Relational entanglement entropies in gauge theory and gravity

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Concepts of Quantum and Spacetime
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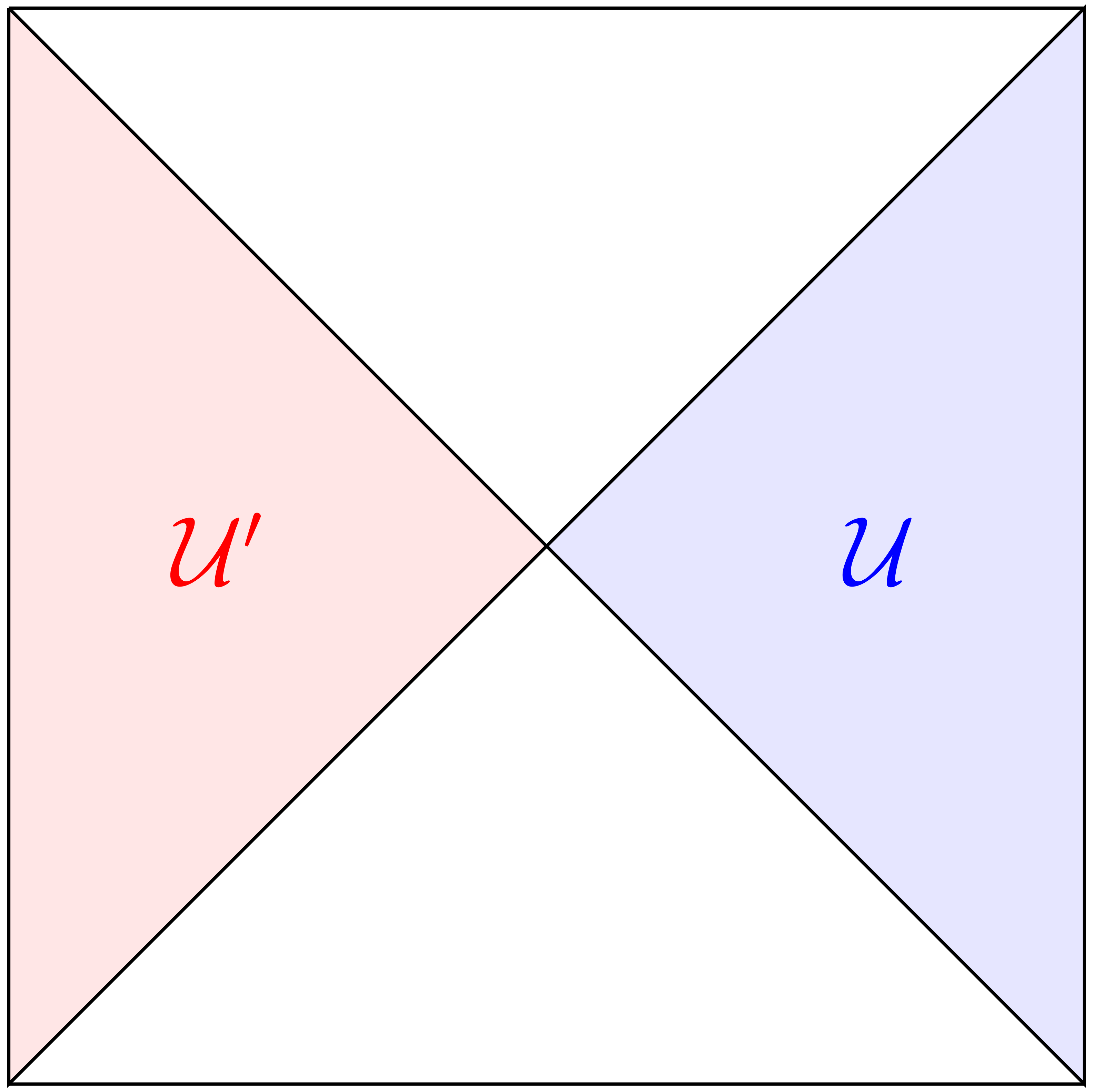
based on: De Vuyst, Eccles, PH, Kirklin, JHEP 07 (2025) 063; JHEP 07 (2025) 146
Araújo-Regado, PH, Sartini, arXiv:2506.23459

Entanglement entropy in QFT



e.g. dS space

2 challenges:

- **UV divergences** (∞ many DoFs around entangling surface)

  $\Rightarrow$ **usually: regulators** (but in tension with covariance/invariance)

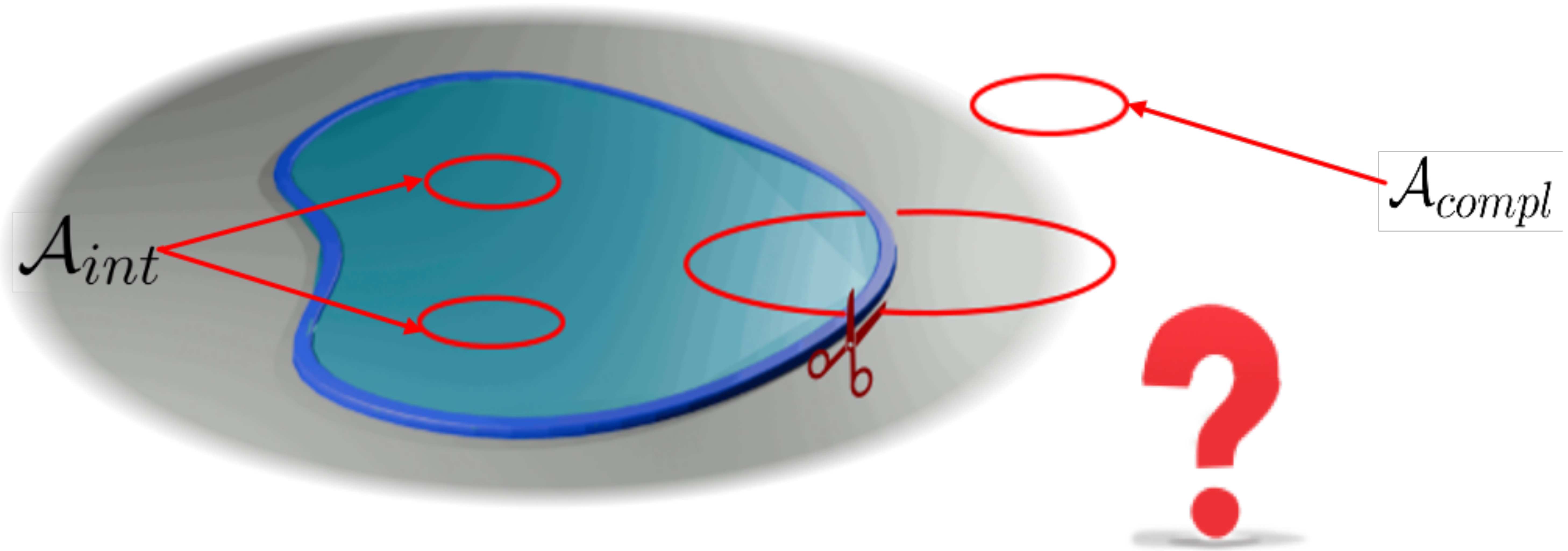
- **Locality in the presence of gauge symmetry**

  $\Rightarrow$ **what do we mean by subsystem?**

Subsystems in gauge systems

Gauge constraints nonlocal: $\mathscr{H}_{\text{phys}} \neq \mathscr{H}_{\text{int}} \otimes \mathscr{H}_{\text{compl}}$

observables nonlocal: e.g. Wilson loops



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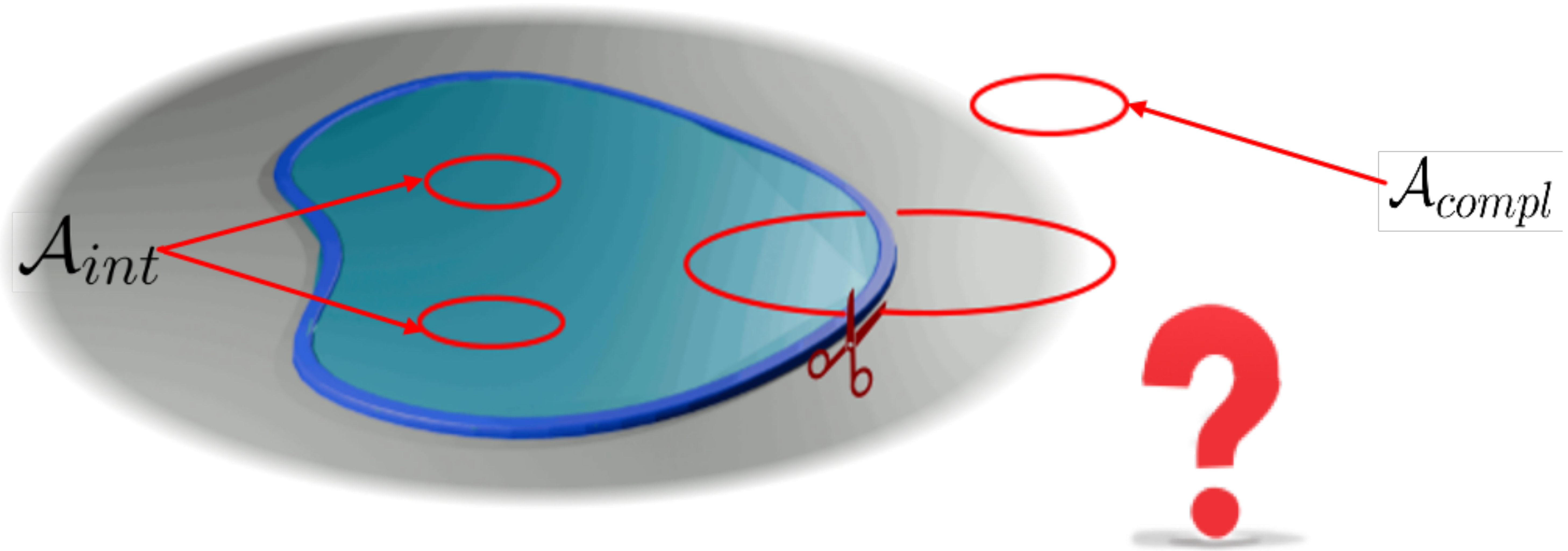

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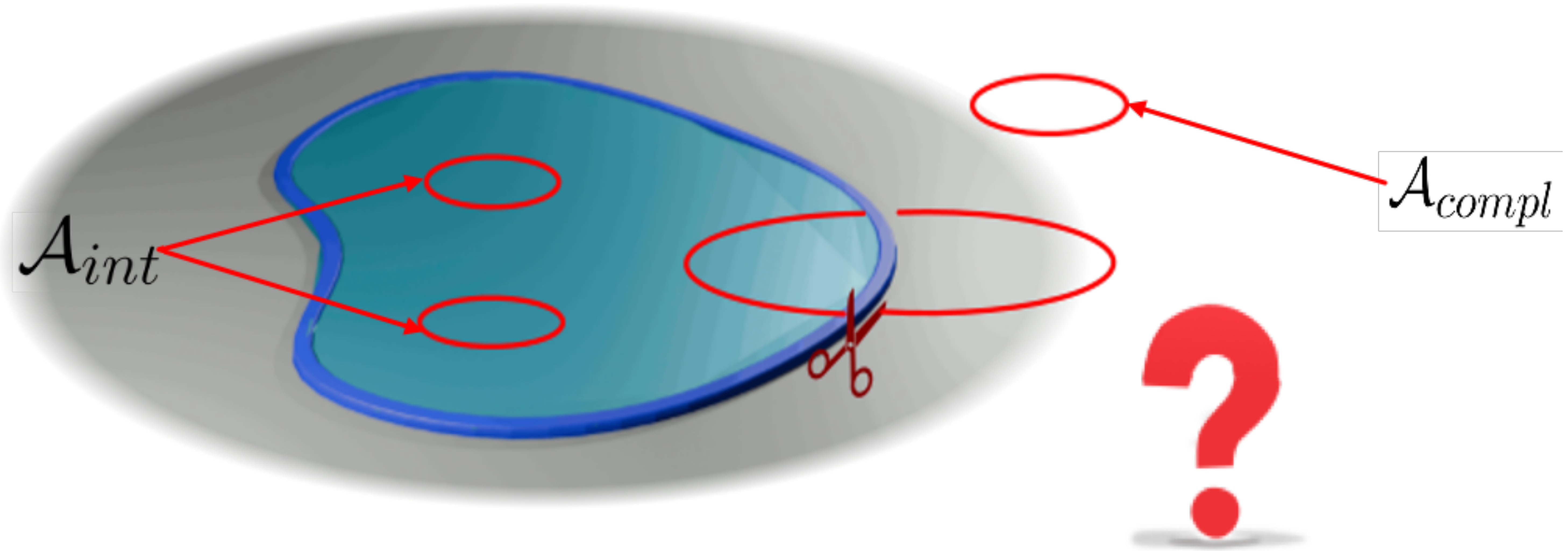

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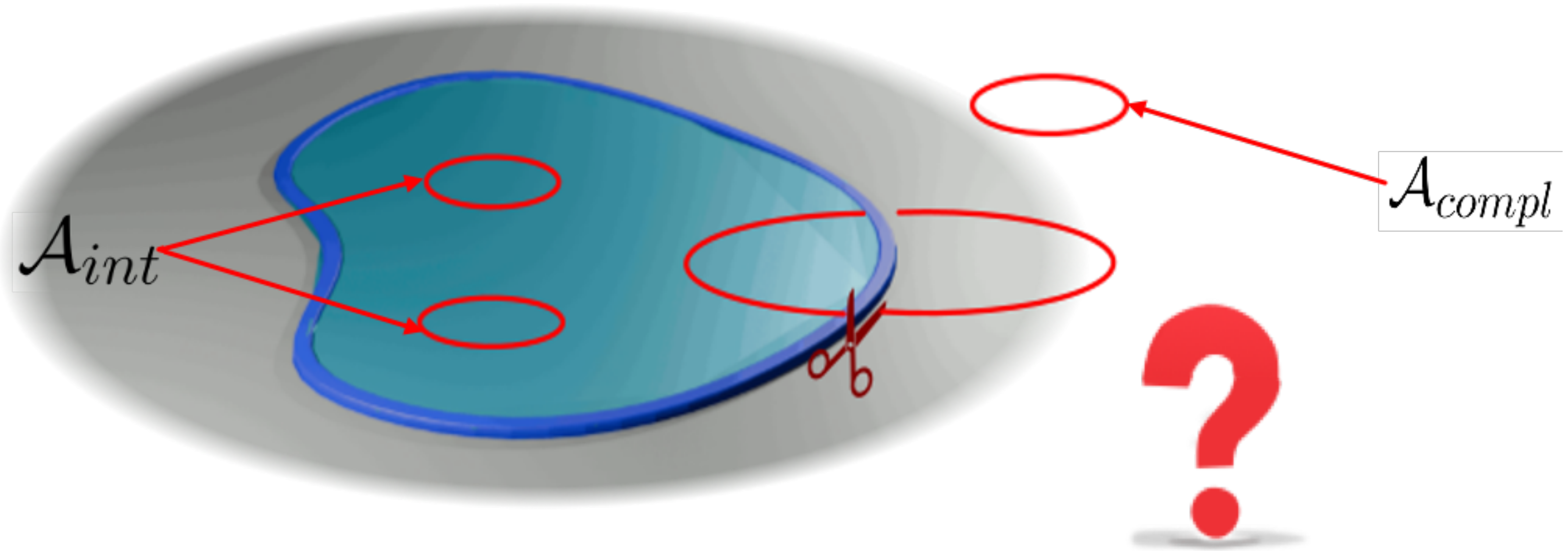

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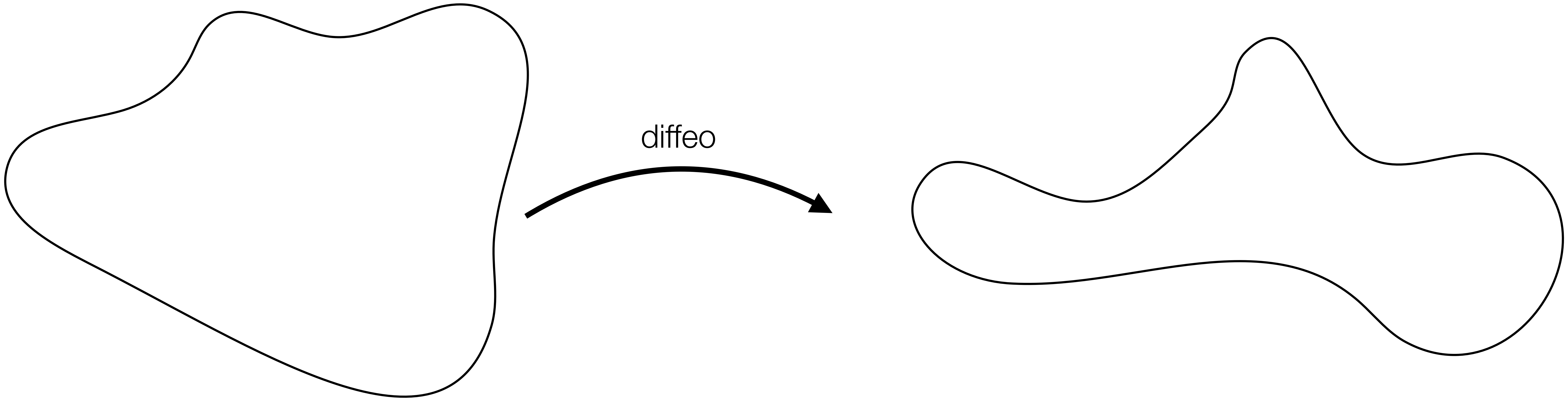
Shannon

$\Rightarrow$ **vN entropy non-distillable** (doesn't measure entanglement) $$S_{\text{vN}}(\rho_{\text{int}}) = \sum_q p_q\, S_{\text{vN}}(\rho^q) + H(\{p_q\}) + \langle \log d_q \rangle$$

[Donnelly '12; Casini, Huerta, Rosabal '13; Sonak, Trivedi '15; van Acoleyen et al. '15; Delcamp, Dittrich, Riello '16; …]

Subsystems in gravity

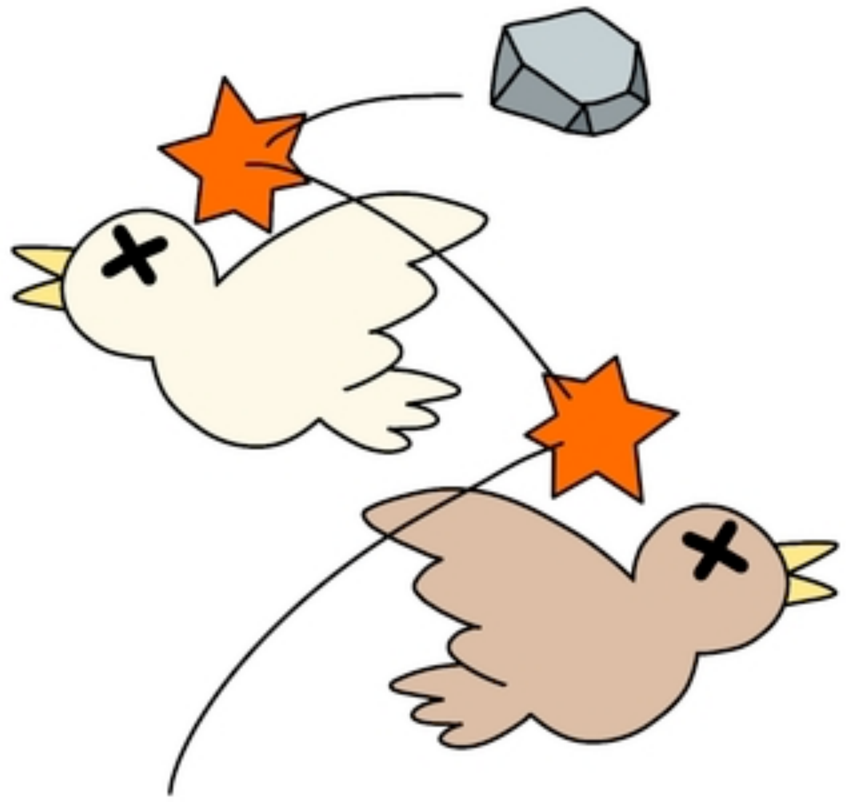
⇒ situation aggravated: gauge diffeos move naive definitions of subregions around



+ also need to worry about what is “in” and “out”

Quantum reference frames …

… offer a new perspective on both challenges



- **intrinsic regularization when describing regional data relative to a QRF** (type conversion of the corresp. vN algebra)

- **QRF perspective $\simeq$ gauge-inv. subsystem decomposition**

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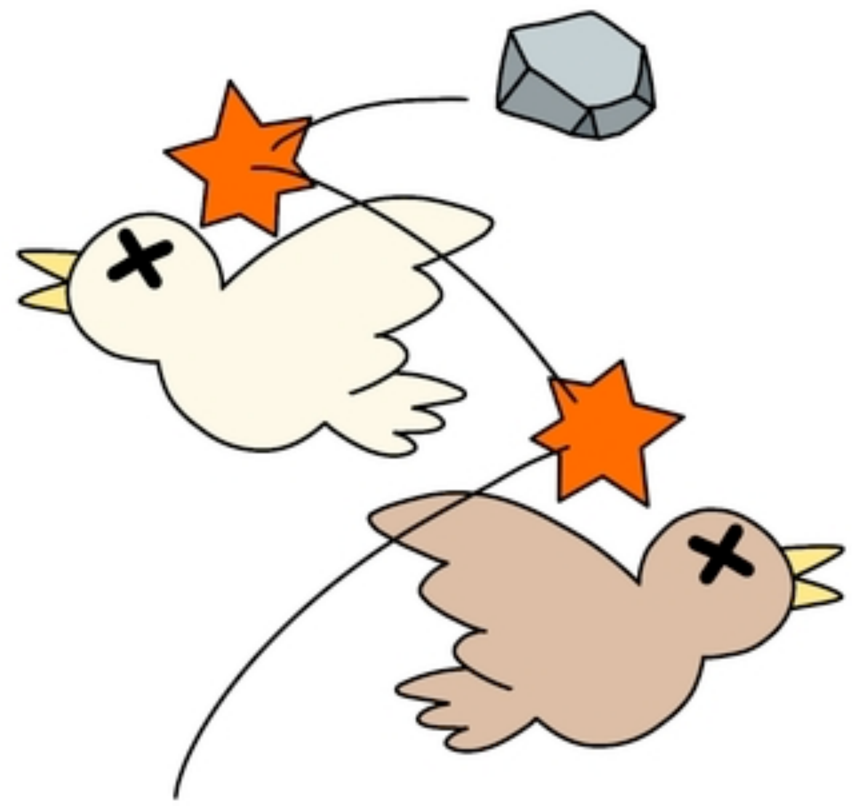

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$\Rightarrow$ embrace that diversity:
entropy and algebra hierarchy + some factor algebras and distillable definitions

Quantum reference frames

[PH, Kotecha, Mele ’13; de la Hamette, Galley, PH, Loveridge, Müller ’21; De Vuyst, Eccles, PH, Kirklin ’24; Araújo-Regado, PH, Sartini ’25;…]

Quantum reference frames and crossed products

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e.g., tetrad valued in the Lorentz group, particle position translation-group-valued,…

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- **ideal QRF: sharp orientations** $\langle g\,|\,g'\rangle = \delta_G(g,g')$ $\Rightarrow \mathscr{H}_R = L^2(G)$
- **nonideal QRF: fuzzy orientations** **e.g.** $\mathscr{H}_R \subsetneq L^2(G)$

Physics from the inside

$\mathscr{H}_R$

Physics from the inside

need a complement S to describe

could be some field

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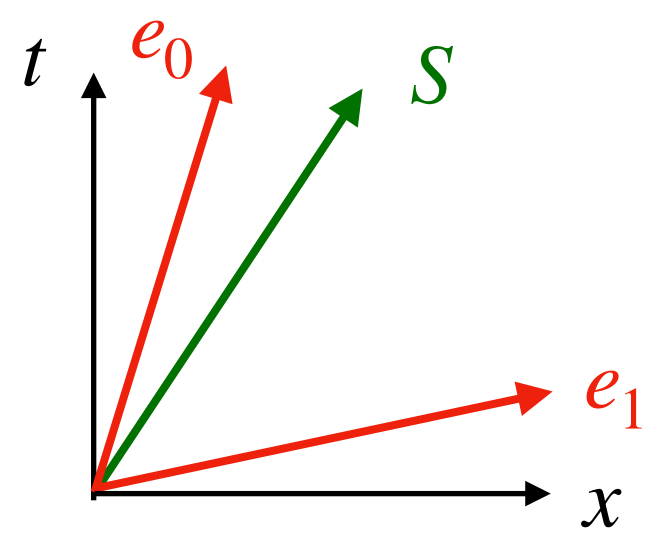

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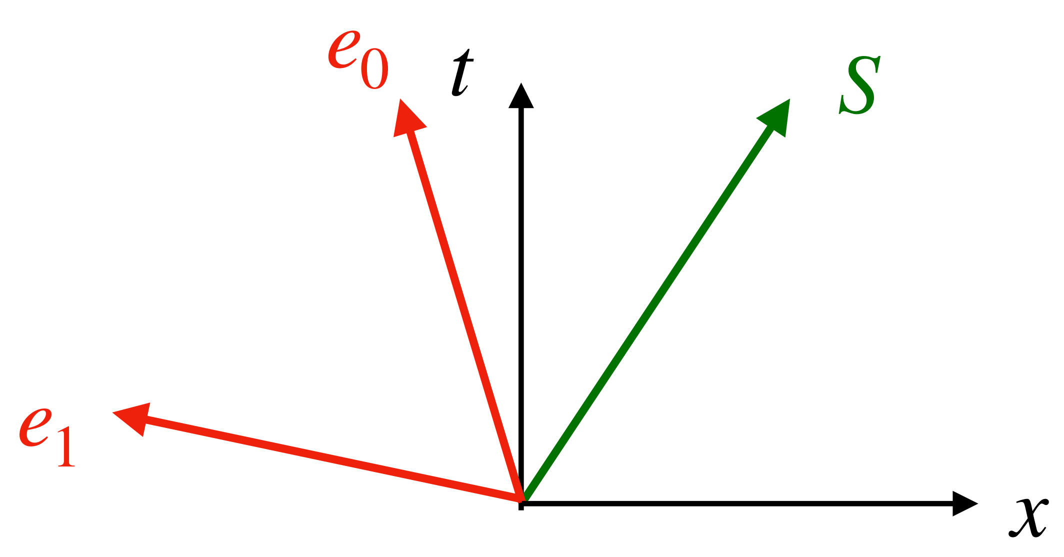

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gauge-inv. description of S relative to R via relational/dressed observables

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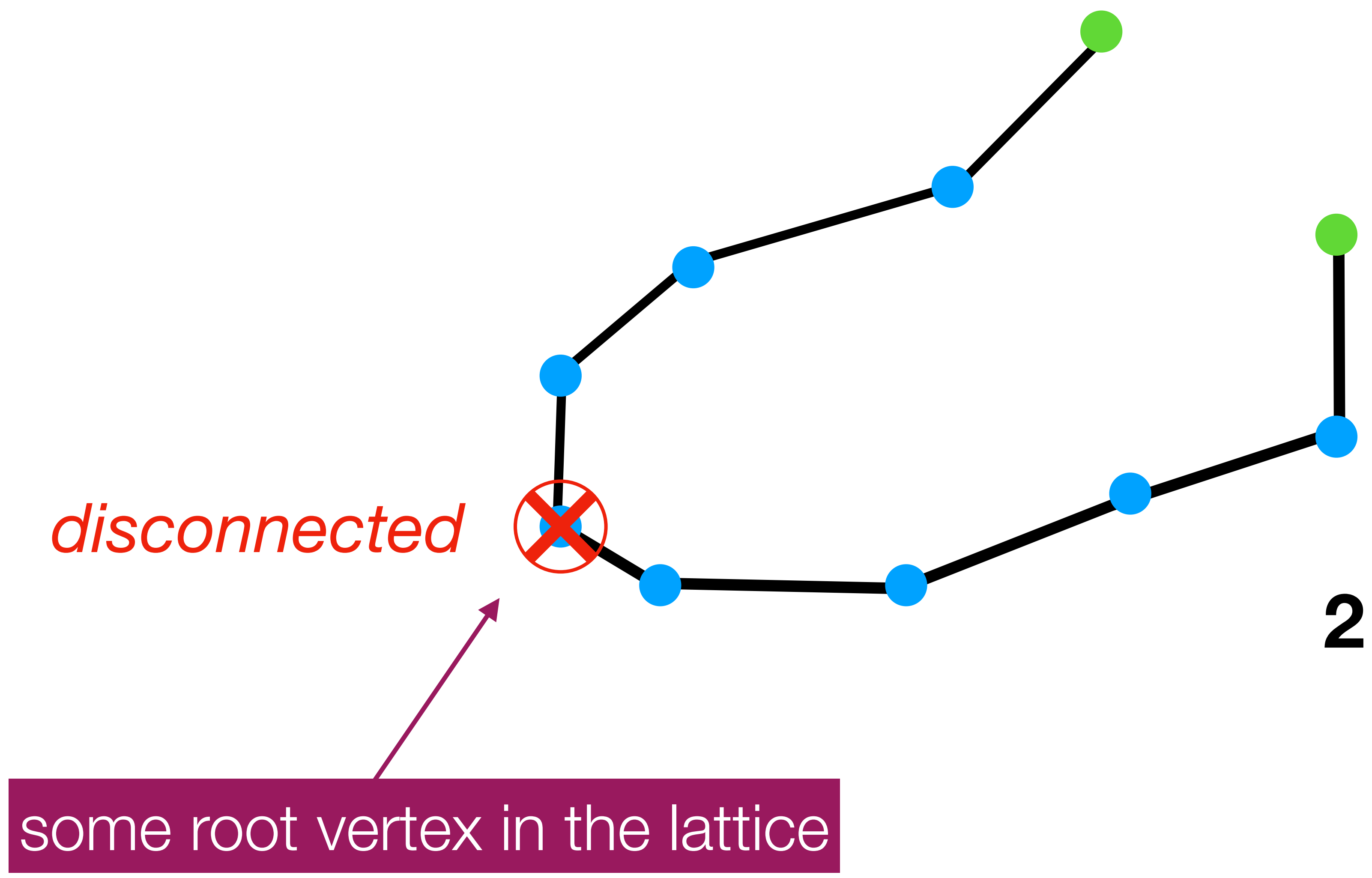

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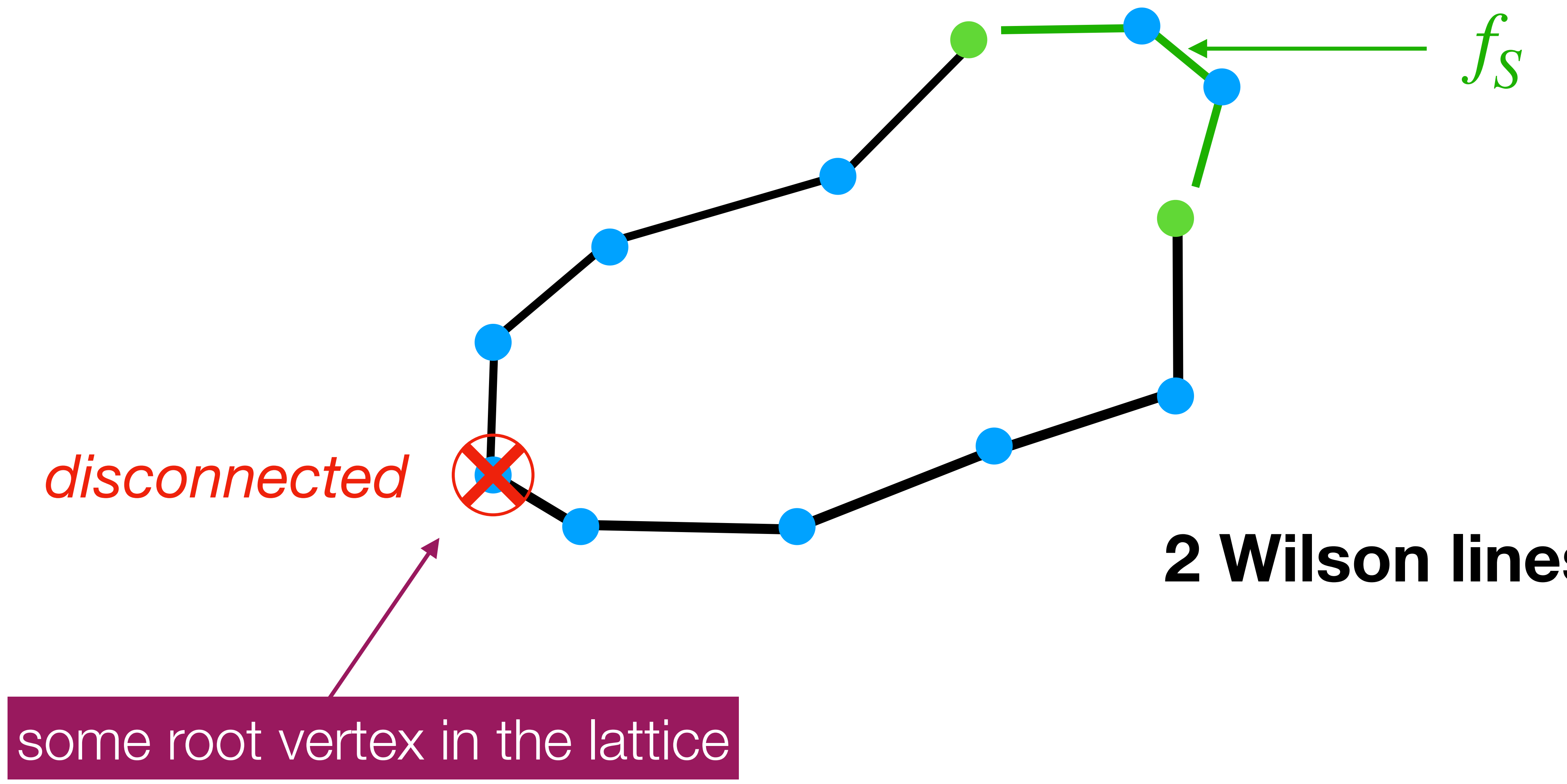

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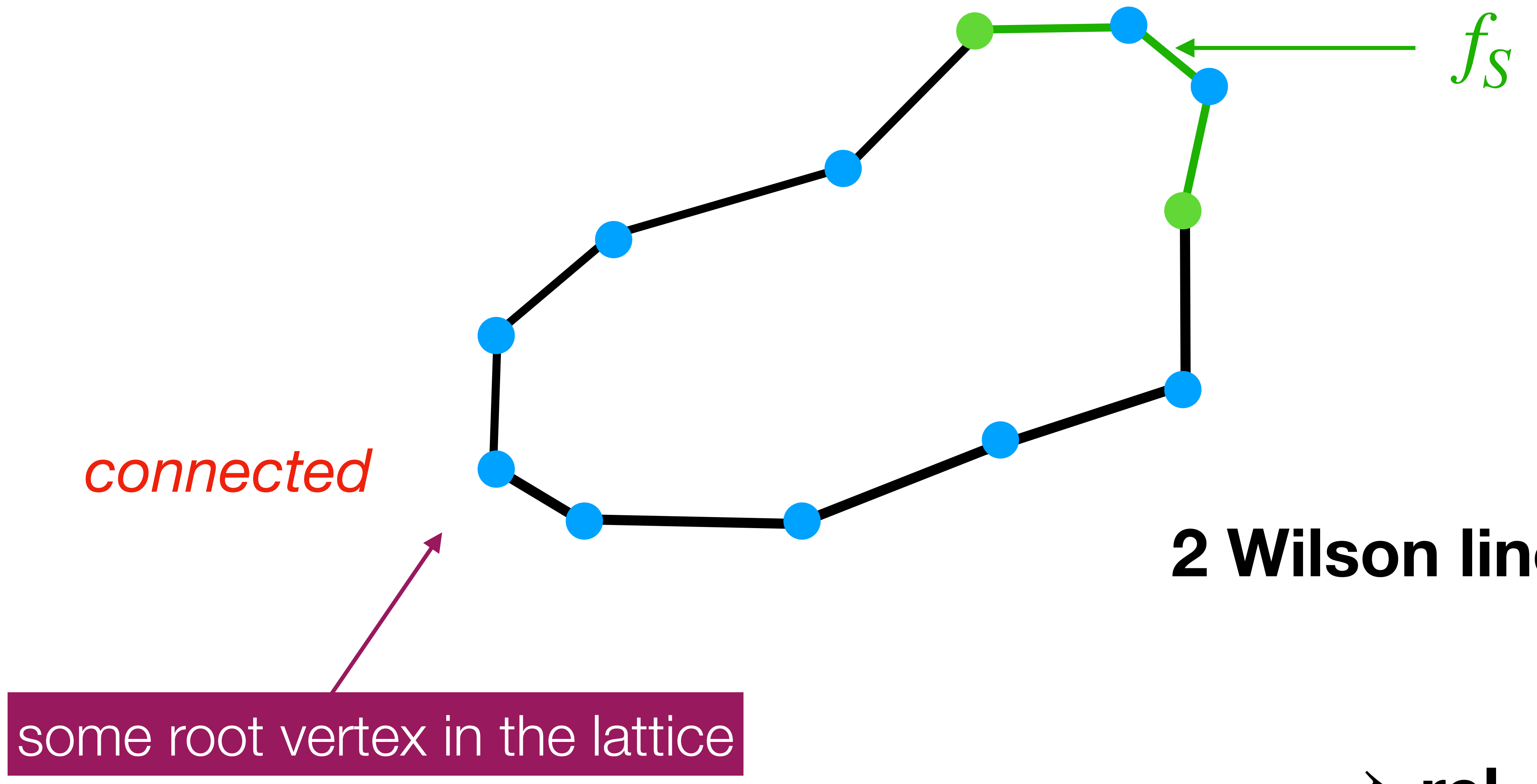

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$\Rightarrow$ **rel. obs. $O^{e}_{|R}(f_S)$ = complete Wilson loop**

Crossed products

Crossed product of system algebra $\mathscr{A}_S$ with reorientations of an ideal QRF

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so far: everything on $\mathscr{H}_{\text{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$

⇒ to compute entropies, proceed to physical Hilbert space: $\mathscr{H}_{\text{phys}} = \Pi(\mathscr{H}_{\text{kin}})$

$$\Pi = \int_G dg\, U_R(g) \otimes U_S(g)$$

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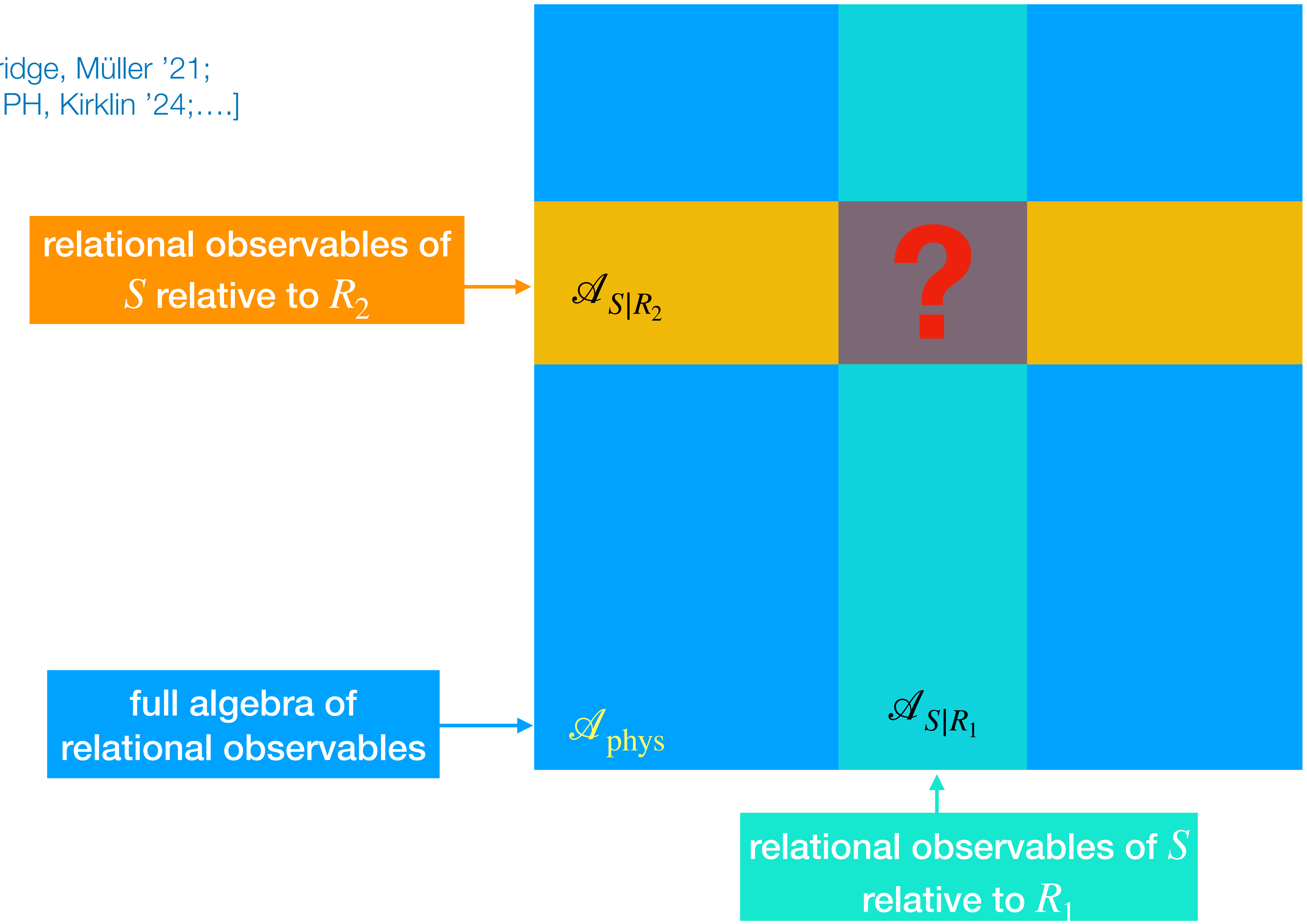
$\mathscr{A}_{S|R_1} \cap \mathscr{A}_{S|R_2} = ?$



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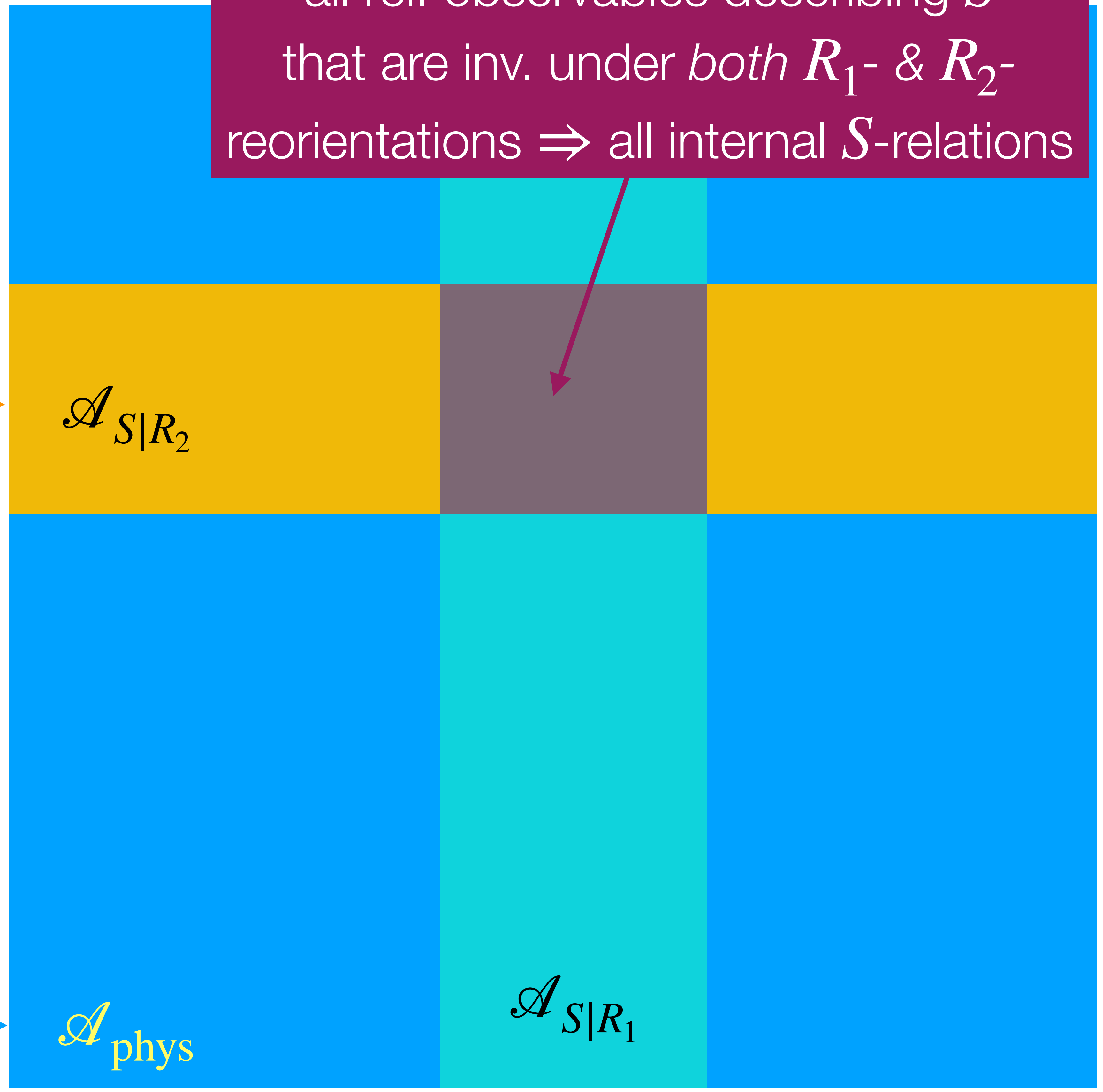
all rel. observables describing S that are inv. under *both* R_1- & R_2-reorientations ⇒ all internal S-relations

$$\mathscr{A}_{S|R_1} \cap \mathscr{A}_{S|R_2} = \mathscr{A}_S^G = \{\text{internal rel. obs. of S}\}$$

relational observables of S relative to R_2



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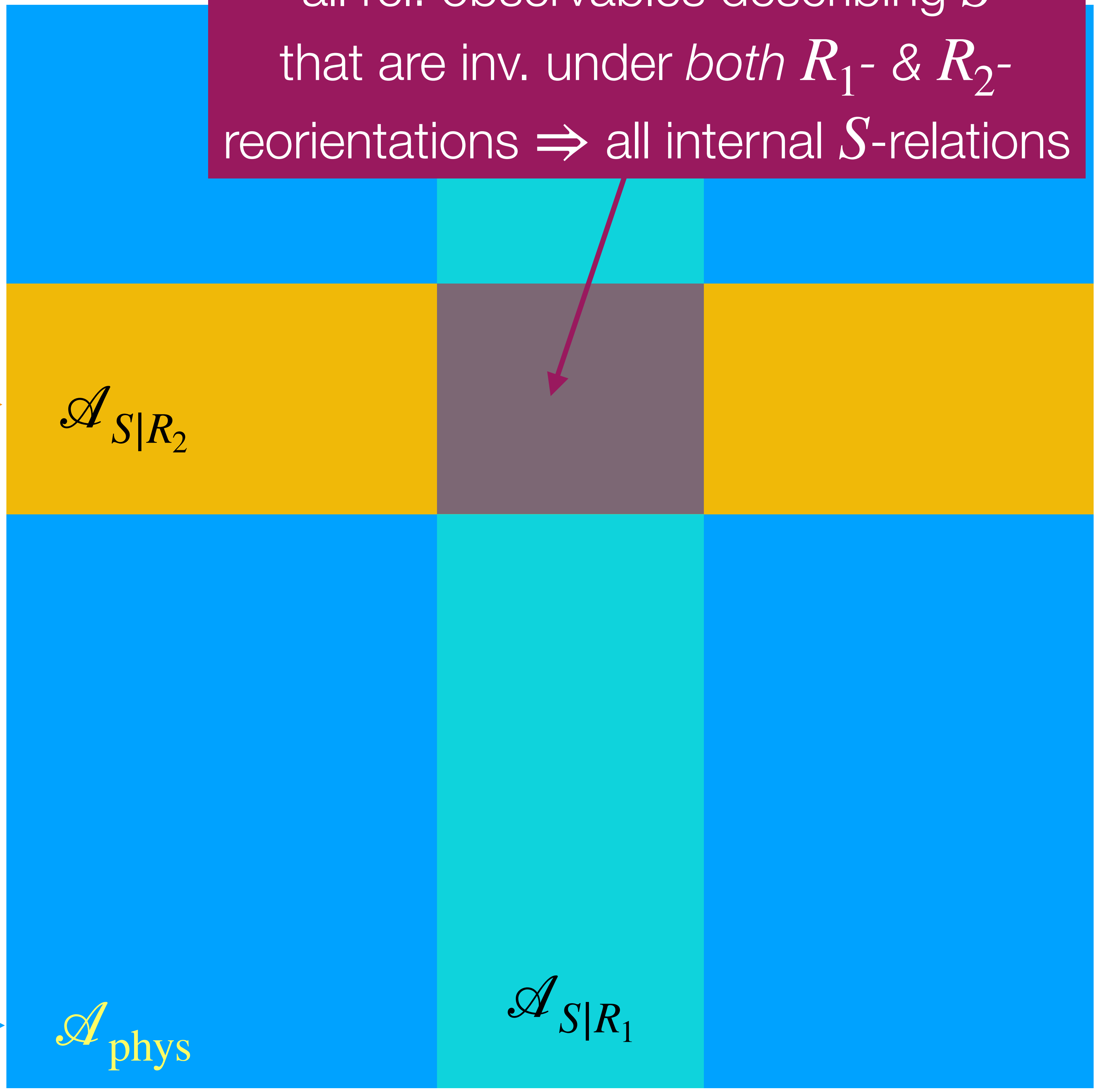

"frames R_1 and R_2 mean different gauge inv. DoFs when they refer to subsystem S"

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$\Rightarrow$ correlations, entropies, thermodynamic properties, … become QRF-dependent [systematic investigation in PH, Kotecha, Mele '23]

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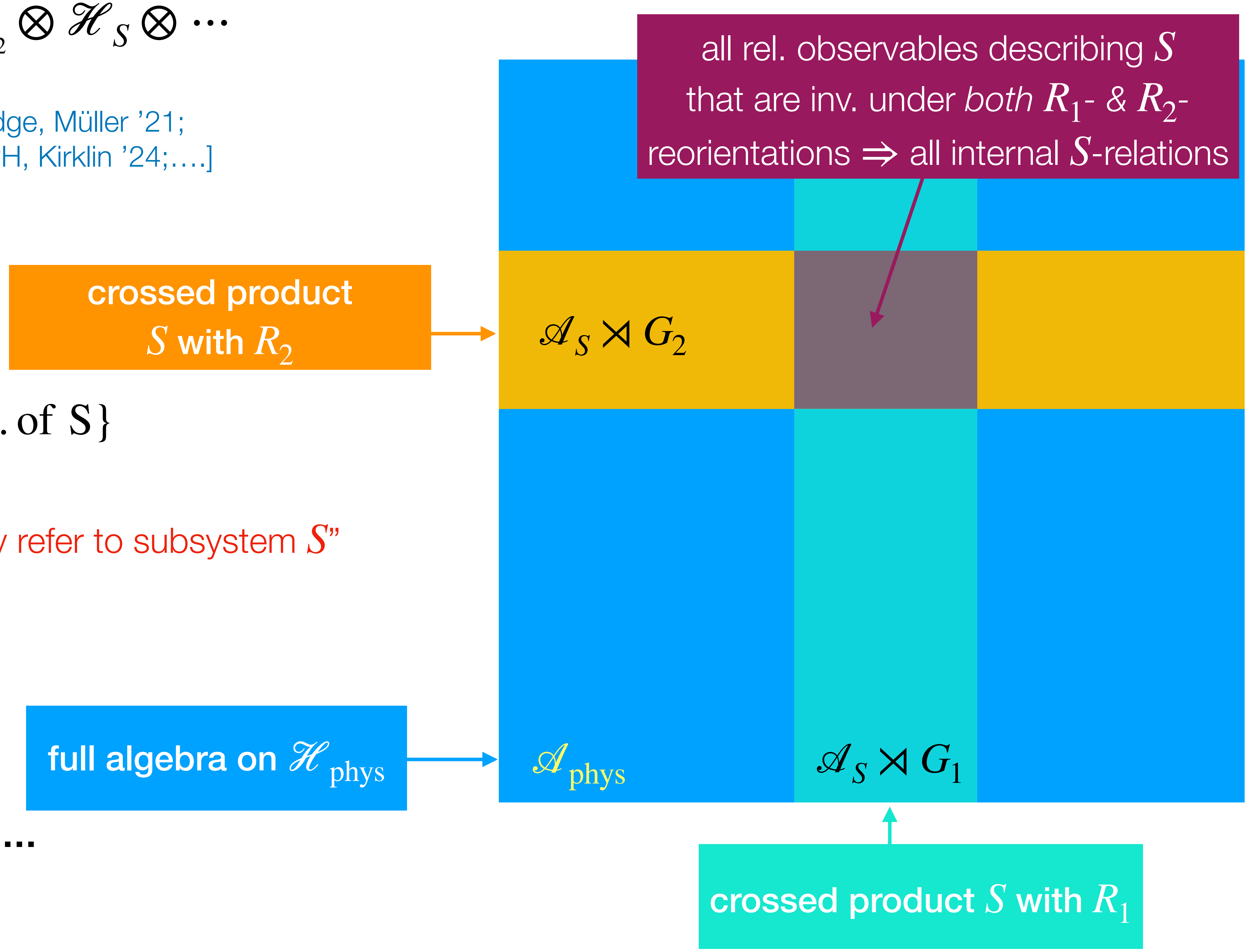

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"frames R_1 and R_2 mean different gauge inv. DoFs when they refer to subsystem S"

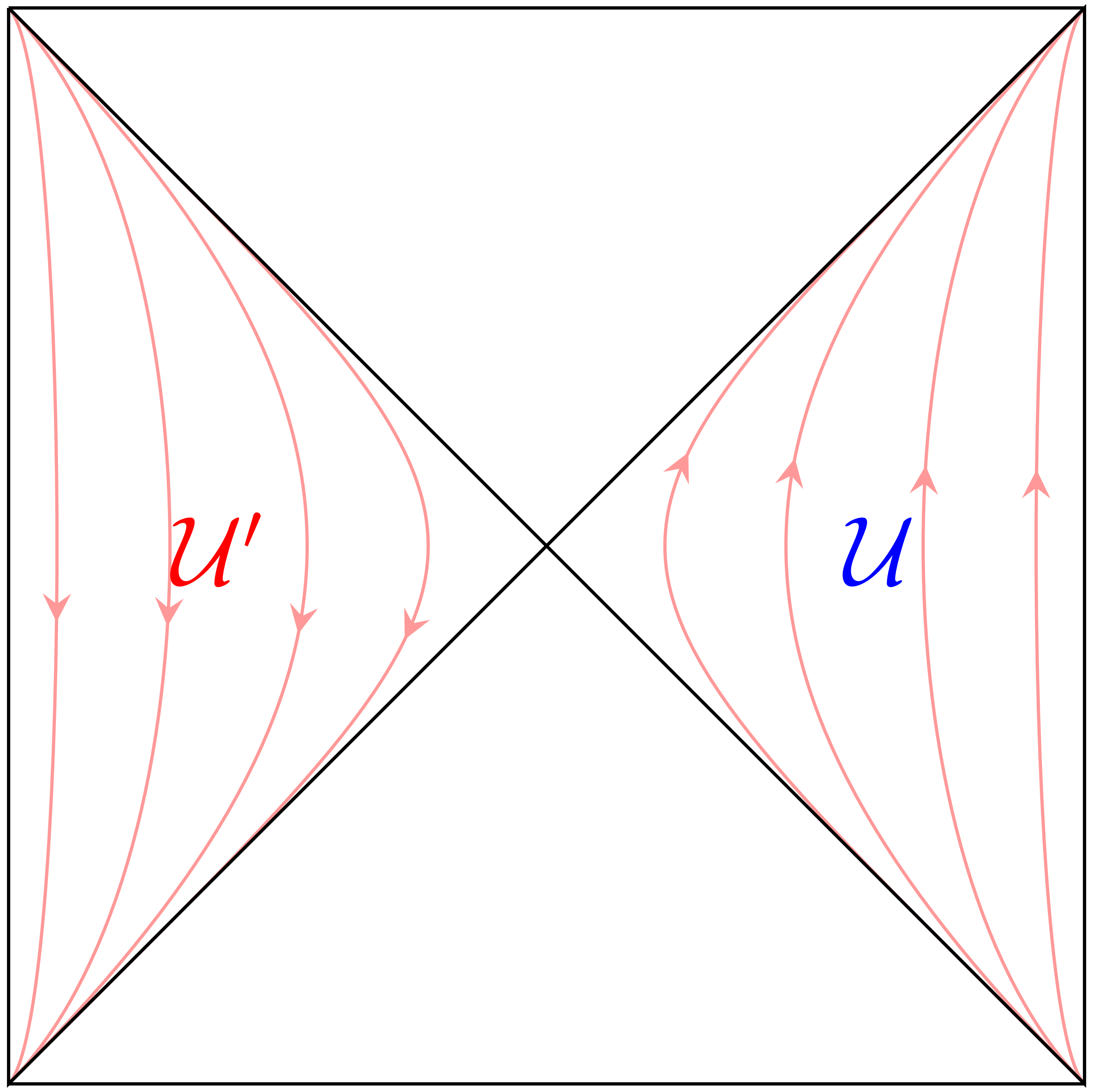
also true on $\mathcal{H}_{\text{phys}}$

$\Rightarrow$ **correlations, entropies, thermodynamic properties, … become QRF-dependent** [systematic investigation in PH, Kotecha, Mele '23]

Relational entanglement entropies in perturbative QG

[Chandrasekaran, Longo, Penington, Witten ’22; Jensen, Sorce, Speranza ’23; Kudler-Flam, Leutheusser, Satishchandran ’23; …
De Vuyst, Eccles, PH, Kirklin ’24 I & II]

Regional algebras in perturbative QG



consider two static patches in dS* filled with matter and gravitons

$\Rightarrow$ what is the EE of the right patch $\mathscr{U}$ in some global state?

* works the same way for any Killing horizon

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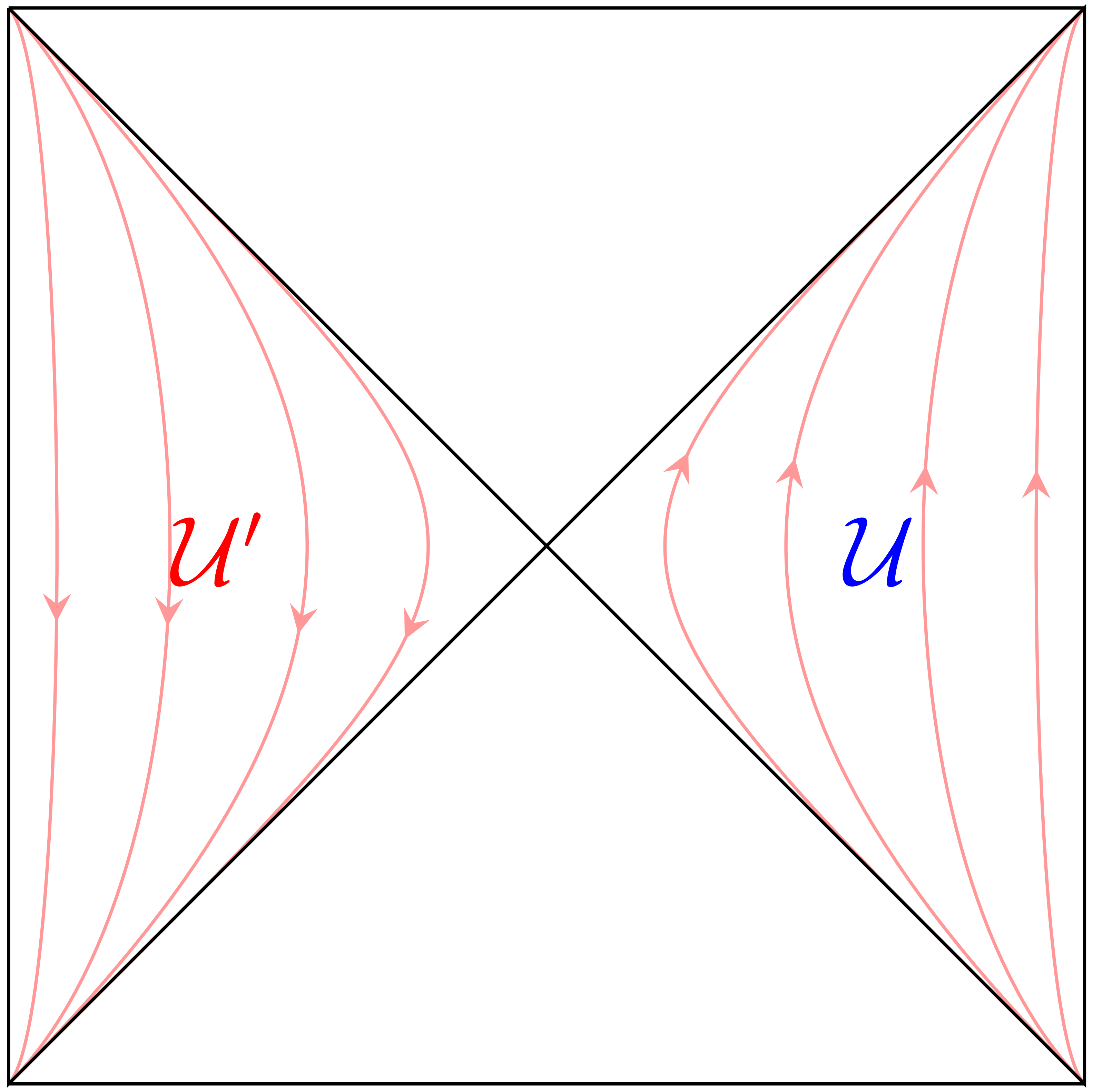

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2 challenges:

- **regional algebras $\mathscr{A}_{\mathscr{U}}, \mathscr{A}_{\mathscr{U}'}$ are type III factors** [Araki '64; Longo '82; Fredenhagen '85]

$\Rightarrow$ **no trace, no density operators, no entropies**

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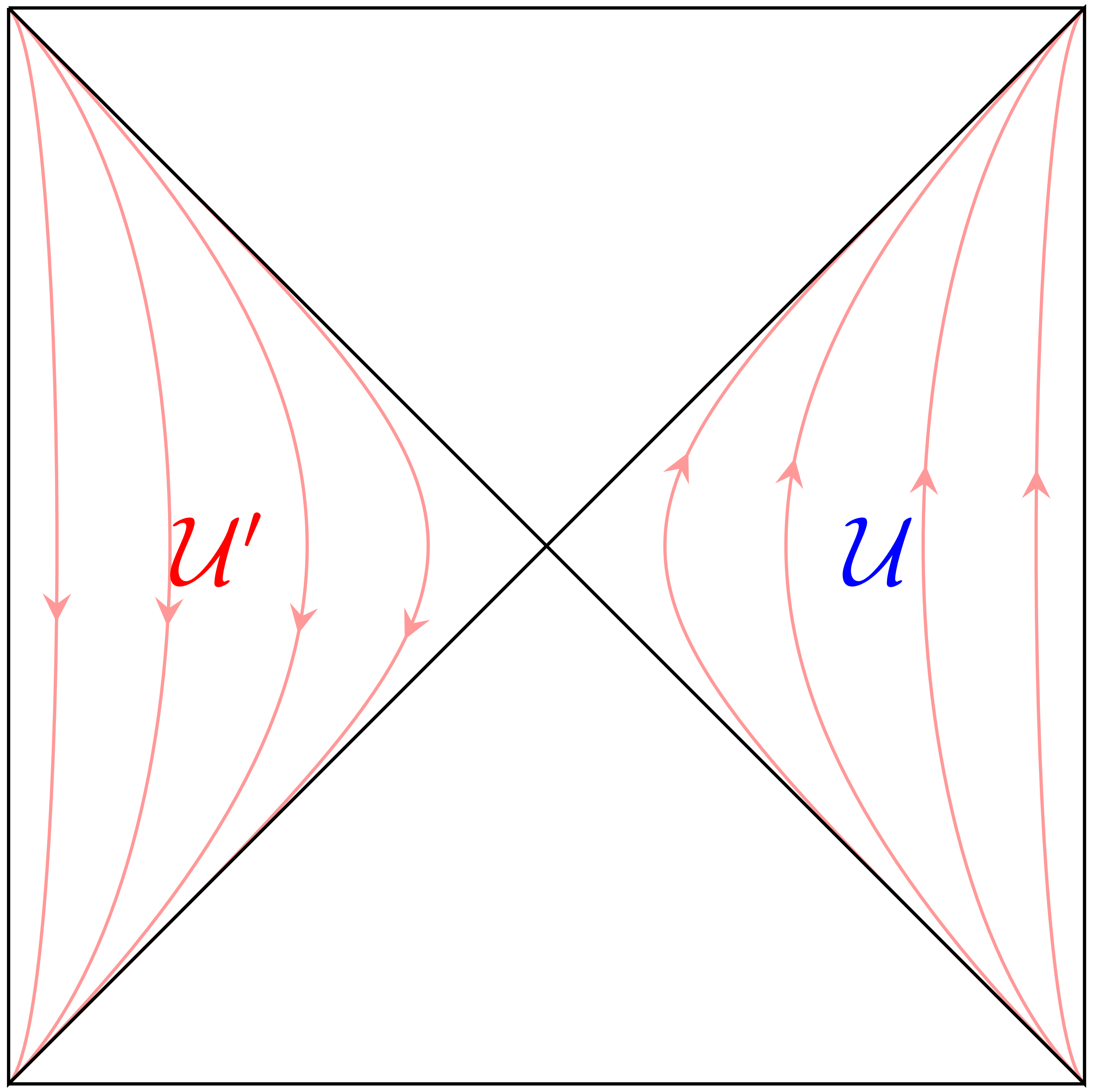
The need for “observers”



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- isometries gauge in dS, so impose boost Hamiltonian H (mod. Ham. of the vacuum) as gauge constraint [Chandrasekaran et al. ’22]

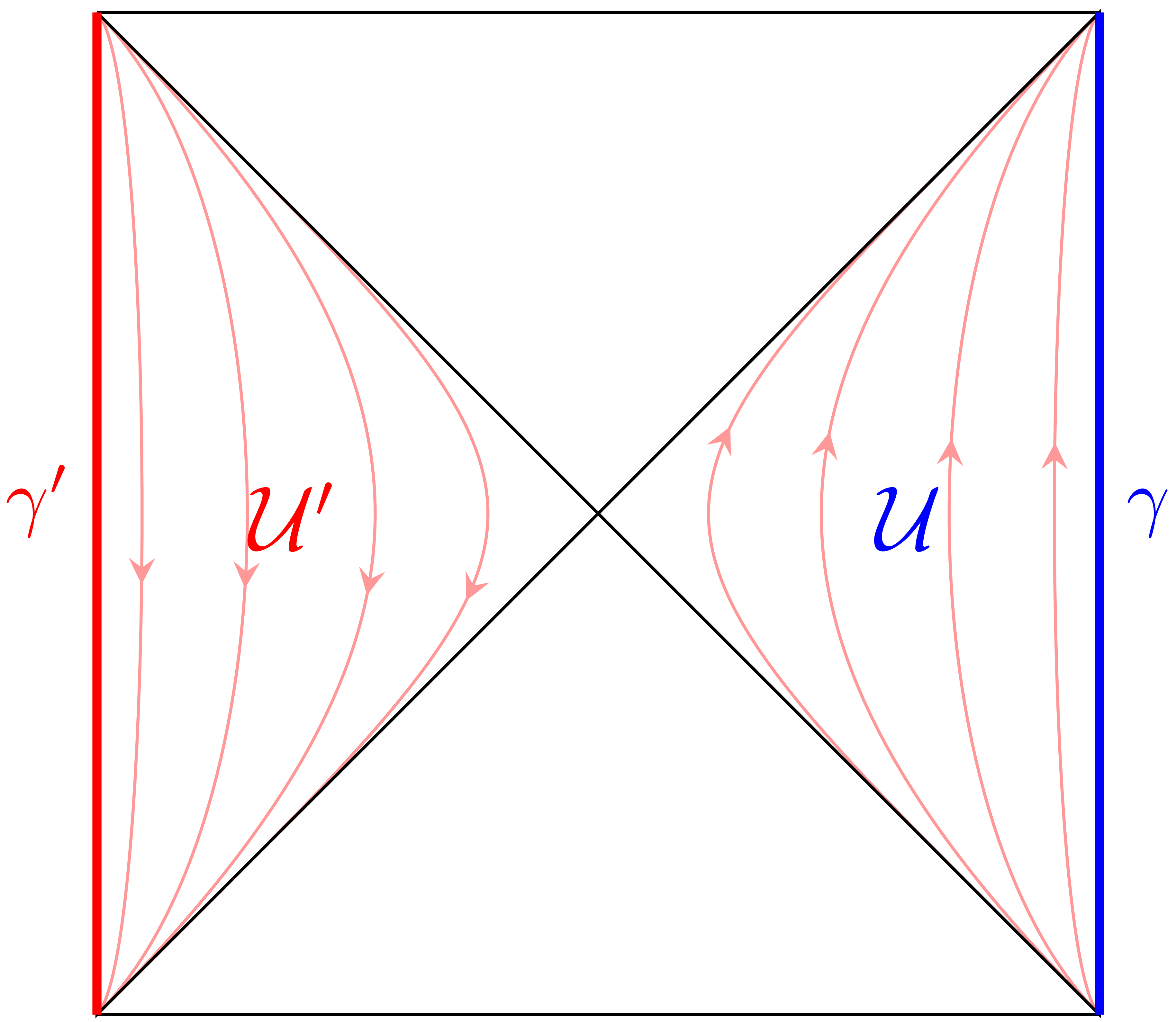
  $\Rightarrow$ acts ergodically on $\mathscr{A}_{\mathscr{U}}, \mathscr{A}_{\mathscr{U}'}$ $\mathscr{A}_{\mathscr{U}}^{H} = \mathbb{C}$

  $\Rightarrow$ no interesting **regional** diff-inv. observables (need additional DoFs)

[see De Vuyst, Eccles, PH, Kirklin JHEP 05 (2025) 211 for how to understand boost constraint in pert. theory (lin. instabilities)]

Observers are QRFs

[Chandrasekaran, Longo, Penington, Witten (CLPW) '22]



remedy: introduce observers with ideal clock in each static patch

$$C = H + p_1 - p_2$$

(perturbative QG $G_N \to 0$)

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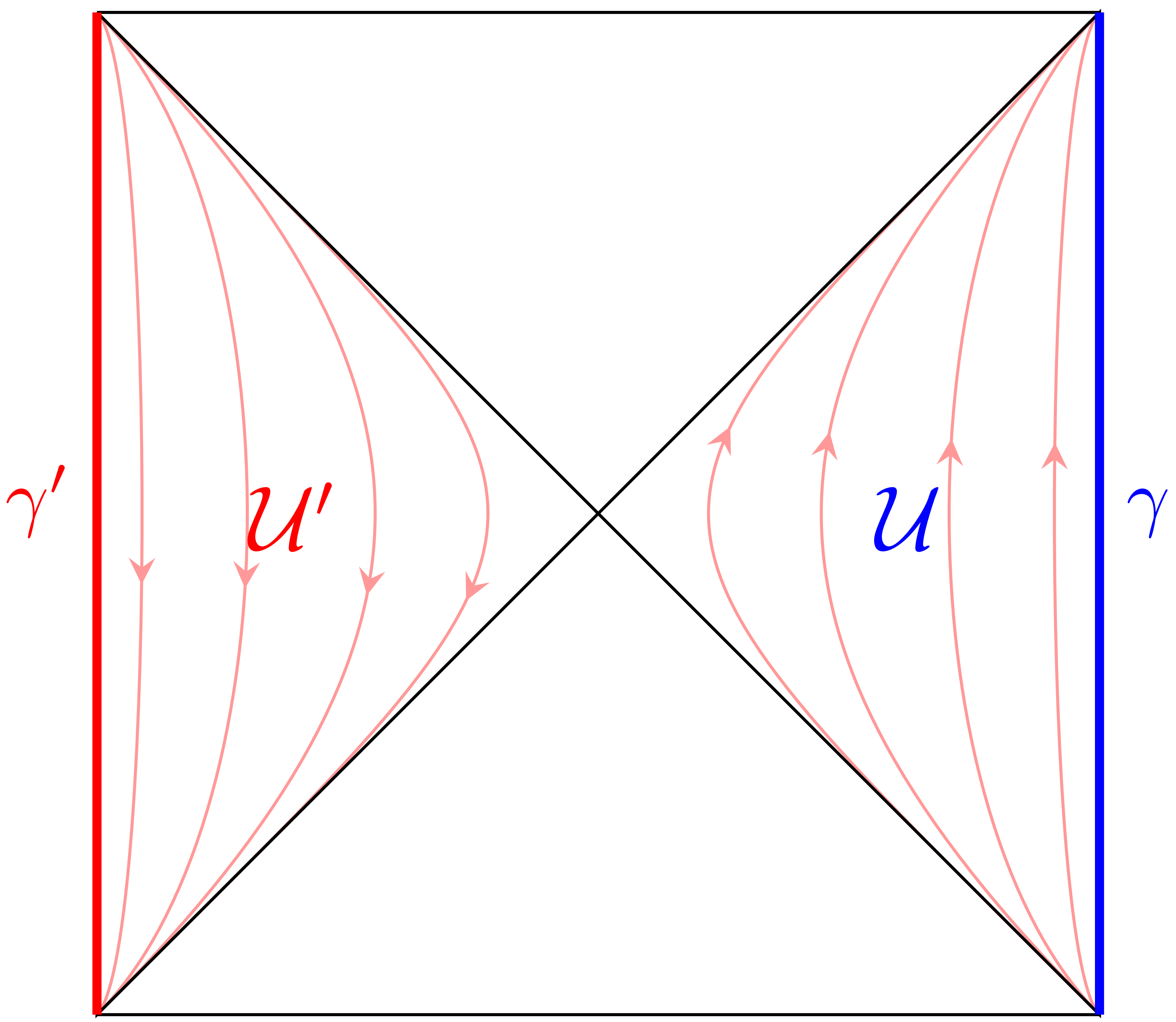

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crossed product = relational observables + clock reorientations

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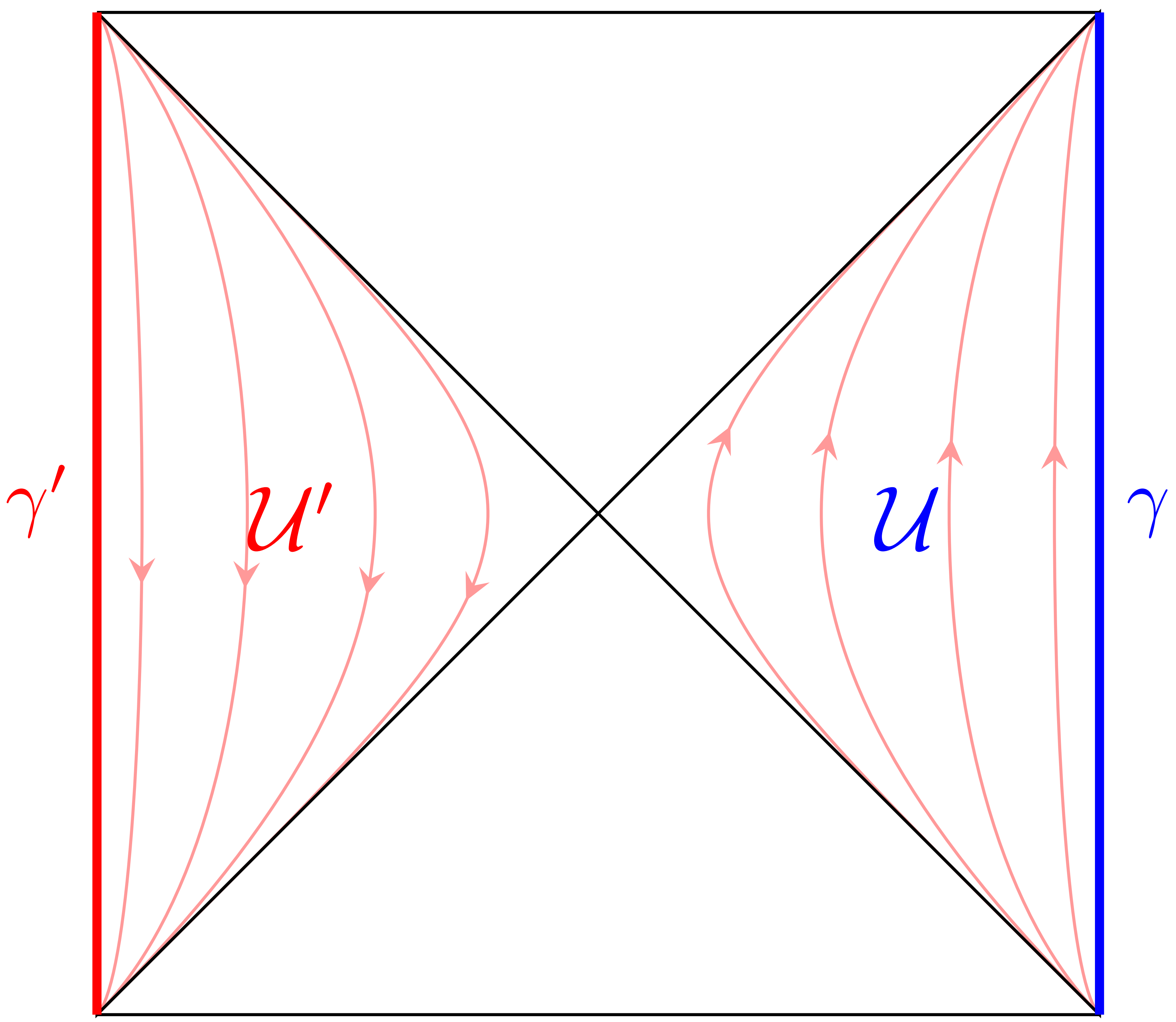

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crux:

regional QFT (incl. graviton) algebra $\mathscr{A}_{\mathscr{U}}$ is a type III factor, but crossed product is a type II factor (possesses trace, density operators and entropies)

Observers are QRFs

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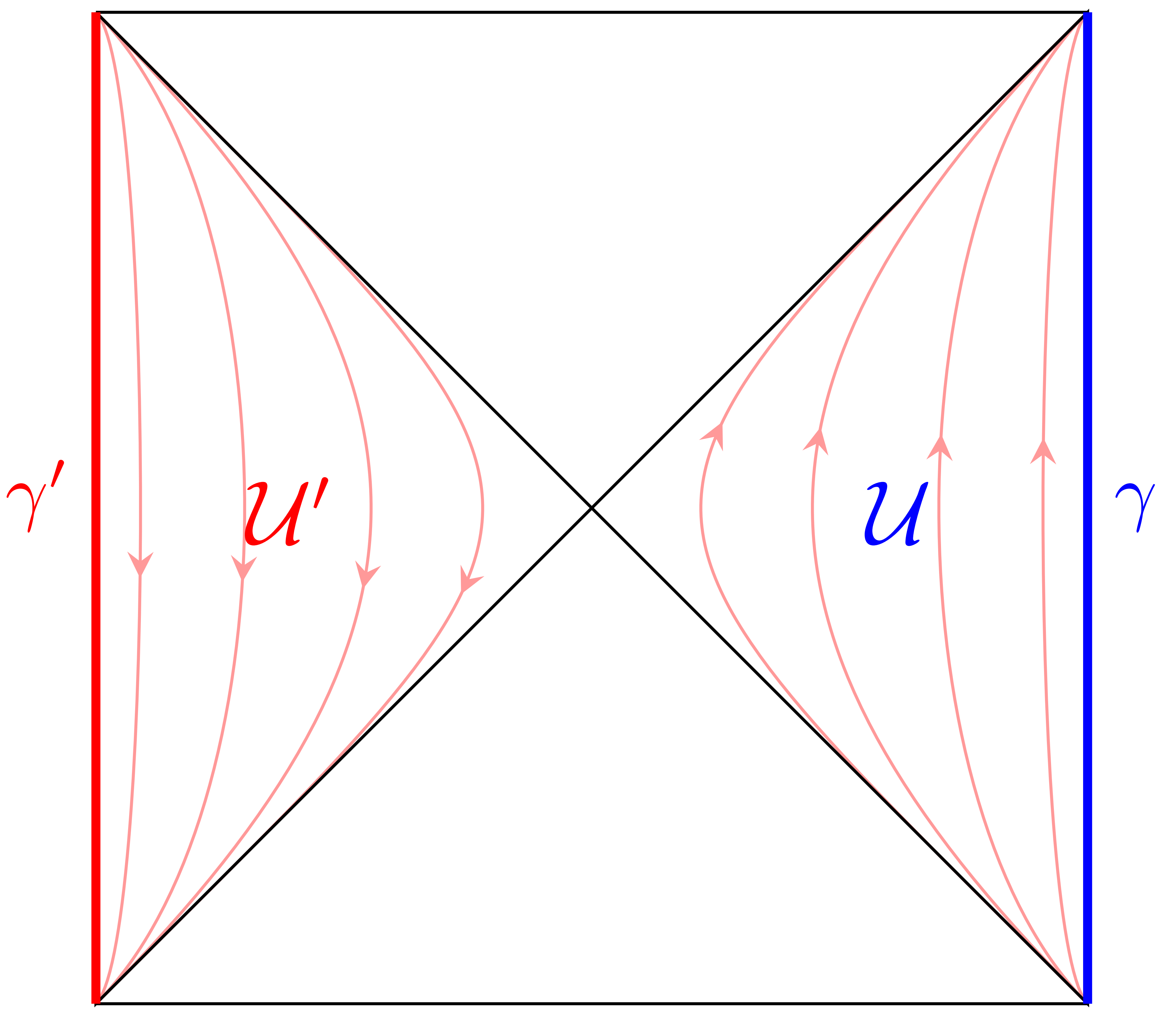

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crux:
regional QFT (incl. graviton) algebra $\mathscr{A}_{\mathscr{U}}$ is a type III factor, but crossed product is a type II factor (possesses trace, density operators and entropies)

(better behaved for non-ideal clocks)

$\Rightarrow$ description *relative to clock QRF* yields well-defined EE ("intrinsic regularization")

Density operators and entropy

[CLPW ’22; Jensen, Sorce, Speranza ’23; …]

- **type II, so trace, density operators and entropies exist**

$\Rightarrow$ **here: crossed product is a factor, so grav. EE distillable**

Density operators and entropy

[CLPW ’22; Jensen, Sorce, Speranza ’23; …]

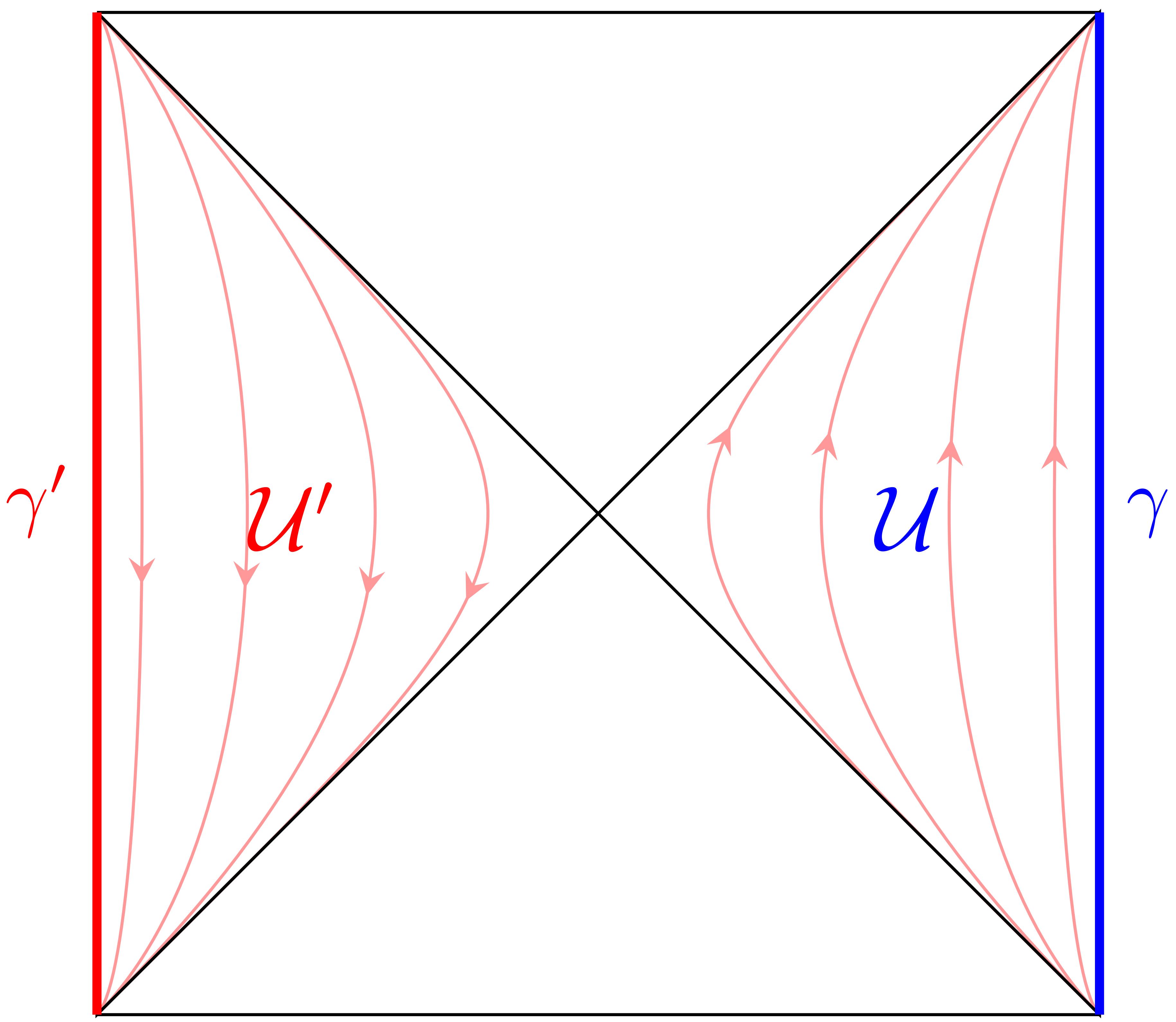
- **type II, so trace, density operators and entropies exist**

  $\Rightarrow$ here: crossed product is a factor, so grav. EE distillable

  $\Rightarrow$ for certain states reproduces generalized entropy

$$S_{\mathcal{U}} = \frac{A_{\partial\mathcal{U}}}{4G} + S_{\mathrm{QFT}}$$

These gravitational EEs are relational

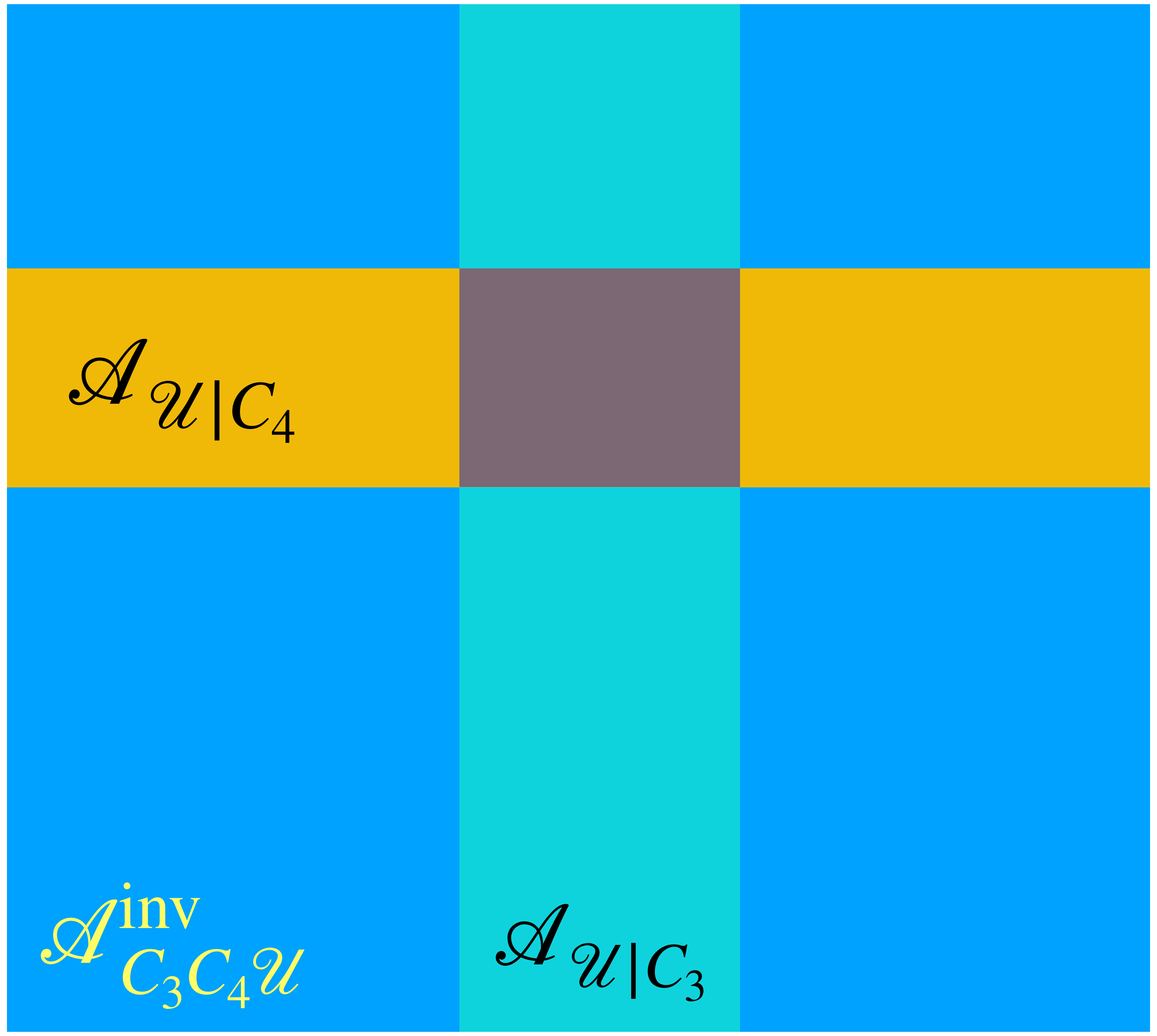


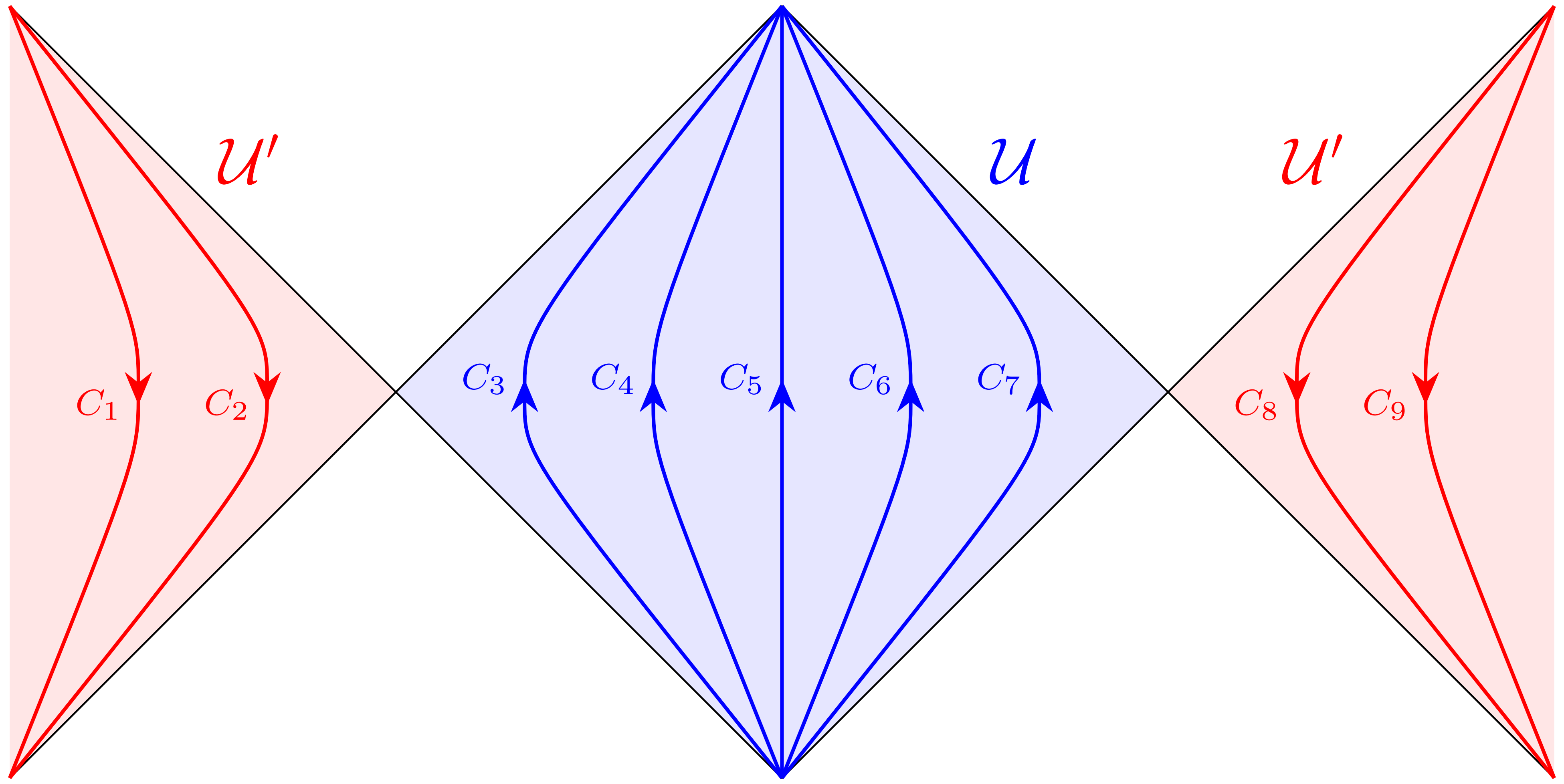
… describing regional QFT DoFs relative to chosen QRF/observer

Gravitational entropy is observer-dependent

- **populating spacetime with arbitrarily many observers/QRFs**

kinematical QFT (incl. gravition) subsystem $\mathscr{A}_{\mathcal{U}}$





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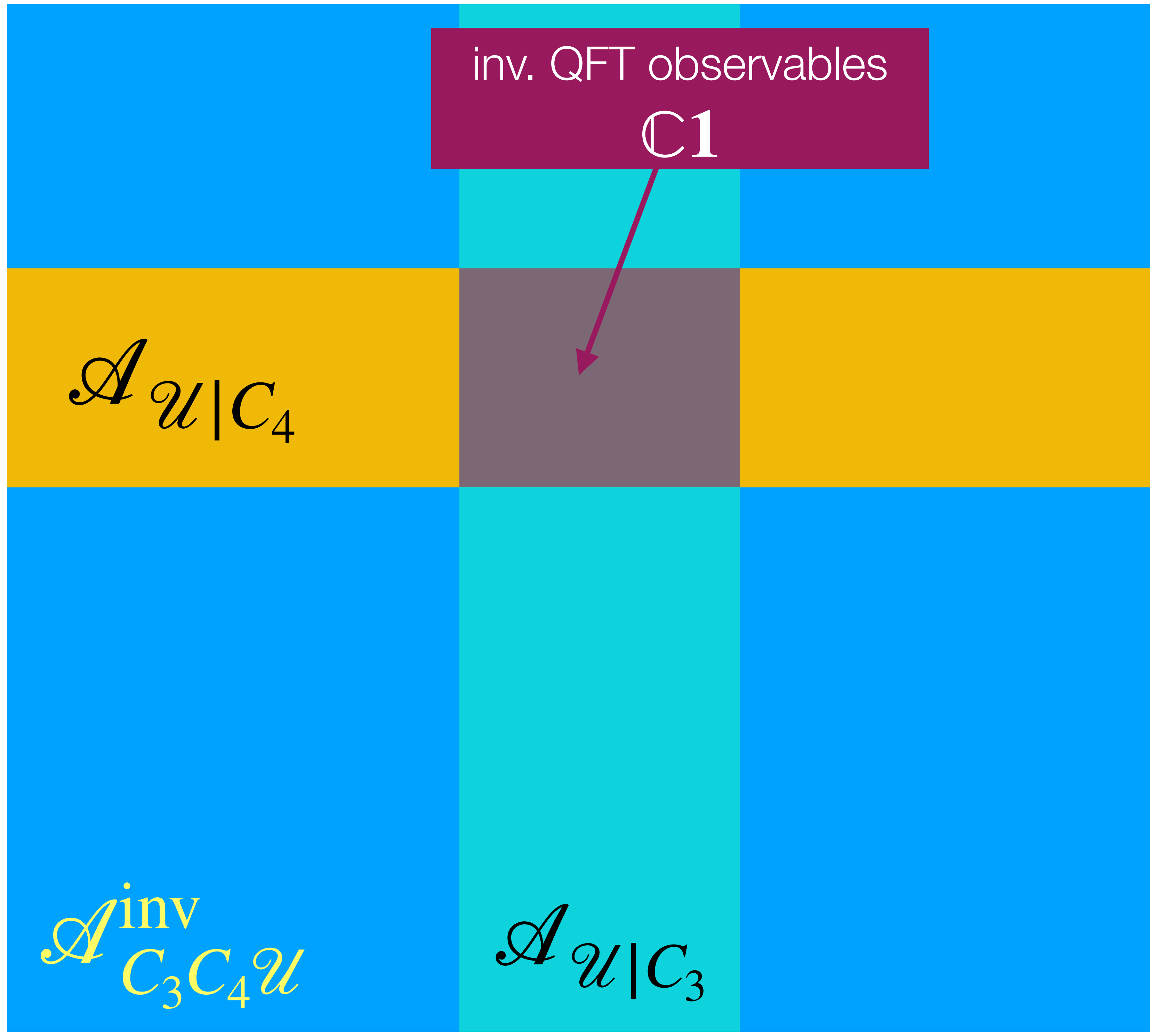
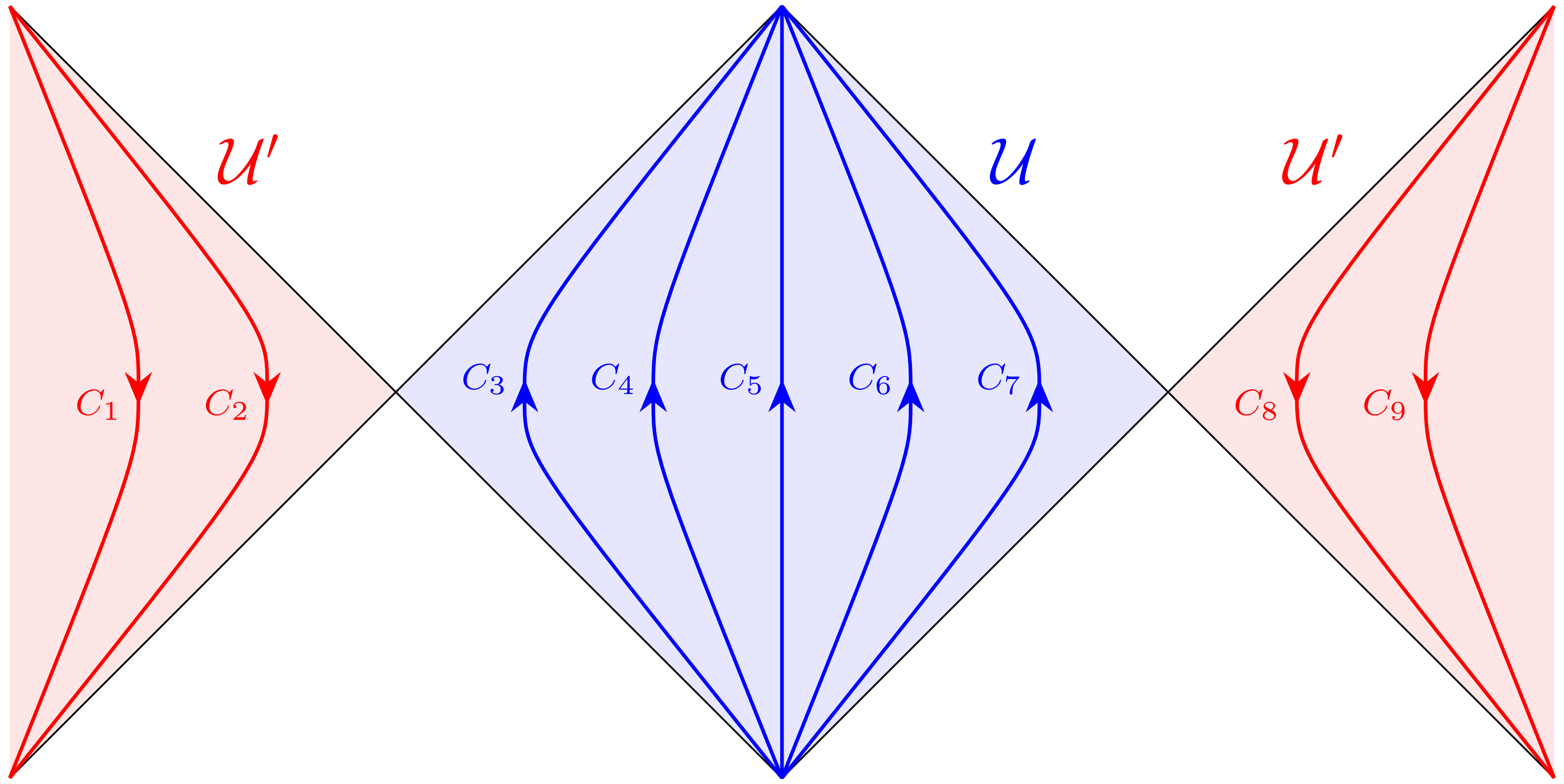




$\Rightarrow$ full regional/horizon entropy invariant, but QFT contributions are not

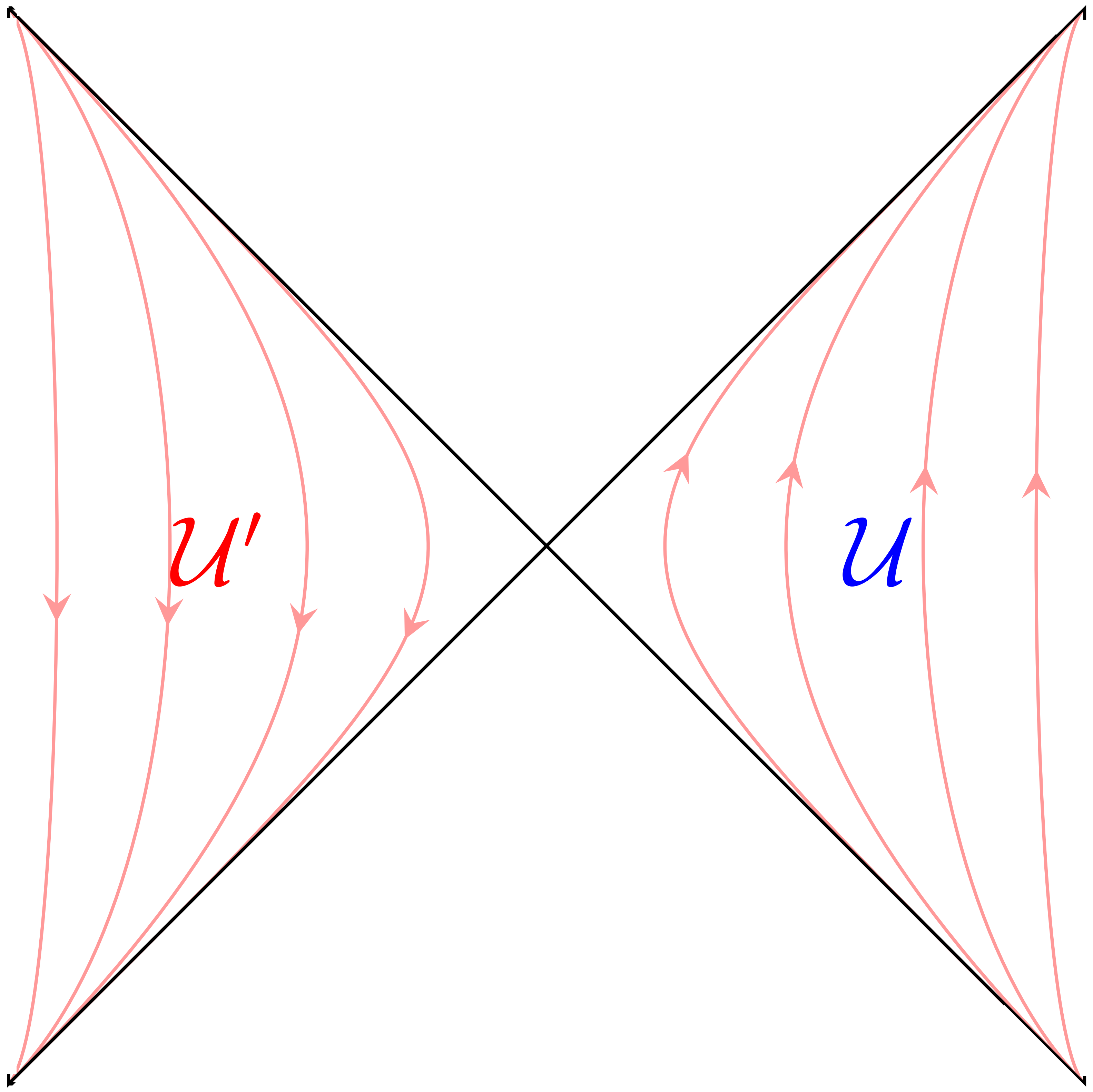
can have $S_{\mathcal{U}|C_3} \neq S_{\mathcal{U}|C_4}$

$\Rightarrow$ many different gravitational entropies per region if multiple observers/QRFs

Type transition beyond gravity

[Fewster, Janssen, Loveridge, Rejzner, Waldron, CMP ’25]

no multiple QRFs, no subsystem relativity



so far gravity, but similar construction applies also in QFT more generally without imposing constraints on states
(within QRF-based measurement scheme in AQFT)

$\Rightarrow$ **need Killing horizon**

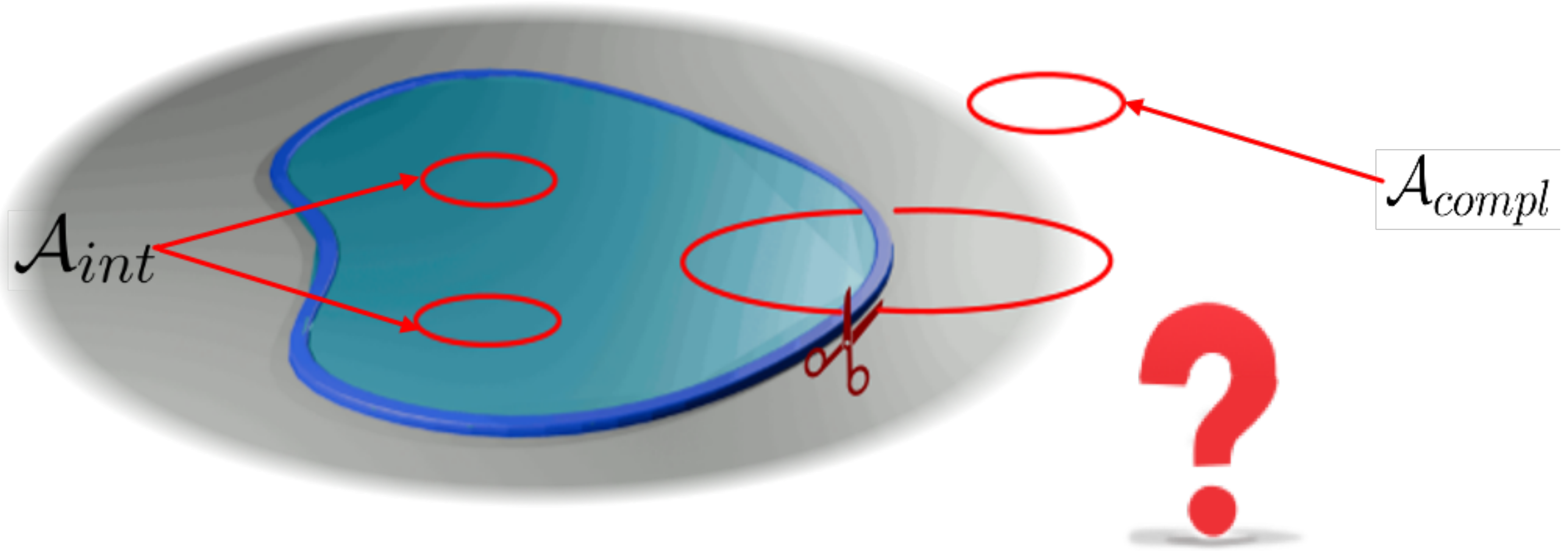
$\Rightarrow$ **more generally: describing QFT relative to QRFs can lead to better behaved entropies**

e.g. scalar QFT across Rindler horizon in Minkowski

Relational entanglement entropies in gauge theory

[Araújo-Regado, PH, Sartini, 2506.23459]

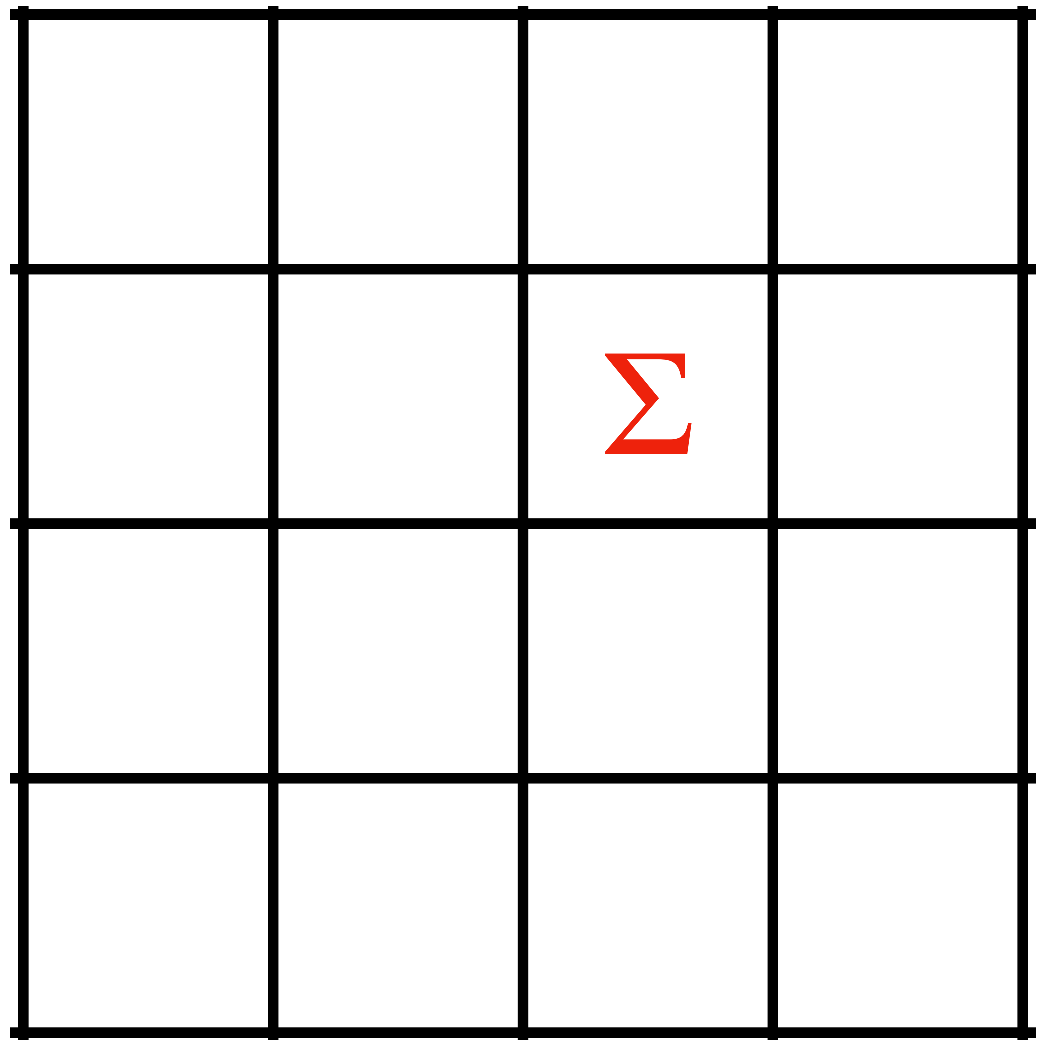
Standard regional center algebras



Standard regional center algebras

$$\mathscr{A}_{\text{phys}} \supset \bigoplus_q \mathscr{A}^q_{\text{int}} \otimes \mathscr{A}^q_{\text{compl}}$$

all regional loops
+ electric fields



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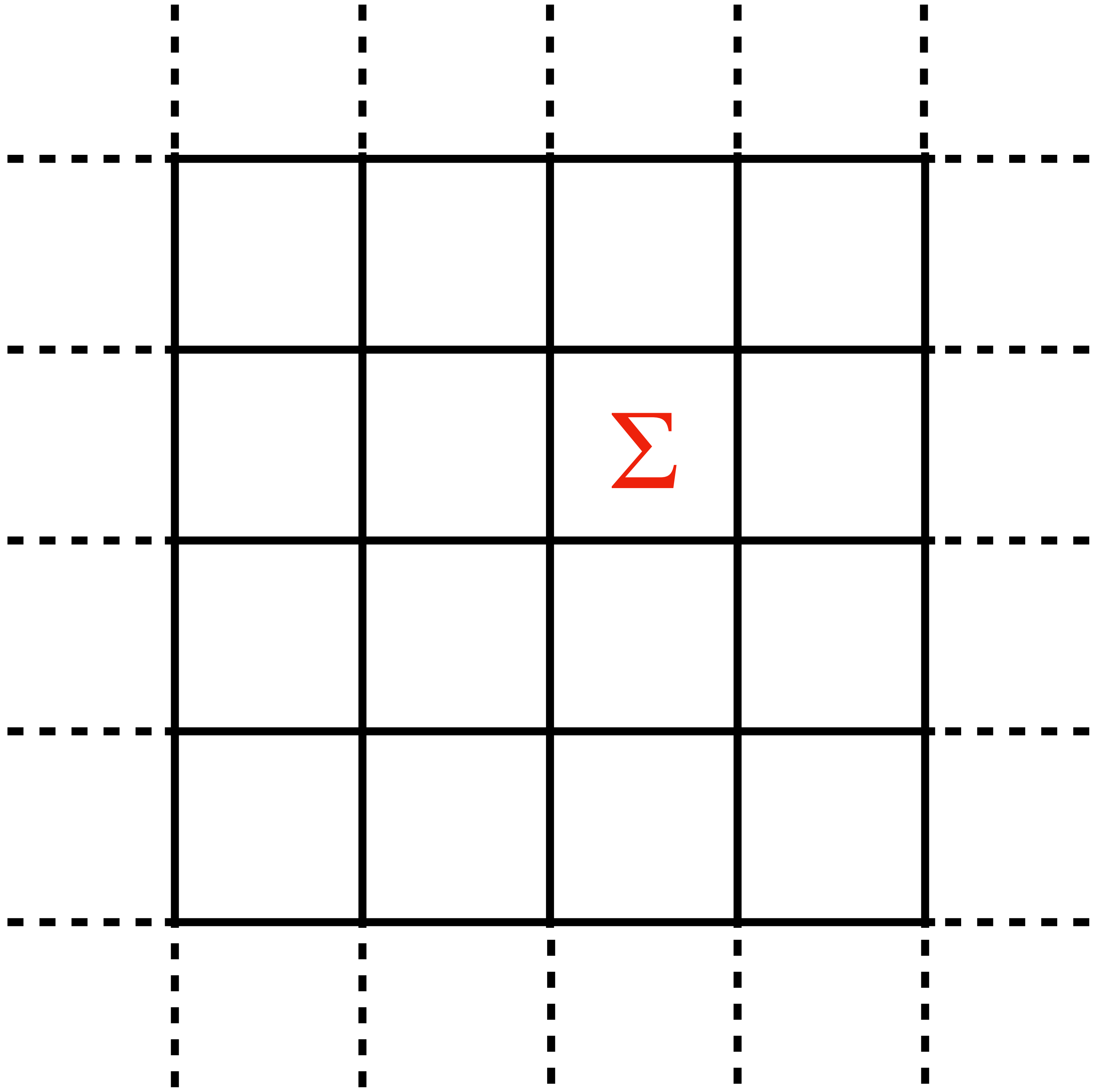

⇒ **by Gauss: get (Casimirs of) normal electric flux as center**

[Casini, Huerta, Rosbal '13; Soni, Trivedi '15; Delcamp, Dittrich, Riello '16]

$$\mathscr{A}_E$$

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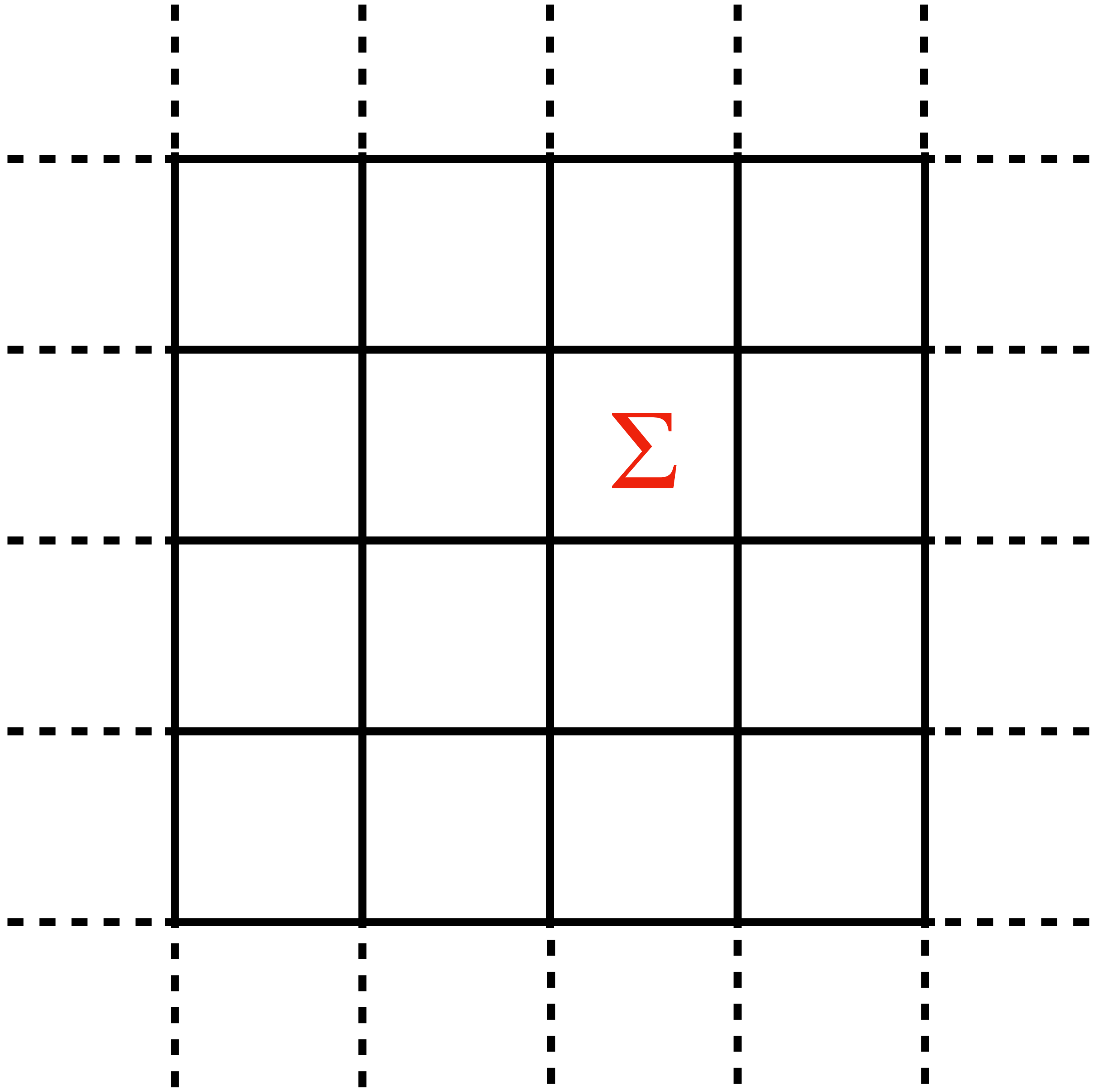

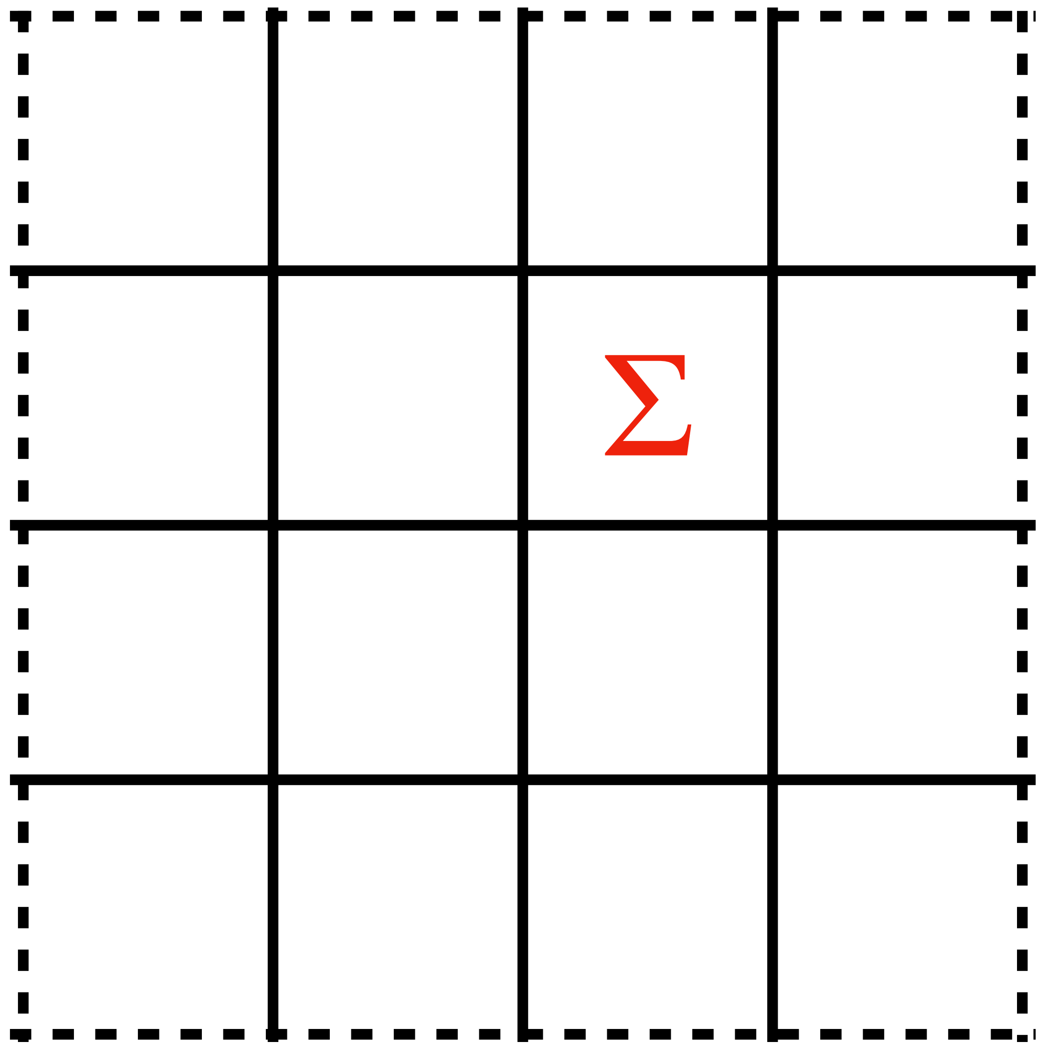

all regional loops
+ bulk electric fields

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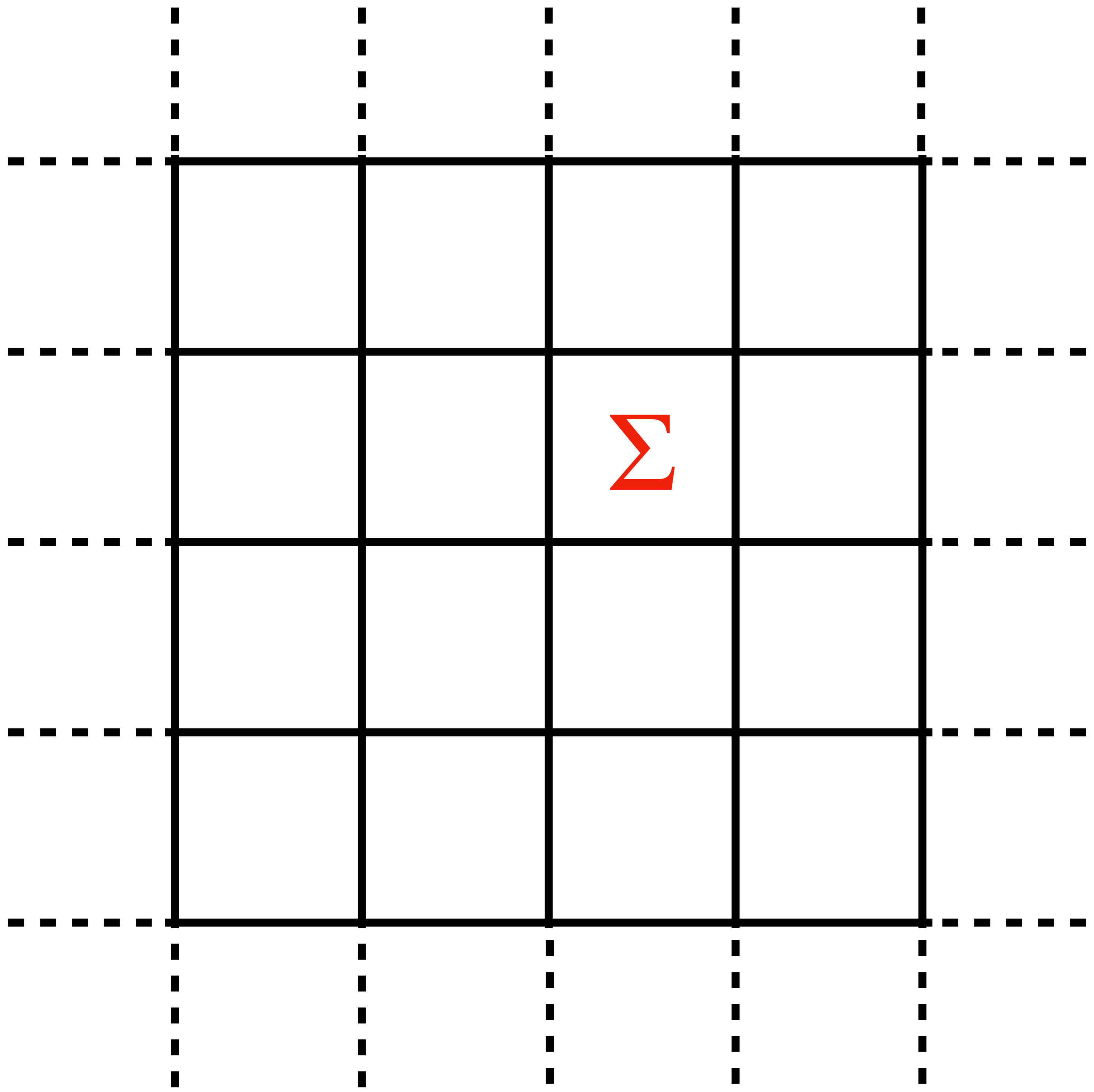

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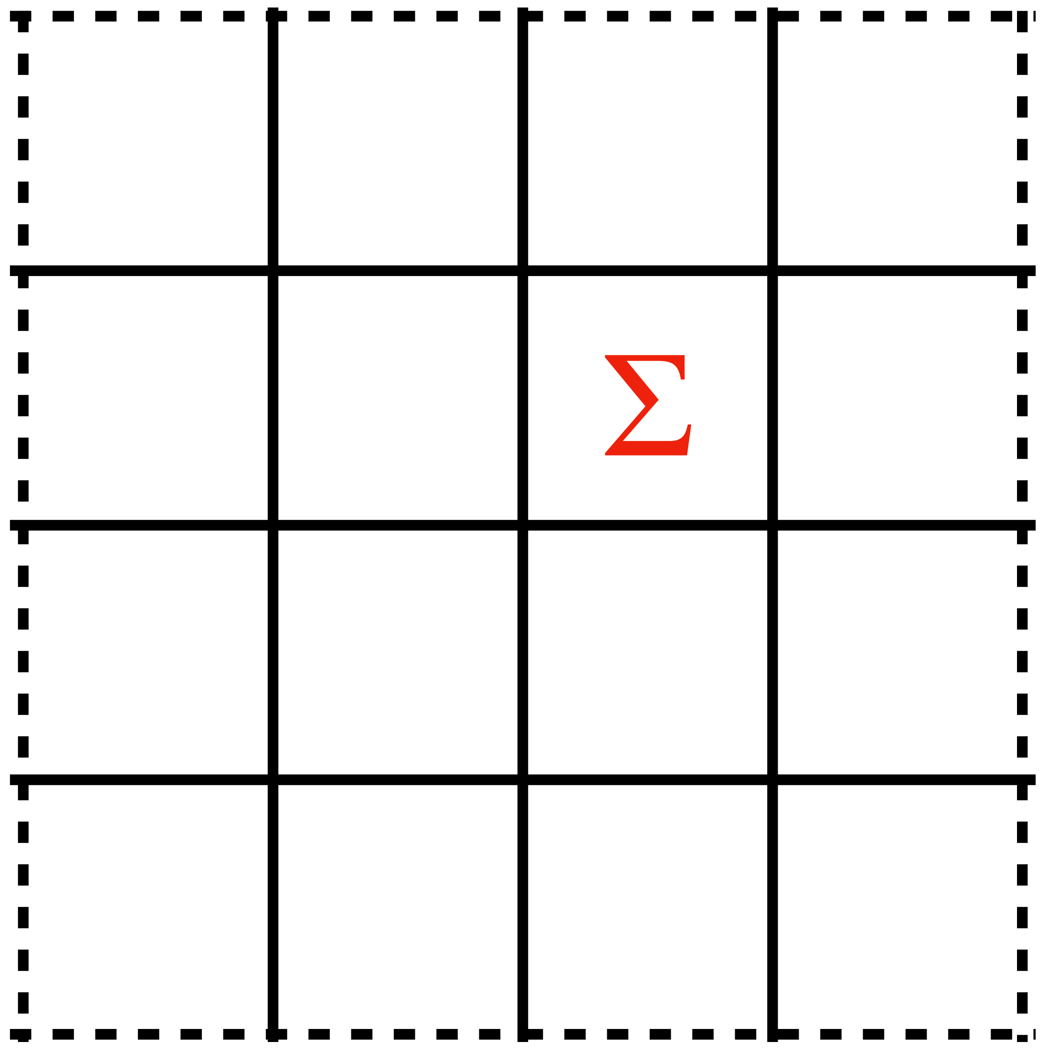
$$\mathscr{A}_E \supsetneq$$



all regional loops
+ bulk electric fields

⇒ **center: corner Wilson loops**

(+ certain loop dressed electric flux for nA theories)

[Casini, Huerta, Rosbal ’13; Delcamp, Dittrich, Riello ’16]

$$\mathscr{A}_M$$

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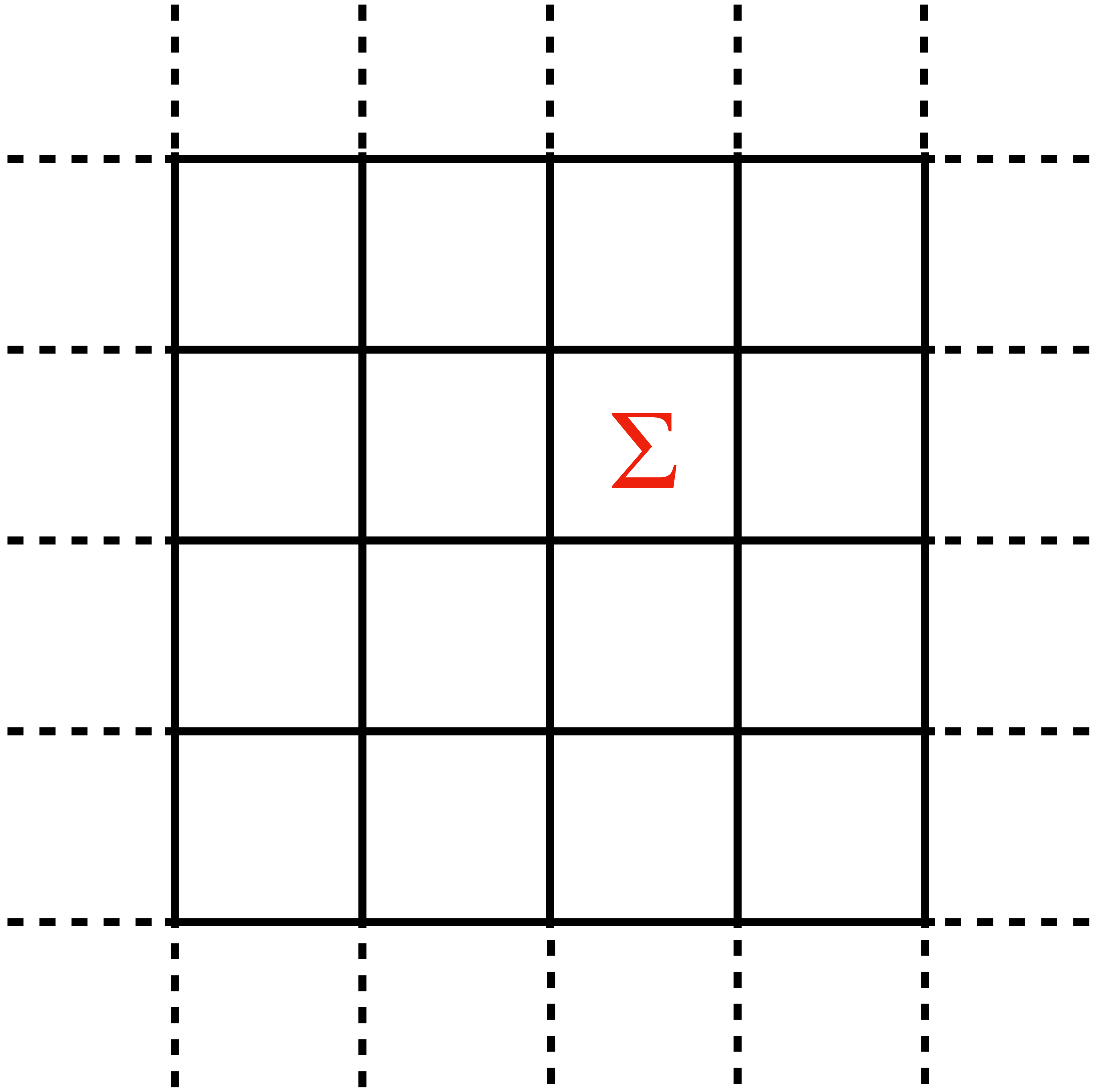
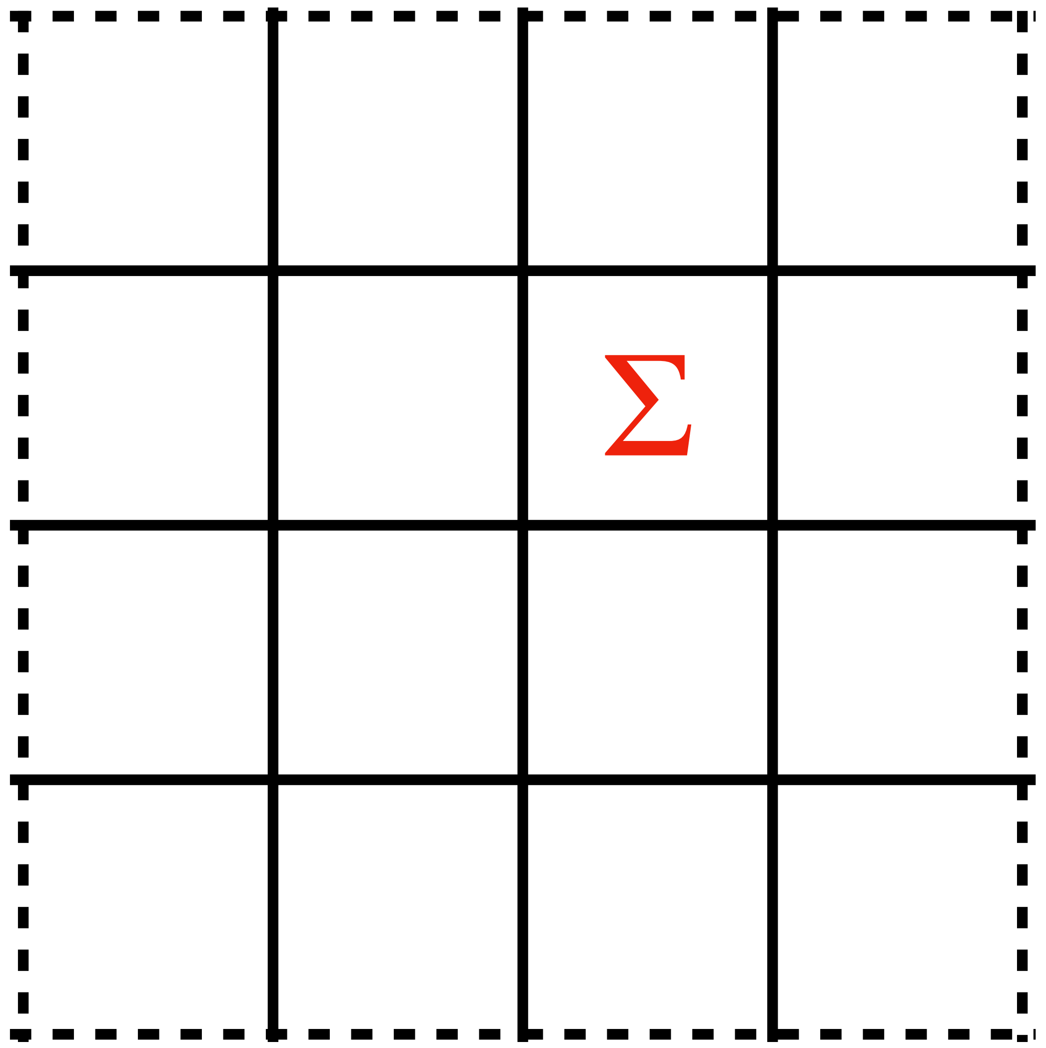
all regional loops
+ electric fields





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⇒ center: corner Wilson loops

(+ certain loop dressed electric flux for nA theories)

[Casini, Huerta, Rosbal ’13; Delcamp, Dittrich, Riello ’16]

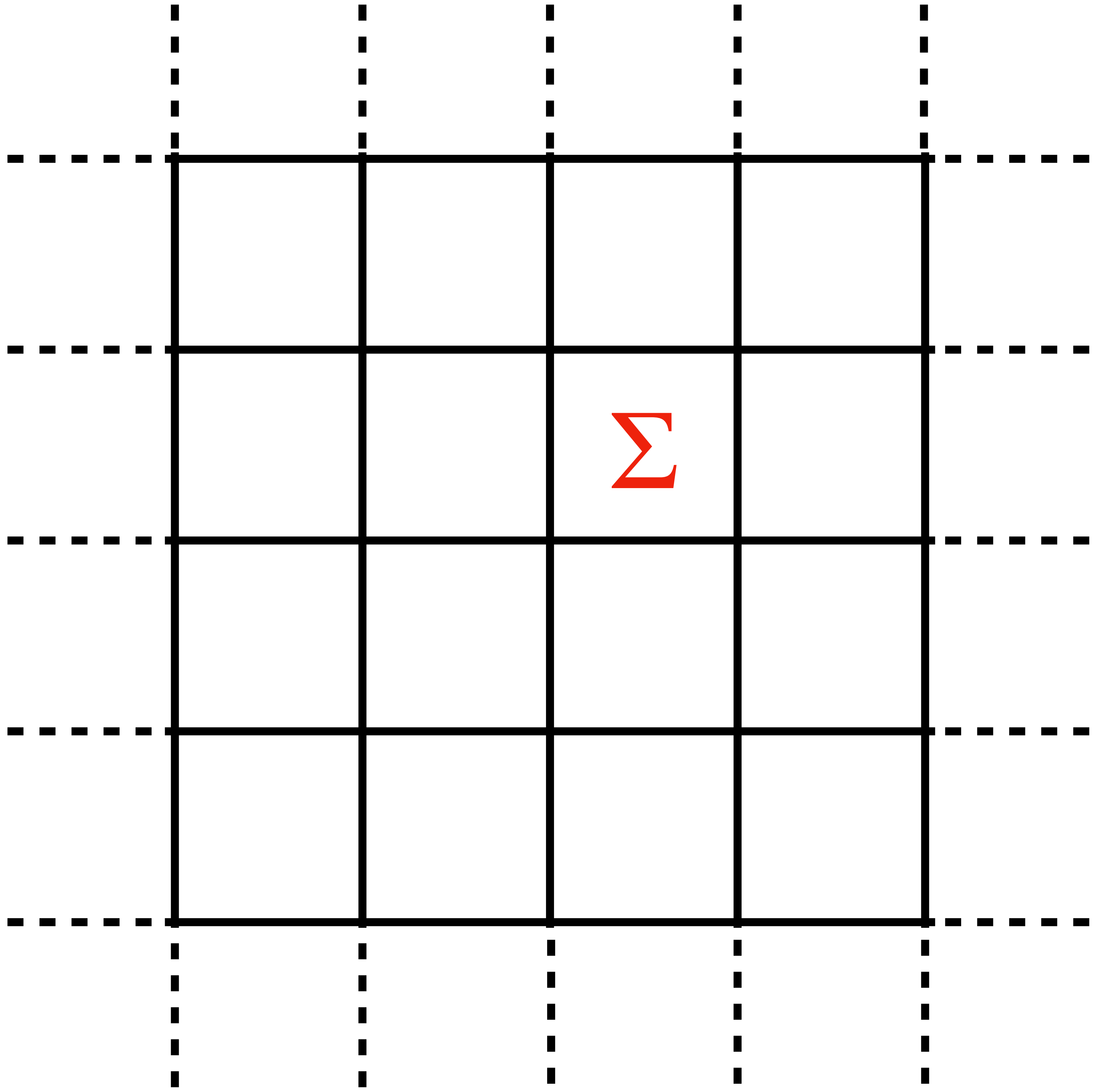
$$\mathscr{A}_E \supsetneq \mathscr{A}_M$$

⇒ vN entropy non-distillable (doesn’t measure entanglement)

$$S_{\text{vN}}(\rho_{\text{int}}) = \sum_q p_q\, S_{\text{vN}}(\rho^q) + H(\{p_q\}) + \langle \log d_q \rangle$$

Including relational regional algebras

all regional loops
+ electric fields



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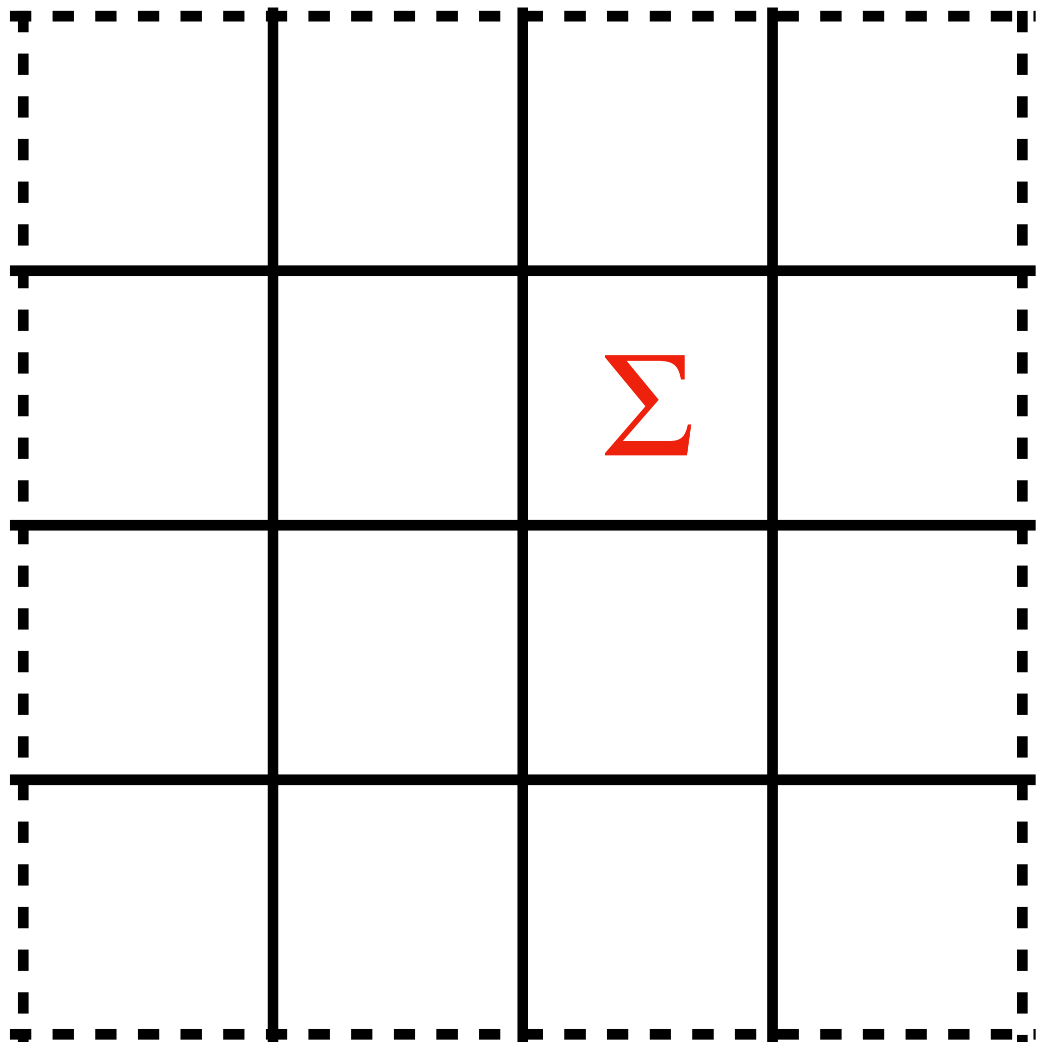

all regional loops
+ **bulk** electric fields

⇒ **center: corner Wilson loops**
(+ certain loop dressed electric flux for nA theories)

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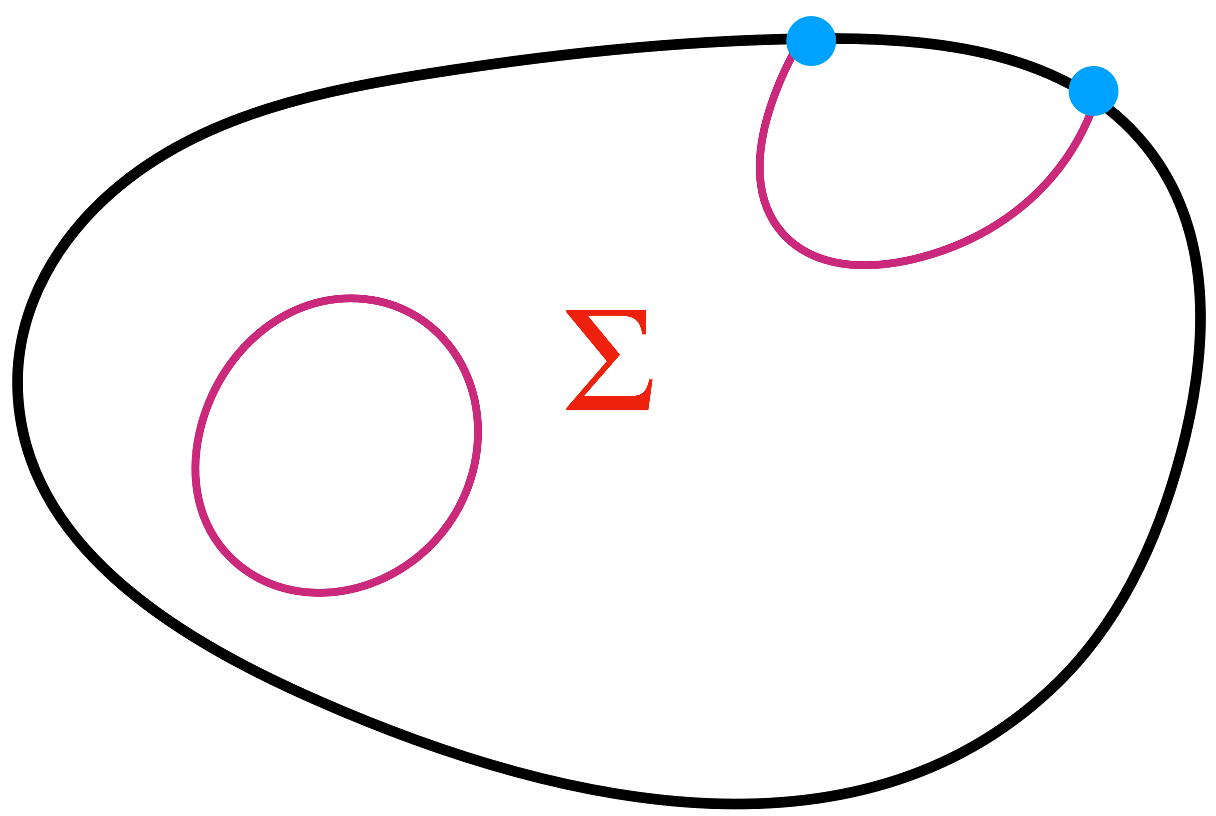
$$\mathscr{A}_{\text{ext}} \supsetneq \mathscr{A}_E \supsetneq \mathscr{A}_{\text{int}} \supsetneq \mathscr{A}_M$$

Edge modes and corner symmetries

electric corner symmetries

$$Q[\rho] = \int_{\partial\Sigma} \mathrm{Tr}(\rho \star \mathrm{F}_\Phi)$$



edge mode Φ

[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

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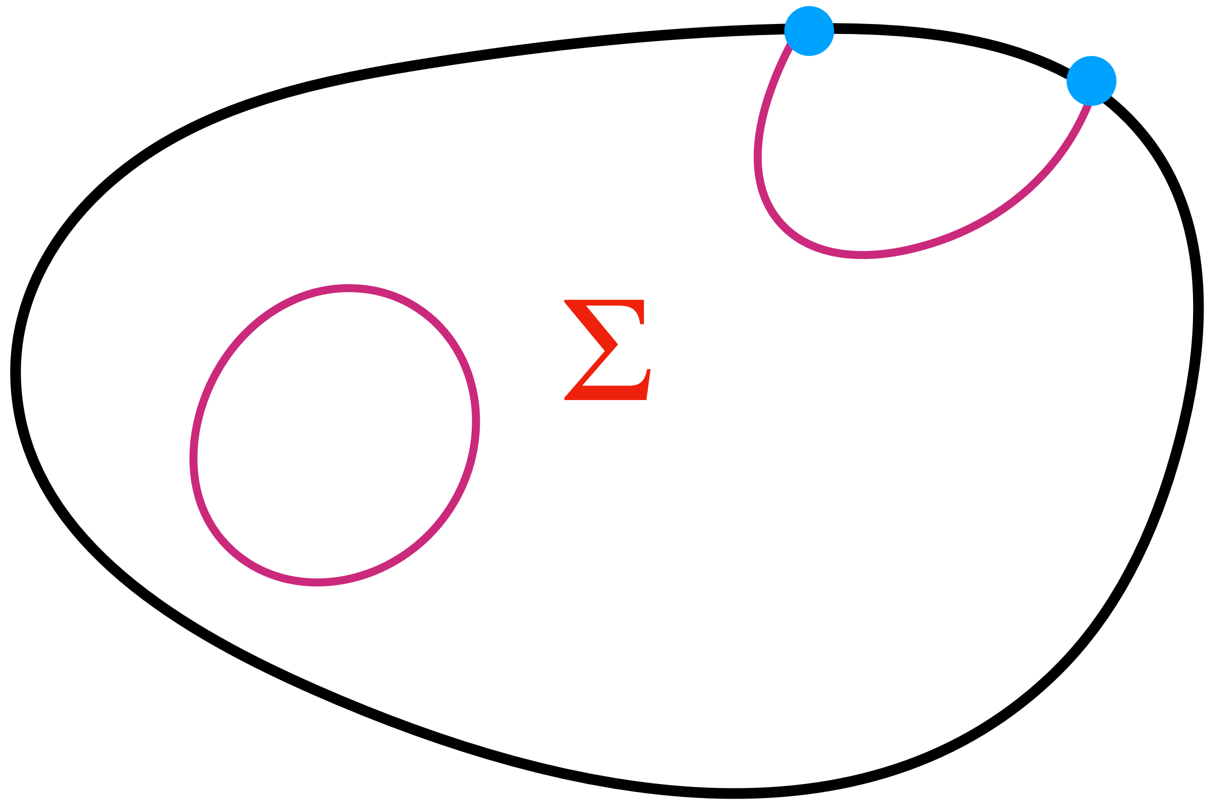

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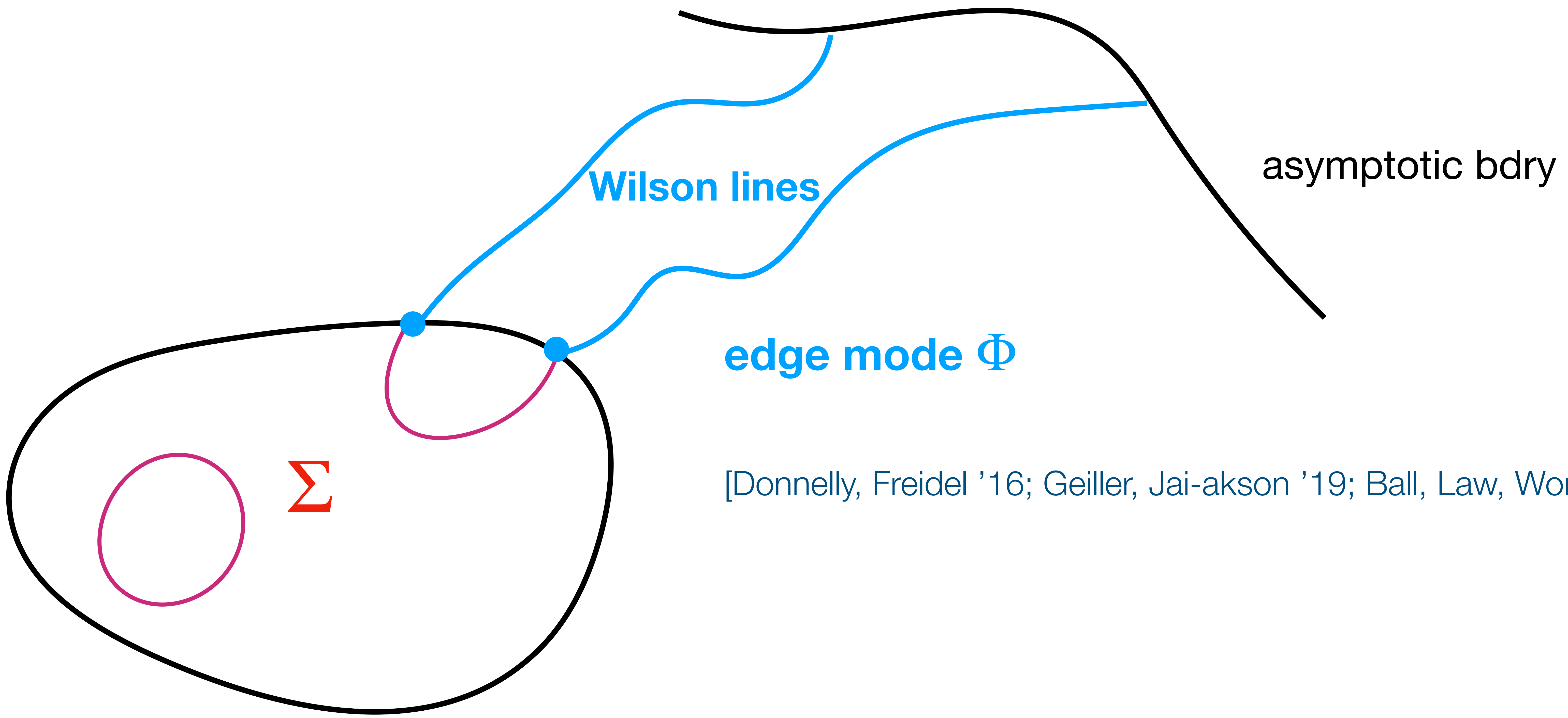
[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

$\Rightarrow$ **how do you realize them?**

Edge modes as QRFs

Francesco’s talk



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$\Rightarrow$ **how do you realize them?** **as QRFs**

extrinsic edge mode QRFs: **e.g. via extrinsic Wilson lines** [Carrozza, PH ’21; Araújo-Regado, PH, Sartini, Tomova ’24]

Edge modes as QRFs

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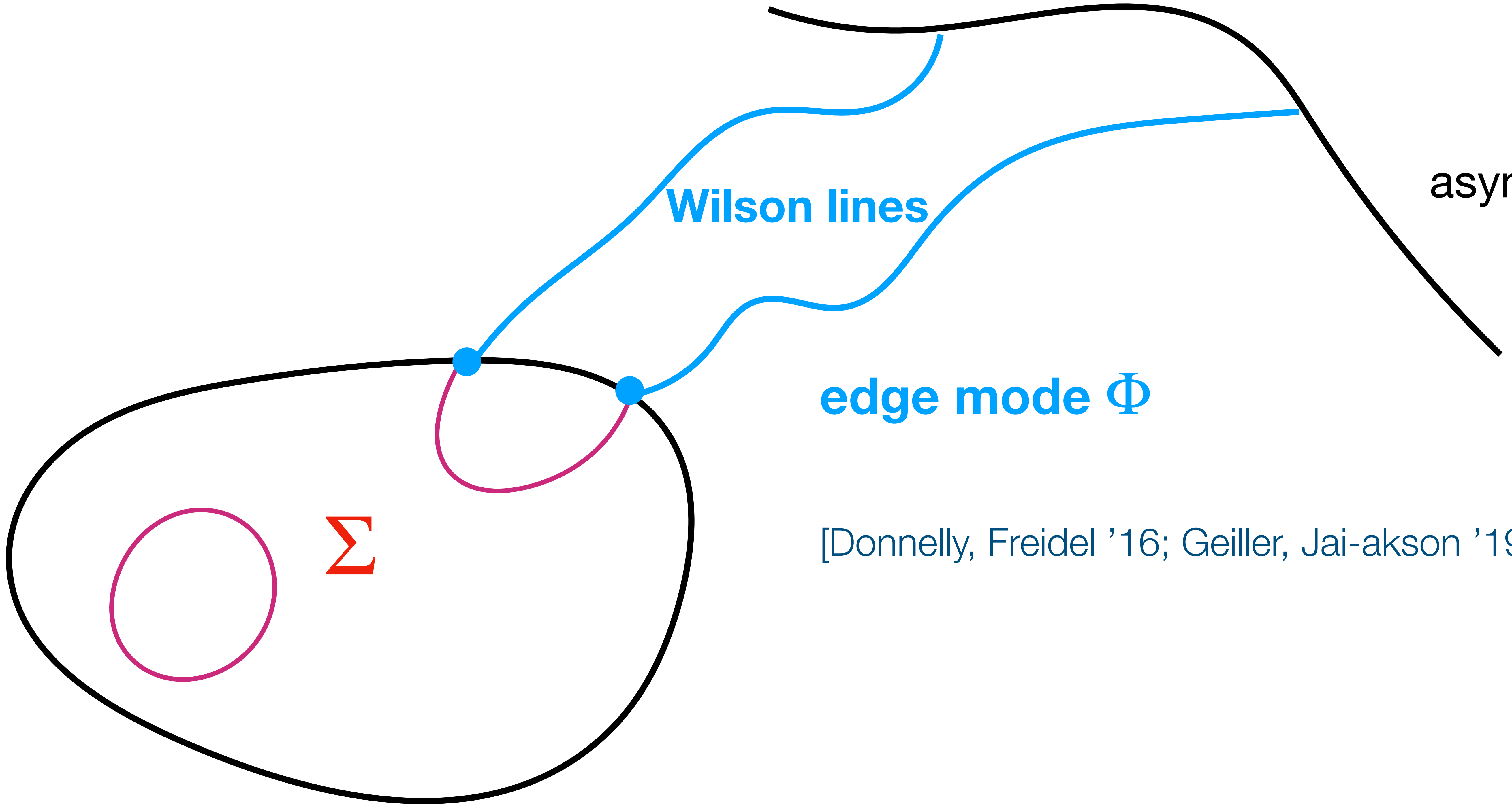


asymptotic bdry

edge mode Φ

[Donnelly, Freidel '16; Geiller, Jai-akson '19; Ball, Law, Wong '24;…]

$\Rightarrow$ **how do you realize them? as QRFs**

extrinsic edge mode QRFs: **e.g. via extrinsic Wilson lines** [Carrozza, PH '21; Araújo-Regado, PH, Sartini, Tomova '24]

intrinsic edge mode QRFs: **e.g. via Wilson lines anchored on the corner $\partial\Sigma$ or a Hodge decomp.**

[Araújo-Regado, PH, Sartini, Tomova '24]

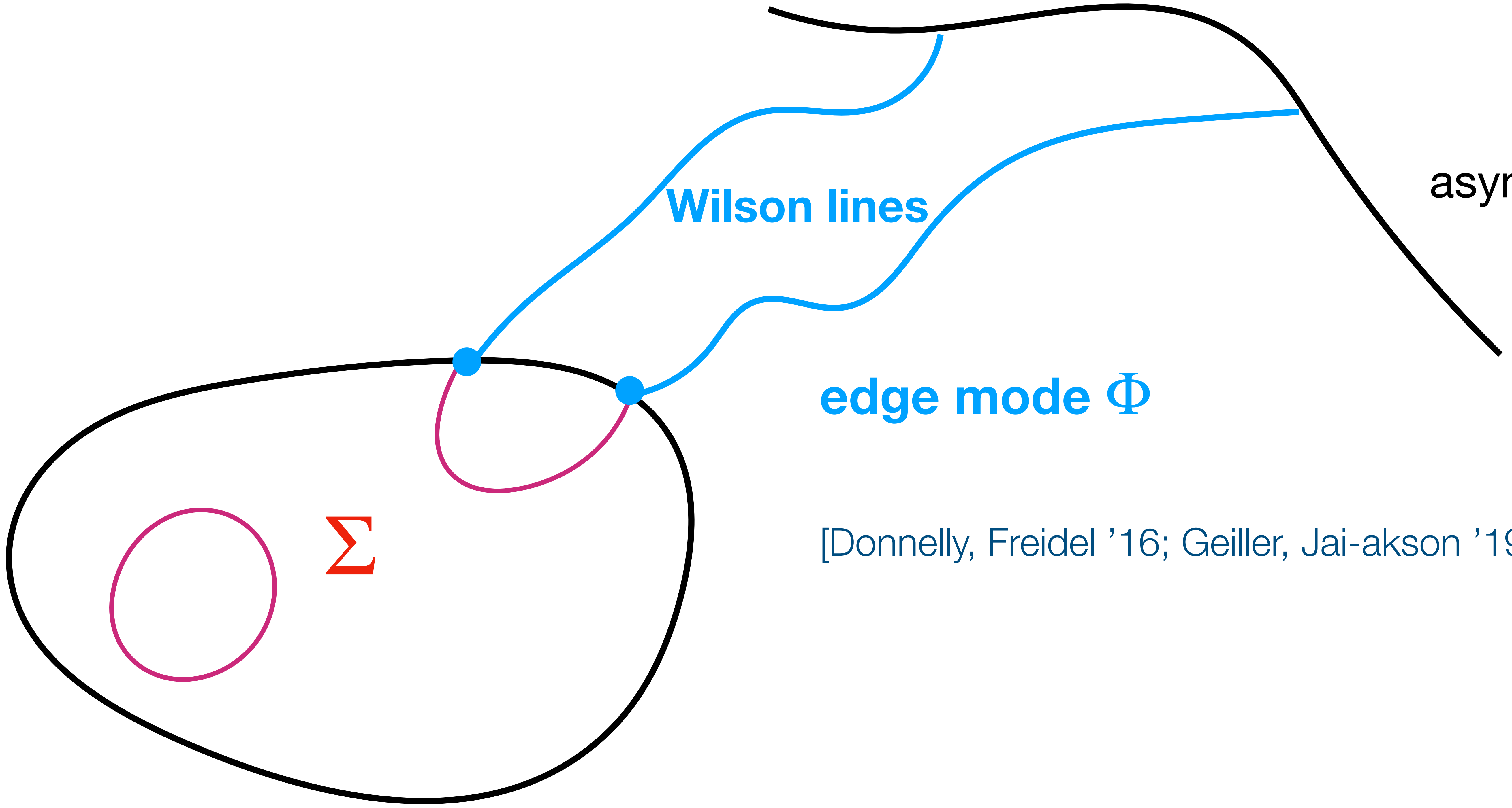
Edge modes as QRFs

Francesco's talk

electric corner symmetries

$$Q[\rho] = \int_{\partial\Sigma} \mathrm{Tr}(\rho \star \mathrm{F}_\Phi)$$

$\Rightarrow$ are reorientations of extrinsic QRFs!



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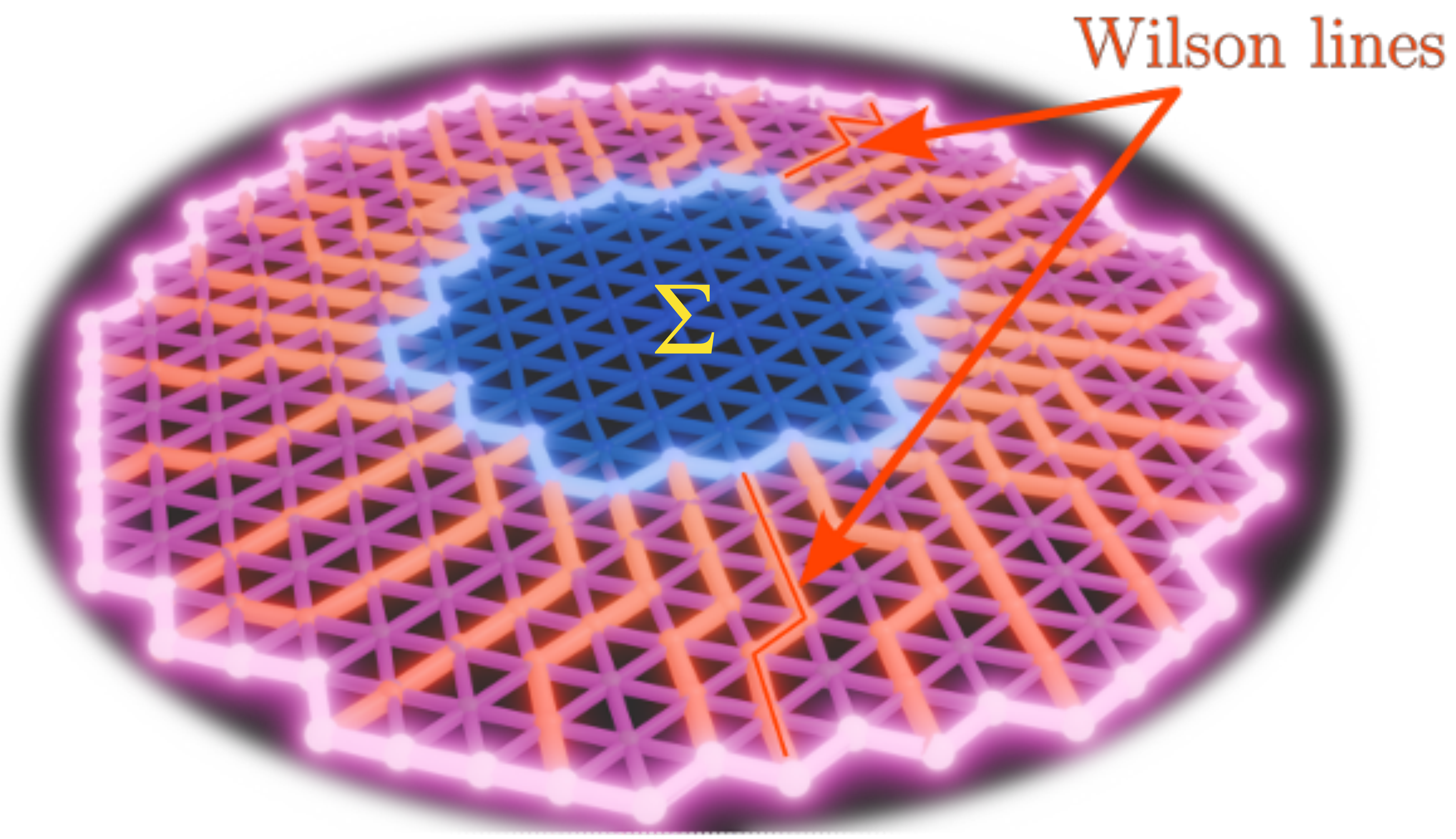
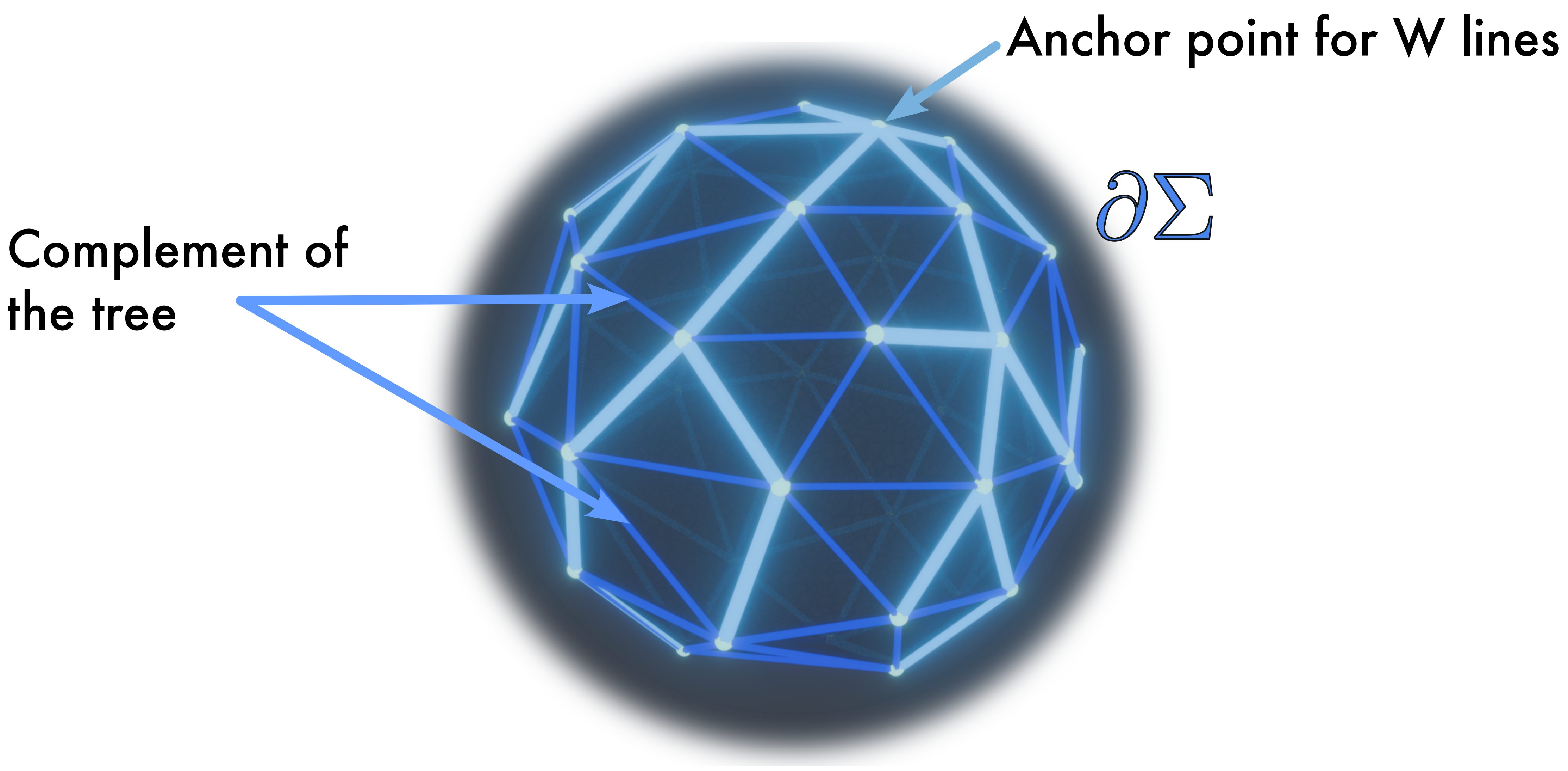
Extrinsic and intrinsic QRFs on the lattice

Extrinsic: congruence of Wilson lines in the complement



Spanning tree anchored on node N of the corner



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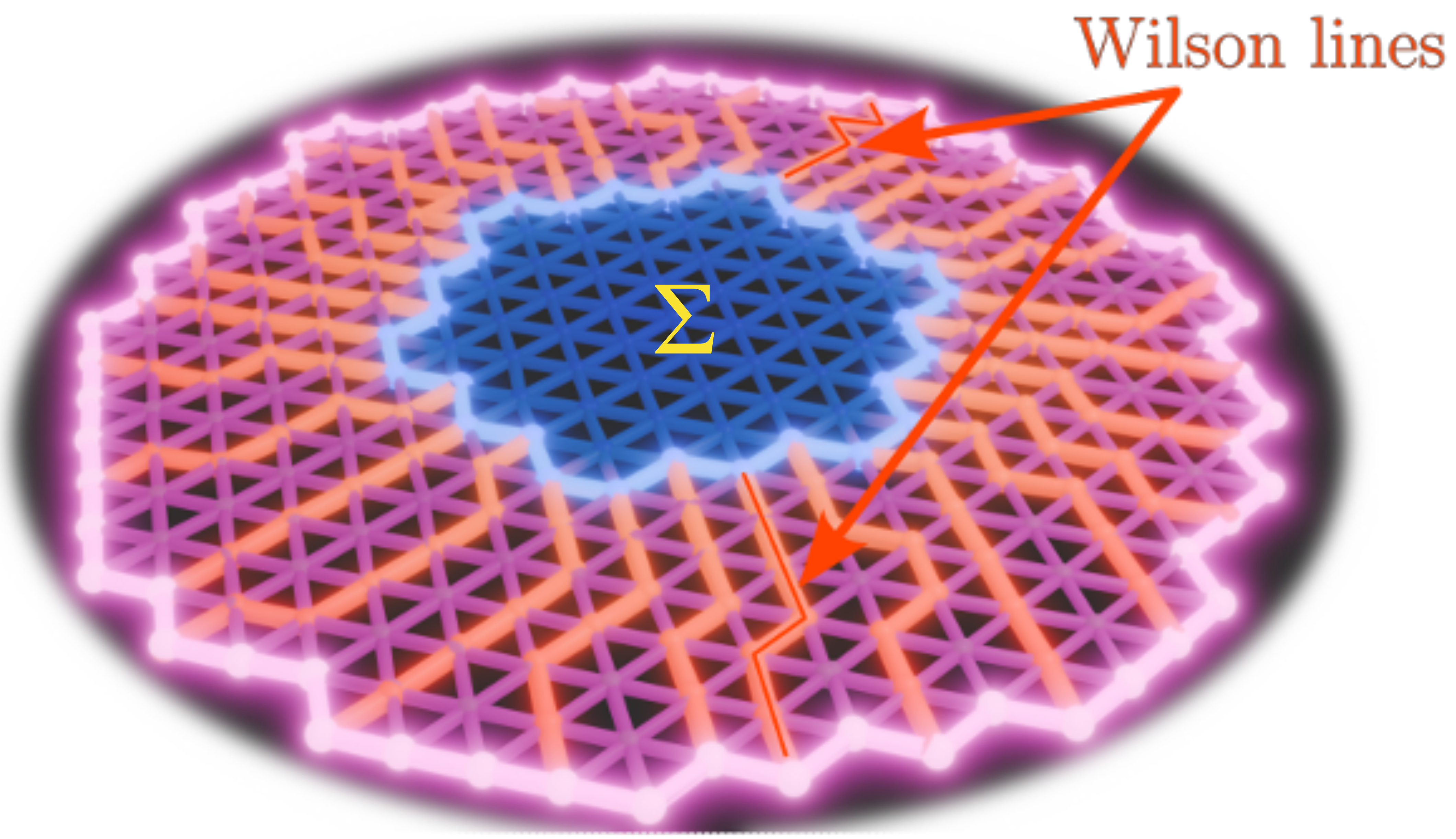
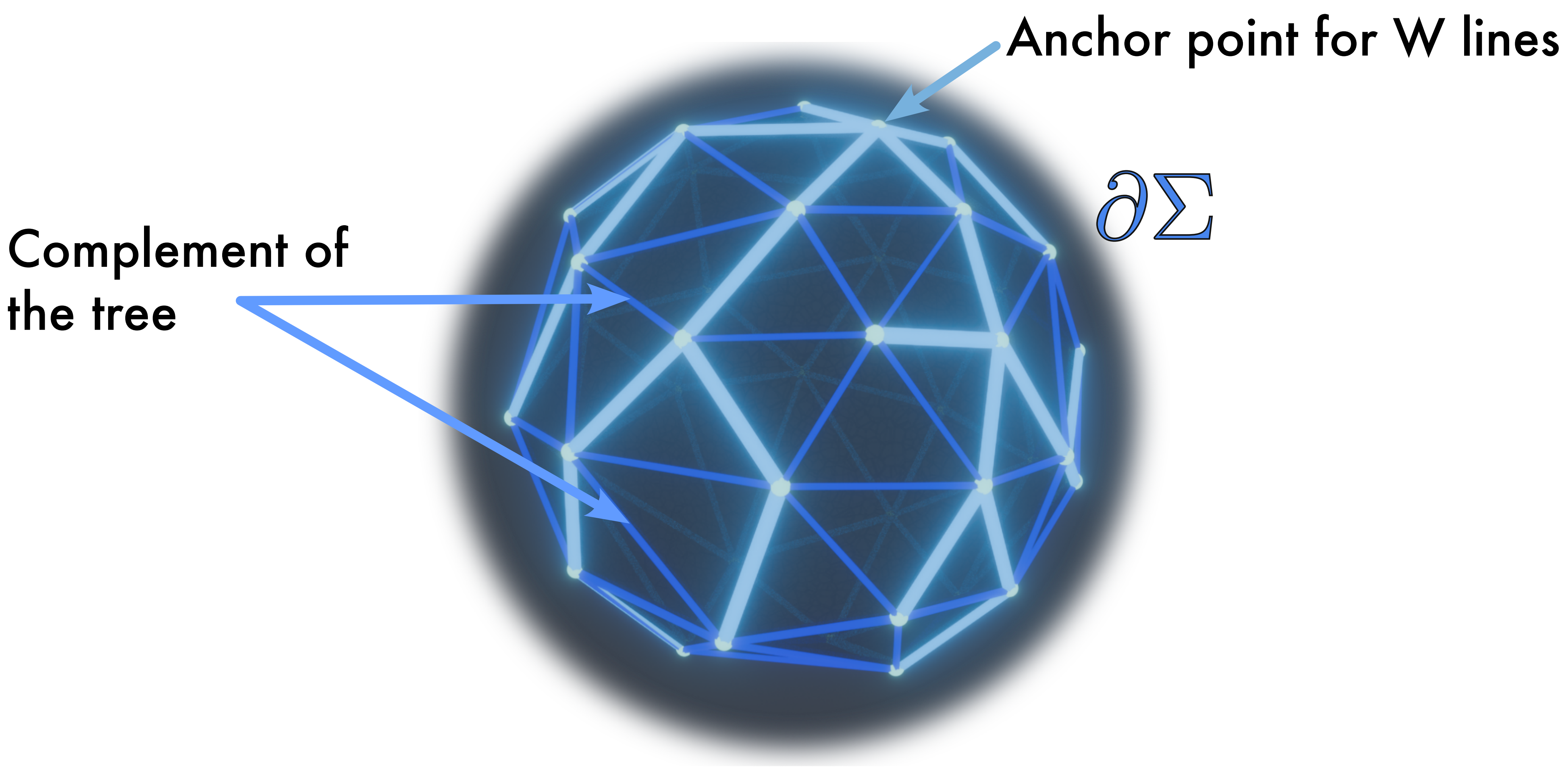


complete ideal QRF: $\mathscr{H}_{\Phi} = L^2(G^{V_{\partial\Sigma}})$

Spanning tree anchored on node N of the corner

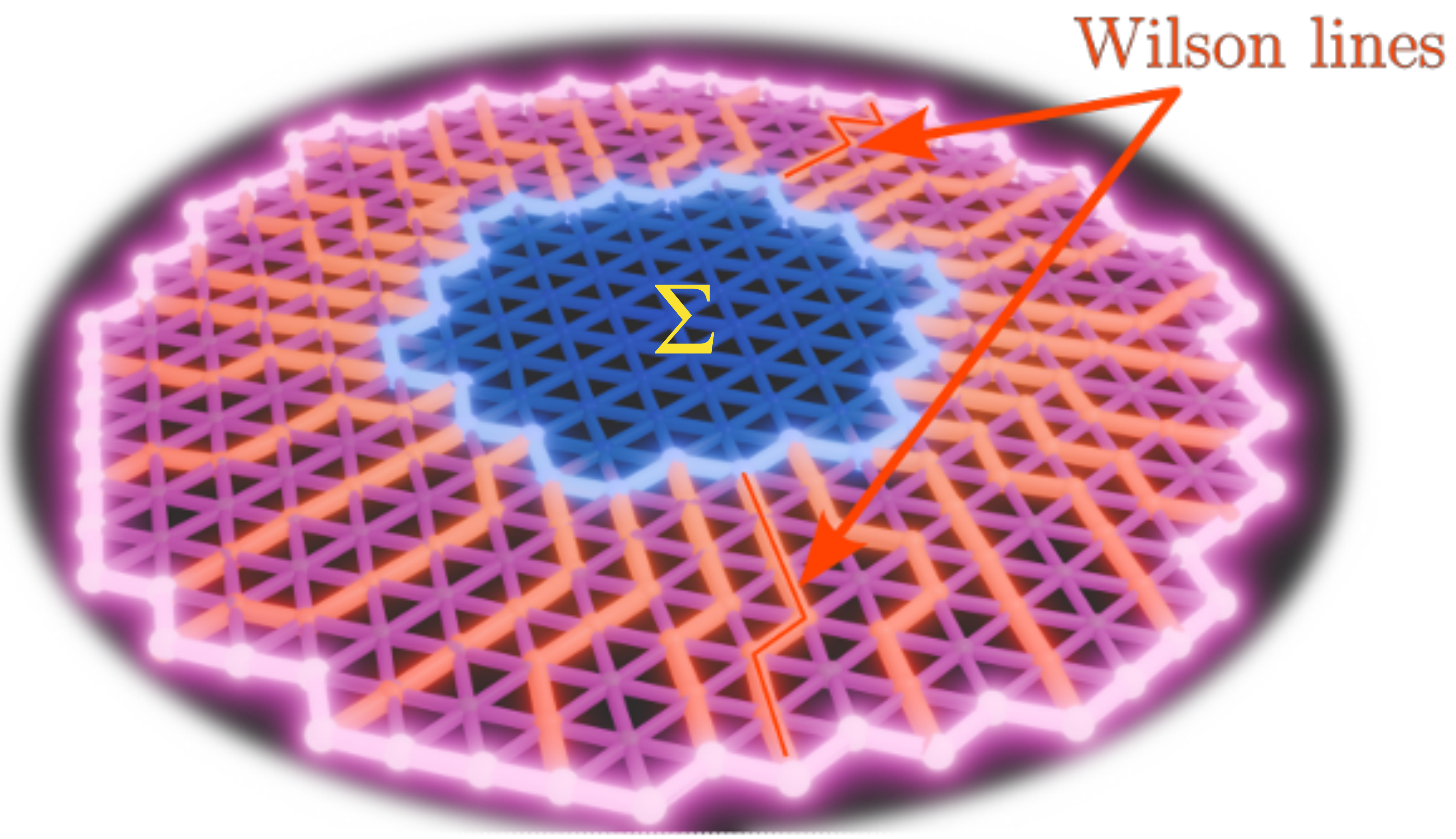


incomplete ideal QRF: $\mathscr{H}_{\tilde{\Phi}} = L^2(G^{V_{\partial\Sigma}-1})$

⇒ can't deparametrize/dress at anchor point N

Extrinsic relational algebras

dress all the regional data in Σ (Wilson lines, electric fields) **with extrinsic Wilson line QRF to asymp. bdry**



$\Rightarrow$ **rel. obs.** $O^g_{|\Phi}(a)$ **with** $a \in \mathscr{A}_\Sigma$

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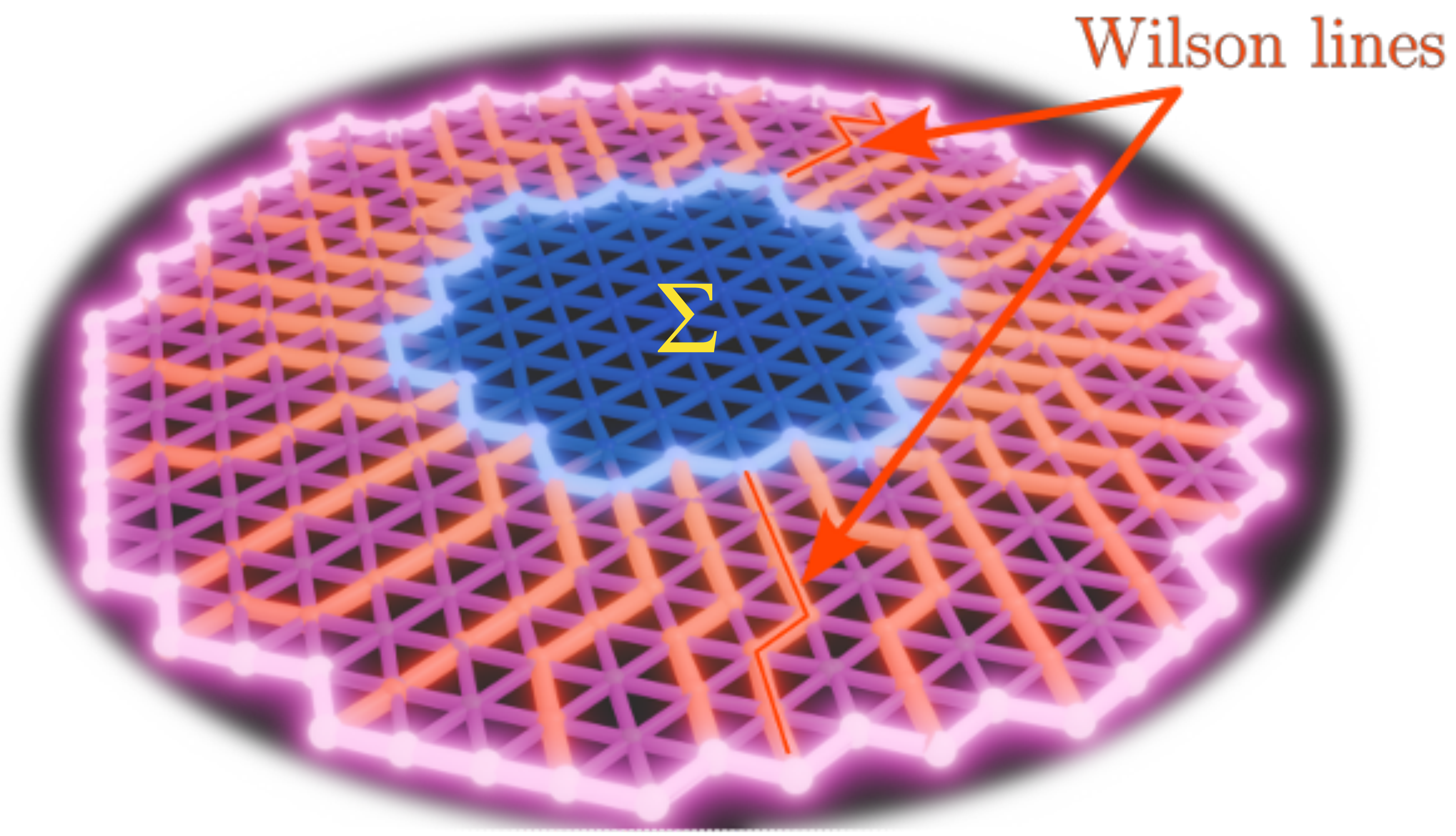

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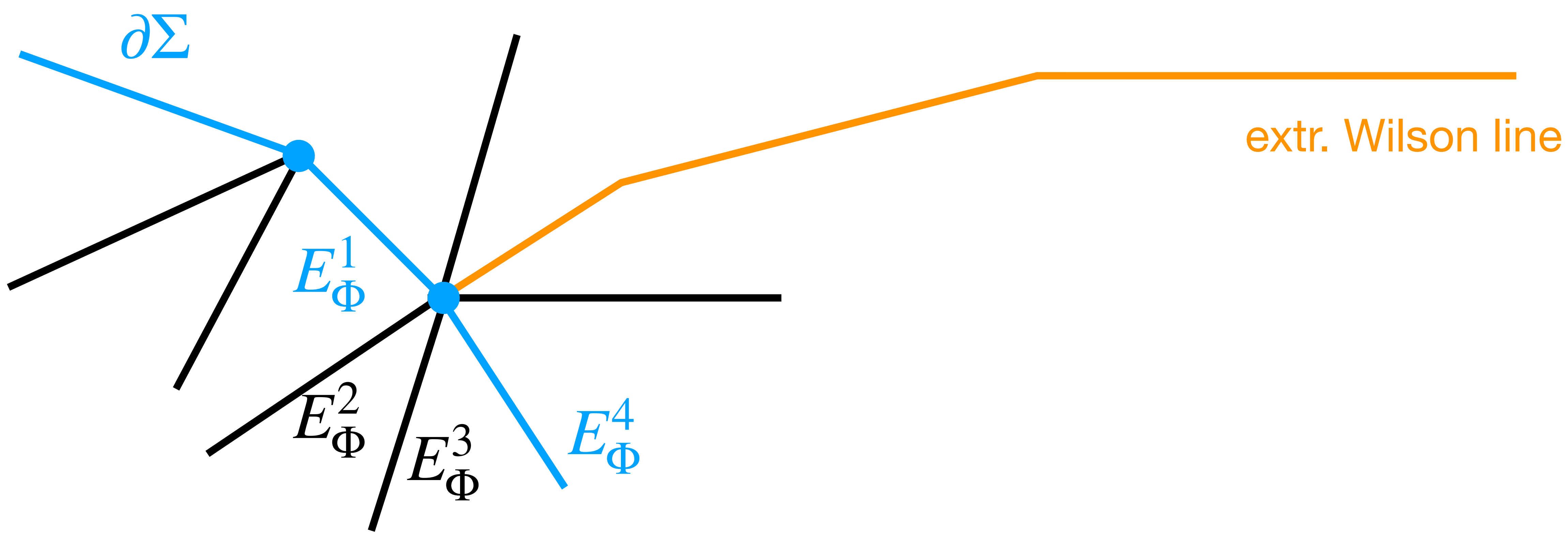

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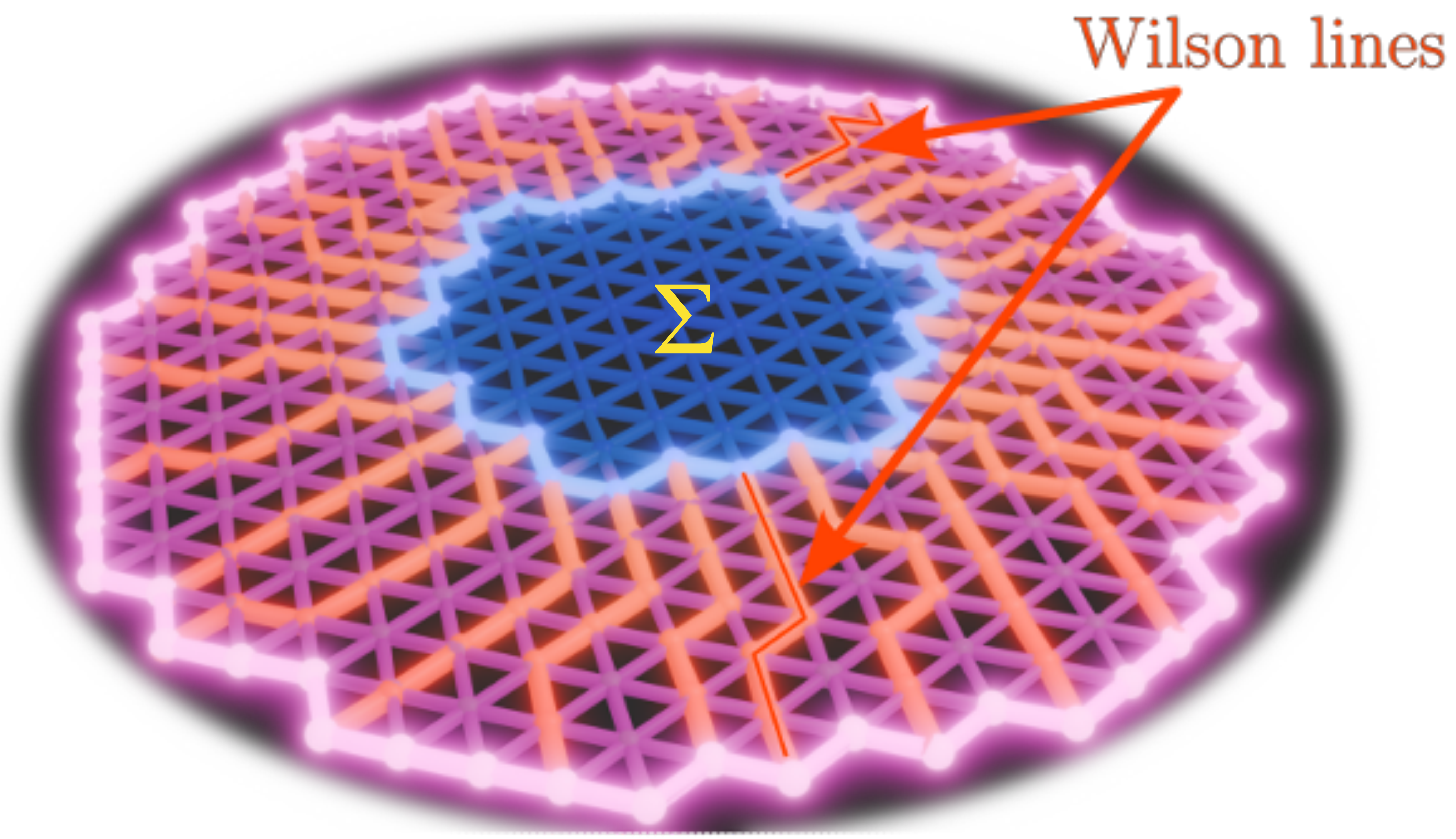

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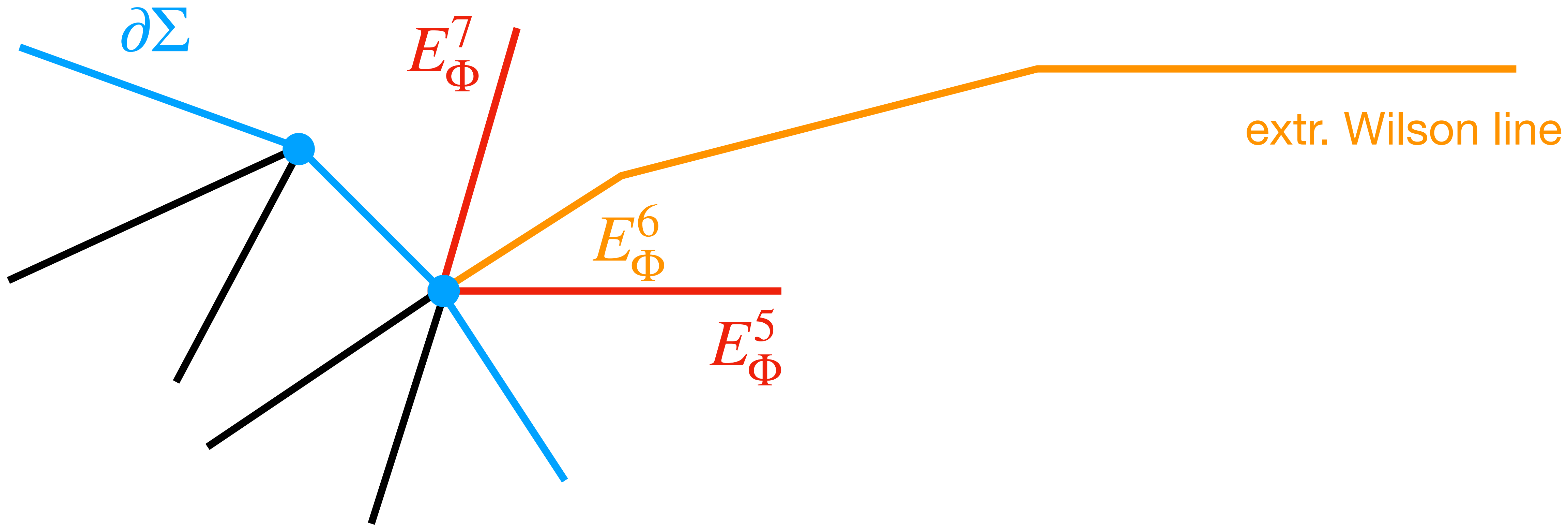

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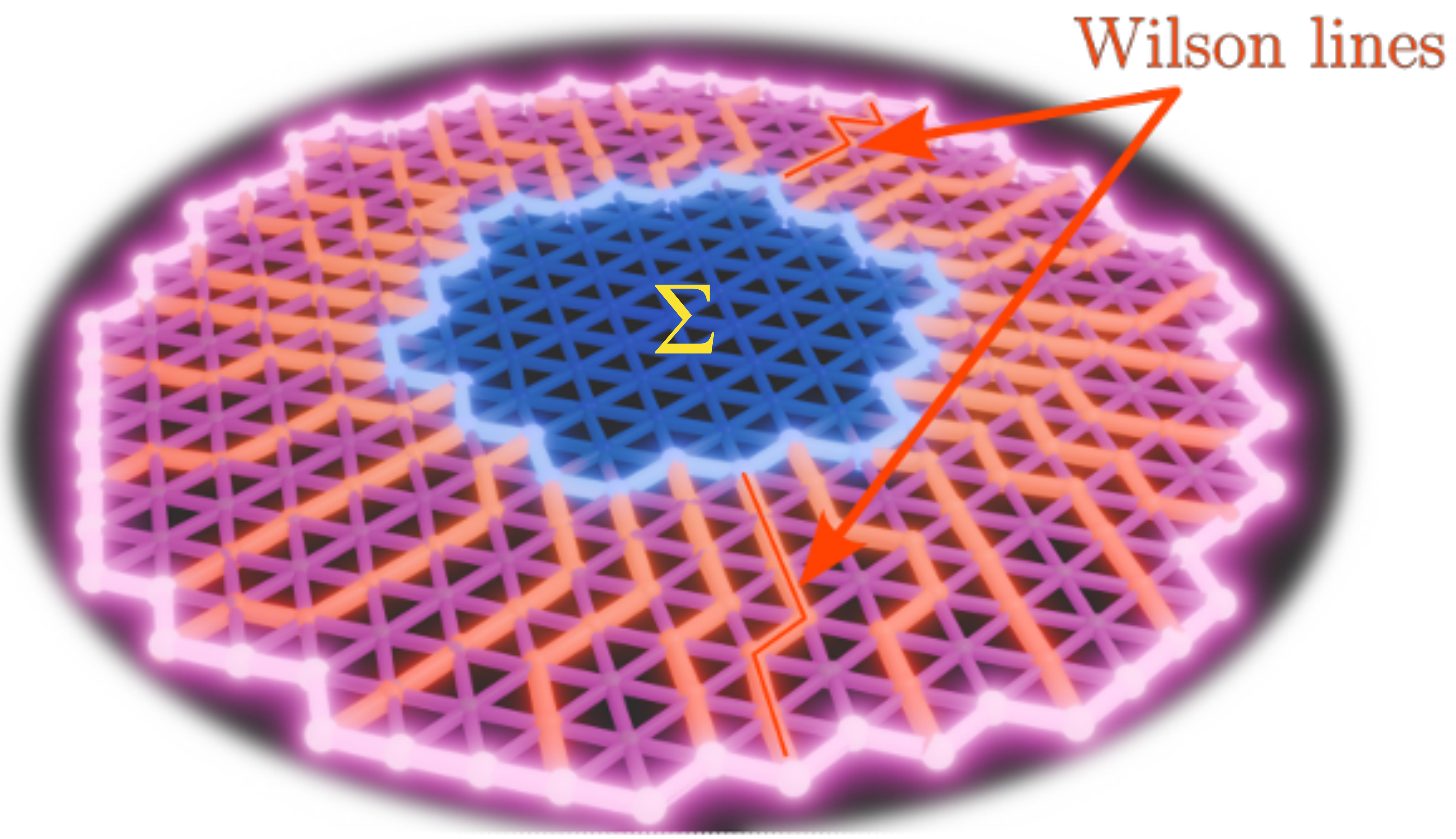

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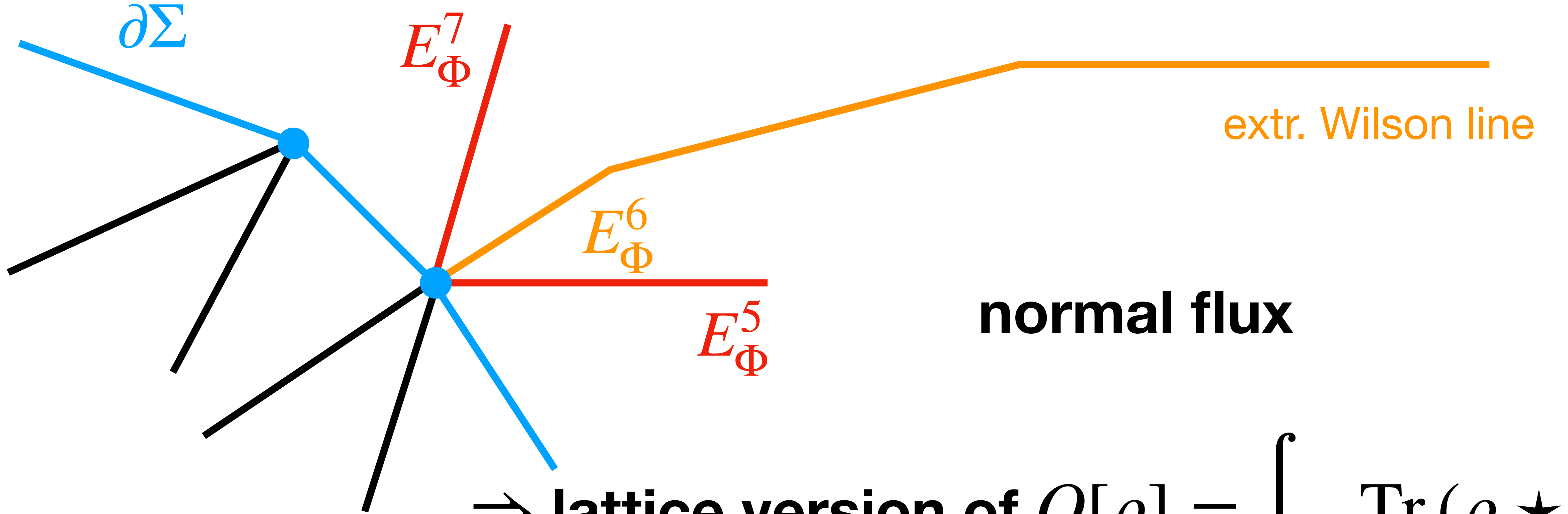

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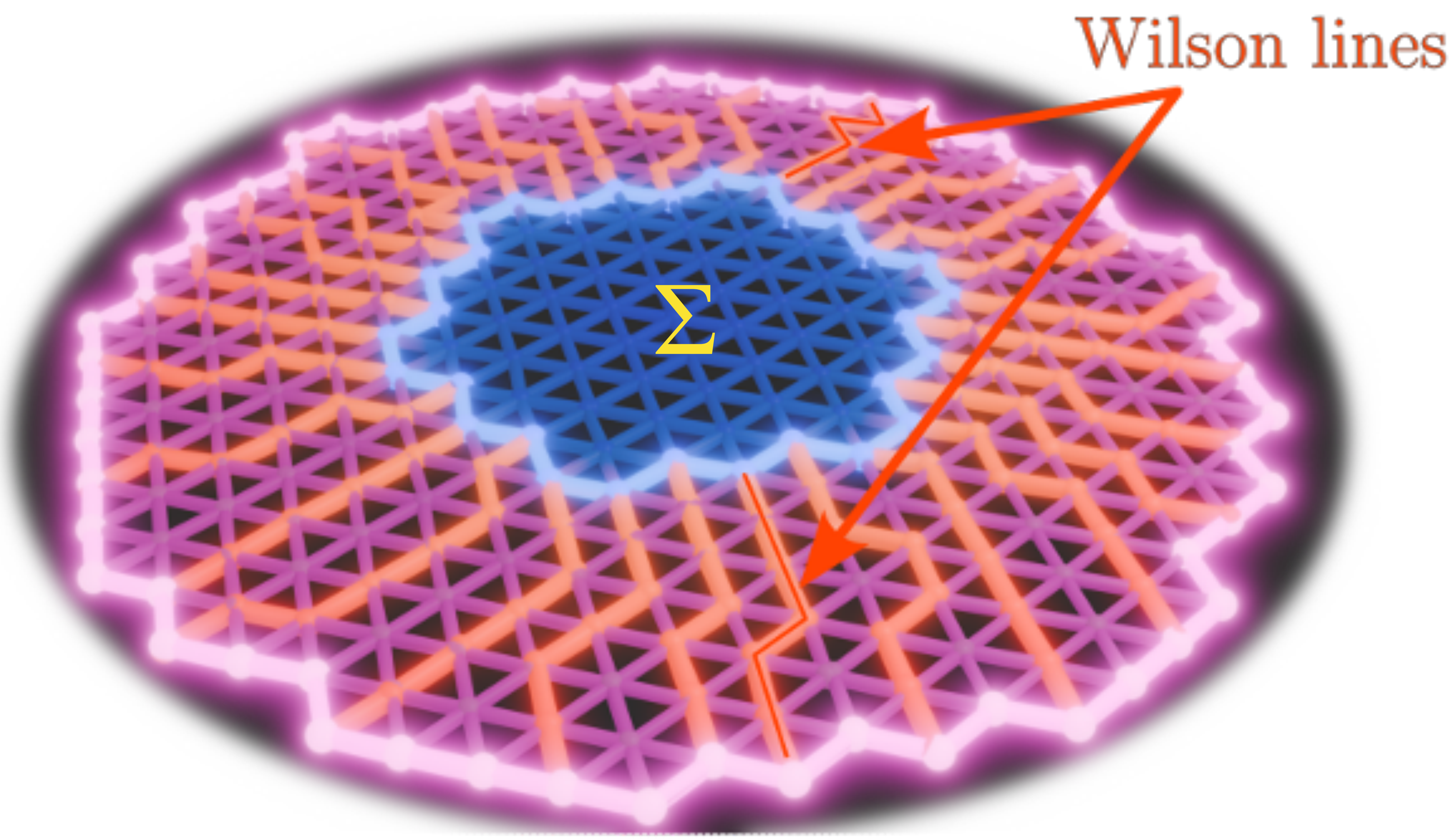

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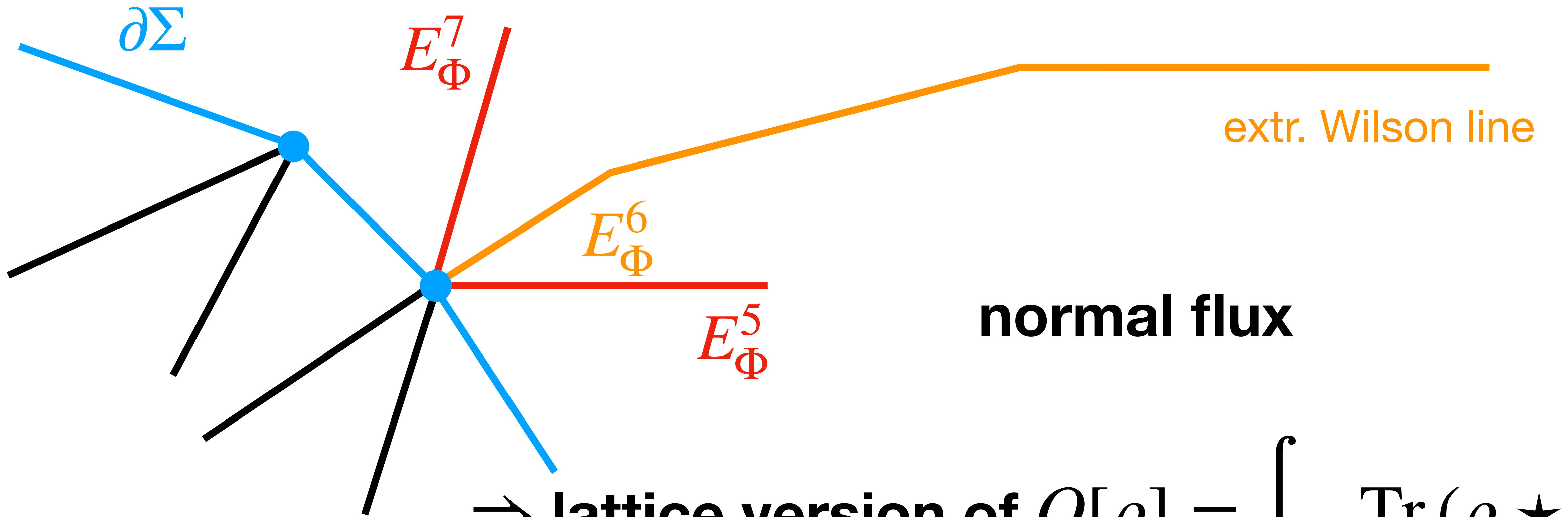
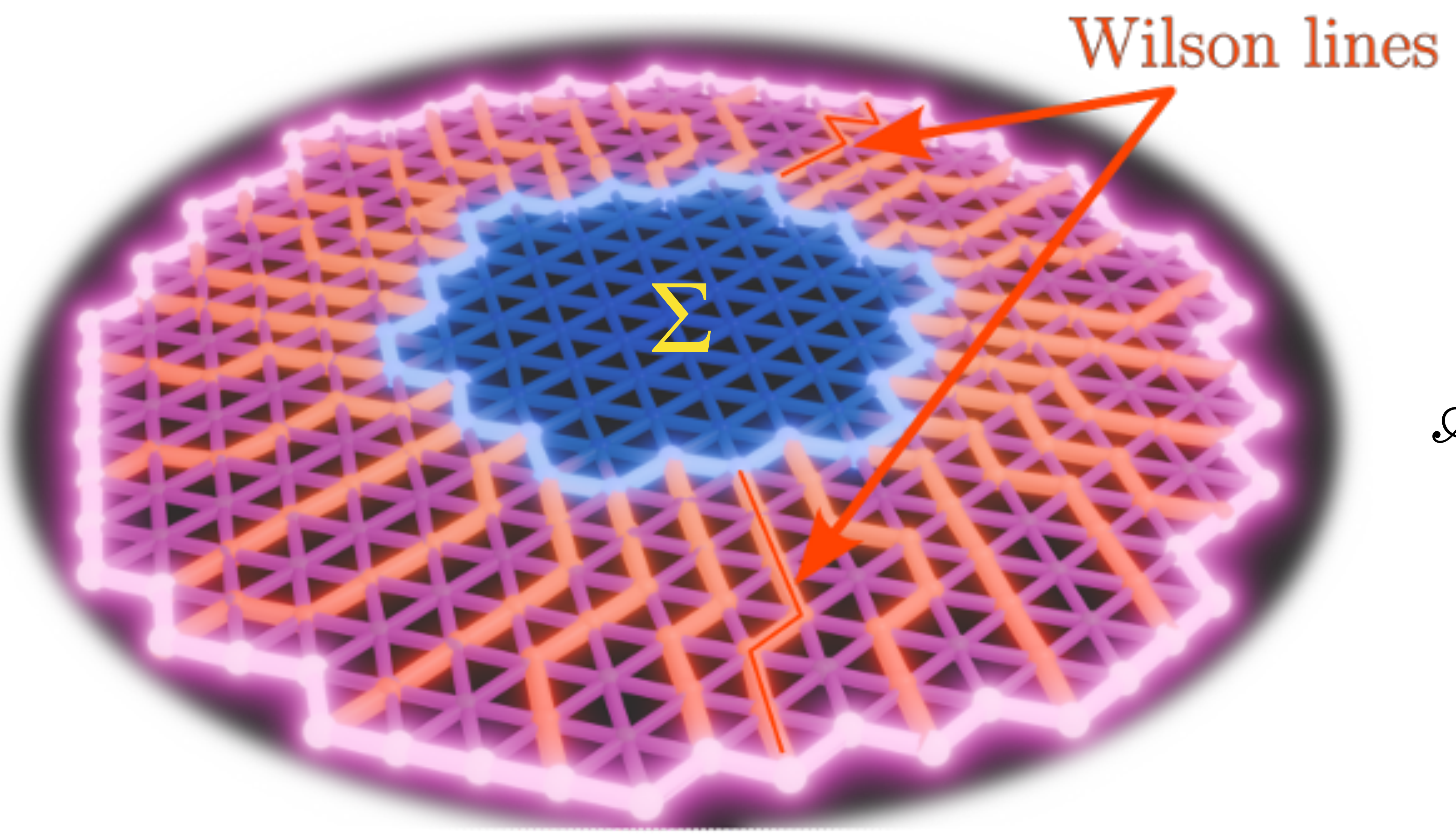


$\Rightarrow$ **lattice version of** $Q[\rho] = \int_{\partial\Sigma} \mathrm{Tr}\,(\rho \star \mathrm{F}_{\Phi})$

$\Rightarrow$ **electric corner symmetries are reorientations of** Φ**:** $\quad |\,g'\rangle_{\Phi} \mapsto V_{\Phi}(g)\,|\,g'\rangle_{\Phi} = |\,g'g^{-1}\rangle_{\Phi}$

Extrinsic relational algebras

dress all the regional data in Σ (Wilson lines, electric fields) **with extrinsic Wilson line QRF to asymp. bdry**



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$\Rightarrow$ **extrinsic relational algebra is a crossed product:**

$$\mathscr{A}_{\text{ext}} = \mathscr{A}_\Sigma \rtimes_{\text{ext}} G^{V_{\partial\Sigma}} = \left(\mathscr{A}_\Sigma \otimes L^2(G^{V_{\partial\Sigma}})\right)^{G^{V_{\partial\Sigma}}}$$

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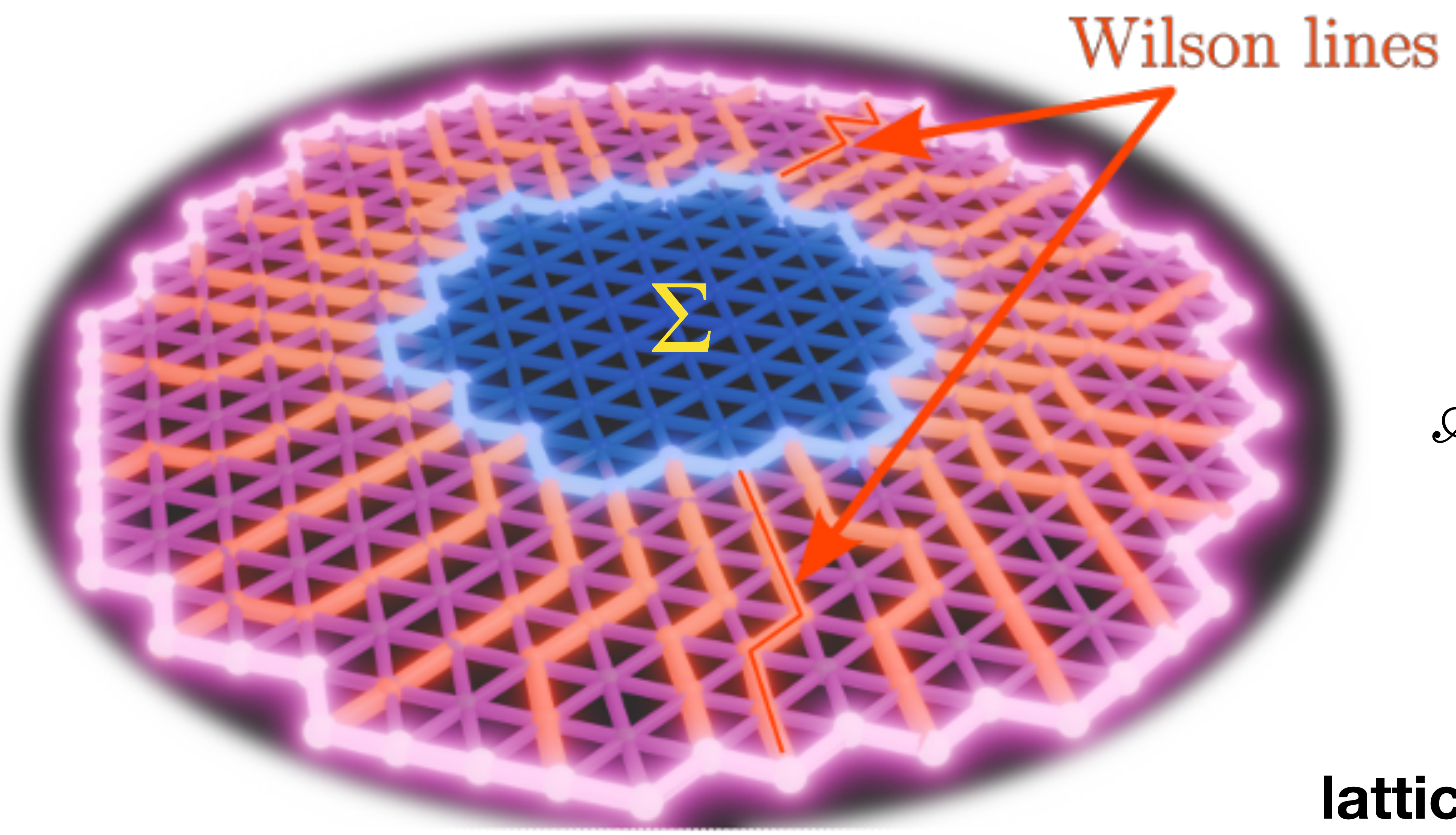
Extrinsic relational algebras

dress all the regional data in Σ (Wilson lines, electric fields) **with extrinsic Wilson line QRF to asymp. bdry**



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lattice gauge theory analog of crossed product in gravity case

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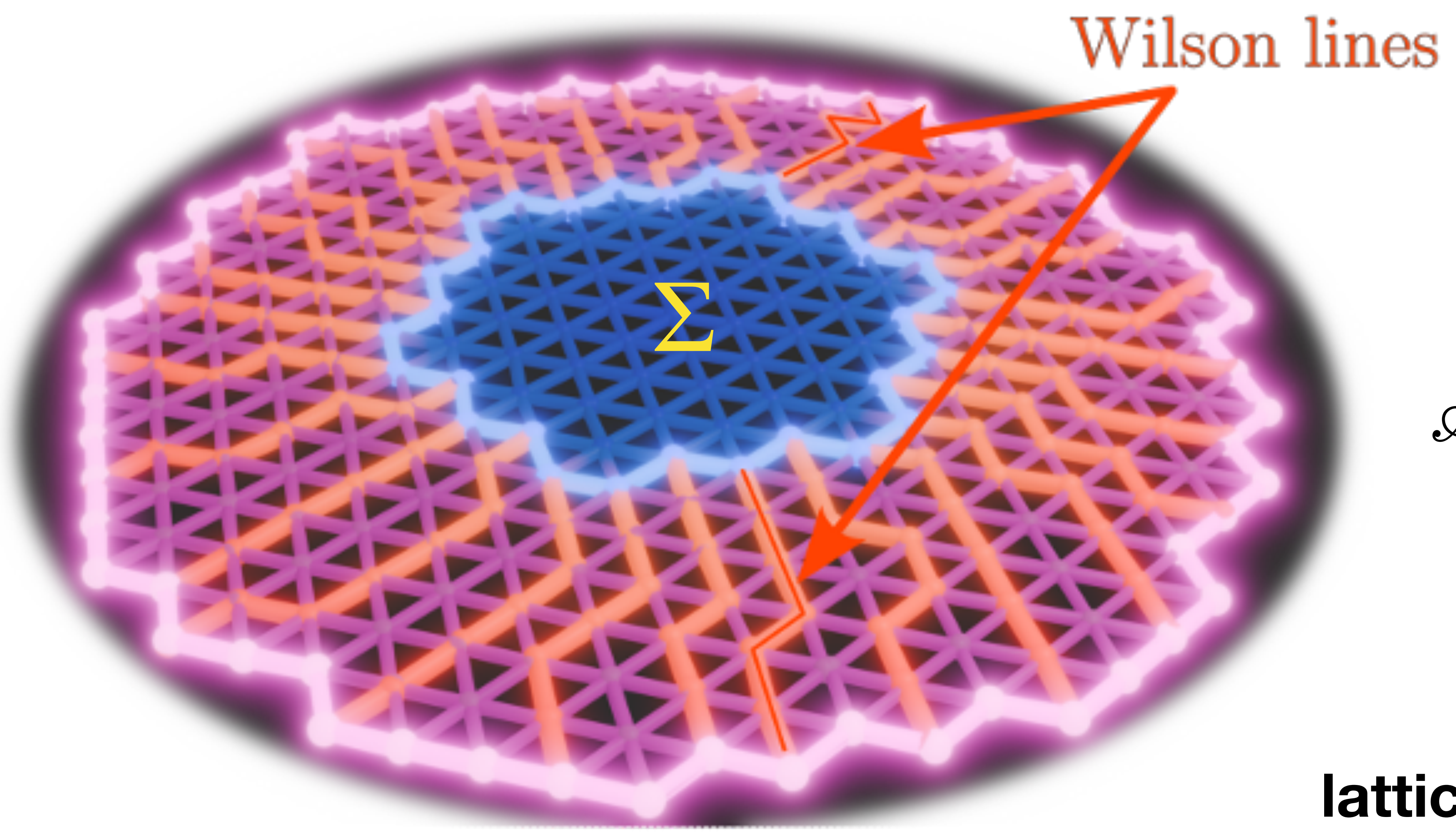

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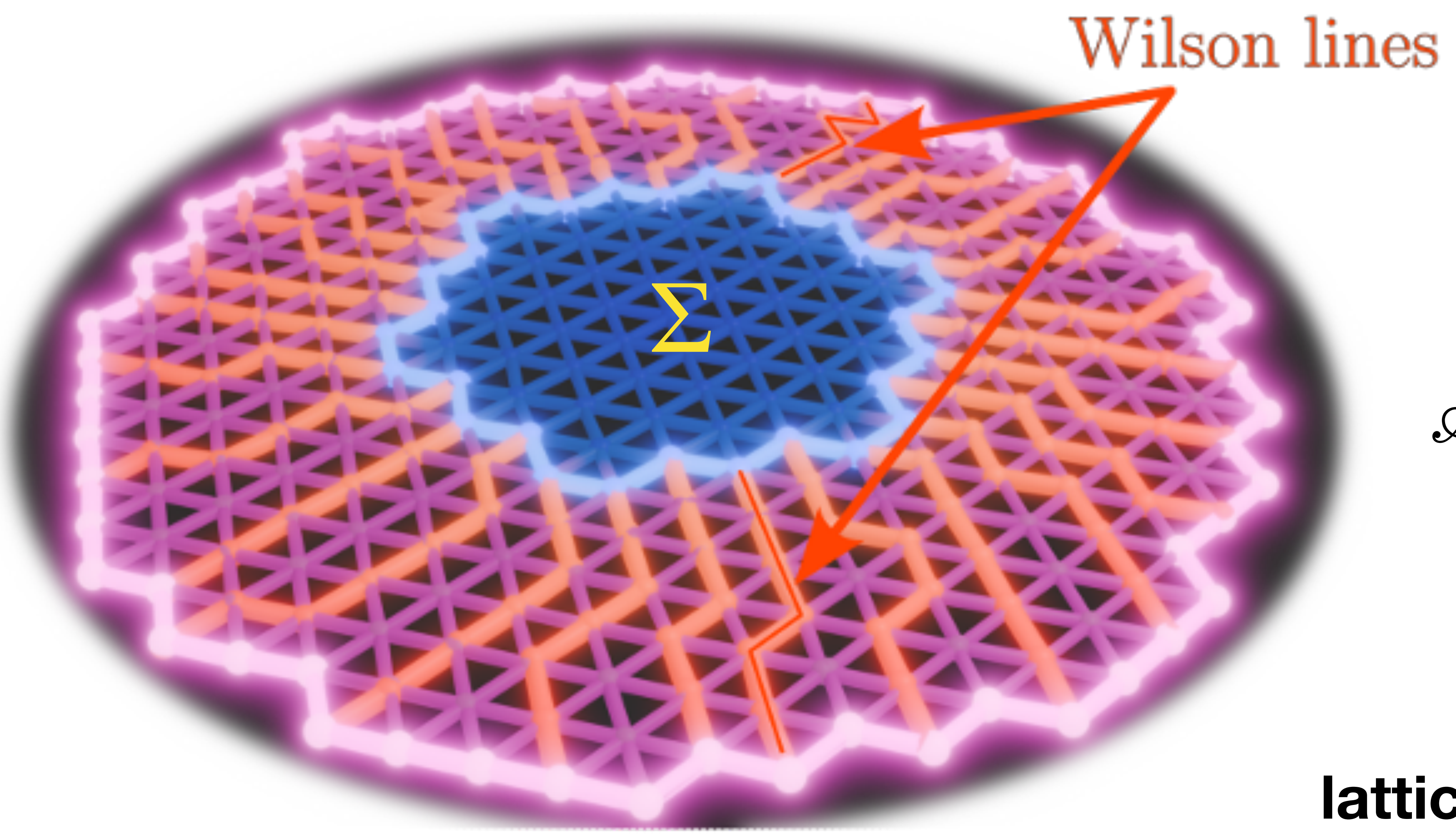

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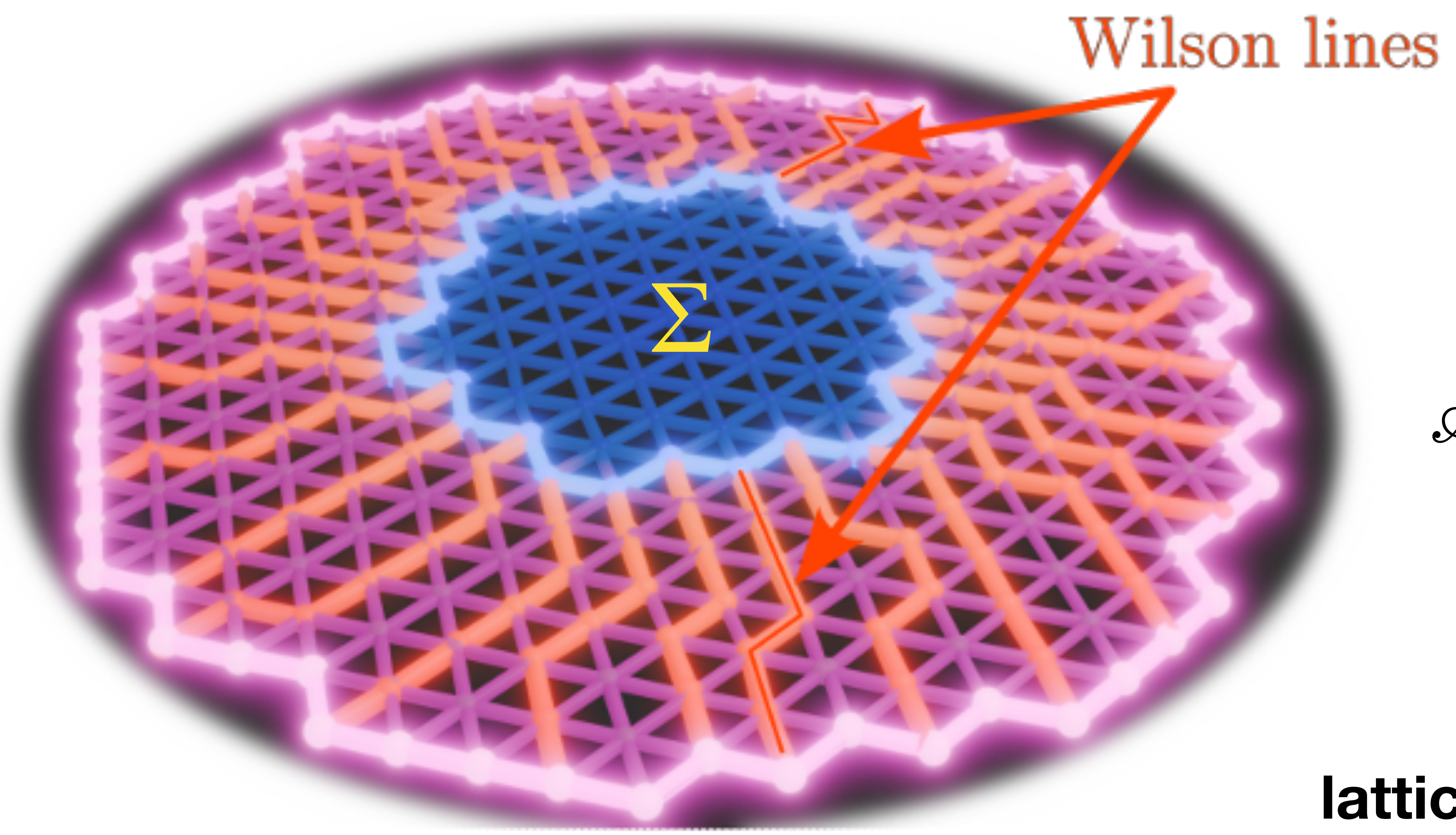
$\Rightarrow$ **relative to extrinsic QRFs get distillable EE!**

Extrinsic EE is relational

dress all the regional data in Σ (Wilson lines, electric fields) **with extrinsic Wilson line QRF to asymp. bdry**



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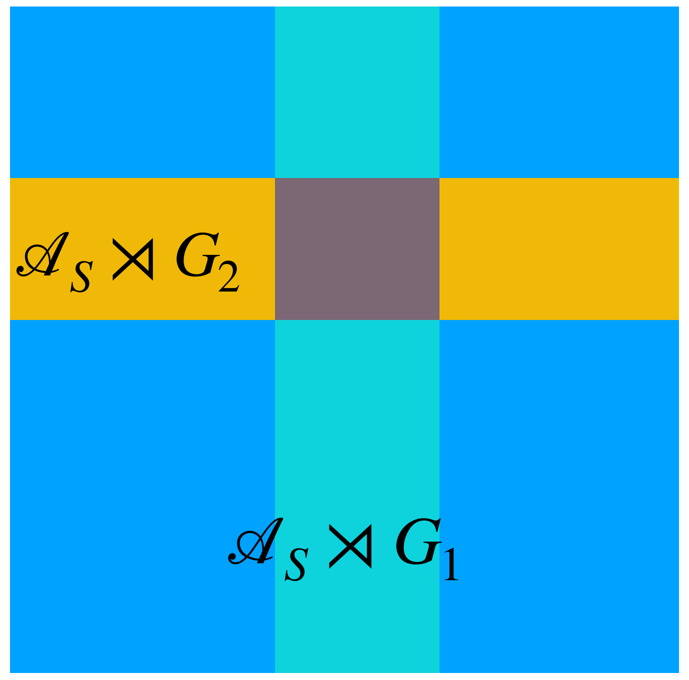


$\Rightarrow$ **relative to extrinsic QRFs get distillable EE!**

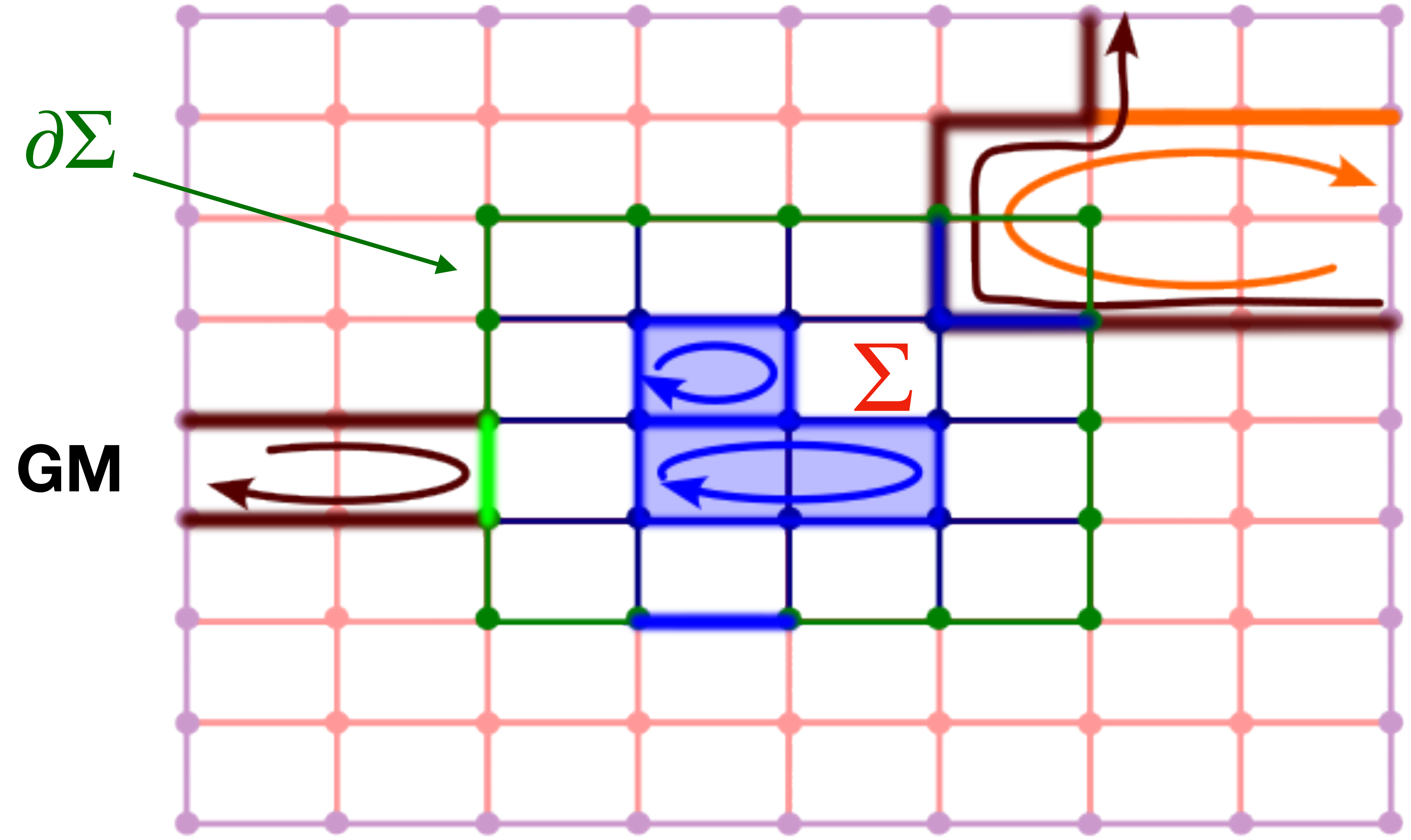
subsystem relativity for diff. ext. QRFs: distillable EE is QRF-dependent!

3 ways to understand the electric center

1. $\mathscr{A}_{\text{ext}}$ **contains a Goldstone mode:**

   rel. obs. describing int. rel. to ext. QRF $O^{g}_{|\Phi}(\tilde{\Phi})$

   $\Rightarrow$ **nonlocal gauge-inv. QRF for the electric corner group**



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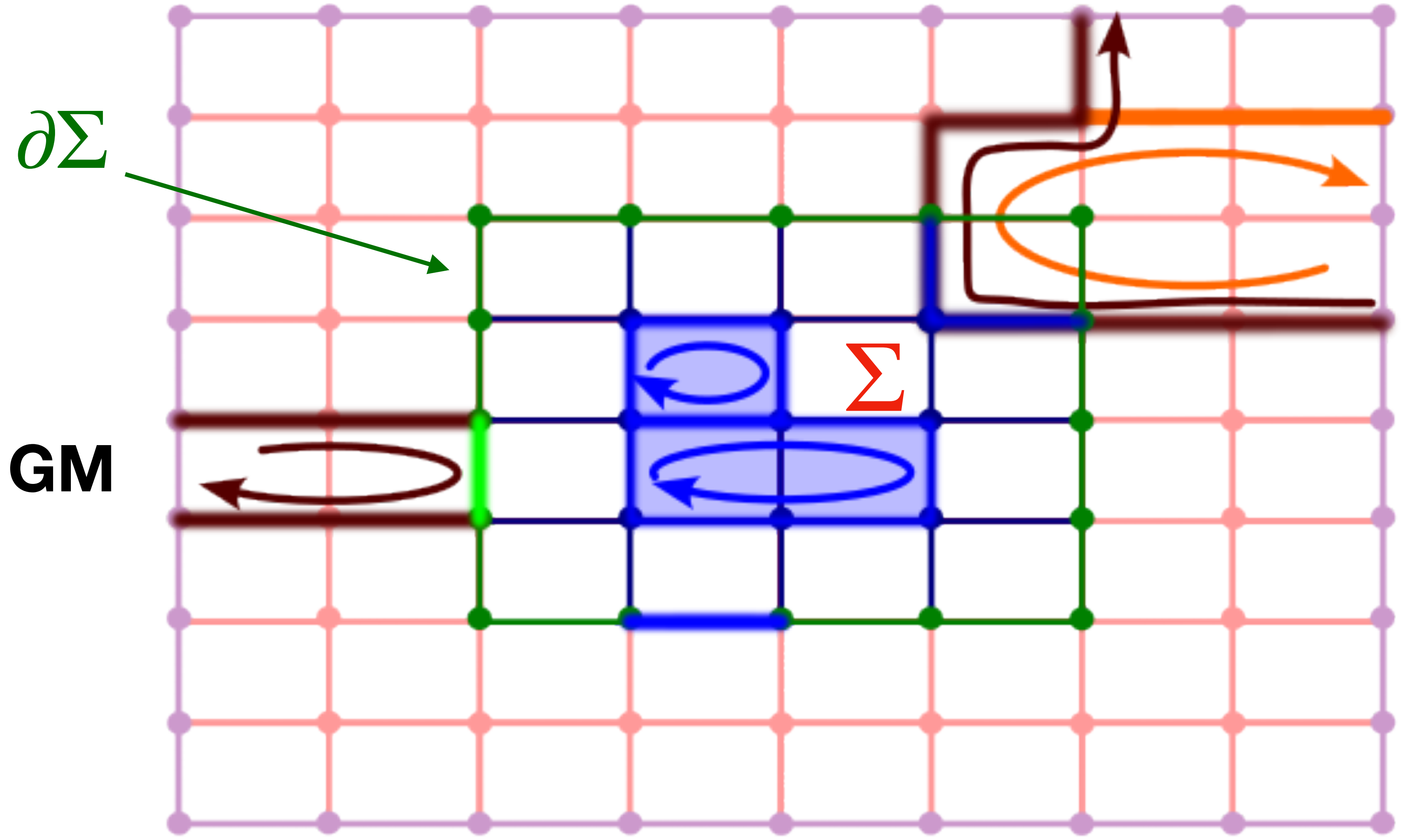
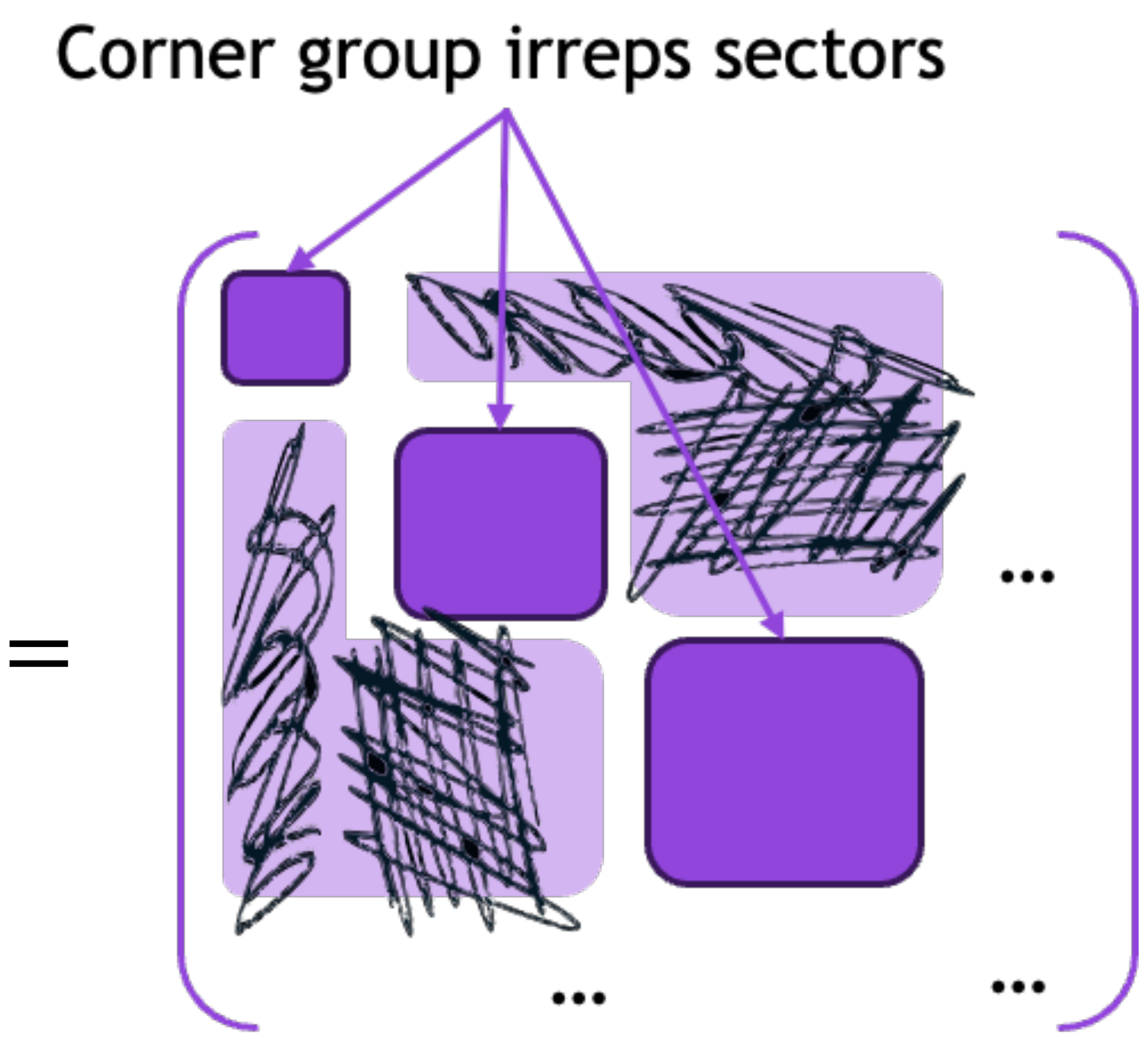
if no access to it $\quad\Rightarrow\quad$ **average over its orientations/corner group**

$$\mathscr{A}_E = \mathbb{G}_{\partial\Sigma}(\mathscr{A}_{\text{ext}}) = \int_{G^{V_{\partial\Sigma}}} dg\, V_\Phi(g)\, \mathscr{A}_{\text{ext}}\, V_\Phi^\dagger(g)$$



electric corner charge superselection:

$\mathscr{A}_E =$



reproduces "electric center" of

[Casini, Huerta, Rosbal '13; Soni, Trivedi '15; Delcamp, Dittrich, Riello '16]

(center: Casimirs of normal flux)

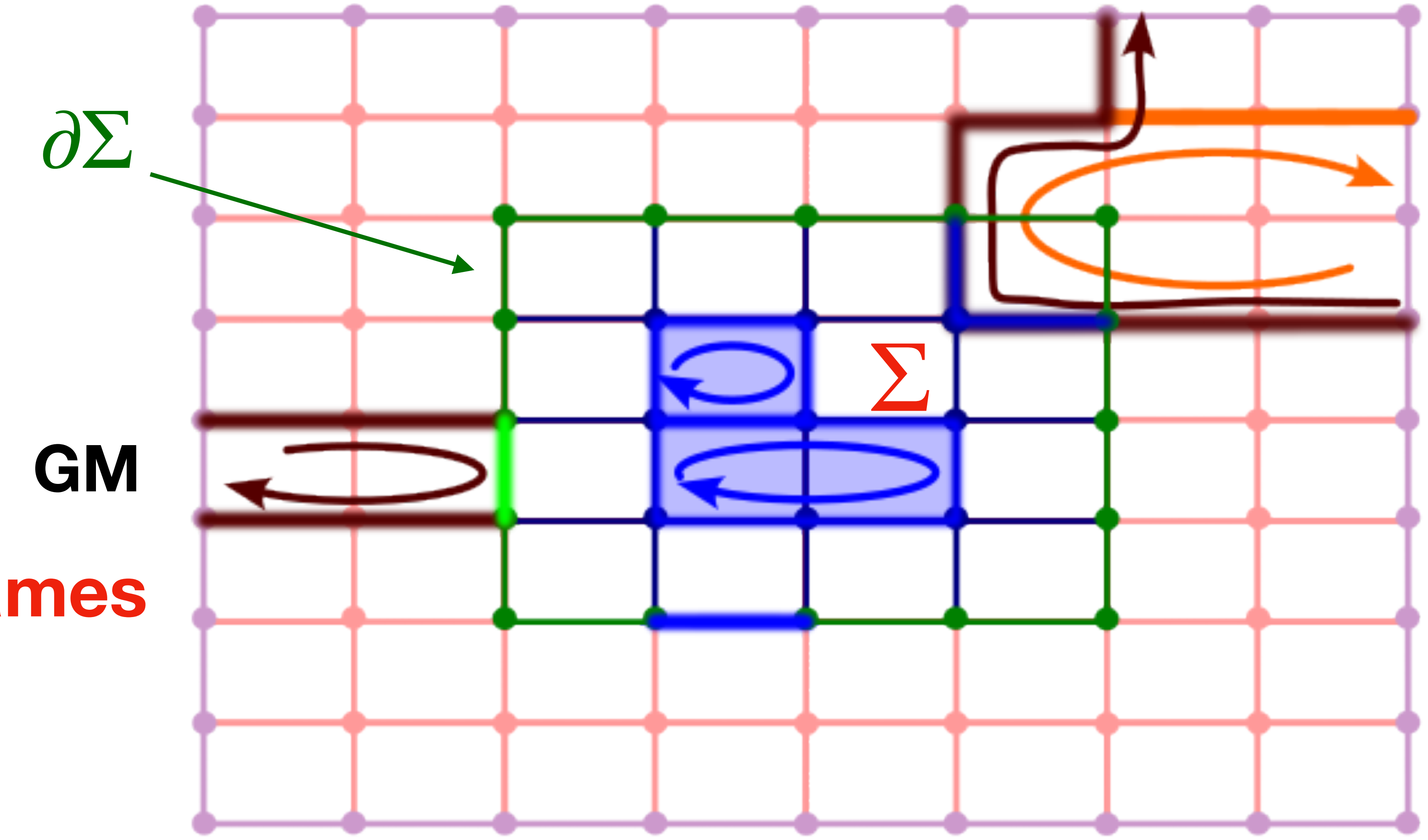
3 ways to understand the electric center

2. what all extr. QRFs agree on:

$$\mathscr{A}_E = \bigcap_{\Phi} \mathscr{A}^{\Phi}_{\text{ext}}$$

(subsystem relativity: agree on what's ext. QRF-indep.)

$\Rightarrow$ electric center is to ext. QRFs what Lorentz scalars are to Lorentz frames



3 ways to understand the electric center

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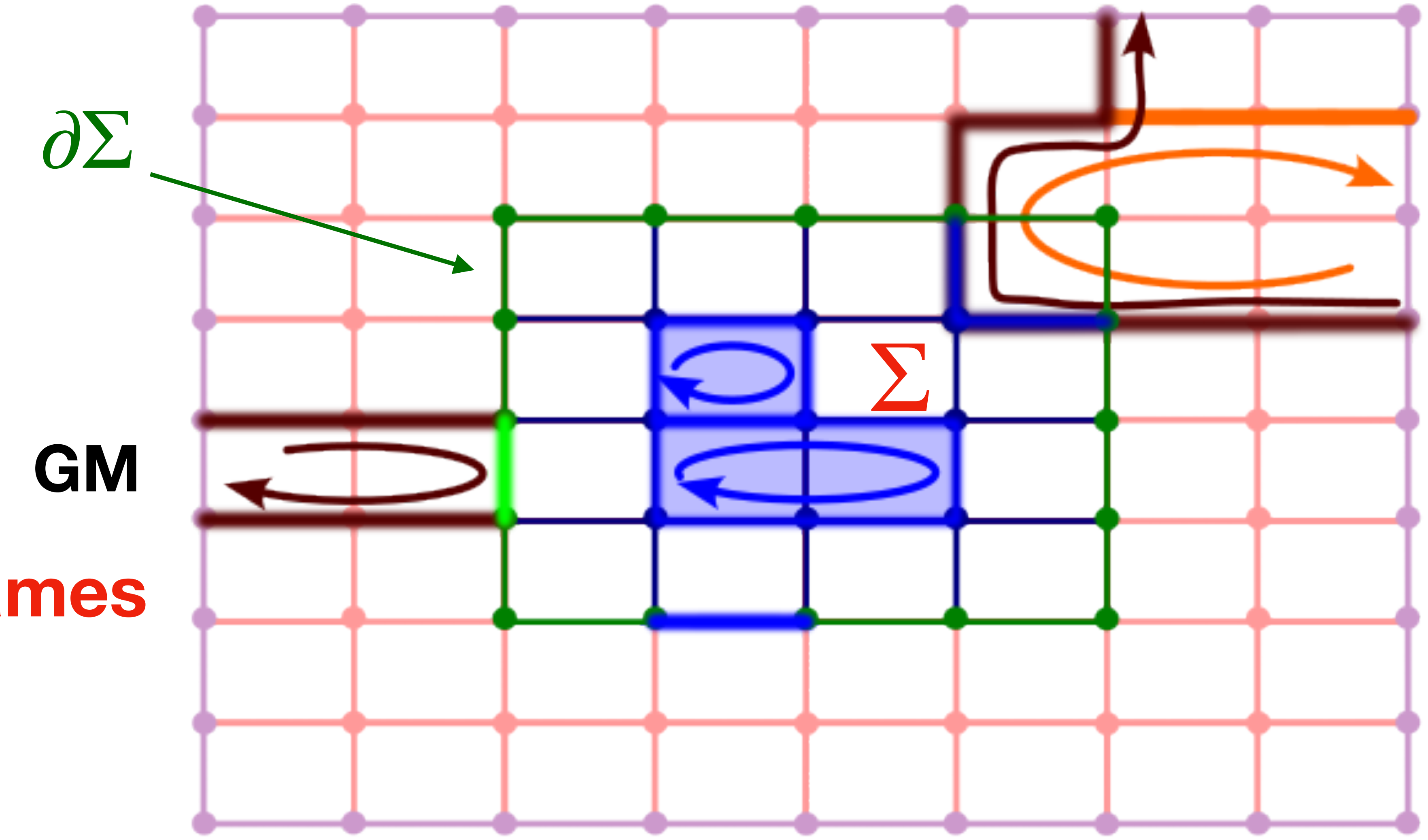
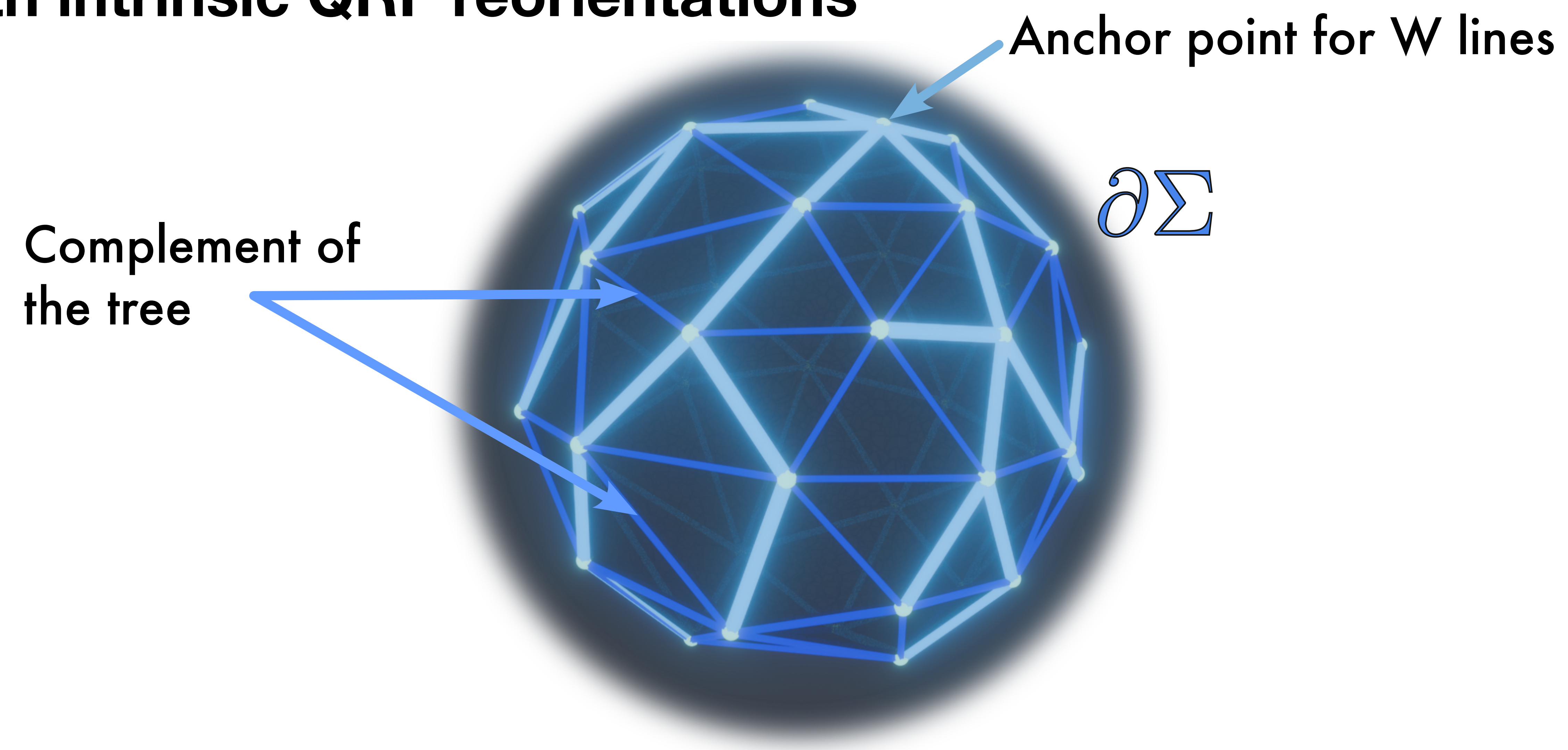


3. as a (projected) crossed product of regional complement with intrinsic QRF reorientations

$$\mathscr{A}_E = \mathscr{A}_\Sigma \rtimes_{\text{int}} G^{V_{\partial\Sigma}-1} = \left(\mathscr{A}_\Sigma \otimes L^2(G^{V_{\partial\Sigma}-1})\right)^{G^{V_{\partial\Sigma}}}$$

$$= \left\langle O^e_{|\tilde{\Phi}}(a), V_{\tilde{\Phi}}(g) \,\middle|\, a \in \mathscr{A}_{\Sigma\backslash\tilde{\Phi}}, g \in G^{V_{\partial\Sigma}-1} \right\rangle''$$

$\Rightarrow$ for Abelian theories: electric center IS a crossed product



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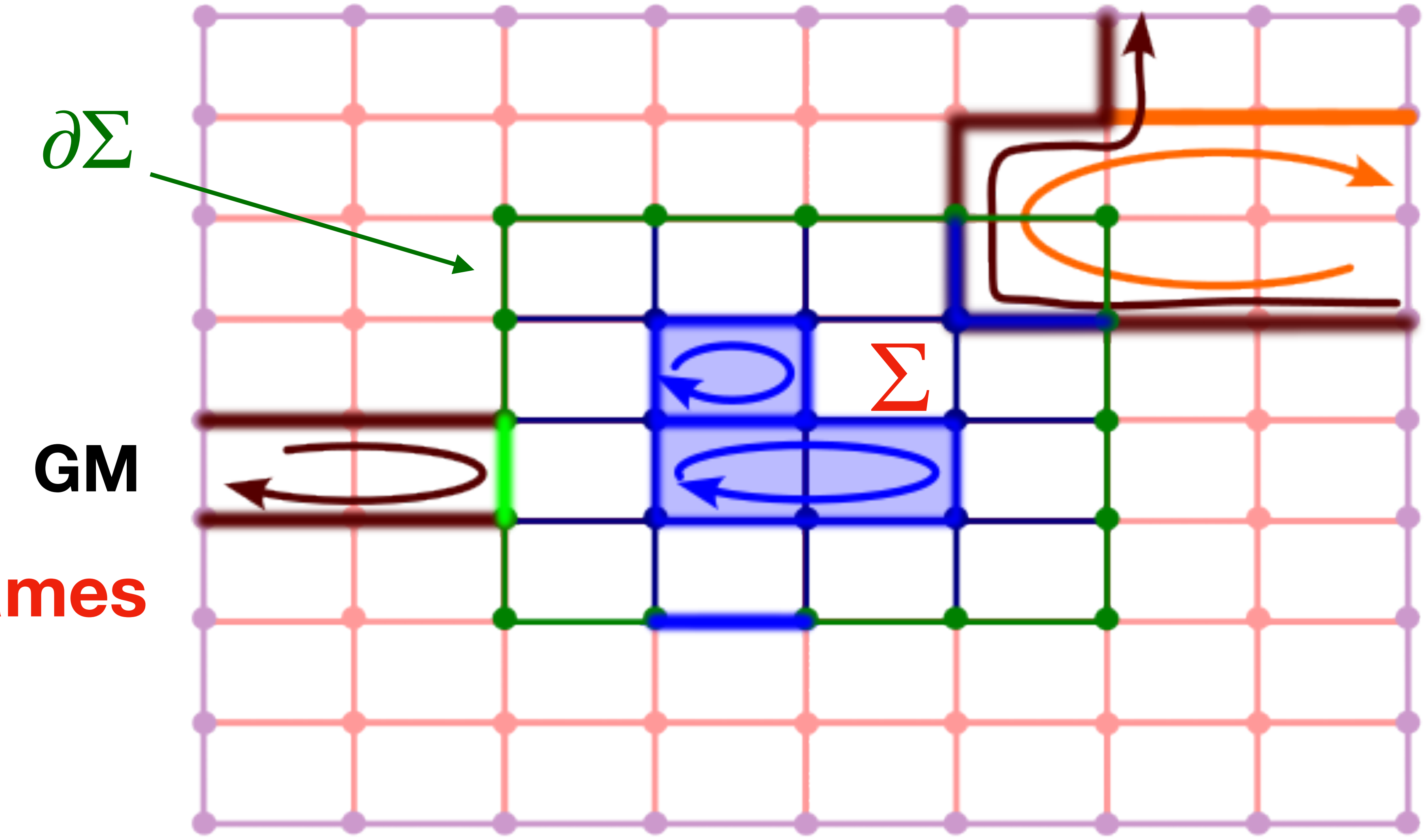


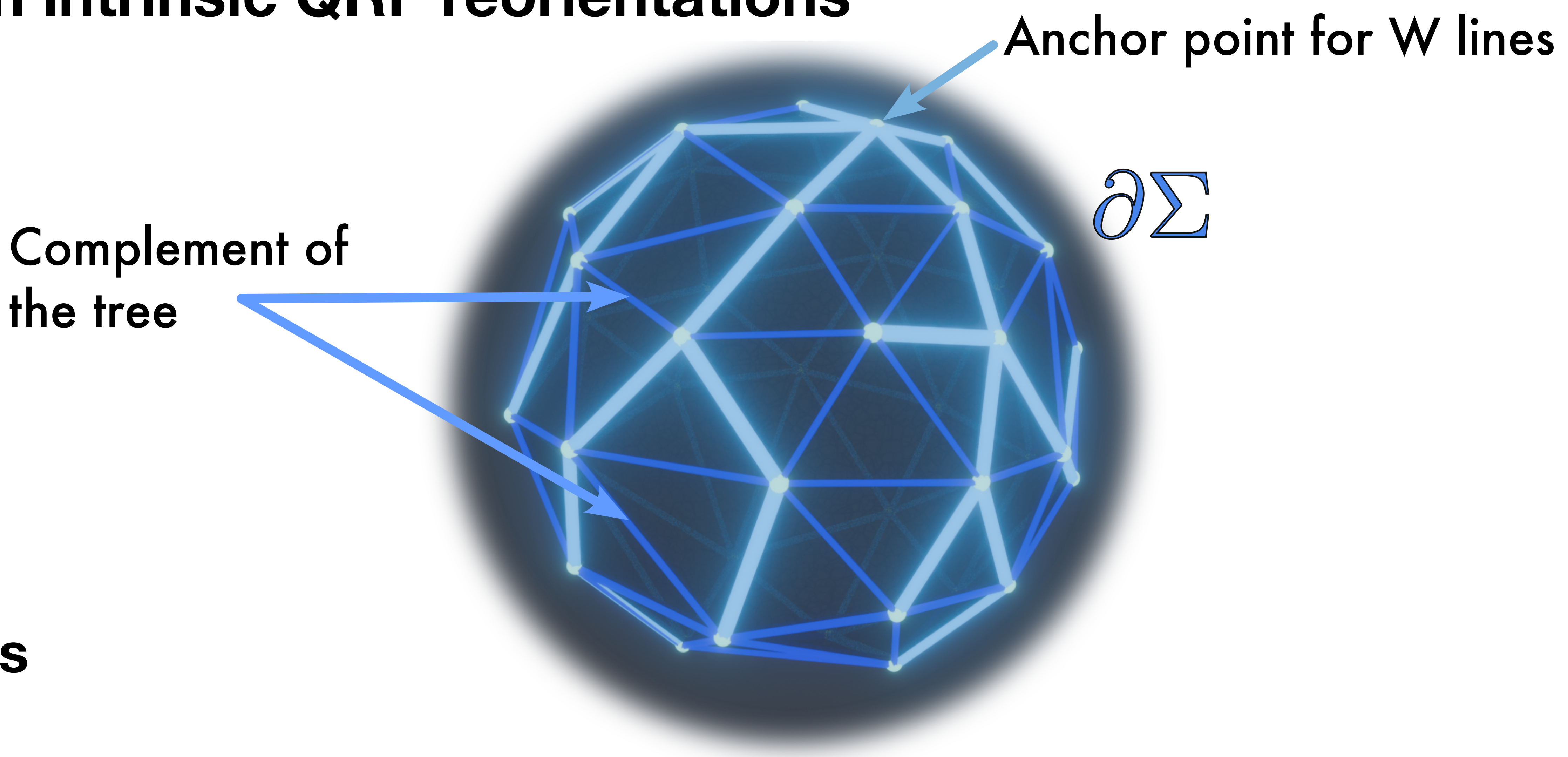
3. as a (projected) crossed product of regional complement with intrinsic QRF reorientations

$$\mathscr{A}_E = \mathscr{G}_N\left(\mathscr{A}_\Sigma \rtimes_{\mathrm{int}} G^{V_{\partial\Sigma}-1}\right) = \left(\mathscr{A}_\Sigma \otimes L^2(G^{V_{\partial\Sigma}-1})\right)^{G^{V_{\partial\Sigma}}}$$

$$= \mathscr{G}_N\left(\left\langle O^e_{|\tilde{\Phi}}(a), \cancel{V_{\tilde{\Phi}}(g)} \,\middle|\, a \in \mathscr{A}_{\Sigma\setminus\tilde{\Phi}}, g \in G^{V_{\partial\Sigma}-1}\right\rangle''\right)$$

incoherent avg. over gauge transf. at N

$$\mathscr{G}_N(\bullet) = \int_G dg\, U_N(g) \bullet U_N^\dagger(g)$$



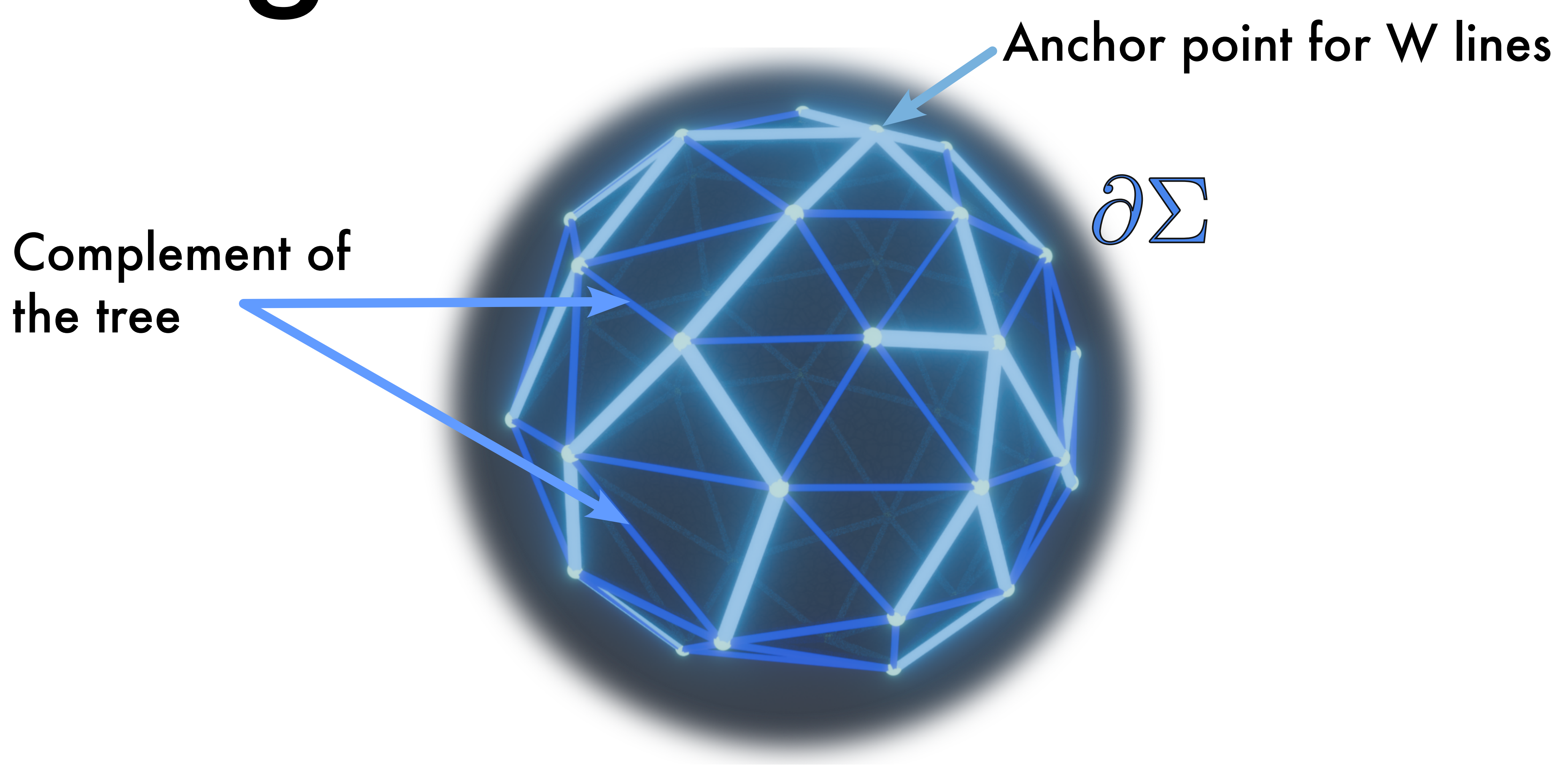
for nA theories:
kills reorientations

Intrinsic relational algebras

only relational observables of regional complement relative to intrinsic QRF $\tilde{\Phi}$

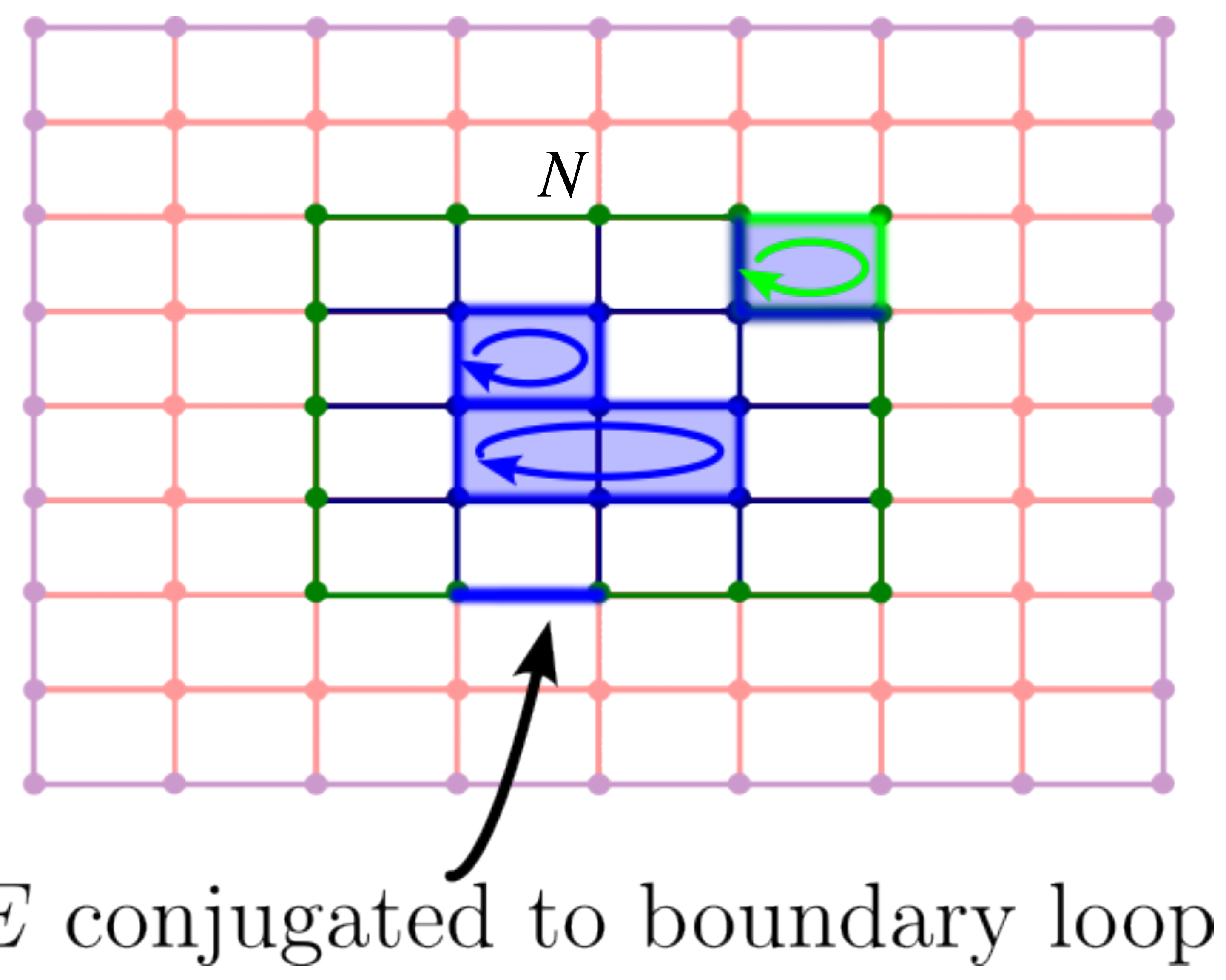
$$\mathscr{A}_{\text{int}} = \left\langle O^g_{|\tilde{\Phi}}(a) \,\middle|\, a \in \mathscr{A}_{\Sigma\backslash\tilde{\Phi}},\, g \in G^{V_{\partial\Sigma}} \right\rangle''$$

(shedding all electric fields tangential to frame tree)



⇒ factor in Abelian case ⇒ distillable EE

⇒ central in non-Abelian case ⇒ vN entropy not fully distillable



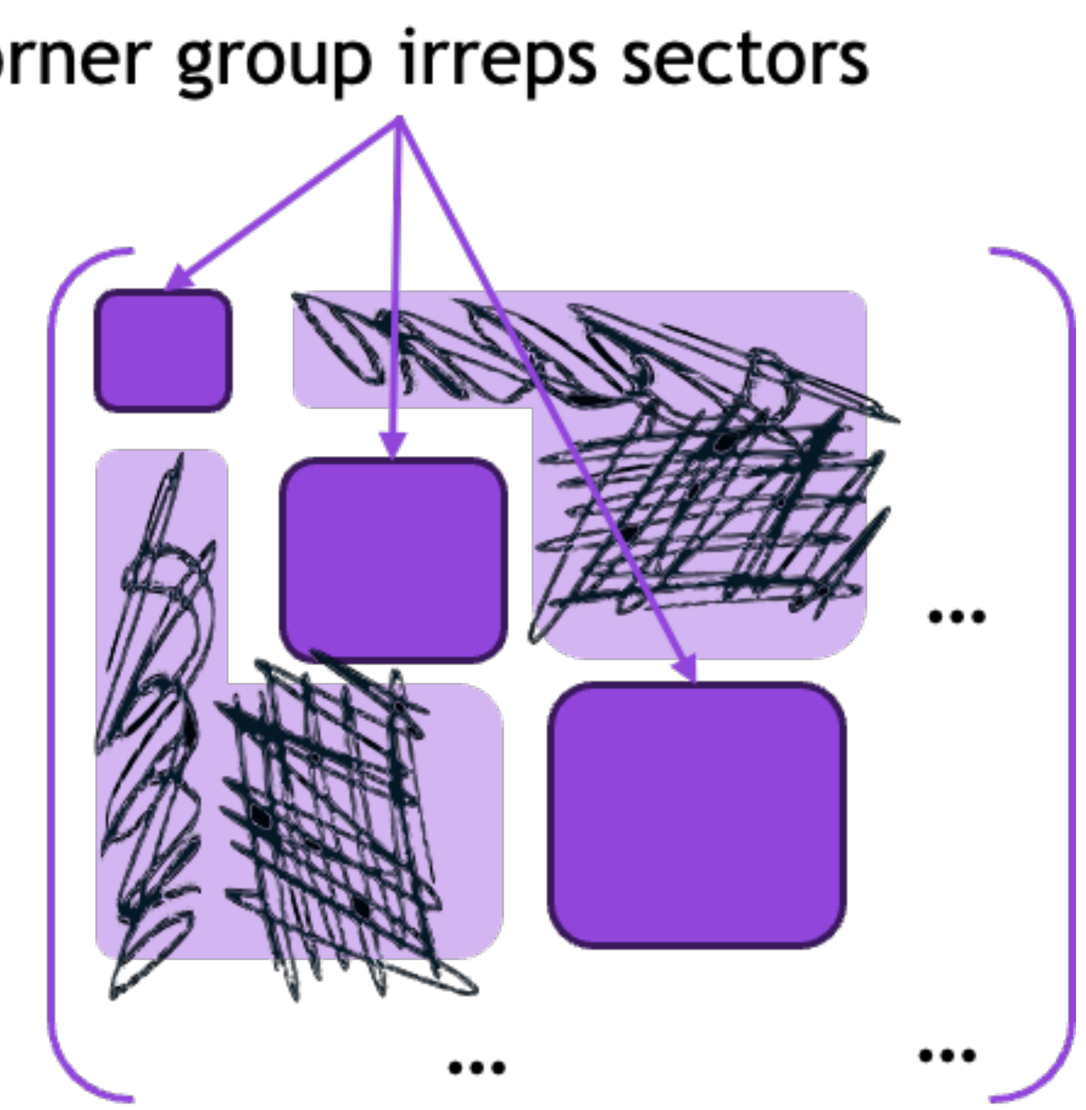
2 ways to understand the magnetic center

1. **in the Abelian case as a twirl over a Pontryagin dual** **magnetic** **corner symmetry group** (generated by corner Wilson loops)

$\Rightarrow$ **magnetic corner charge superselection**

lattice version of $\quad Q_M[\kappa] = \int_{\partial\Sigma} \kappa F$

$$\mathscr{A}_M = \hat{G}_{\partial\Sigma}(\mathscr{A}_{\mathrm{int}}) =$$



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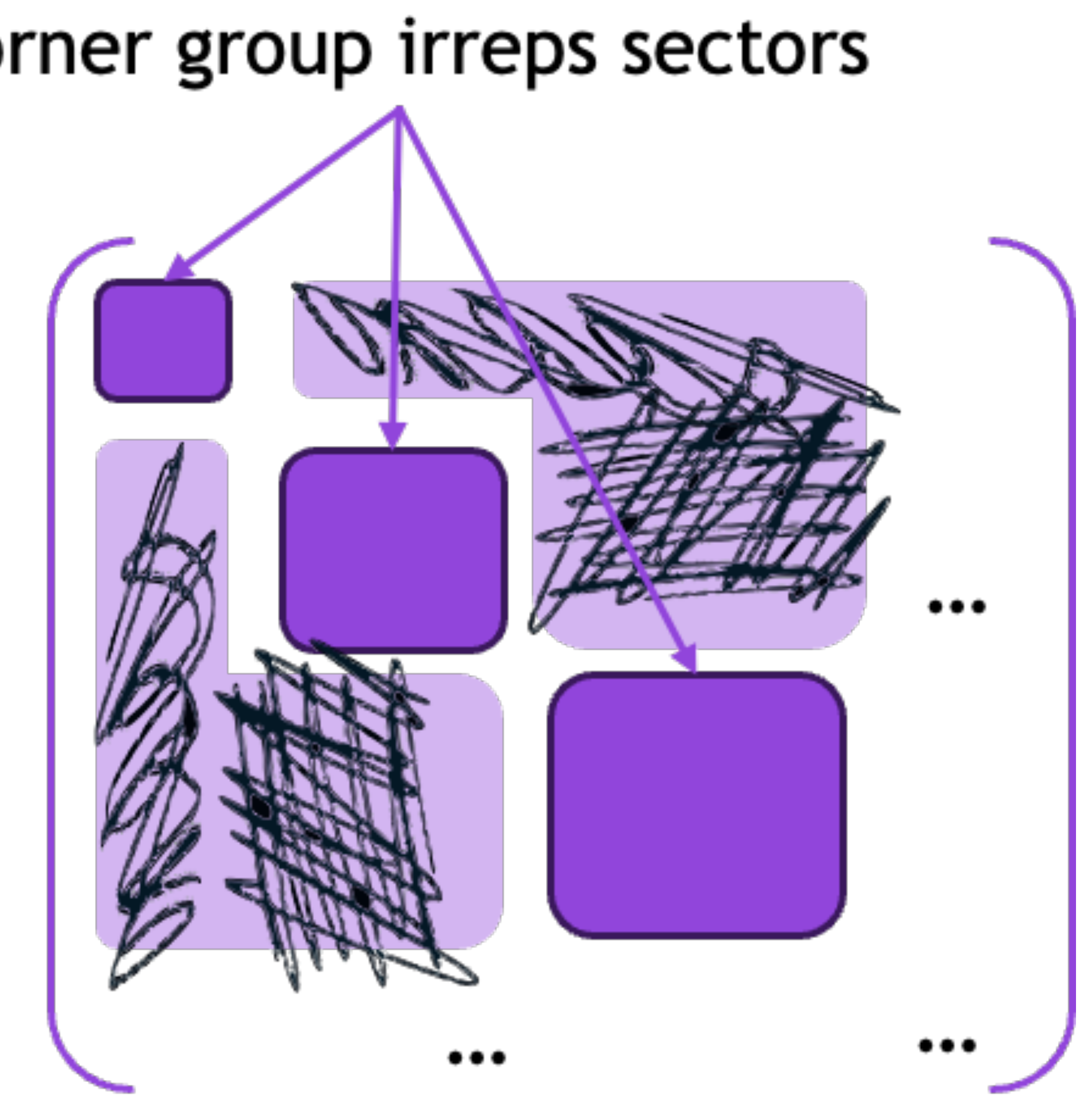

2. **in general: what all intr. QRFs agree on:**

$$\mathscr{A}_M = \bigcap_{\tilde{\Phi}} \mathscr{A}^{\tilde{\Phi}}_{\text{int}}$$ (subsystem relativity: agree on what's int. QRF-indep.)

$\Rightarrow$ magnetic center is to int. QRFs what Lorentz scalars are to Lorentz frames

(generalizes magnetic center beyond finite groups, recovers [Casini, Huerta, Rosabal '13; Delcamp, Dittrich, Riello '16] **as special cases)**

fininte Abelian

finite non-Abelian, 2+1d

Algebra and entropy hierarchy

algebra	entanglement entropy	distillable
$\mathscr{A}_{\text{ext}}$	$S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\vert\Phi})$	✓
$\mathcal{A}_E$	$H(\{p_{\mathfrak{e}}\}) + \sum_{\mathfrak{e}} p_{\mathfrak{e}} \left(\log d_{\mathfrak{e}} + S_{\text{vN}}(\rho_{\mathfrak{e}})\right)$	×
$\mathscr{A}_{\text{int}}$	A: $S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\setminus\tilde{\Phi}\vert\tilde{\Phi}})$	✓
	nA: $H(\{p_{\hat{\mathfrak{e}}}\}) + \sum_{\hat{\mathfrak{e}}} p_{\hat{\mathfrak{e}}} \left(\log d_{\hat{\mathfrak{e}}} + S_{\text{vN}}(\rho_{\hat{\mathfrak{e}}})\right)$	×
$\mathcal{A}_M$	$H(\{p^m_{\bar{\mathfrak{e}}}\}) + \sum_{m,\bar{\mathfrak{e}}} p^m_{\bar{\mathfrak{e}}} \left(\log d^m_{\bar{\mathfrak{e}}} + S_{\text{vN}}(\rho^m_{\bar{\mathfrak{e}}})\right)$	×

$$\mathscr{A}^{\Phi}_{\text{ext}} \supsetneq \mathscr{A}_E \supsetneq \mathscr{A}^{\tilde{\Phi}}_{\text{int}} \supsetneq \mathscr{A}_M$$

relational

QRF-indep.

Algebra and entropy hierarchy

algebra	entanglement entropy	distillable
$\mathscr{A}_{\text{ext}}$	$S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\|\Phi})$	✓
$\mathcal{A}_E$	$H(\{p_{\mathfrak{e}}\}) + \sum_{\mathfrak{e}} p_{\mathfrak{e}} \left(\log d_{\mathfrak{e}} + S_{\text{vN}}(\rho_{\mathfrak{e}})\right)$	×
$\mathscr{A}_{\text{int}}$	A: $S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\setminus\tilde{\Phi}\|\tilde{\Phi}})$	✓
	nA: $H(\{p_{\hat{\mathfrak{e}}}\}) + \sum_{\hat{\mathfrak{e}}} p_{\hat{\mathfrak{e}}} \left(\log d_{\hat{\mathfrak{e}}} + S_{\text{vN}}(\rho_{\hat{\mathfrak{e}}})\right)$	×
$\mathcal{A}_M$	$H(\{p^m_{\bar{\mathfrak{e}}}\}) + \sum_{m,\bar{\mathfrak{e}}} p^m_{\bar{\mathfrak{e}}} \left(\log d^m_{\bar{\mathfrak{e}}} + S_{\text{vN}}(\rho^m_{\bar{\mathfrak{e}}})\right)$	×

$$\mathscr{A}^{\Phi}_{\text{ext}} \supsetneq \mathscr{A}_E \supsetneq \mathscr{A}^{\tilde{\Phi}}_{\text{int}} \supsetneq \mathscr{A}_M$$

relational

QRF-indep.

rel. entropy hierarchy: $S(\rho_{|\Phi}||\sigma_{|\Phi}) \geq S(\rho_E||\sigma_E) \geq S(\rho_{|\tilde{\Phi}}||\sigma_{|\tilde{\Phi}}) \geq S(\rho_M||\sigma_M)$ (same global states)

Algebra and entropy hierarchy

algebra	entanglement entropy	distillable
$\mathscr{A}_{\text{ext}}$	$S_{\text{vN}}(\rho^{\text{phys}}_{\Sigma\|\Phi})$	✓
$\mathcal{A}_E$	$H(\{p_{\mathfrak{e}}\}) + \sum_{\mathfrak{e}} p_{\mathfrak{e}} \left(\log d_{\mathfrak{e}} + S_{\text{vN}}(\rho_{\mathfrak{e}})\right)$	✕
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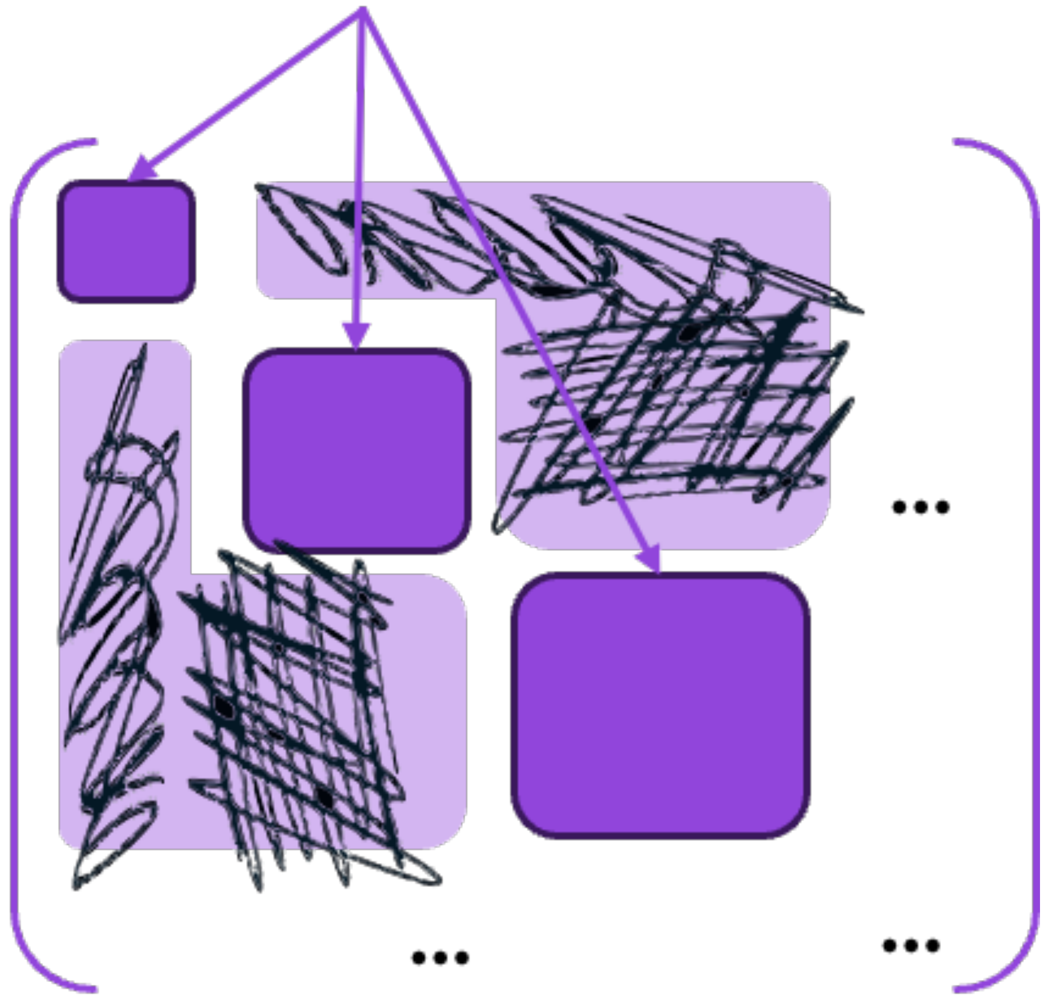
Corner group irreps sectors



$$\mathscr{A}^{\Phi}_{\text{ext}} \supsetneq \mathscr{A}_E \supsetneq \mathscr{A}^{\tilde{\Phi}}_{\text{int}} \supsetneq \mathscr{A}_M$$

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QRF-indep.

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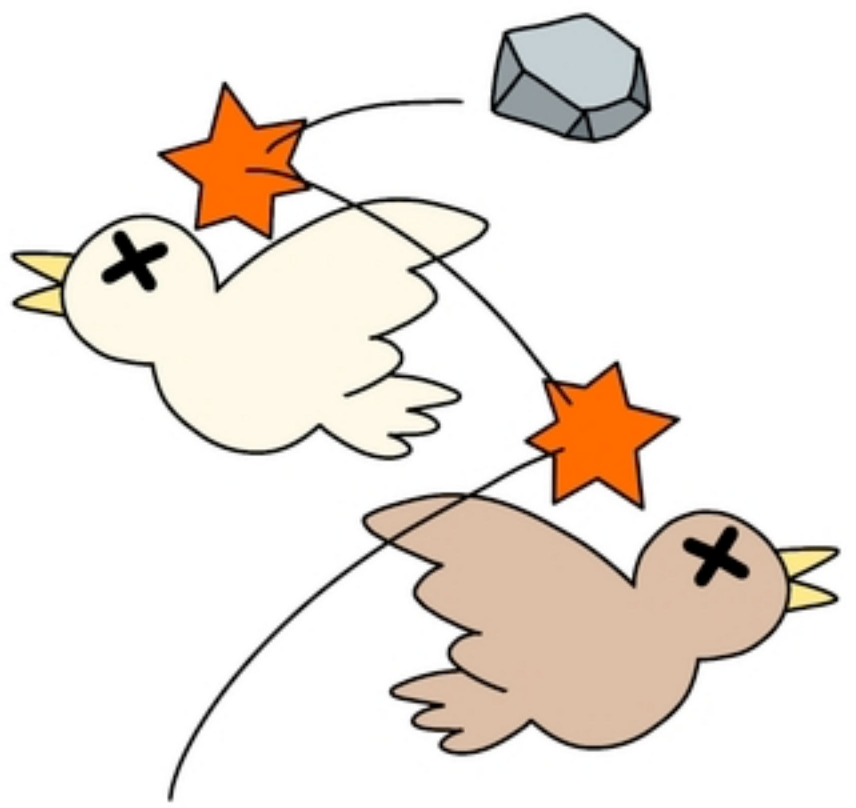
EE inequalities from incoherent corner twirls

(charge superselection)

$S_{\text{vN}}(\rho_{|\Phi}) \leq S_{\text{vN}}(\rho_E)$ $S_{\text{vN}}(\rho_{|\tilde{\Phi}}) \leq S_{\text{vN}}(\rho_M)$

finite Abelian groups

Summary



- **QRFs help with intrinsically regulating UV divergences in EEs**
  (von Neumann algebra type transition)

- **multitude of possible gauge-inv. subsystems associated with a subregion in gauge theories**
  (algebra and entropy hierarchy)

- **QRFs perspectives as gauge-inv. subsystem partitions**
  (may be factor algebras ⇒ distillable entanglement)

- **relational entanglement entropies**
  (relational entropies are QRF/observer-dependent)

- **electric/magnetic center algebra is what all ext./int. QRFs agree on**