations as genuinely quantum coordinates
Bengyat, Brukner, MC (soon to appear)