Variational quantum sensing using Unruh-DeWitt detectors

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P. A. LeMaitre, T. Rick Perche, **MK**, H. J. Briegel, *Universal Quantum Computer From Relativistic Motion*, PRL **134**, 190601 (2025), arXiv:2411.00105

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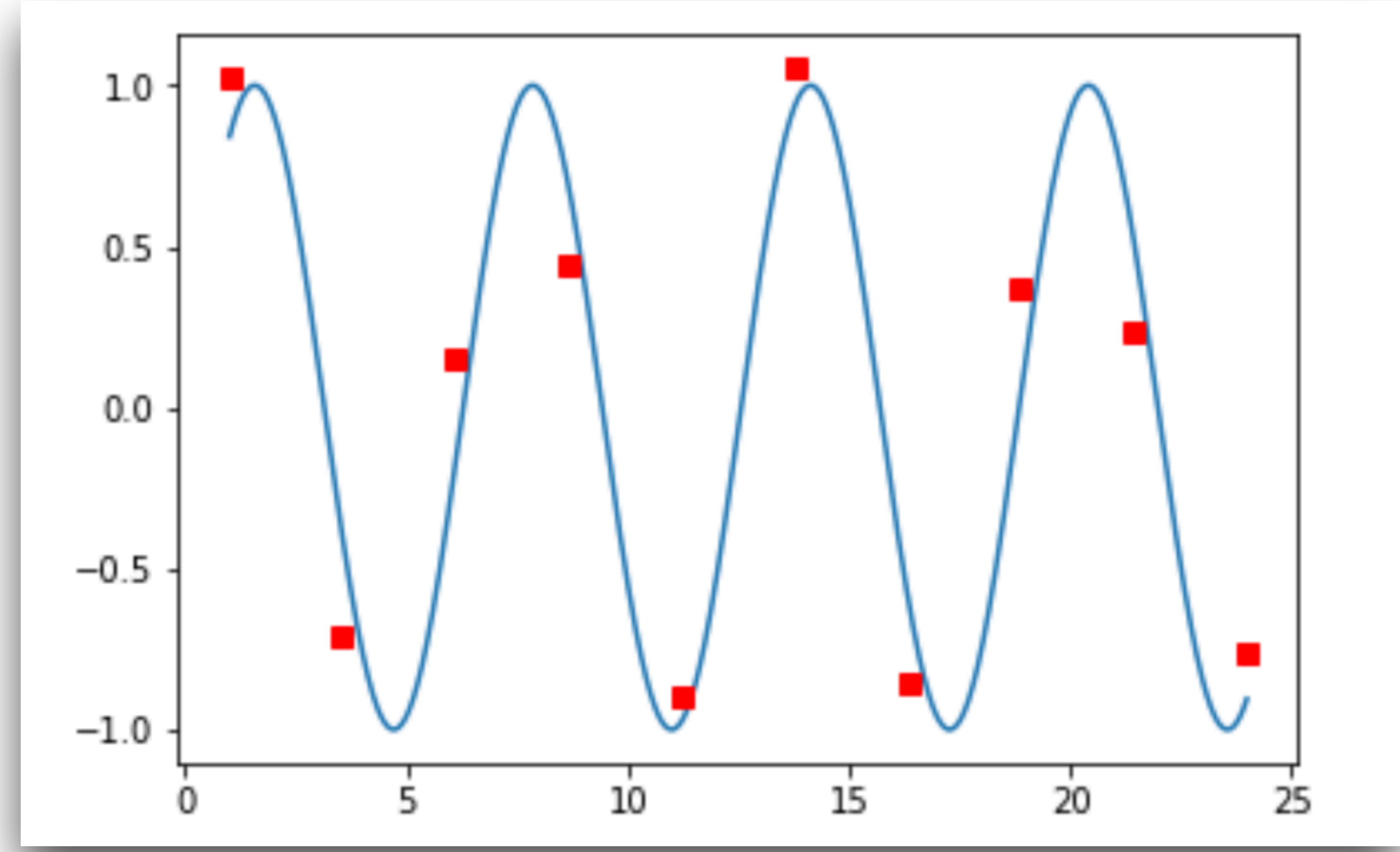
(Supervised) Machine Learning

- **Given:** raw **data** samples $(x_j, y_j) \in \mathcal{T}$
- **Task: function approximation** a.k.a. **curve fitting**

  Tune the parameters $\vec{\theta}$ of a **parametrized function ansatz** $f_{\vec{\theta}}(x) = y$ to **reproduce data** points as well as possible

  $\rightarrow$ Trained function $f_{\vec{\theta}}$ will be used to **interpolate** and **extrapolate** to new data



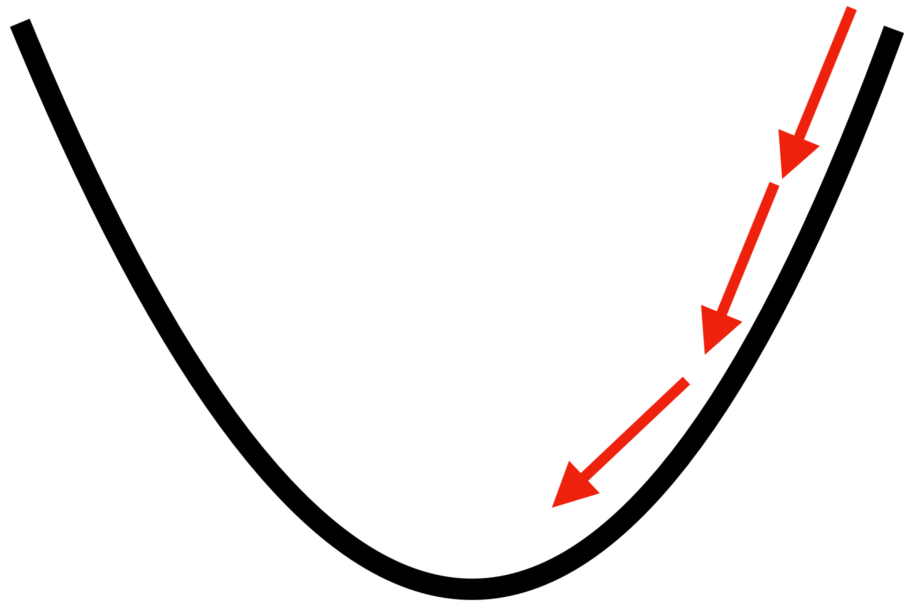
- **Loss function:** Differentiable error measure of the approximation

$$L(\vec{\theta}) = \sum_j \left(f_{\vec{\theta}}(x_j) - y_j \right)^2$$

- **Optimization a.k.a. training:**

  Many small **gradient descent** steps: $\vec{\theta}_{\text{new}} = \vec{\theta}_{old} - \eta \nabla_{\vec{\theta}} L(\theta_{\text{old}})$

  η learning rate, step size



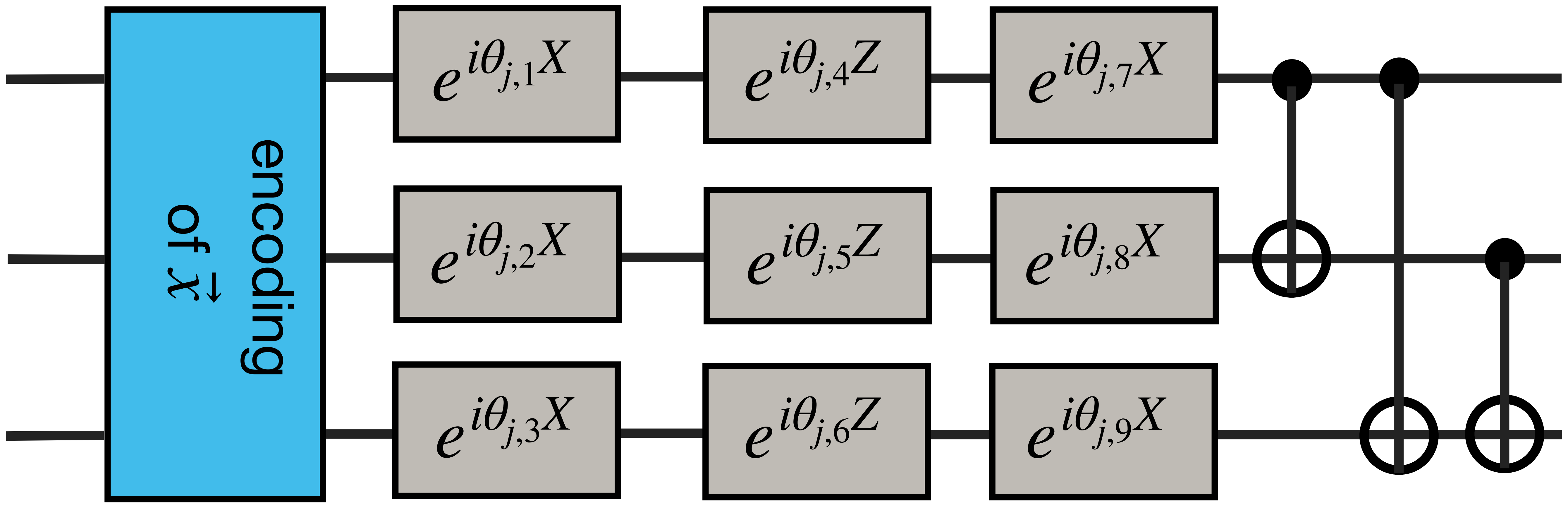
Variational Quantum Circuits (VQC)

- **Basic Idea:** Replace artificial neural network with a **Parametrized Quantum Circuit (PQC)**
- Most popular choice: **Hardware Efficient Ansatz**

$$U(\vec{\theta},\vec{x}) = U_L(\vec{\theta}_L,\vec{x}) \cdot \ldots \cdot U_1(\vec{\theta}_1,\vec{x})$$

(followed by measurement in computational basis)

$$U_j(\vec{\theta}_j,\vec{x}) =$$



$\vec{x}$ input

$\vec{\theta}$ trainable parameters

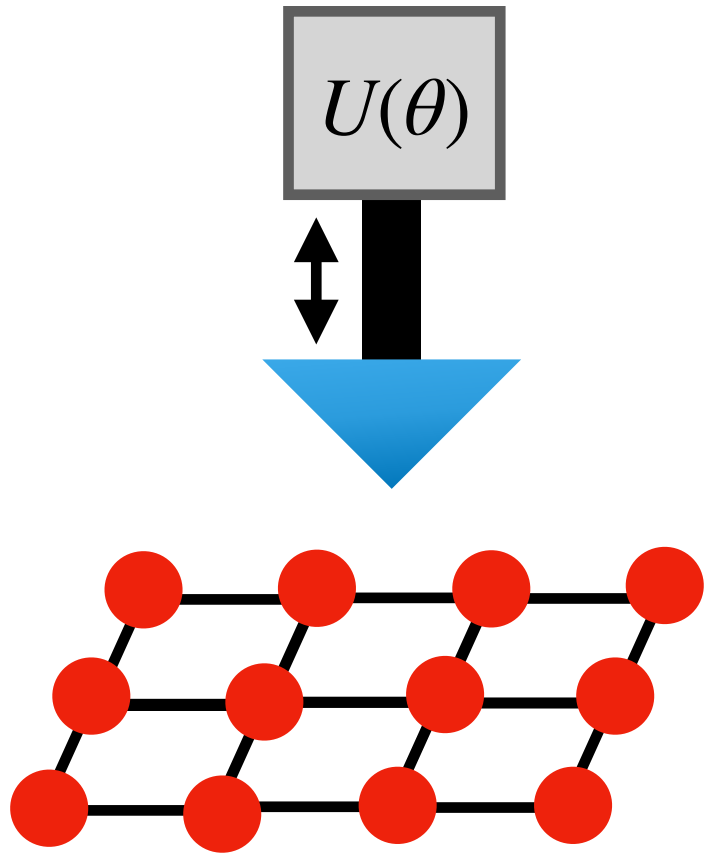
Because of its layered structure, this is often called a **Quantum Neural Network (QNN)**

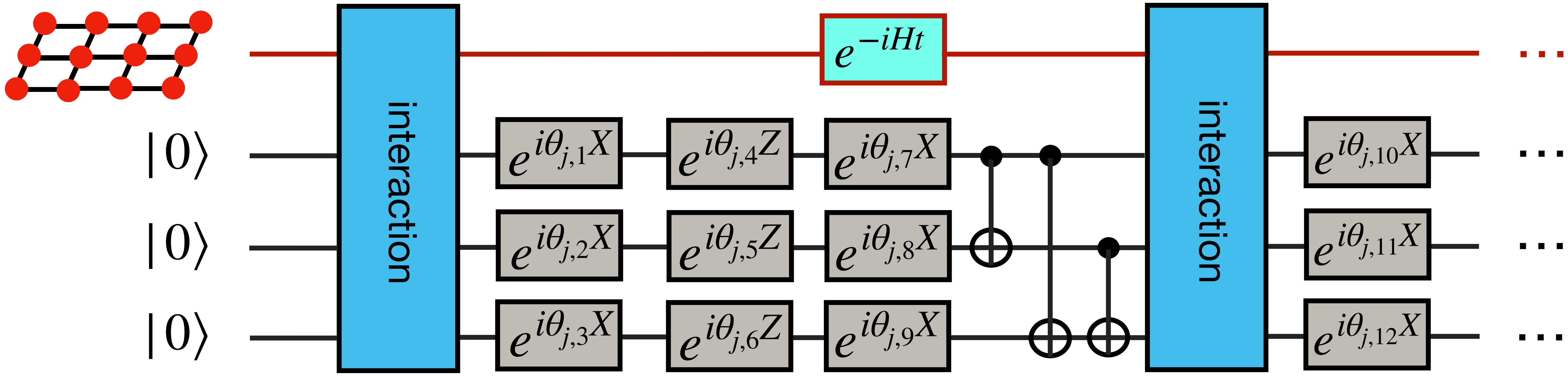
- **Measure gradient** of input-output statistics, update parameters $\vec{\theta}$ via **classical optimizer**
- Machine learning's **focus on input-output** statistics makes this well suited for **noisy quantum computers**

Can already be applied to some real-world problems!

Quantum sensing in variational quantum circuits

- Instead of usual data uploading step, have interaction with environment
- Allows to **delay measurement collapse** as long as possible





- **Learn** to **extract** quantum many body properties if **no hand-crafted protocol is known**

Flexible quantum toolbox for quantum experiments!

Less prone to barren plateau problems: Intermediate number of qubits might be enough!

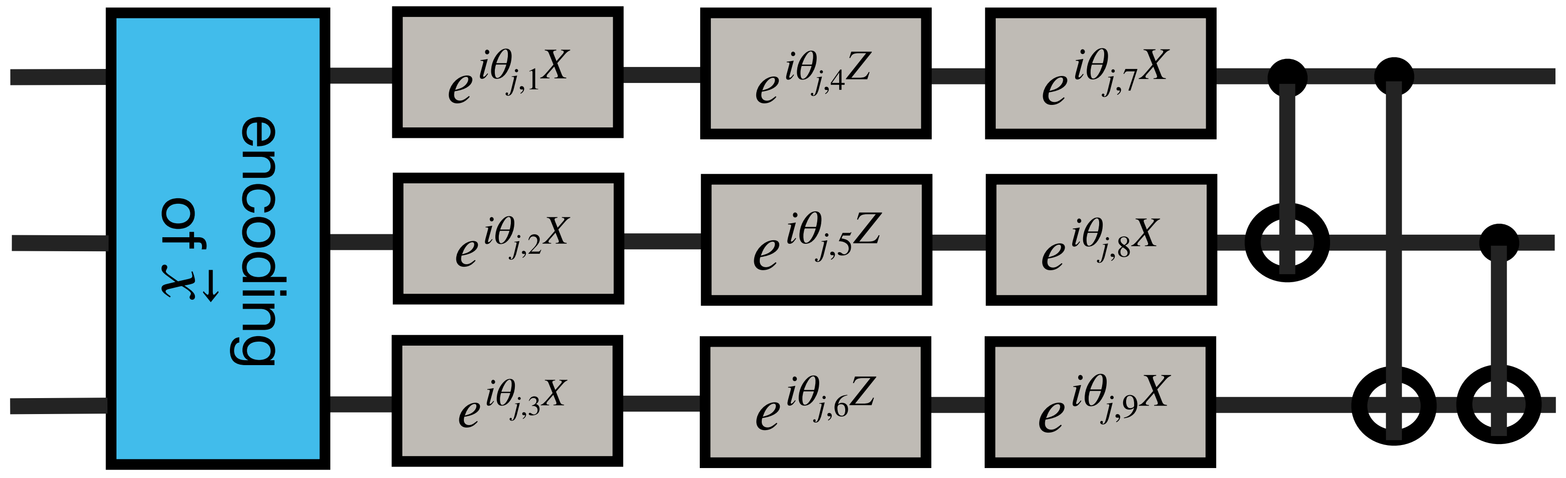
D. Aharonov, J. Cotler, X.-L. Qi, *Quantum algorithmic measurement,* Nature Comm. **13**, 887 (2022)

R. R. Allen et al., *Quantum Computing Enhanced Sensing*, arXiv:2501.07625

H.-Y. Huang et al., *Quantum advantage in learning from experiments*, Science **376**, 1182 (2022)

S. A. Khan et al., *Quantum computational sensing using quantum signal processing, quantum neural networks, and Hamiltonian engineering*, arXiv:2507.15845

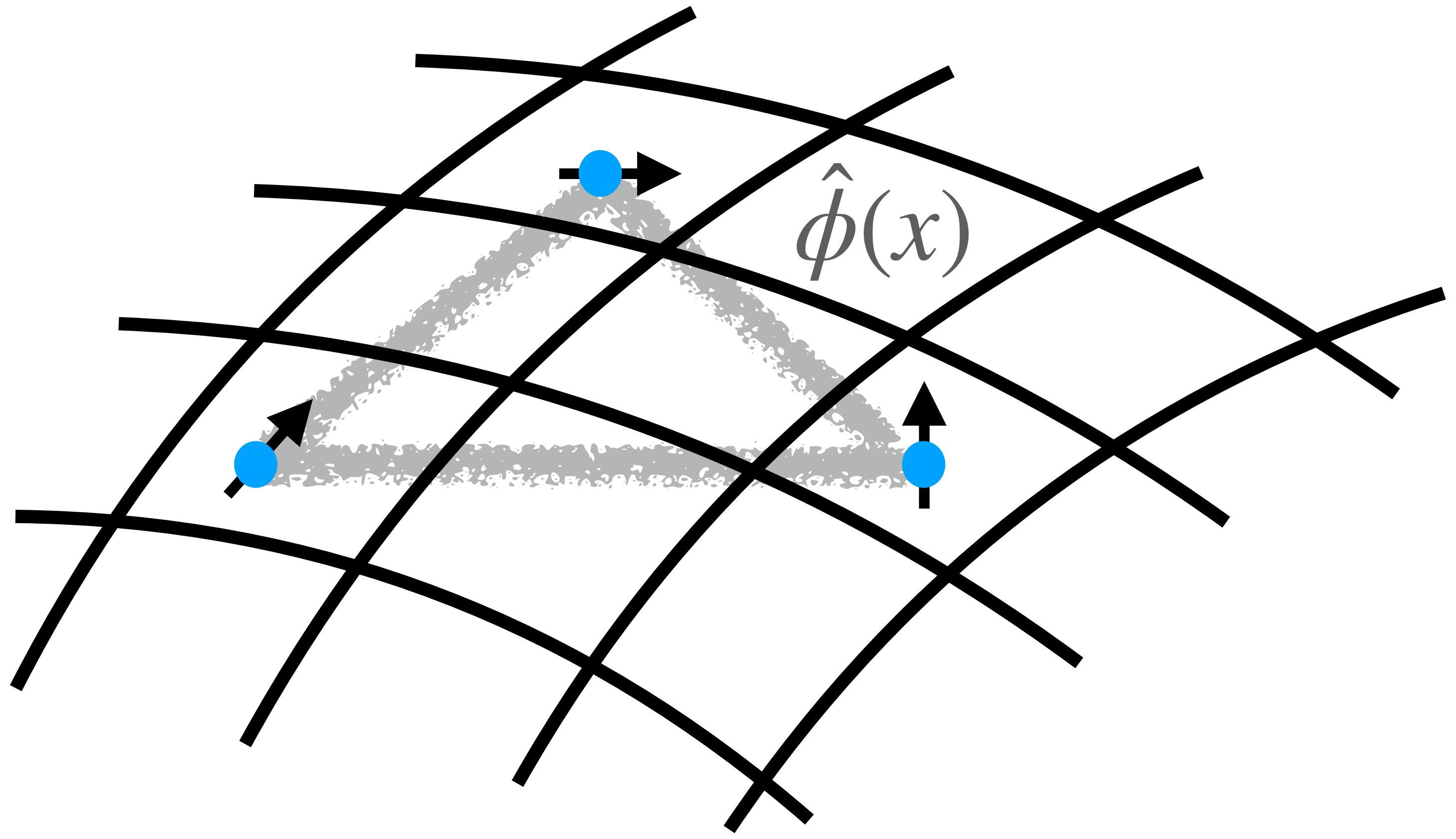
Single qubit rotations



- Assume **resource**:

  Can **control axis** of free qubit evolution

$$\vec{n} \rightarrow \vec{n}(\varphi, \theta)$$

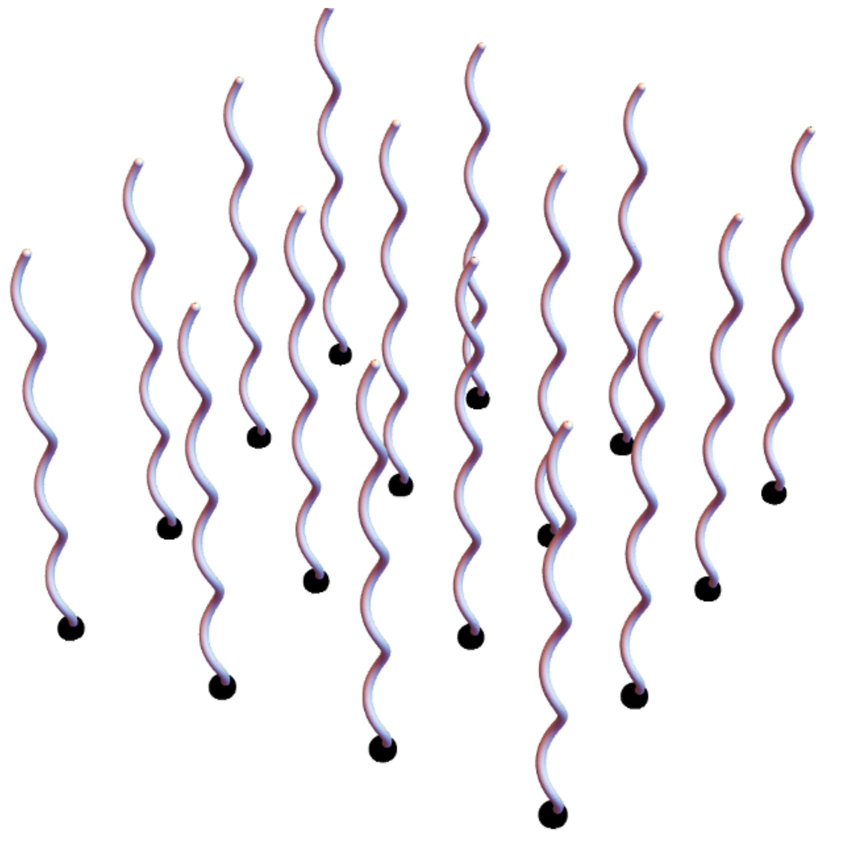


- Rotation **angles** φ, θ are **tunable** parameters (τ **proper time**)

$$U_{\text{free}}(\theta, \varphi, \Delta\tau) = \exp\left\{-i\left(\Delta\tau\frac{\Omega}{2}\right)\vec{n}(\varphi, \theta)\cdot\begin{pmatrix}X\\Y\\Z\end{pmatrix}\right\}$$

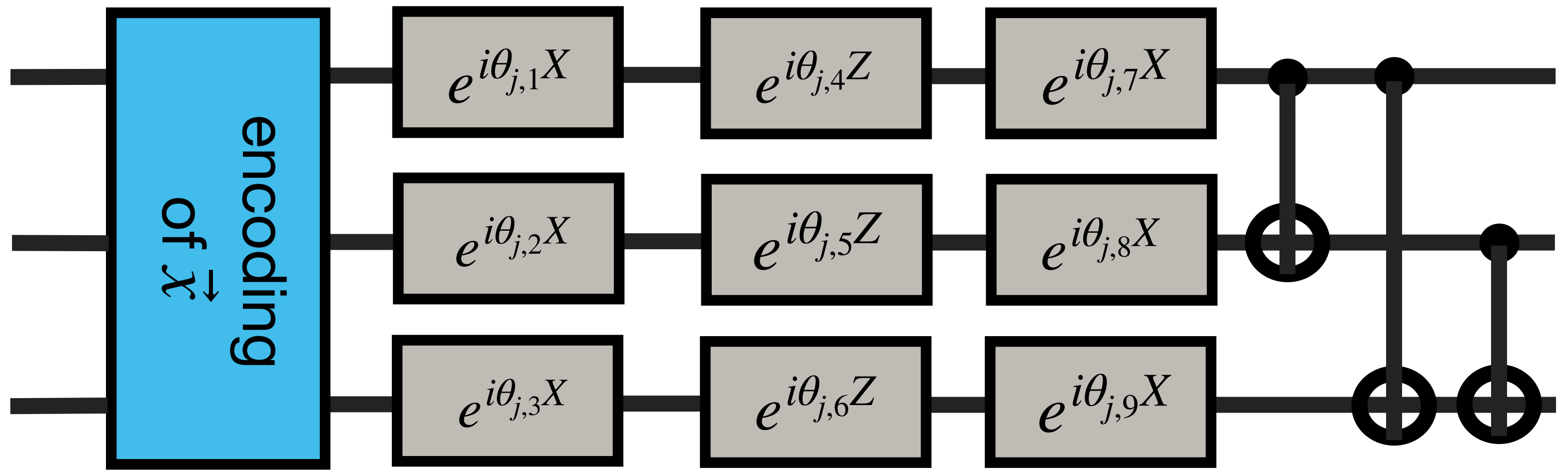
- Relativistic method for choosing $\Delta\tau$:

  Use motion to pick **time-dilation factor**!



- **Circular** motion: $\Delta\tau = \Delta t \cdot \sqrt{1 - R^2\omega^2}$

Interaction schedule



- **Coupling to field:**

$$H_{\text{int}}(x) = \lambda\,\Lambda(x)\,Z \otimes \phi(x)$$

Spacetime smearing: size of detector
time dependence of coupling strength

- **During** trainable single **qubit rotations:** $\Lambda = 0$, **no coupling**

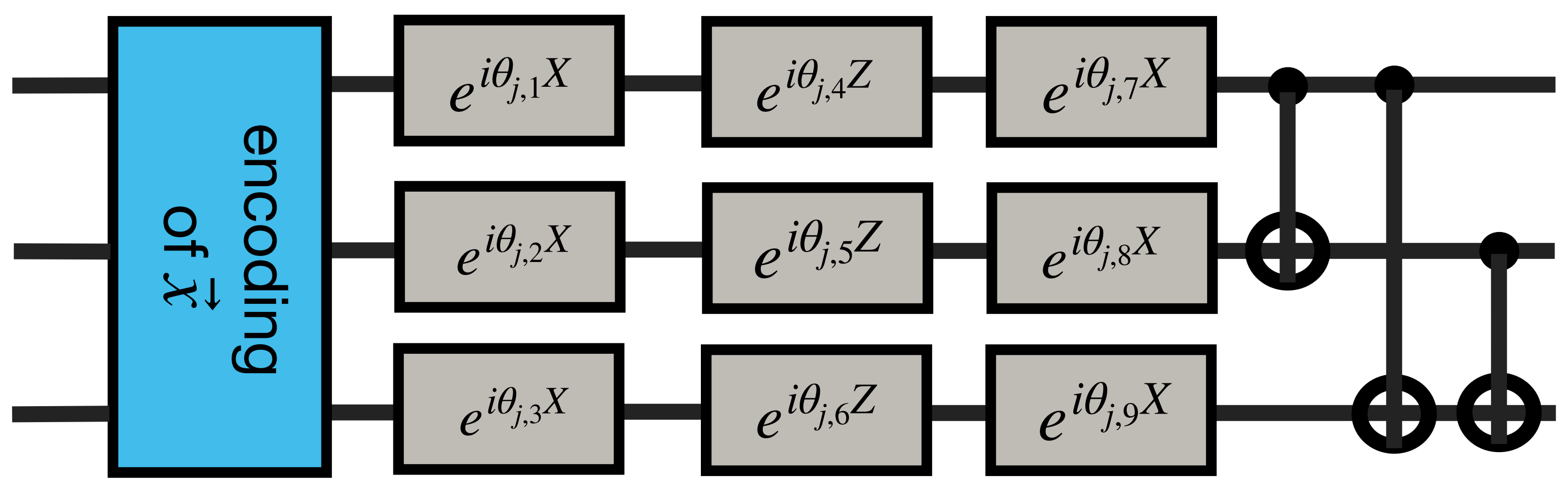
- **During interaction:** $\vec{n} = \sigma_Z \;\Rightarrow\; [H_{\text{free}}^{\text{qubits}}, H_{\text{int}}] = 0$ or $\Omega = 0 \;\Rightarrow\; H_{\text{free}}^{\text{qubits}} = 0$

- Assume field starts in **quasi-free state**: Correlators $\mathrm{Tr}[\rho\,\phi(x_1)\ldots\phi(x_M)]$ can be calculated from $\mathrm{Tr}[\rho\,\phi(x_1)\,\phi(x_2)]$ via **Wick's rule**

- Examples: **Vacuum, thermal states**

- Assume: After interaction, **field relaxes back** into quasi-free state

Entangling gates



- **Time evolution** of qubits **decomposes**:

$$\mathscr{E}(\rho_{\text{qubits}}) = U_c\,\mathscr{E}_\phi(\rho_{\text{qubits}})U_c^\dagger$$

contains **noise**, traces out **field**

- **Unitary interaction** part:

$$U_c = \exp\left\{ -\frac{i\lambda^2}{2}\sum_{j<k}\Delta_{jk}Z^{(j)}Z^{(k)}\right\}$$

Δ_{jk} Symmetric Propagator

$$\Delta_{jk} := \int \mathrm{d}V\,\mathrm{d}V'\Lambda_j(x)\Lambda_k(x')\Delta(x,x')$$

$\Lambda_k(x)$ spacetime smearing of qubit k

$$i\Delta(x,x') = \theta(t-t')[\phi(x),\phi(x')] + \theta(t'-t)[\phi(x'),\phi(x)]$$

Noise analysis

- **Fidelity** between **unitary interaction** and **noisy interaction**:

Fidelity: $$F\Big(\mathscr{E}(|\psi_0\rangle\langle\psi_0|), U_c|\psi_0\rangle\langle\psi_0|U_c^\dagger\Big) \geq \exp(-4N_{\text{couple}}^2\lambda^2|W|)$$

↑ #qubits coupled simultaneously

- **No noise:** $\lambda^2 N_{\text{couple}}^2 |W| \ll 1$ $W \approx \dfrac{1}{4\pi}$ for Gaussians

$\Rightarrow$ make **interaction strength λ small** enough

- Example: $\lambda \sim 10^{-4}, \quad N = 100$ qubits $F \geq 0.998,$ 100 layers

- **Relevant entanglement:** $|\lambda^2 \Delta_{jk}| \sim 1$ $U_c = \exp\Big\{-\dfrac{i\lambda^2}{2}\sum_{j<k}\Delta_{jk}Z^{(j)}Z^{(k)}\Big\}$

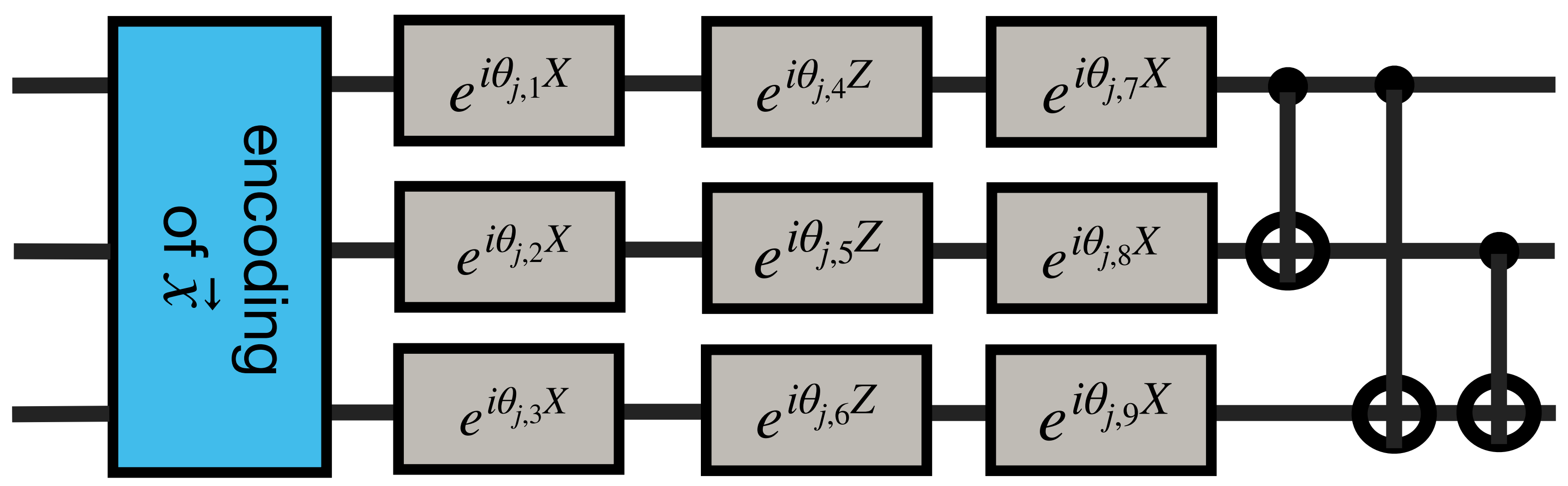
- **Gaussians:**

$$\Delta_{jk} = -\frac{1}{2\sqrt{\pi}}\frac{T}{L_{jk}}$$ can be **arbitrarily large**!

$T \sim$ interaction time

L_{jk} distance between qubits

A relativistic hardware-efficient ansatz



- Single-qubit rotations:

$$U_{\text{free}}(\theta, \varphi, \Delta\tau) = \exp\left\{-i\Delta\tau\frac{\Omega}{2}\vec{n}(\theta,\varphi)\cdot\vec{\sigma}\right\}$$

- Entangling layer: $$U_c = \exp\left\{-\frac{i\lambda^2}{2}\sum_{j<k}\Delta_{jk}Z^{(j)}Z^{(k)}\right\}$$

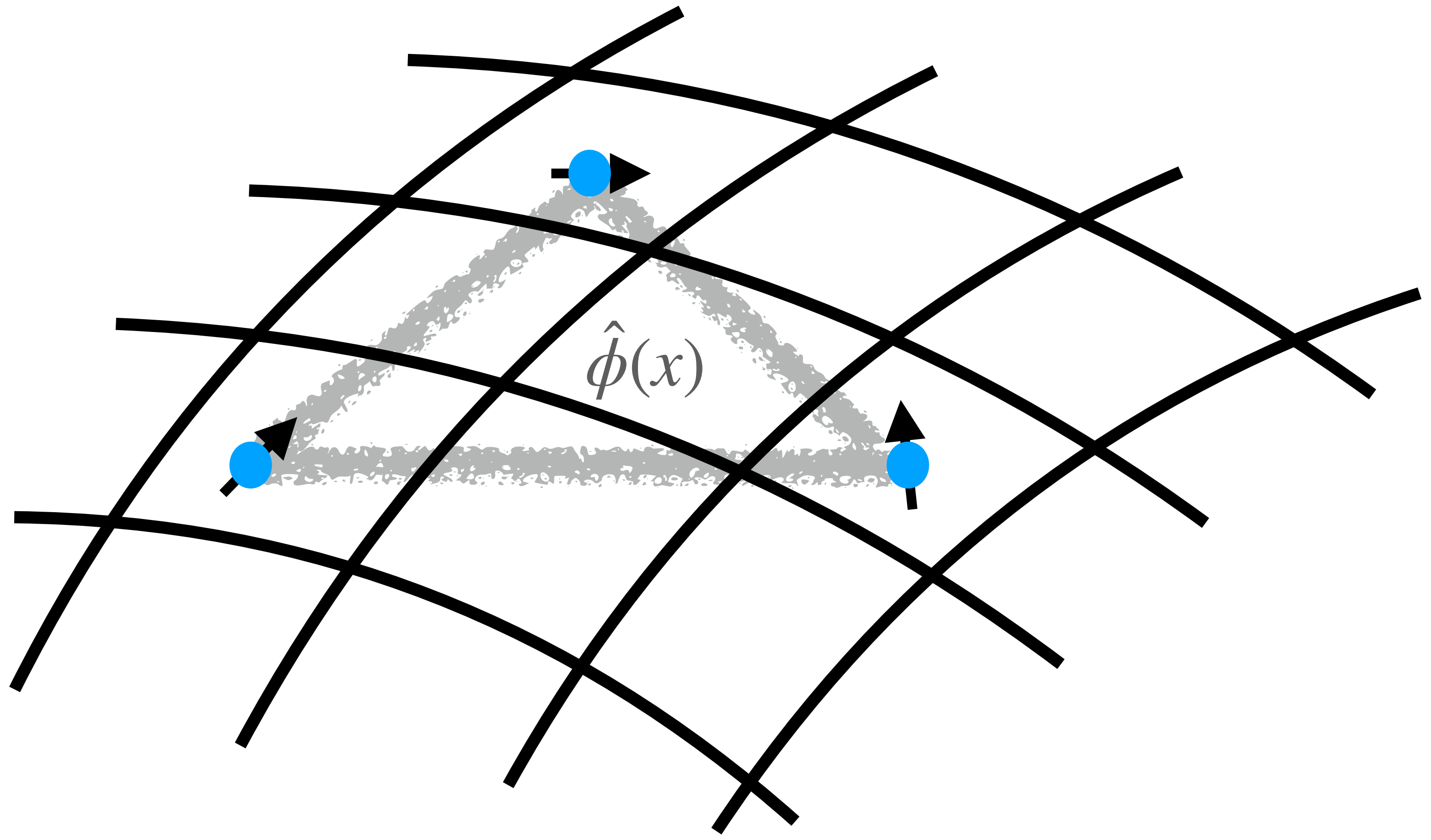
- It is **universal** for quantum computation!

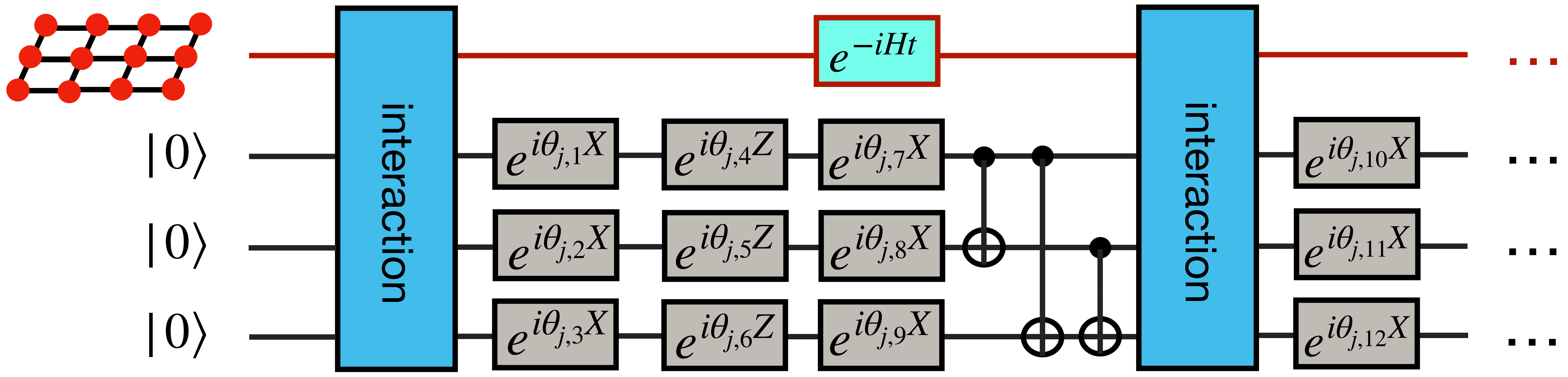
- **Proof** sketch:
  - Need to show that we can get **2-qubit entangling** gates
  - Clever placement of X gates via U_{free} allows to remove qubits from two consecutive U_c

$$X^{(1)}\cdot e^{\lambda Z^{(1)}\otimes Z^{(2)}}\cdot X^{(1)} = e^{-\lambda Z^{(1)}\otimes Z^{(2)}}$$

  - Need $\mathcal{O}(N)$ layers for one two-qubit entangling gate

Thanks for your attention!





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Gaussian spacetime smearings

- **Gaussian** spacetime smearing $\Lambda_k(x)$

$$\Lambda_k(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left\{ -\frac{(t-t_0)^2}{2T^2} \right\} \exp\left\{ -\frac{(\vec{x}-\vec{x}^{(k)})^2}{2\sigma^2} \right\}$$

$T \sim$ interaction time

L_{jk} distance between qubits

$\sigma \sim$ “radius” of qubit

- Flat spacetime (**Minkowski** space), **massless** field

- **Long interaction times:** $T \gg L_{jk} \gg \sigma$

$$\Delta_{jk} = -\frac{1}{2\sqrt{\pi}}\frac{T}{L_{jk}}$$

$$W(\Lambda_k, \Lambda_k) = \frac{1}{4\pi}$$

Wightman function $W(x,x') = \mathrm{Tr}[\phi(x)\,\phi(x')\,\rho]$

$$W(\Lambda_j, \Lambda_k) = \int \mathrm{d}V\,\mathrm{d}V'\,\Lambda_j(x)\,W(x,x')\,\Lambda_k(x')$$