2026/3/11 Concepts of Quantum and Spacetime @ KEK

A Cosmic Cat State and Decoherence in Early Universe

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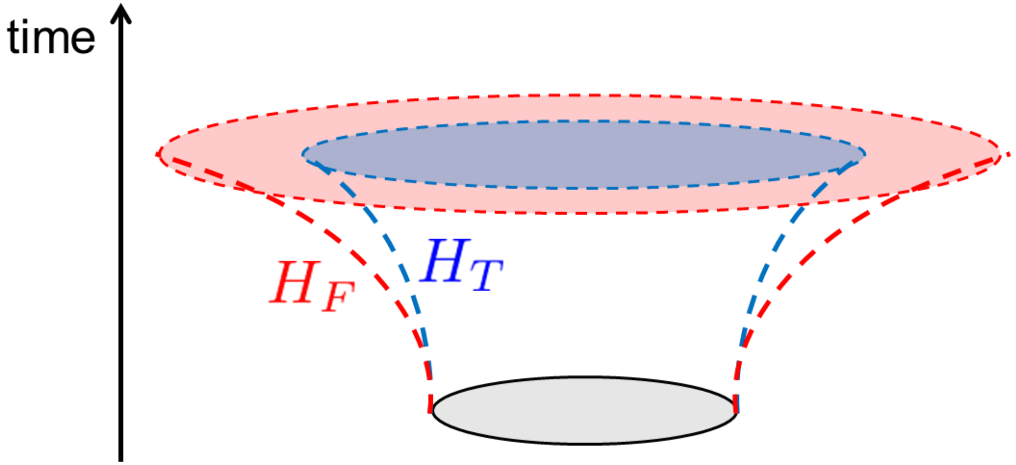
Collaborator: Asuka Ito, Jiro Soda

On-going work



Introduction

Quantum superposition of spacetime

“Does gravitational field superpose when the mass is in quantum superposition?”

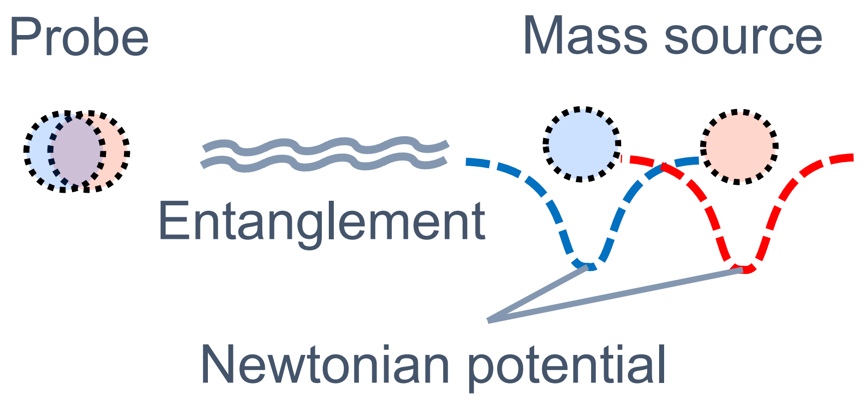
by Feynman

Non-relativistic ⟷ Relativistic

BMV proposals

Bose+ (2017), Marletto, Vedral (2017)

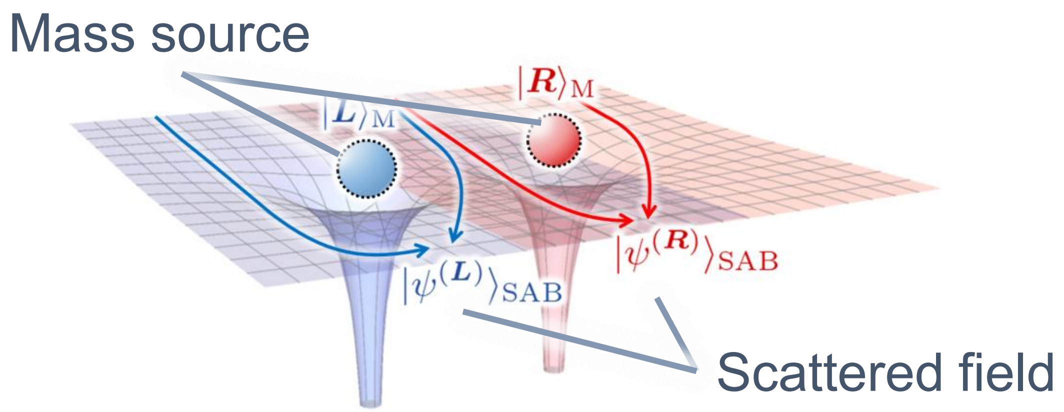
Experimental proposal
What happens if Newtonian gravity is induced by superposed mass?



Spacetime cat state

Foo+ (2020), Arrasmis+ (2019), YK+ (2025)

Gedanken experiment
What happens if background spacetime is in quantum superposition?



Quantum superposition of spacetime

"Does gravitational field superpose when the mass is in quantum superposition?"

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Non-relativistic ⟷ Relativistic

Our interest

BMV proposals

Bose+ (2017), Marletto, Vedral (2017)

Experimental proposal
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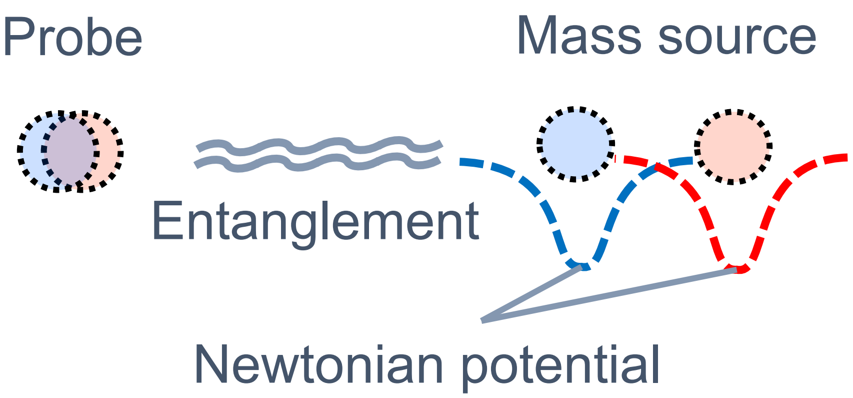

Spacetime cat state

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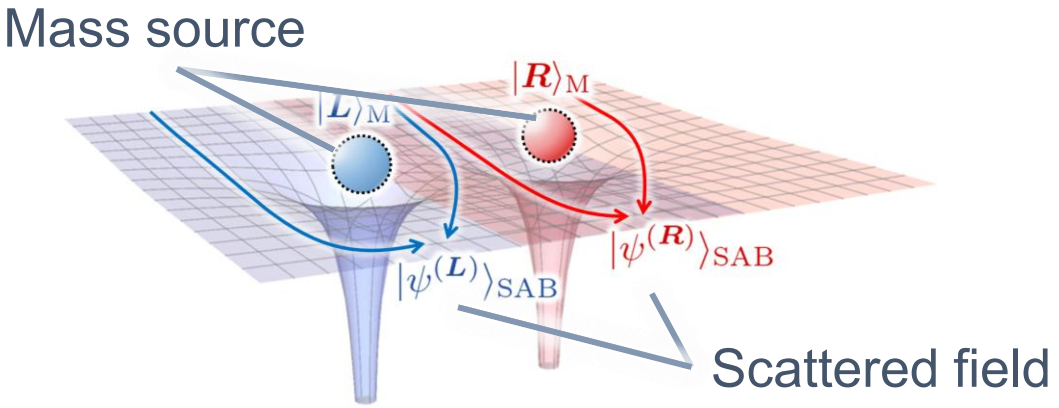
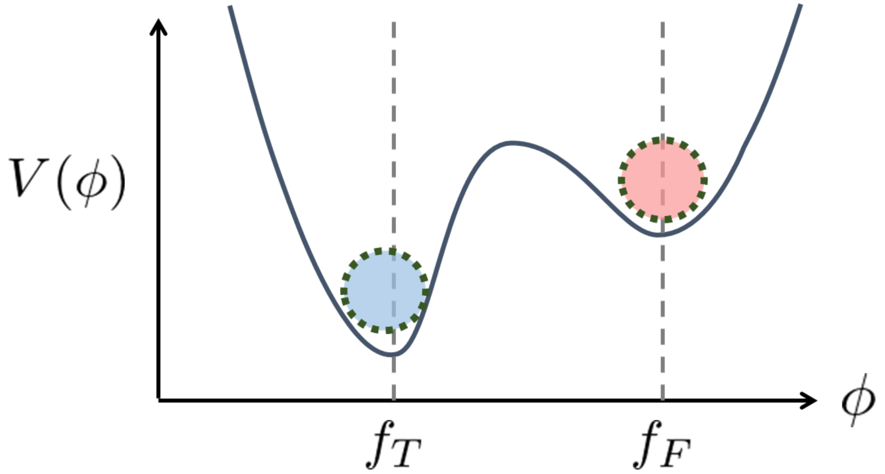
What happens if background spacetime is in quantum superposition?



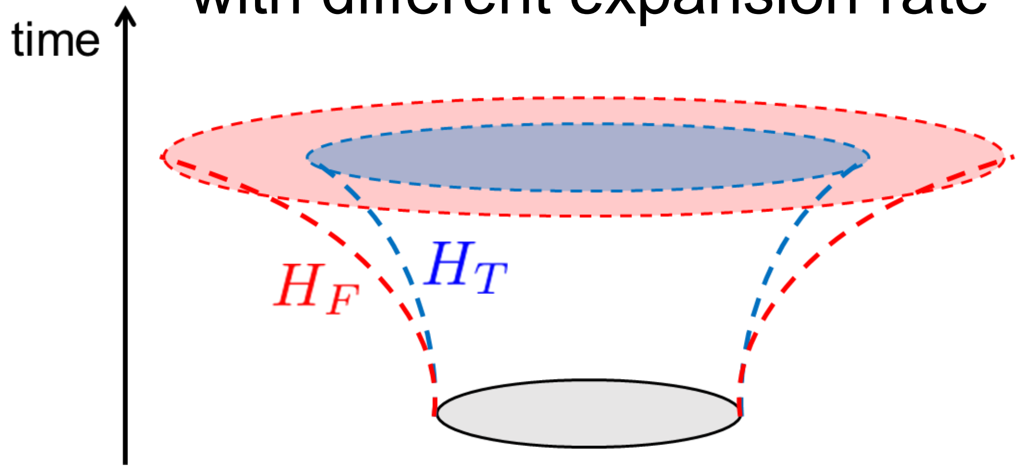
Our motivation

- Let's consider superposed spacetime in the early universe!

Superposition of matter with different energy



Superposition of FLRW universe with different expansion rate

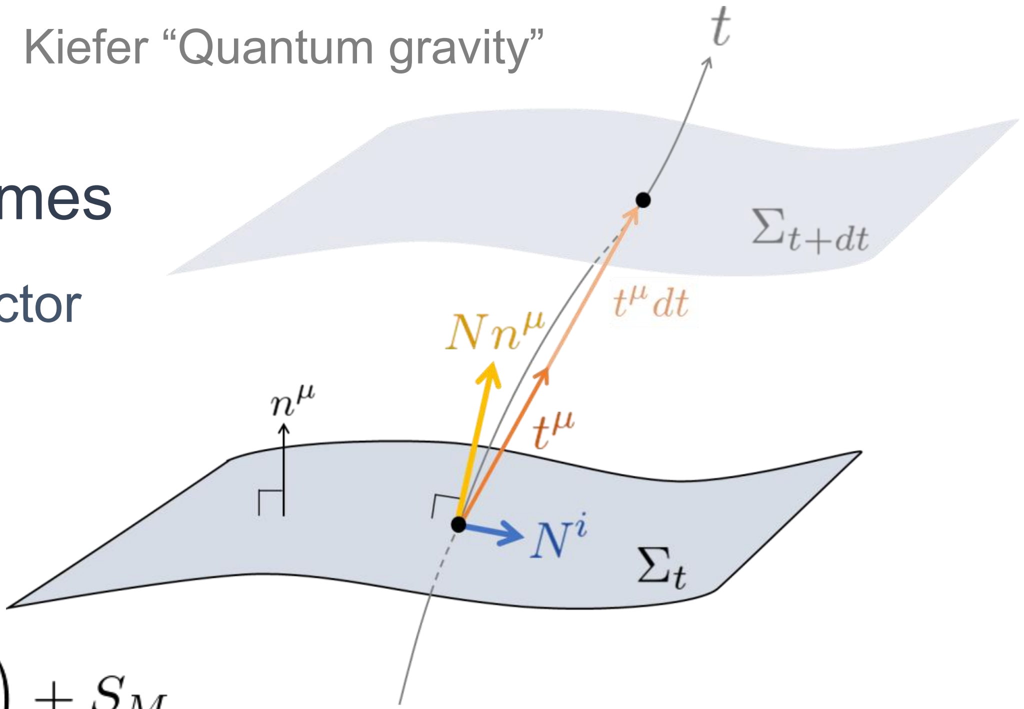


- How can we explicitly write down this cosmic cat state?
  - ▸ Solution of Wheeler-DeWitt equation
- How does this cosmic cat state decohere?
  - ▸ Decoherence due to inhomogeneous matter field fluctuation

Wheeler-DeWitt equation

: Framework to describe superposed spacetimes

Kiefer "Quantum gravity"



- 3+1 decomposition
  - Time evolution vector: $t^\mu = \underbrace{Nn^\mu}_{\text{Lapse function}} + \underbrace{N^\mu}_{\text{Shift vector}}$
  - Metric: $g_{\mu\nu} = \begin{pmatrix} N_i N^i - N^2 & N_i \\ N_i & h_{ij} \end{pmatrix}$
  - Action: $S = \int d^4x \frac{N\sqrt{h}}{16\pi G}\left({}^{(3)}R - K^2 + K_{ij}K^{ij}\right) + S_M$

Independent on $\dot{N}$ $\rightarrow$ $p_N = 0$: Primary constraint

We get secondary constraint by requiring $p_N = 0$ to be preserved in time:

$0 \approx \{p_N, H\} =: H_\perp$: Hamiltonian constraint

Quantize

$H_\perp \Psi(h_{ij}, \phi) \approx 0$: **Wheeler-DeWitt equation**

Constraint that the joint state of gravity& matter should satisfy

(rather than a time evolution of each subsystem)

A Cosmic Cat State and Decoherence in Early Universe

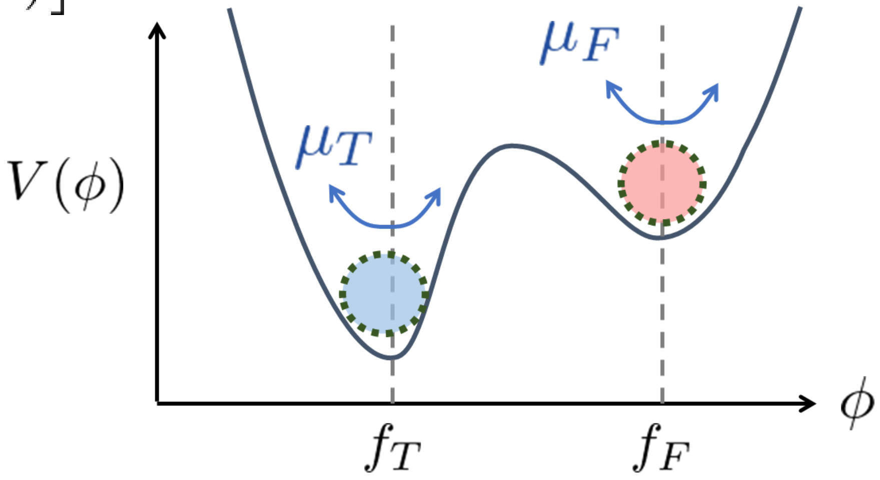
Setup

- Spatially flat FLRW universe $a(t)$ + scalar field $\phi(t,\boldsymbol{x}) = \bar{\phi}(t) + \delta\phi(t,\boldsymbol{x})$

$$ds^2 = -N(t)^2 dt^2 + a(t)^2\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$V(\phi) \simeq \begin{cases} \frac{\mu_F}{2}(\phi - f_F)^2 + V(f_F) & (\phi \simeq f_F) \\ \frac{\mu_T}{2}(\phi - f_T)^2 + V(f_T) & (\phi \simeq f_T) \end{cases}$$

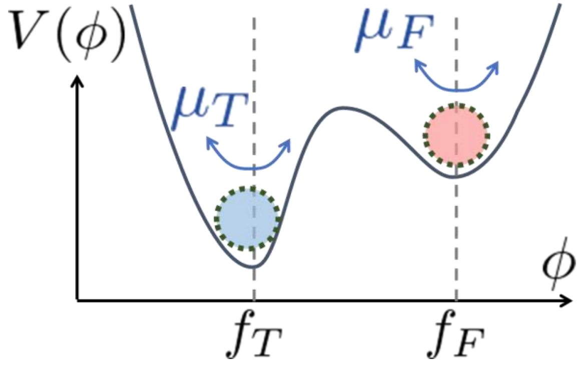


Superposed scalar field at f_T and f_F ▸ Superposed FLRW universe

- – Probe: background fields a and $\bar{\phi}$
- – Environment: inhomogeneous scalar field $\delta\phi$

How does the superposed FLRW universe decohere due to $\delta\phi$?

WDW equation



- Physical variables

$$a(t)\,,\quad \phi(t,\boldsymbol{x}) = \bar{\phi}(t) + \delta\phi(t,\boldsymbol{x}) \longleftrightarrow \phi_{\boldsymbol{k}}$$

Fourier trf

- WDW equation

$$H_\perp \Psi(a,\bar{\phi},\{\phi_{\boldsymbol{k}}\}) = \left[\frac{2\pi G}{3va^3}\left(a\frac{\delta}{\delta a}\right)^2 + \frac{va^3}{8\pi G}\Lambda + \mathcal{H}_{\bar{\phi}} + \mathcal{H}_{\delta\phi}\right]\Psi(a,\bar{\phi},\{\phi_{\boldsymbol{k}}\}) \approx 0$$

Hamiltonian for $\bar{\phi}$: $\mathcal{H}_{\bar{\phi}} = -\frac{1}{2va^3}\left(\frac{\delta}{\delta\bar{\phi}}\right)^2 + va^3\left\{\frac{1}{2}\mu_{T/F}^2(\bar{\phi} - f_{T/F})^2 + V(f_{T/F})\right\}$

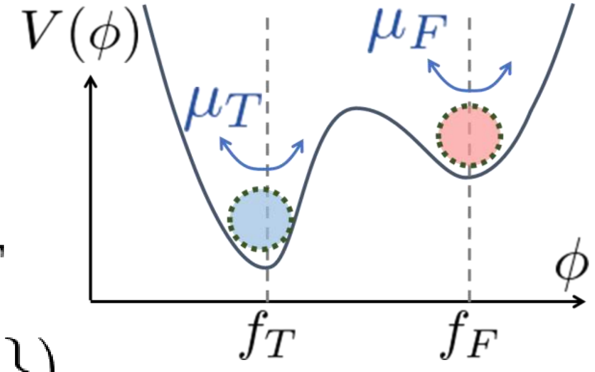
Hamiltonian for $\delta\phi$: $\mathcal{H}_{\delta\phi} = \int d^3k\left[-\frac{1}{2a^3}\left|\frac{\delta}{\delta\phi_{\boldsymbol{k}}}\right|^2 + \frac{a^3}{2}\left(\frac{k^2}{a^2} + \mu_{T/F}^2\right)|\phi_{\boldsymbol{k}}|^2\right]$

*Mass difference *Vacuum energy difference of scalar field

Using WKB expansion for $G \ll 1$, we obtain Gaussian wave function @ $\bar{\phi} \simeq f_{T/F}$

Cosmic cat state solution

- Superposition of two different WDW solution around $\bar{\phi} \simeq f_{T/F}$



$$\Psi(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) \propto \sqrt{p}\,\Psi_T(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) + \sqrt{1-p}\,\Psi_F(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\})$$

where, for $j = T, F$

$$\Psi_j(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) \propto \exp\left[-\underbrace{\frac{iv}{4\pi G}H_j a^3}_{\text{Leading WKB solution of } a} - \underbrace{\frac{va^3}{2}\bar{\kappa}_j(\bar{\phi} - f_j)^2}_{\text{Gaussian of } \bar{\phi}} - \underbrace{\int d^3k\, \frac{a^3}{2}\kappa_{j,k}|\phi_k|^2}_{\text{Gaussian of } \phi_k}\right]$$

$$\bar{\kappa}_j = H_j\left(\nu_{I,j} + \frac{3}{2}i\right) \qquad \kappa_{j,k} = H_j\left(-ik\tau_j\frac{H^{(2)\prime}_{\nu_j}(-k\tau_j)}{H^{(2)}_{\nu_j}(-k\tau_j)} + \frac{3}{2}i\right)$$

: Inverse of Gaussian variance

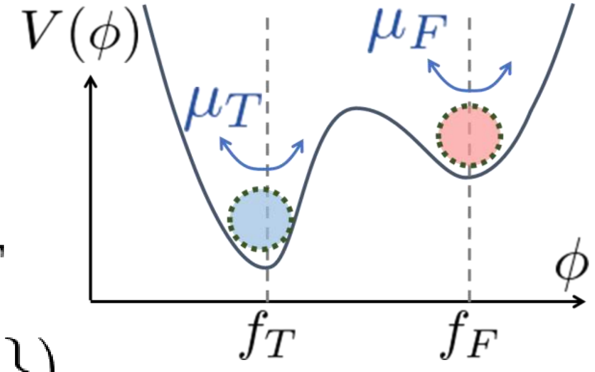
$$H_j = \sqrt{\frac{\Lambda + 8\pi G V(f_j)}{3}}$$

*Vacuum energy difference

$$\nu_{I,j} = \sqrt{\frac{\mu_j^2}{H_j^2} - \frac{9}{4}}$$

*Mass difference

Cosmic cat state solution



- Superposition of two different WDW solution around $\bar{\phi} \simeq f_{T/F}$

$$\Psi(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) \propto \sqrt{p}\,\Psi_T(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) + \sqrt{1-p}\,\Psi_F(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\})$$

where, for $j = T, F$

$$\Psi_j(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) \propto \exp\left[-\frac{iv}{4\pi G} H_j a^3 - \frac{va^3}{2}\bar{\kappa}_j(\bar{\phi} - f_j)^2 - \int d^3k\, \frac{a^3}{2}\kappa_{j,k}|\phi_k|^2\right]$$

Leading WKB solution of a — Gaussian of $\bar{\phi}$ — Gaussian of ϕ_k

$$\bar{\kappa}_j = H_j\left(\nu_{I,j} + \frac{3}{2}i\right) \qquad \kappa_{j,k} = H_j\left(-ik\tau_j \frac{H^{(2)\prime}_{\nu_j}(-k\tau_j)}{H^{(2)}_{\nu_j}(-k\tau_j)} + \frac{3}{2}i\right)$$

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*Vacuum energy difference

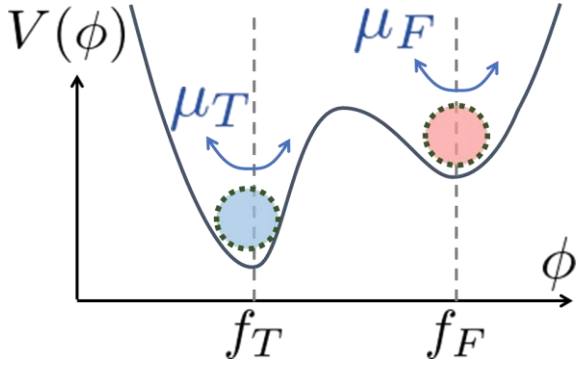
$$\nu_{I,j} = \sqrt{\frac{\mu_j^2}{H_j^2} - \frac{9}{4}}$$

*Mass difference

$H_T - H_F$ or $\nu_T - \nu_F$ indicates the separation of cosmic cat state.

These difference appears in the Gaussian variance.

Decoherence



- Reduced density matrix by tracing out environmental noise ϕ_k

$$\rho_{\rm bg}(a,\bar\phi|a',\bar\phi') := \int \left(\prod_k \mathcal{D}\phi_k\right) \Psi^*(a',\bar\phi',\{\phi_{\boldsymbol k}\})\,\Psi(a,\bar\phi,\{\phi_{\boldsymbol k}\})$$

$$\propto p\rho^{TT}(a,\bar\phi|a',\bar\phi') + (1-p)\rho^{FF}(a,\bar\phi|a',\bar\phi') + \sqrt{p(1-p)}\left\{\rho^{TF}(a,\bar\phi|a',\bar\phi') + (T\leftrightarrow F)\right\}$$

Let's focus on this term, which is unique to cosmic cat state

- Interference visibility and Coherence "length"

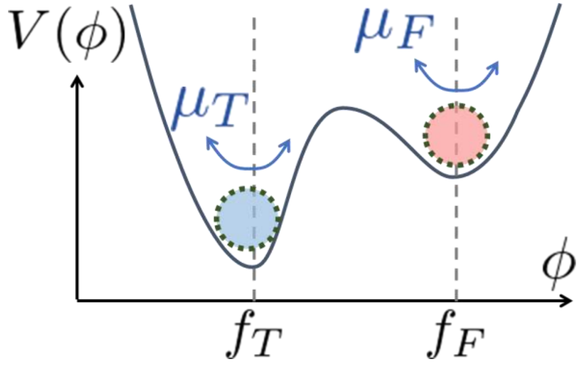
$$\mathcal{V}(a|a') := |\rho^{TF}(a,\bar\phi|a',\bar\phi')| \simeq \prod_k \exp\left[-\left(\frac{\Delta a}{\ell_k}\right)^2\right], \qquad \Delta a := \frac{a-a'}{a+a'}$$

Envelope of interference between a and a'

Coherence length: how far coherence persists in Δa

Let's see how the coherence length ℓ_k changes with respect to $\Delta H := \dfrac{H_T - H_F}{H_T + H_F}$

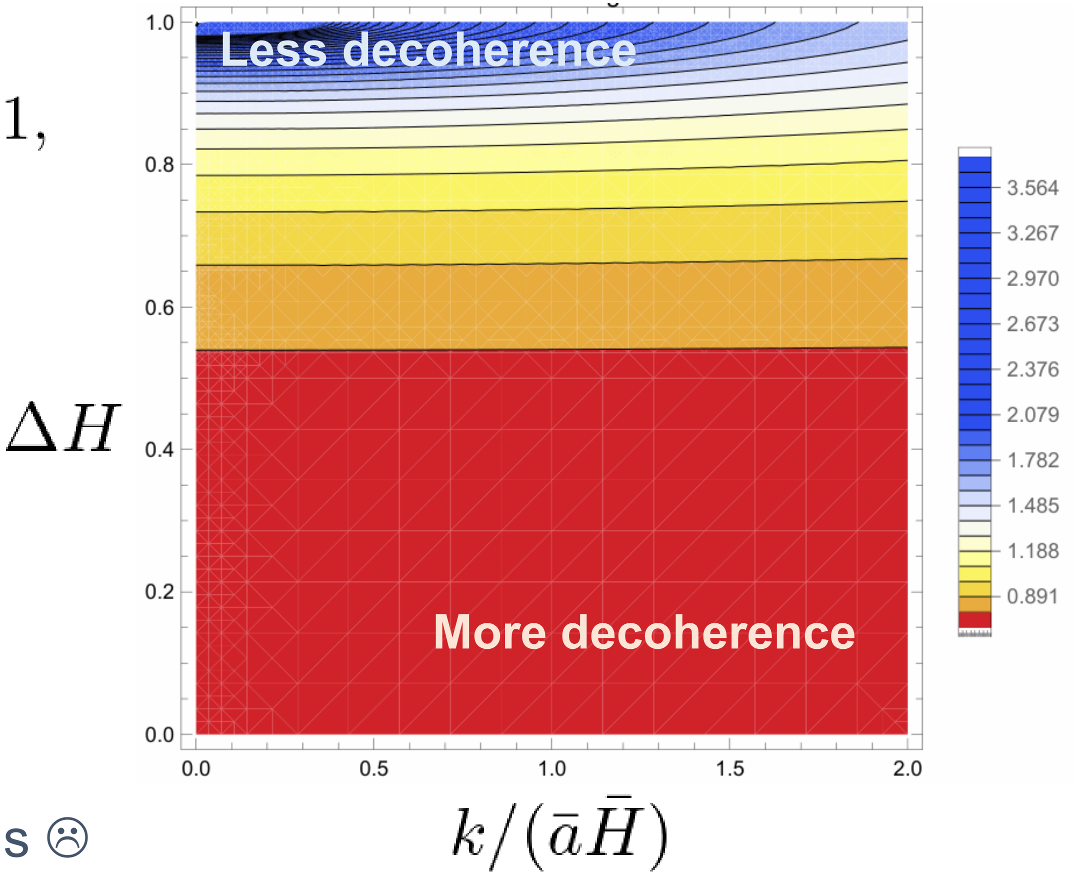
Results



- Coherence length $\bar{a} = 0.5,\ \bar{H} = 10,\ \bar{\nu_I} = 10,\ \bar{H}t = -1,\ \Delta\nu_I = 0$

$$\ell_k := \sqrt{2}\left(\frac{\partial^2 \log \mathcal{V}}{\partial(\Delta a)^2}\right)^{-1/2}_{\Delta a=0}$$

$$\simeq \begin{cases} \frac{2}{3}\left(1 + \frac{\Delta H^2}{2}\right) & \text{for} \quad |k\tau| \ll 1,\ \frac{\mu}{H} \gg 1, \\ \frac{2}{3}\left\{1 + \frac{(Ht)^2 \Delta H^2}{2}\right\} & \text{for} \quad |k\tau| \gg 1 \end{cases}$$



Decoherence relaxes as the cosmic cat state separation ΔH grows.

- Environment $\delta\phi$ becomes less sensitive to Δa if there is cosmic cat state.

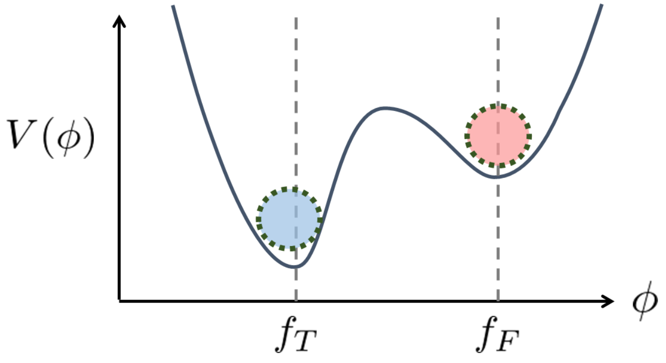
We still don't have clear physical understanding for this ☹

Summary

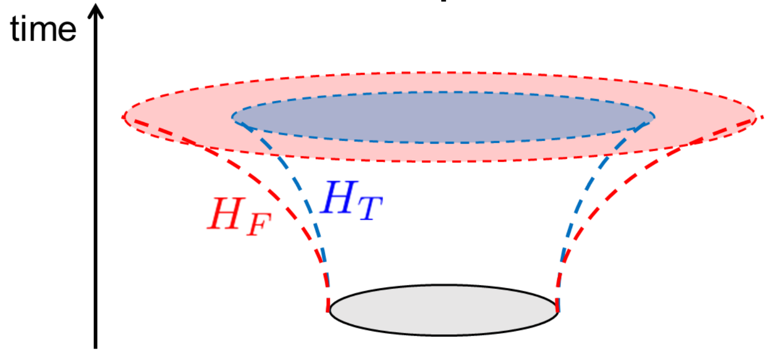
Summary

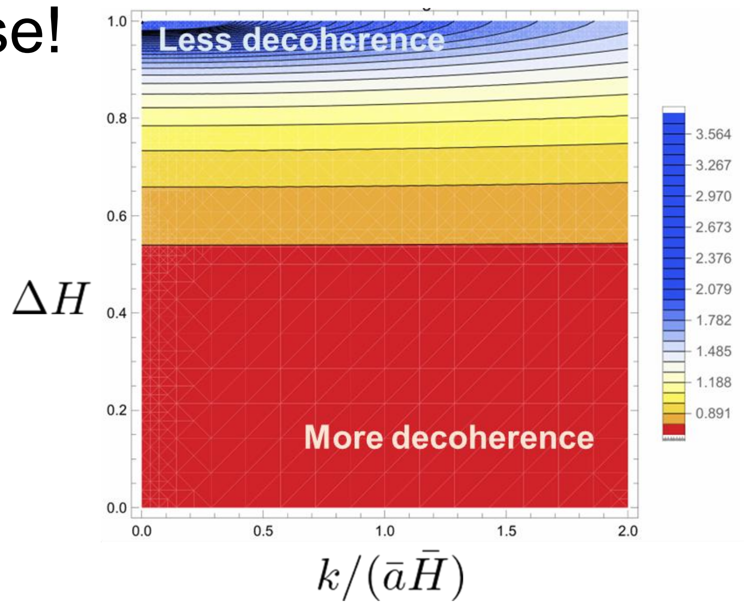
- Let's consider superposed spacetime in the early universe!

Superposition of matter with different energy



Superposition of FLRW universe with different expansion rate



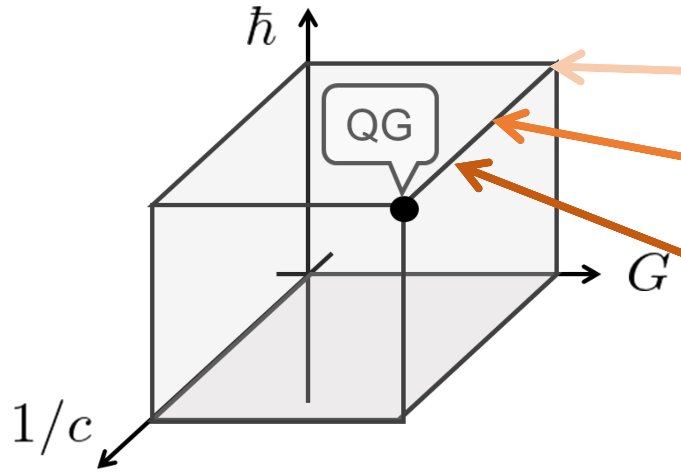


- How does this cosmic cat state decohere due to inhom. scalar field?
  - As the cosmic cat state separation grows, decoherence relaxes.
- Future work
  - Superposed WDW solutions for BH spacetime
  - Gravity-induced entangled state in WDW solution

Backup

Works inspired by BMV proposals

Relativistic extension of BMV-like proposals



Newton limit (More feasible)

Quantum aspects of Newtonian potential: BMV proposals

Quantum aspects of non-dynamical curved spacetime

YK, Maeda, Nambu, Osawa (2022), Zych, Mann (2022)

Quantum aspects of dynamical gravity (GW, gravitons)

Kanno, Soda, Tokuda (2021), Mehdi, Hope, Haine (2023)

Relativistic (Less feasible)

- Motivations

1. Understand quantum feature unique to GR phenomena by exceeding Newtonian limit
2. Fill a gap between BMV-like proposal and quantum gravity theory

Today's talk: **Gravitational lensing & Einstein ring image in superposed curved spacetime**

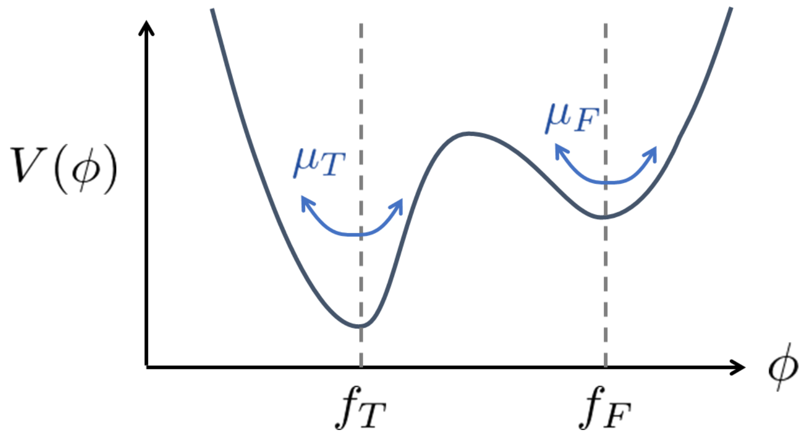
Setup & Abstract

- Spatially flat FLRW universe $a(t)$ + scalar field $\phi(t,\boldsymbol{x}) = \bar{\phi}(t) + \delta\phi(t,\boldsymbol{x})$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2 dt^2 + a(t)^2\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

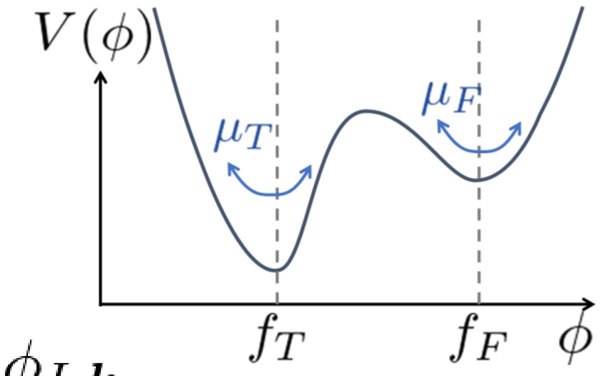
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$V(\phi) \simeq \begin{cases} \frac{\mu_F}{2}(\phi - f_F)^2 + V(f_F) & (\phi \simeq f_F) \\ \frac{\mu_T}{2}(\phi - f_T)^2 + V(f_T) & (\phi \simeq f_T) \end{cases}$$



- What to do?
  1. The WdW equation for this setup
  2. WKB approximation ($G \ll 1$) to solve WdW equation
  3. Gaussian wavefunction of the true and false vacuum states
  4. Reduced density matrix of the FLRW universe by tracing out $\phi(t,\boldsymbol{x})$
     - How does the superposed universe decohere due to scalar field coupling?

WdW equation



- Dynamical variables:

$$a(t)\,,\quad \phi(t,\boldsymbol{x}) = \bar{\phi}(t) + \underline{\delta\phi(t,\boldsymbol{x})} \longleftrightarrow \underline{\phi_{R,\boldsymbol{k}} + i\,\phi_{I,\boldsymbol{k}}}$$

Fourier trf

- WdW equation

$$H_\perp \Psi(a,\bar{\phi},\{\phi_{\lambda,\boldsymbol{k}}\}) = \left[\frac{2\pi G}{3va^3}\left(a\frac{\delta}{\delta a}\right)^2 + \frac{va^3}{8\pi G}\Lambda + \mathcal{H}_{\bar{\phi}} + \mathcal{H}_{\delta\phi}\right]\Psi(a,\bar{\phi},\{\phi_{\lambda,\boldsymbol{k}}\}) \approx 0$$

where

$$\mathcal{H}_{\bar{\phi}} = -\frac{1}{2va^3}\left(\frac{\delta}{\delta\bar{\phi}}\right)^2 + va^3\left\{\underline{\frac{1}{2}\mu_{T/F}^2(\bar{\phi} - f_{T/F})^2 + V(f_{T/F})}\right\} \quad \text{at } \bar{\phi}(t) \simeq f_{T/F}$$

Quadratic potential around each potential minimum

$$\mathcal{H}_{\delta\phi} = \sum_{\lambda=R,I}\int_{\frac{1}{2}} d^3k\left[-\frac{1}{2a^3}\left(\frac{\delta}{\delta\phi_{\lambda,\boldsymbol{k}}}\right)^2 + \underline{\frac{a^3}{2}\left(\frac{k^2}{a^2} + \mu^2\right)\phi_{\lambda,\boldsymbol{k}}^2}\right]$$

Scalar field perturbation around each potential minimum

WKB approximation

- WKB approximation with expansion parameter G to solve WdW equation

$$\Psi = e^{i(G^{-1}S_0 + S_1 + GS_2 + \cdots)}$$

$$H_\perp \Psi = \left[\frac{2\pi G}{3va^3}\left(a\frac{\delta}{\delta a}\right)^2 + \frac{va^3}{8\pi G}\Lambda + \mathcal{H}_{\bar{\phi}} + \mathcal{H}_{\delta\phi}\right] \approx 0$$

  - Order G^{-2} : $\dfrac{\delta S_0}{\delta \phi_{\lambda,\boldsymbol{k}}} = 0$
  - Order G^{-1} : $\left(\dfrac{\delta S_0}{\delta a}\right)^2 = \left(\dfrac{\sqrt{3\Lambda}v}{4\pi}a^2\right)^2$

  We obtain S_0

  - Order G^0 : Separation of variables as $e^{iS_1} = \bar{\chi}(a, \bar{\phi})\,\chi(a, \{\phi_{\lambda,\boldsymbol{k}}\})/\mathcal{D}(a)$

    ▶ $$\left[-2a\frac{\delta S_0}{\delta a}\frac{\delta}{\delta a} + \left(\frac{\delta S_0}{\delta a} + a\frac{\delta^2 S_0}{\delta a^2}\right)\right]\mathcal{D} = 0$$

    $$\left[\frac{4\pi i}{3va}\frac{\delta S_0}{\delta a}\frac{\delta}{\delta a} + \mathcal{H}_{\bar{\phi}}\right]\bar{\chi} = 0, \qquad \left[\frac{4\pi i}{3va}\frac{\delta S_0}{\delta a}\frac{\delta}{\delta a} + \mathcal{H}_{\delta\phi}\right]\chi = 0$$

WKB approximation

- WKB approximation with expansion parameter G to solve WdW equation

$$\Psi = e^{i(G^{-1}S_0+S_1+GS_2+\cdots)}$$

$$H_\perp \Psi = \left[\frac{2\pi G}{3va^3}\left(a\frac{\delta}{\delta a}\right)^2 + \frac{va^3}{8\pi G}\Lambda + \mathcal{H}_{\bar{\phi}} + \mathcal{H}_{\delta\phi}\right] \approx 0$$

By using $p_a = \dfrac{1}{G}\dfrac{\delta S_0}{\delta a}$

  - Order G^{-2} : $\dfrac{\delta S_0}{\delta \phi_{\lambda,\boldsymbol{k}}} = 0$
  - Order G^{-1} : $\left(\dfrac{\delta S_0}{\delta a}\right)^2 = \left(\dfrac{\sqrt{3\Lambda}v}{4\pi}a^2\right)^2$
  - Order G^0 : Separation of variables as $e^{iS_1} = \bar{\chi}(a,\bar{\phi})\,\chi(a,\{\phi_{\lambda,\boldsymbol{k}}\})/\mathcal{D}(a)$
    - ▸ $\left[-2a\dfrac{\delta S_0}{\delta a}\dfrac{\delta}{\delta a} + \left(\dfrac{\delta S_0}{\delta a} + a\dfrac{\delta^2 S_0}{\delta a^2}\right)\right]\mathcal{D} = 0$

$$\left[\frac{4\pi i}{3va}\frac{\delta S_0}{\delta a}\frac{\delta}{\delta a} + \mathcal{H}_{\bar{\phi}}\right]\bar{\chi} = 0, \qquad \left[\frac{4\pi i}{3va}\frac{\delta S_0}{\delta a}\frac{\delta}{\delta a} + \mathcal{H}_{\delta\phi}\right]\chi = 0$$

WKB approximation

- WKB approximation with expansion parameter G to solve WdW equation

$$\Psi = e^{i(G^{-1}S_0+S_1+GS_2+\cdots)}$$

$$H_\perp \Psi = \left[\frac{2\pi G}{3va^3}\left(a\frac{\delta}{\delta a}\right)^2 + \frac{va^3}{8\pi G}\Lambda + \mathcal{H}_{\bar{\phi}} + \mathcal{H}_{\delta\phi}\right] \approx 0$$

- Order G^{-2} : $\dfrac{\delta S_0}{\delta \phi_{\lambda,\boldsymbol{k}}} = 0$
- Order G^{-1} : $\left(\dfrac{1}{N}\dfrac{\dot{a}}{a}\right)^2 = \dfrac{\Lambda}{3}$ → Friedmann eq.

By using $p_a = \dfrac{1}{G}\dfrac{\delta S_0}{\delta a}$

- Order G^0 : Separation of variables as $e^{iS_1} = \bar{\chi}(a,\bar{\phi})\,\chi(a,\{\phi_{\lambda,\boldsymbol{k}}\})/\mathcal{D}(a)$
  - ▸ $\left[-2a\dfrac{\delta S_0}{\delta a}\dfrac{\delta}{\delta a} + \left(\dfrac{\delta S_0}{\delta a} + a\dfrac{\delta^2 S_0}{\delta a^2}\right)\right]\mathcal{D} = 0$

$$\frac{i}{N}\frac{d}{dt}\bar{\chi} = \mathcal{H}_{\bar{\phi}}\,\bar{\chi}, \qquad \frac{i}{N}\frac{d}{dt}\chi = \mathcal{H}_{\delta\phi}\,\chi$$ → Schrödinger eq.

Gaussian ansatz for scalar field
▸ Obtain S_1

Gaussian wavefunction of true and false vacua

- Gaussian wavefunction of vacuum states in each potential minimum ($J = T, F$)

Phase shift due to different vacuum energy

$$\Psi_J \sim \exp\left[-\frac{iv}{4\pi G}\sqrt{\frac{\Lambda_J}{3}}a^3 - \frac{1}{4}\log\left(\frac{\pi}{va^3\mathrm{Re}[\bar{\sigma}_J]}\right) - \frac{va^3}{2}\bar{\sigma}_J(\bar{\phi} - f_J)^2 \right.$$

Gaussian at each potential minimum

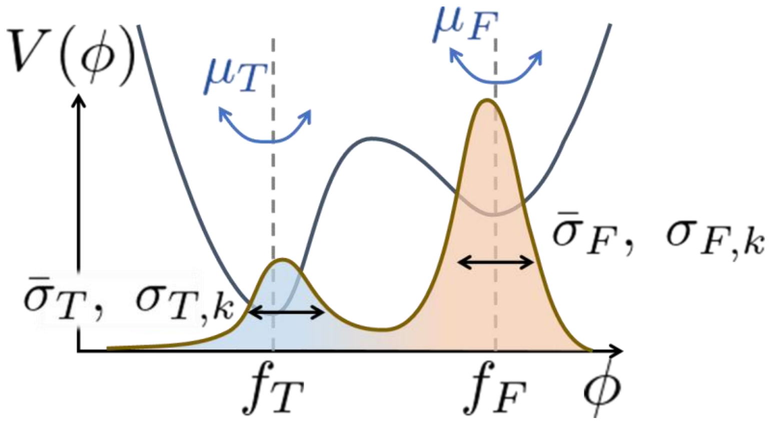
$$\left. - \int d^3k \left\{\frac{1}{4}\log\left(\frac{\pi}{a^3\mathrm{Re}[\sigma_{J,k}]}\right) + \frac{a^3}{2}\sigma_{J,k}|\phi_k|^2\right\}\right]$$

Gaussian of scalar field pert.

where

$$\bar{\sigma}_J = iH_J\left(-\nu_J\tanh[H_J\nu_J(t-t_0)] + \frac{3}{2}\right)$$

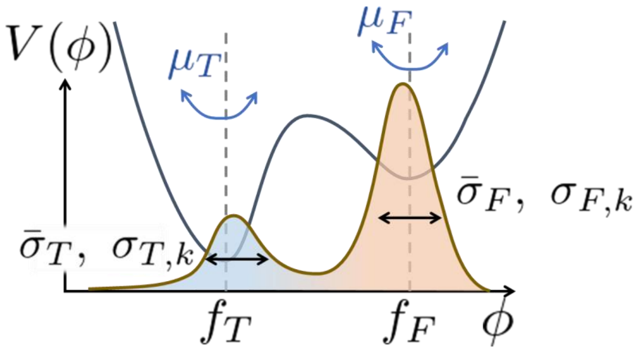
$$\sigma_{J,k} = iH_J\left(k\tau_J\frac{H_{\nu_J}^{(2)\prime}(k\tau_J)}{H_{\nu_J}^{(2)}(k\tau_J)} + \frac{3}{2}\right) \quad \nu_J = \sqrt{\frac{9}{4} - \frac{\mu_J^2}{H_J^2}}, \quad \tau_J = -\frac{e^{-H_J t}}{H_J}$$



- Quantum superposition of true + false vacuum states

$$\Psi(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) = n_{\mathrm{cat}}\left(\sqrt{p}\,\Psi_T(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) + \sqrt{1-p}\,\Psi_F(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\})\right)$$

Decoherence of true + false vacua



- The off-diagonal element of density matrix for FLRW metric

$$\rho_{aa'} = \int \mathcal{D}\bar{\phi} \int \mathcal{D}(\delta\phi)\ \Psi^*(a', \bar{\phi}, \{\phi_{\boldsymbol{k}}\})\Psi(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\})$$

$$= n_{\rm cat}(a) n_{\rm cat}(a') \left[p\ \rho^{TT}_{aa'} + (1-p)\rho^{FF}_{aa'} + \sqrt{p(1-p)}\left(\rho^{TF}_{aa'} + \rho^{FT}_{aa'}\right)\right]$$

Quantum coherence of true + false vacuum states

$$\rho^{IJ}_{aa'} = \int \mathcal{D}\bar{\phi} \int \mathcal{D}(\delta\phi)\ \Psi^*_J(a', \bar{\phi}, \{\phi_{\boldsymbol{k}}\})\Psi_I(a, \bar{\phi}, \{\phi_{\boldsymbol{k}}\}) = \exp[-\gamma^{IJ}_{aa'} + i\omega^{IJ}_{aa'}]$$

Phase shift due to $\Lambda_T - \Lambda_F$ (1st term) and $f_T - f_F$ (2nd term):

$$\omega^{IJ}_{aa'} = -\frac{v}{4\pi G}\left(\sqrt{\frac{\Lambda_I}{3}}a^3 - \sqrt{\frac{\Lambda_J}{3}}a'^3\right) + \frac{iv}{2}\frac{(f_I - f_J)^2}{(a^3\bar{\sigma}_I)^{-1} + (a'^3\bar{\sigma}_J)^{-1}}$$

Decoherence of superposed FLRW universe due to $H_T - H_F$ and $\mu_T - \mu_F$:

$$\gamma^{IJ}_{aa'} = \frac{1}{4}\log\left(\frac{(a^3\bar{\sigma}_I + a'^3\bar{\sigma}^*_J)^2}{4a^3a'^3{\rm Re}[\bar{\sigma}_I]{\rm Re}[\bar{\sigma}_J]}\right) + \frac{1}{4}\int d^3k \log\left(\frac{(a^3\sigma_{I,k} + a'^3\sigma^*_{J,k})^2}{4a^3a'^3{\rm Re}[\sigma_{I,k}]{\rm Re}[\sigma_{J,k}]}\right)$$

Decoherence in Quantum mechanics

- Interferometry within environment noise in quantum mechanics
  - Probe + Env. $|\Psi\rangle = \sqrt{p}|\psi(s_1)\rangle|\mathcal{E}(s_1)\rangle + \sqrt{1-p}|\psi(s_2)\rangle|\mathcal{E}(s_2)\rangle$
  - Visibility of interference $\mathcal{V}(s_1, s_2) := \dfrac{|\langle\psi(s_1)|\Psi\rangle\langle\Psi|\psi(s_2)\rangle|}{|\langle\psi(s_1)|\Psi\rangle|\ |\langle\psi(s_2)|\Psi\rangle|} = |\langle\mathcal{E}(s_1)|\mathcal{E}(s_2)\rangle|$

  Suppose that environment changes its state with respect to s as

$$|\mathcal{E}(s)\rangle = e^{-is\hat{G}}|\mathcal{E}(0)\rangle$$

  - Visibility ▸ $\mathcal{V}(s_1, s_2) = \exp\left[-\dfrac{\mathrm{Var}[\hat{G}]\,\bar{s}^2}{2}(\Delta s)^2\right]$ $\quad \bar{s} = \dfrac{s_1+s_2}{2},\ \Delta s = \dfrac{s_1-s_2}{s_1+s_2}$
  - Coherence length

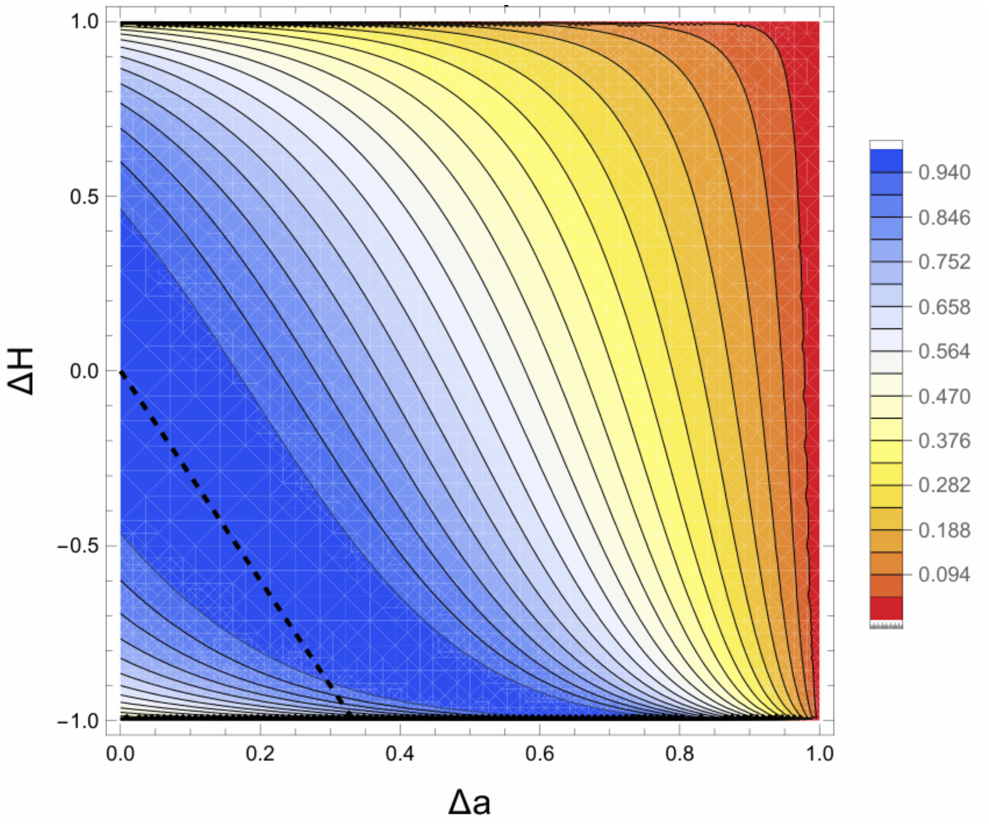
$$\ell_{\Delta s} := \sqrt{2}\left(-\frac{\partial^2 \log \mathcal{V}(s_1, s_2)}{\partial(\Delta s)^2}\right)^{-1/2}_{\Delta s=0} = \sqrt{\frac{2}{\mathrm{Var}[\hat{G}]\,\bar{s}^2}}$$

Results

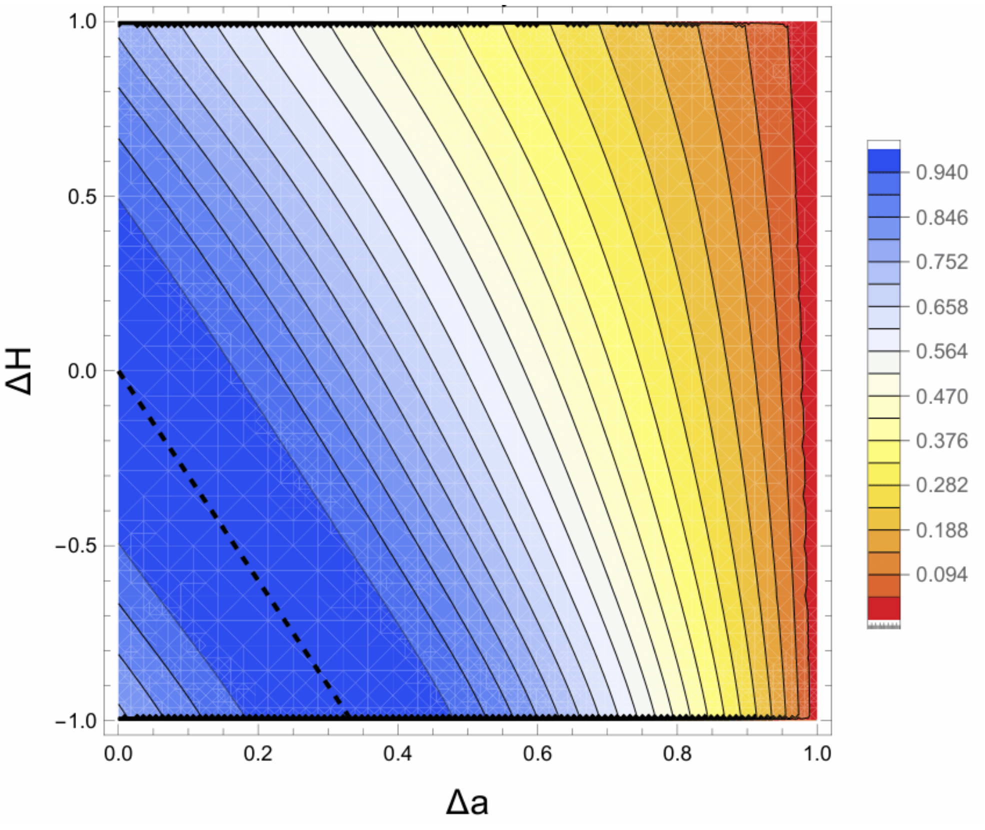
- Hubble difference ΔH dependence of Visibility

$$\mathcal{V} \simeq \begin{cases} \exp\left[-\frac{9}{4}\left\{\Delta a(1+(\Delta H+\Delta\nu_I))+\frac{\Delta H+\Delta\nu_I}{3}\Delta H\right\}\left\{\Delta a(1-(\Delta H+\Delta\nu_I))-\frac{Ht}{3}\Delta H\right\}+\cdots\right] & \text{for} \quad |k\tau|\ll 1,\ \mu/H\gg 1, \\ \exp\left[-\frac{9}{4}\left\{\Delta a(1+Ht\,\Delta H)-\frac{Ht}{3}\Delta H\right\}\left\{\Delta a(1-Ht\,\Delta H)-\frac{Ht}{3}\Delta H\right\}+\cdots\right] & \text{for} \quad |k\tau|\gg 1 \end{cases}$$

$k/(\bar{a}\bar{H}) = 0.2$



$k/(\bar{a}\bar{H}) = 10$

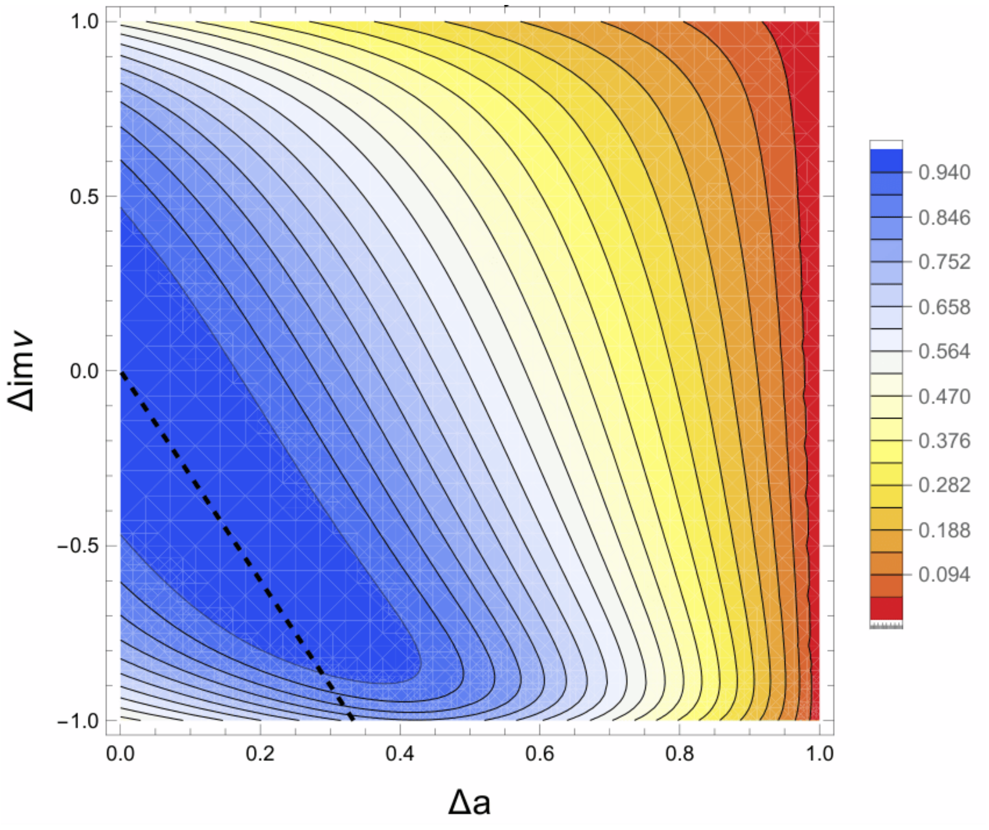


Results

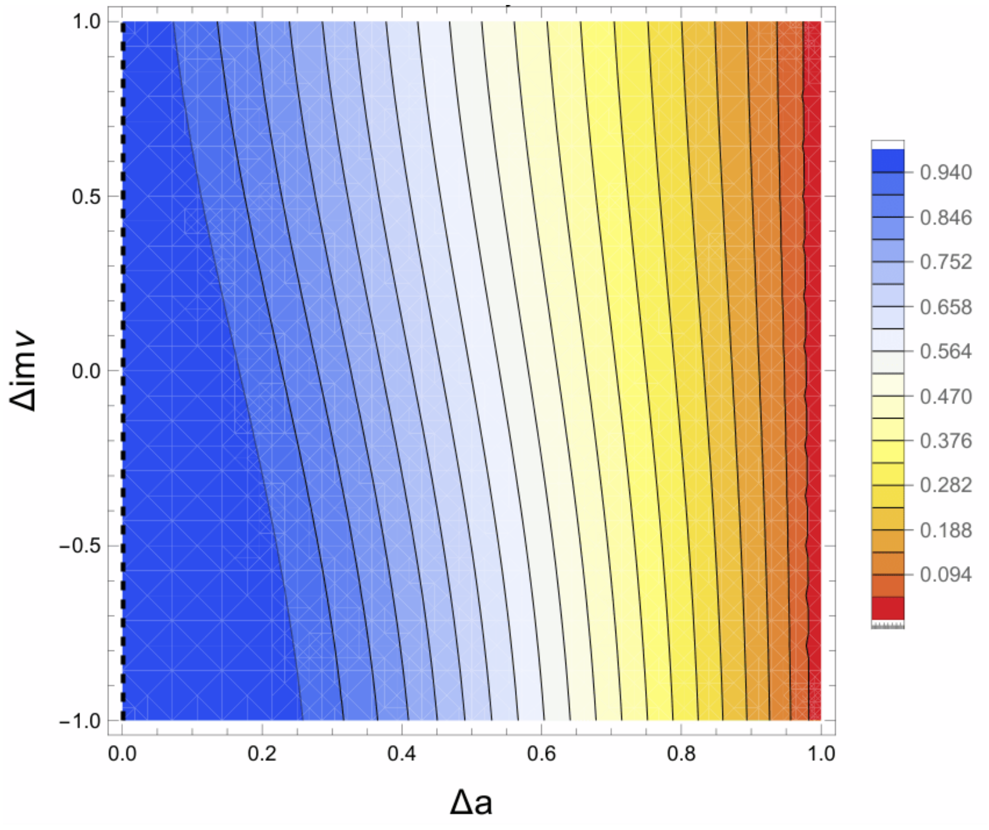
- Mass difference $\Delta\nu_I$ dependence of Visibility

$$\mathcal{V} \simeq \begin{cases} \exp\left[-\frac{9}{4}\left\{\Delta a(1+(\Delta H+\Delta\nu_I))+\frac{\Delta H+\Delta\nu_I}{3}\Delta H\right\}\left\{\Delta a(1-(\Delta H+\Delta\nu_I))-\frac{Ht}{3}\Delta H\right\}+\cdots\right] & \text{for} \quad |k\tau|\ll 1,\ \mu/H\gg 1, \\ \exp\left[-\frac{9}{4}\left\{\Delta a(1+Ht\,\Delta H)-\frac{Ht}{3}\Delta H\right\}\left\{\Delta a(1-Ht\,\Delta H)-\frac{Ht}{3}\Delta H\right\}+\cdots\right] & \text{for} \quad |k\tau|\gg 1 \end{cases}$$

$k/(\bar{a}\bar{H}) = 0.2$



$k/(\bar{a}\bar{H}) = 10$



Results

- Hubble, Mass difference dependence of Coherent length

$$\ell_k := \sqrt{2}\left(\frac{\partial^2 \log \mathcal{V}}{\partial(\Delta a)^2}\right)^{-1/2}_{\Delta a=0} \simeq \begin{cases} \frac{2}{3}\left\{1+\frac{(\Delta H+\Delta\nu_I)^2}{2}\right\} & \text{for} \quad |k\tau| \ll 1,\ \mu/H \gg 1, \\ \frac{2}{3}\left\{1+\frac{(Ht)^2\Delta H^2}{2}\right\} & \text{for} \quad |k\tau| \gg 1 \end{cases}$$