DE2008@KEK 12. Dec. 2008

Toward understanding the light propagation in clumpy universe

--- Perturbation theory of N point mass gravitational lens

Hideki Asada (Hirosaki Univ.)

Today's Menu

1. Intro:
   Obs in Clumpy Universe

2. Pertubation Theory:
   Arbitrary N
   point mass lens

3. Summary

§1. Introducion

cosmic observations

||

propagation of

light (+ν, CR, GW··)

through our clumpy univ.

(Long-standing) Problem

Obs. in FLRW univrse

Obs. in "averaged" universe

1) Both agree or not ?

2) If not,
what's difference?

(not a complete list,)

Cosmological Perturbation

Gravitational Instability

g=b.g.+h **δ**

Lifshitz (1946)

cosmological Newtonian

Nariai and Ueno (1960), Irvine (1965)

$$|\Phi| \ll 1,\ (v/c)^2 \ll 1,\ L/L_{\rm H} \ll 1$$

Progress of Theoretical Physics, Vol. 23, No. 2, February 1960

On a New Approach to Cosmology. II

——*The Problem of Local Gravitation*——

Hidekazu NARIAI and Yoshio UENO

Research Institute for Theoretical Physics, Hiroshima University
Takehara-shi, Hiroshima-ken

(Received October 8, 1959)

As a sequel to the previous paper, an attempt is made to develop a general method for attacking at the problem of local gravitational field due to such a large scale aggregation of matter that the effect of the cosmic expansion cannot be ignored. The formalism of this paper will provide us with a basis for treating the dynamical motion of galaxies within the Supergalaxy, together with the reexamination of the velocity-distance relation of galaxies.

Topic is modern still now!

Light propagation through inhomogeneity

Zeldovich (1964)

Dashevskii, Slysh (1964)

Kantowski (1969)

Dyer, Roeder (1972,73)

OBSERVATIONS IN A UNIVERSE HOMOGENEOUS IN THE MEAN

Ya. B. Zel'dovich

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 1,
pp. 19-24, January-February, 1964
Original article submitted June 12, 1963

A local nonuniformity of density due to the concentration of matter of the universe into separate galaxies produces a significant change in the angular dimensions and luminosity of distant objects as compared to the formulas for the Friedman model.

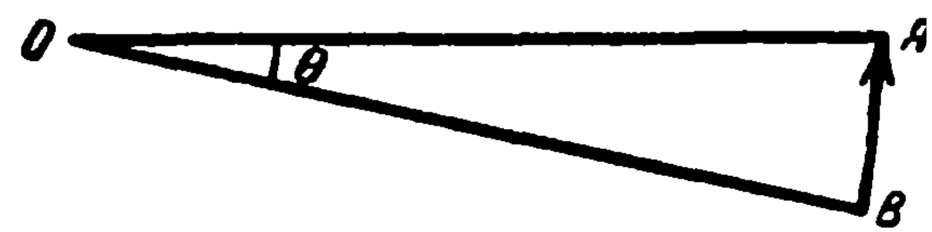


Fig. 1.

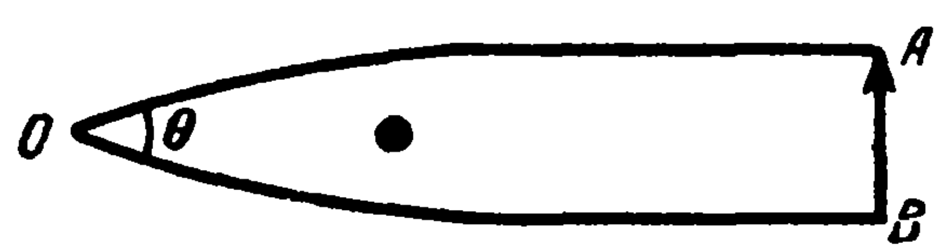


Fig. 2.

“Dyer-Roeder” distance

Dyer, Roeder (1972,73)

Raychaudhuri Eq

$$\frac{d\theta}{dv} = \frac{1}{2} k^{\mu}_{;\mu\nu} k^{\nu} = -\frac{1}{2}\mathcal{R} - (\theta^2 + \sigma^2)$$

Assumption:

1) R = α ρ_{FLRW} (clumpiness)

2) σ^2 = Negligible

$$\frac{d^2}{dw^2} D + \frac{3}{2} (1 + z)^5 \alpha \Omega D = 0$$

$$\frac{dz}{dw} = (1 + z)^2 \sqrt{\Omega z (1 + z)^2 - \lambda z (2 + z) + (1 + z)^2}$$

FLRW

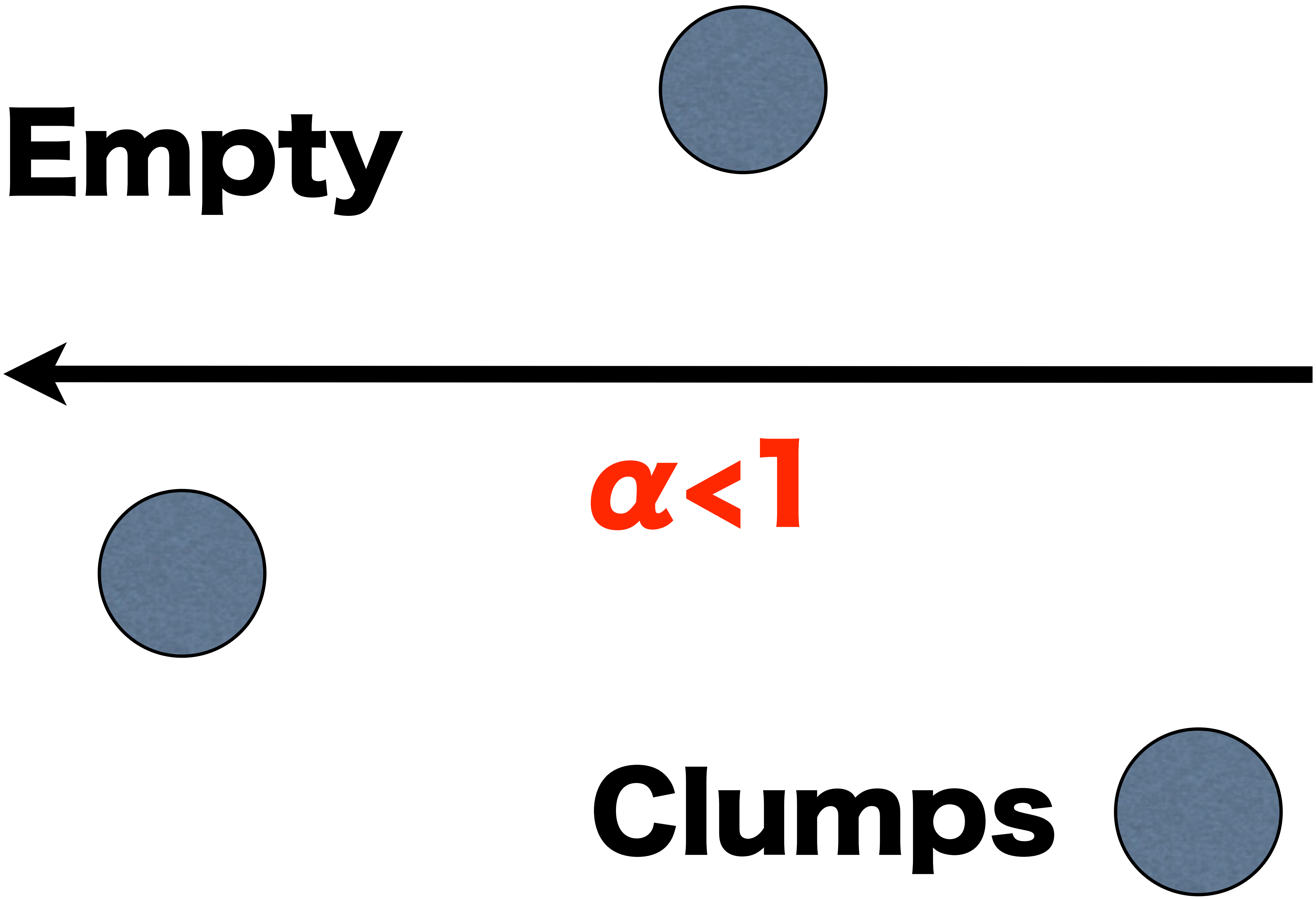
Homogeneous & Isotropic

←

α=1

feel **average density**

←


Empty
α<1
Clumps

Inequalities

in Observables

HA, ApJ **485**, 460 (1997); **501**, 473 (98)

Monotonicity:

Dyer, Roeder

$$D_{\mathrm{OL}}(\alpha_1) > D_{\mathrm{OL}}(\alpha_2)$$

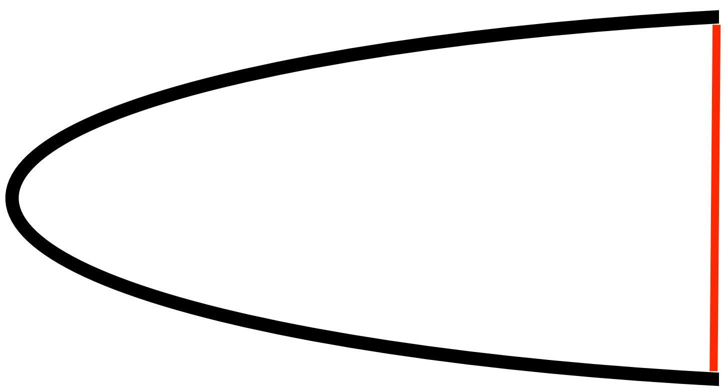
$$\text{for} \quad \alpha_1 < \alpha_2.$$

R=Source size fixed

smaller α

weaker Ricci focus

smaller ϕ



larger α

larger ϕ

$D_A = R/\phi$ decrease with α

THE ASTROPHYSICAL JOURNAL, 501:473–477, 1998 July 10

OBSERVATION OF GRAVITATIONAL LENSING IN THE CLUMPY UNIVERSE

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Received 1997 August 7; accepted 1998 February 17

ABSTRACT

We discuss how inhomogeneities of the universe affect observations of the gravitational lensing: (1) the bending angle, (2) the lensing statistics, and (3) the time delay. In order to take account of the inhomogeneities, the so-called Dyer-Roeder distance is used, which includes a parameter representing the clumpiness of the matter along the line of sight. It is shown analytically that all three combinations of distances appearing in the above observations, (1)–(3), are monotonic with respect to the clumpiness in general for any given set of the density parameter, cosmological constant, and redshifts of the lens and the source. Some implications of this result for the observations are presented; the clumpiness decreases both the bending angle and the lensing event rate, while it increases the time delay. We also discuss cosmological tests using the gravitational lensing in the clumpy universe.

1) Bending angle for $\alpha_1 < \alpha_2$

$$\frac{D_{\rm LS}}{D_{\rm OS}}(\alpha_1) < \frac{D_{\rm LS}}{D_{\rm OS}}(\alpha_2)$$

2) Lens statistics

$$\frac{D_{\rm OL}D_{\rm LS}}{D_{\rm OS}}(\alpha_1) < \frac{D_{\rm OL}D_{\rm LS}}{D_{\rm OS}}(\alpha_2)$$

3) Time delay

$$\frac{D_{\rm OL}D_{\rm OS}}{D_{\rm LS}}(\alpha_1) > \frac{D_{\rm OL}D_{\rm OS}}{D_{\rm LS}}(\alpha_2)$$

Monotonic in Lambda-term --- Competing with clumpliness

$$\frac{D_{\rm LS}}{D_{\rm OS}}(\lambda_1) < \frac{D_{\rm LS}}{D_{\rm OS}}(\lambda_2)$$

$$\frac{D_{\rm OL}D_{\rm LS}}{D_{\rm OS}}(\lambda_1) < \frac{D_{\rm OL}D_{\rm LS}}{D_{\rm OS}}(\lambda_2)$$

Effects by

Clumpiness

(Inhomogeneity)

and

Lamda term

(Dark energy)

FLRW distance is valid?

"Average"

"Yes"

e.g., Tomita, HA, Hamana (1999)

Numerical Simulation approaches

$\alpha=1$

if $z>1$

In reality ---

Need of 3D distribution

$$\alpha = \alpha(z, \theta, \phi)$$

Distances in Inhomogeneous Cosmological Models

Kenji Tomita,[1,*)] Hideki Asada[2,**)] and Takashi Hamana[3,***)]

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[3] *Astronomical Institute, Tohoku University, Sendai 980-8578, Japan*

(Received February 10, 1999)

Distances play important roles in cosmological observations, especially in gravitational lens systems, but there is a problem in determining distances because they are defined in terms of light propagation, which is influenced gravitationally by the inhomogeneities in the universe. In this paper we first give the basic optical relations and the definitions of different distances in inhomogeneous universes. Next we show how the observational relations depend quantitatively on the distances. Finally, we give results for the frequency distribution of different distances and the shear effect on distances obtained using various methods of numerical simulation.

Tomita et al. (99)

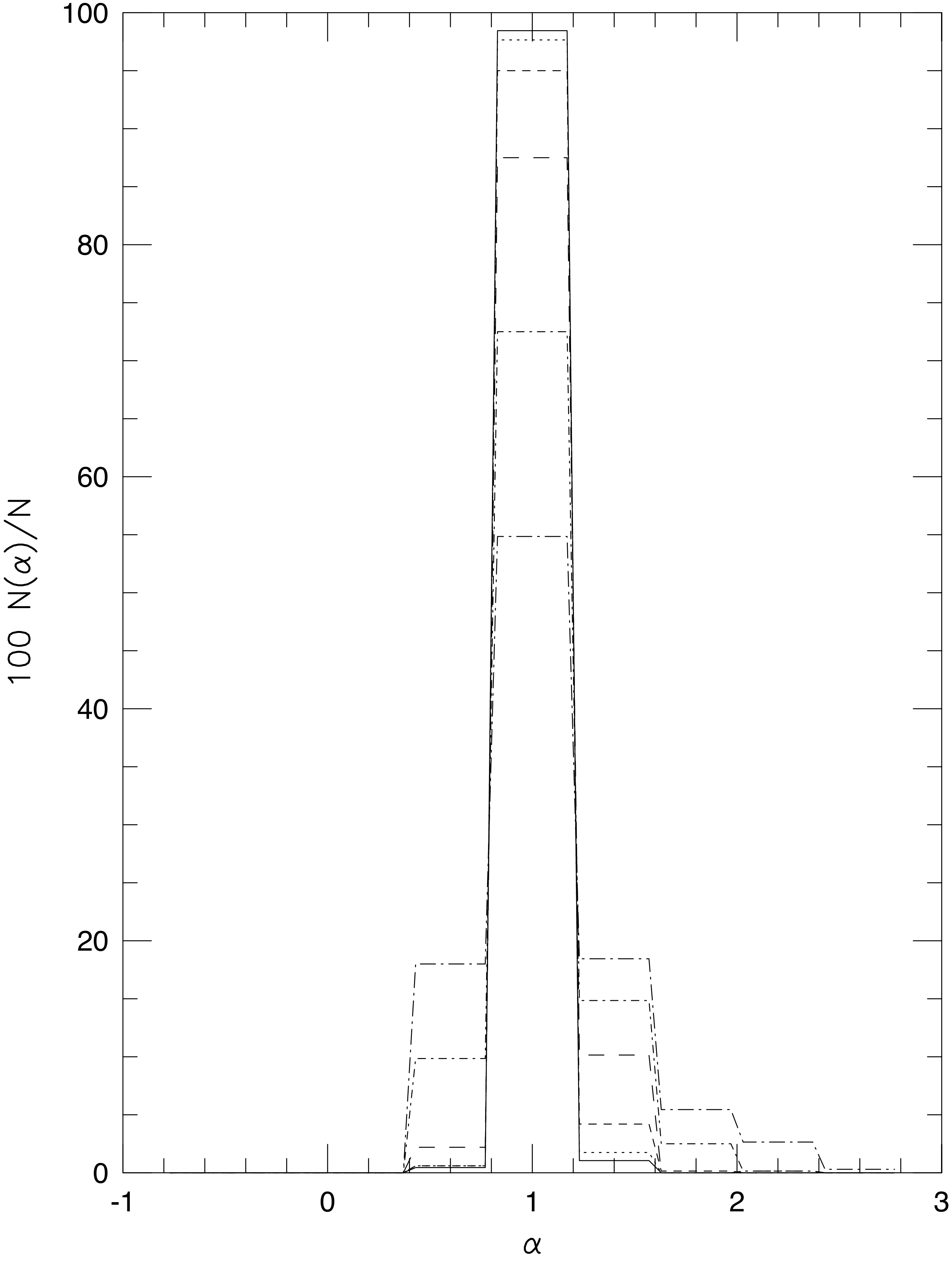


Fig. 1. The percentage $(100N(\alpha)/N)$ of the distribution of α in bins with the interval $\Delta\alpha = 0.4$, for $D_{\rm lA}$ in the lens model 1 and model S with $(\Omega_0, \lambda_0) = (1.0, 0)$. Results for $z = 0.5, 1, 2, 3, 4$ and 5 are denoted by dot-long dashed, dot-short dashed, long dashed, short dashed, dotted and solid lines, respectively.

“Variance”

“No”

Monte-Carlo Simulations

e.g., Rauch (1991)

Holz, Wald (1998)

Metcalf, Silk (1999)

Barber (2000)

Porciani, Madau (2000)

Valageas (2000)

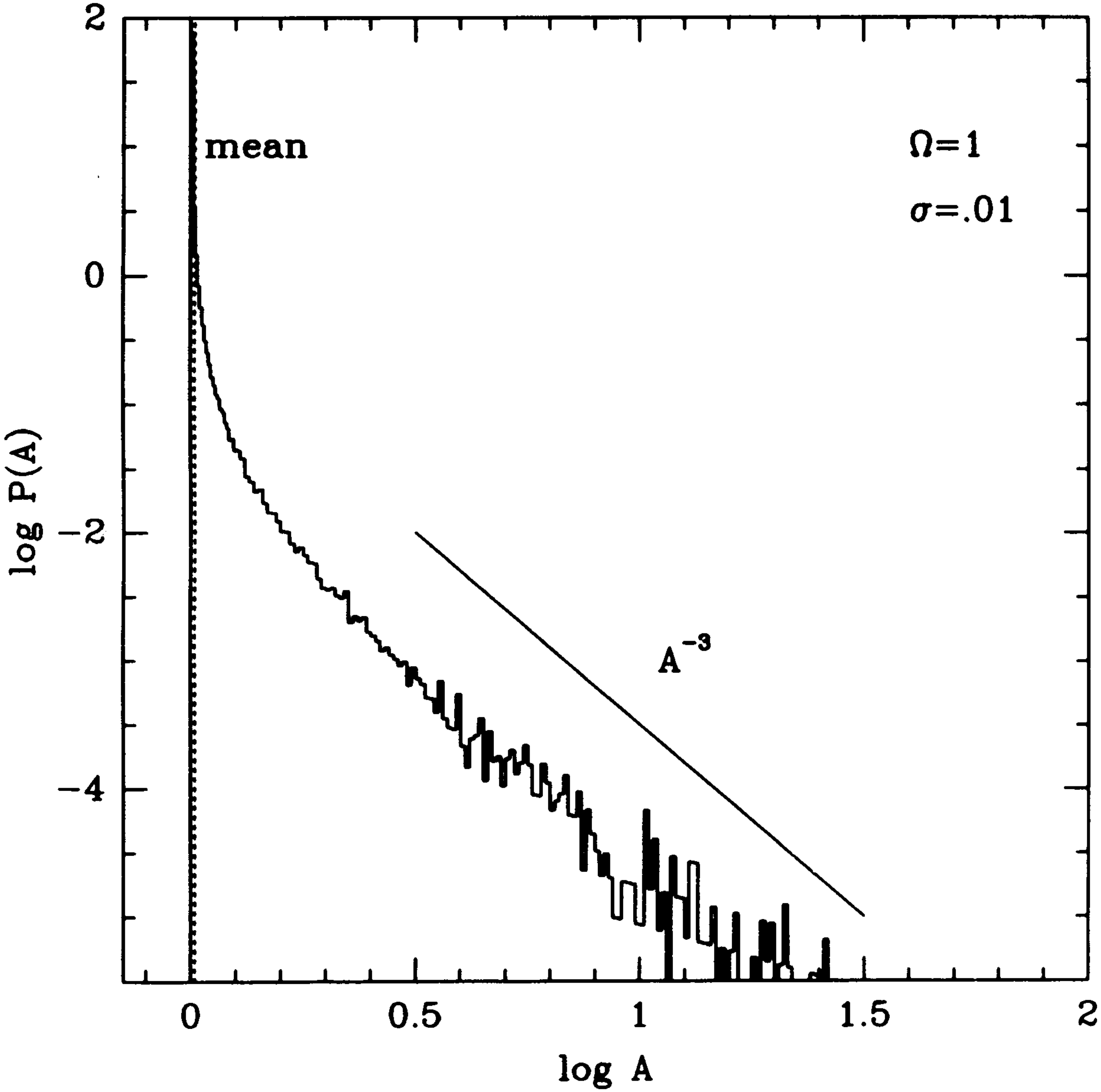
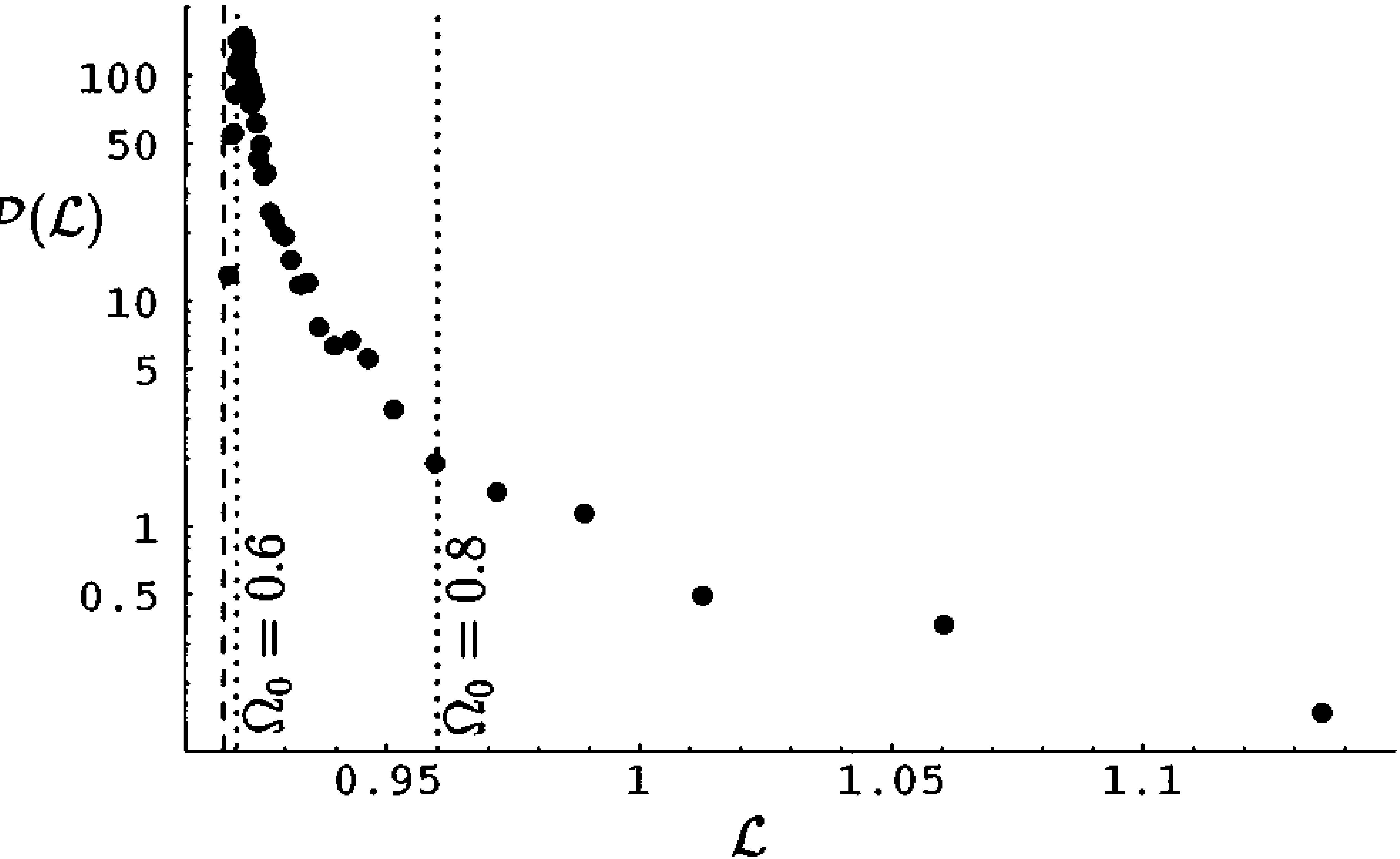


FIG. 2.—Amplification probability distribution of a compact object–dominated $\Omega = 1$ universe for $\sigma = 0.01$ (corresponding to a redshift of $z_{src} = 0.22$). A line proportional to A^{-3} has been drawn for comparison. The curve for the $\Lambda \neq 0$, $\sigma = 0.01$ case is nearly identical (see Fig. 5).

Holz, Wald (1998)



though more are demagnified...

Recent, more sophistcated work

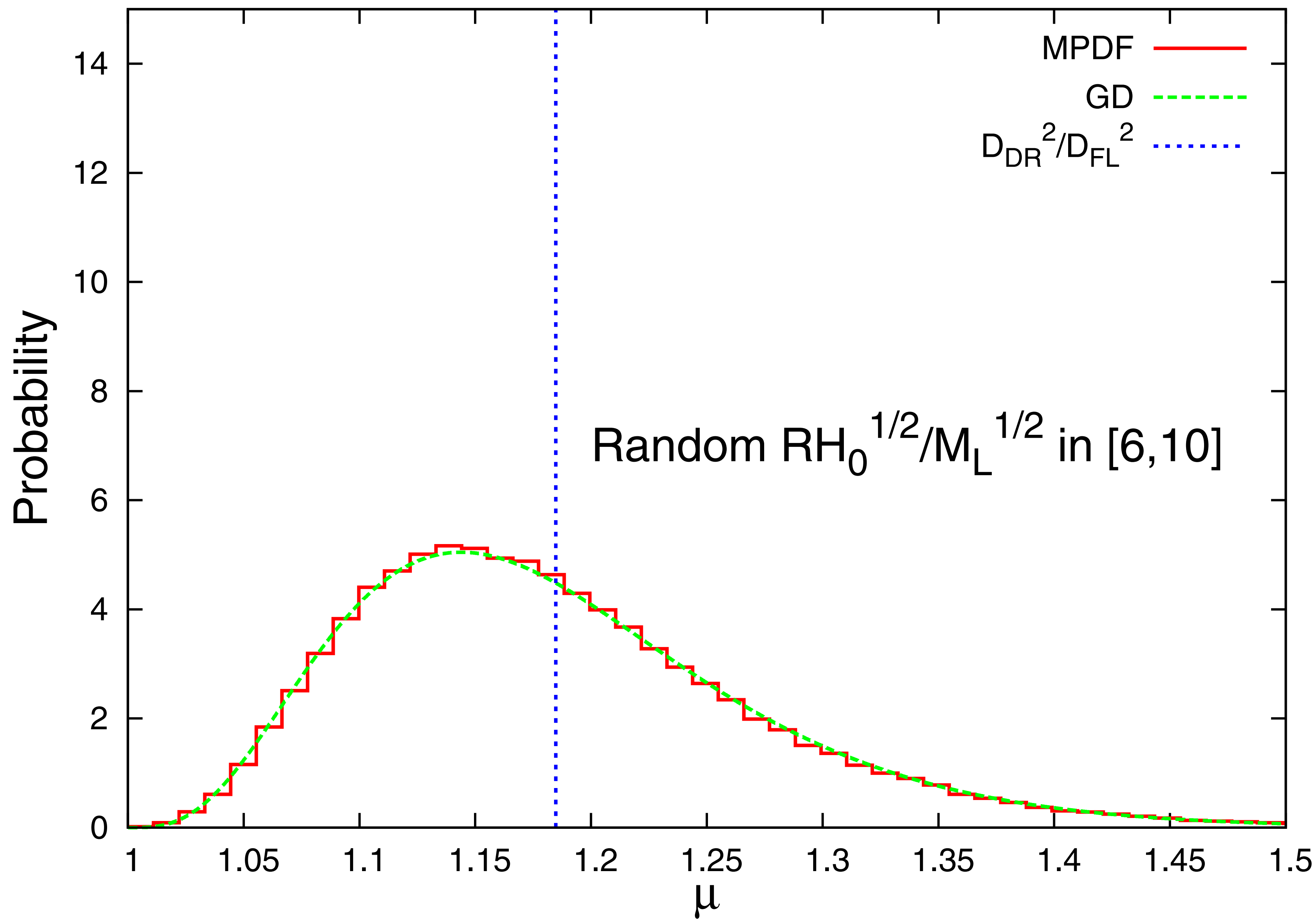
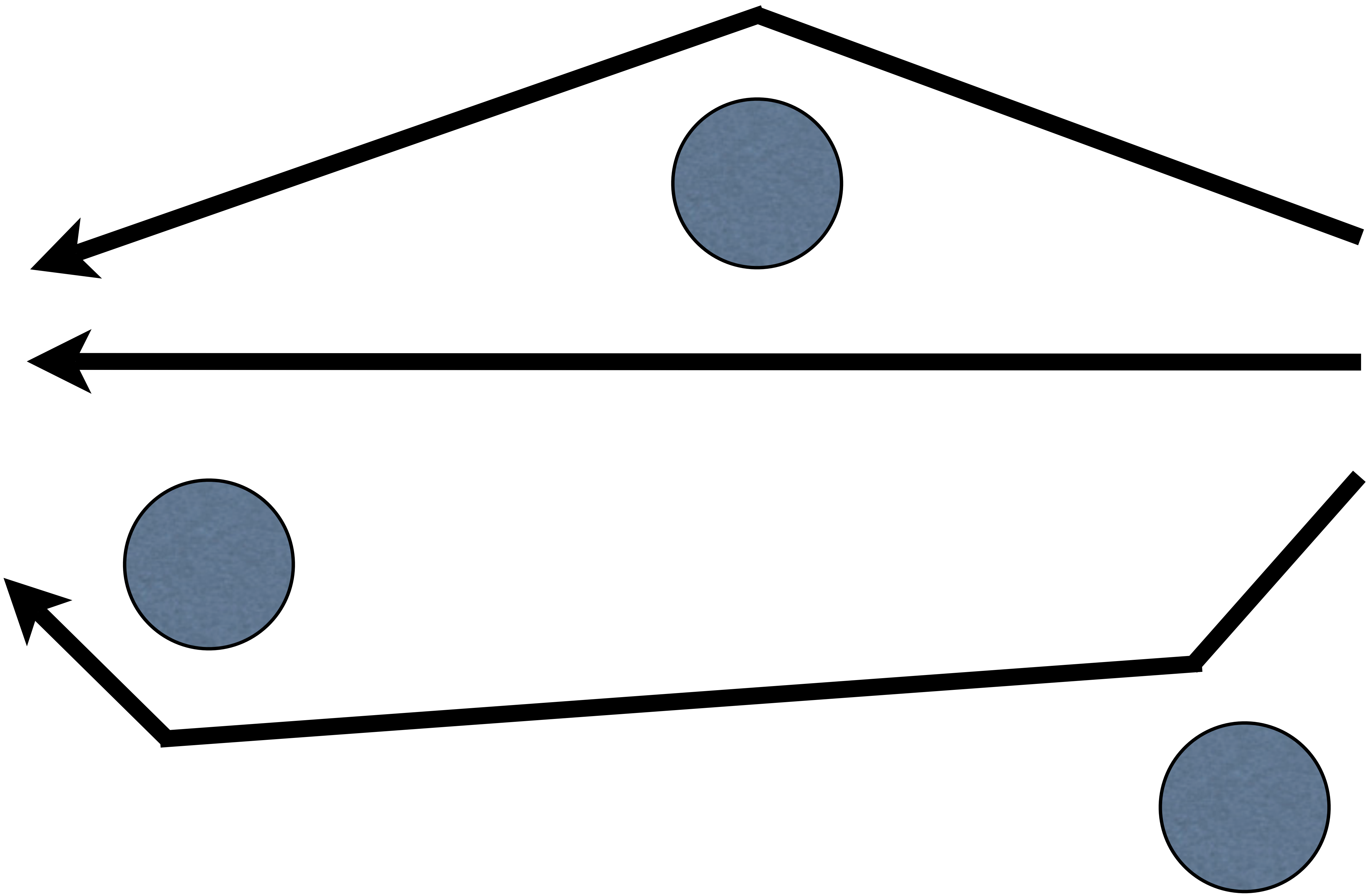


Fig. 10. MPDF for the lens model (b) at $z_S = 1.2$ is shown. Values of $R\sqrt{H_0/M_L}$ of clumps are distributed uniformly within $6 \leq R\sqrt{H_0/M_L} \leq 10$. The smooth lines are the gamma distributions that fit the MPDFs.

Yoo, Ishihara, Nakao, Tagoshi (2008)

In Reality, Strong Lensing



Multiple paths

§2-1. Intro to N pt.

Gravitational Lens (GL)

Direct Probe
of Gravity (Mass)

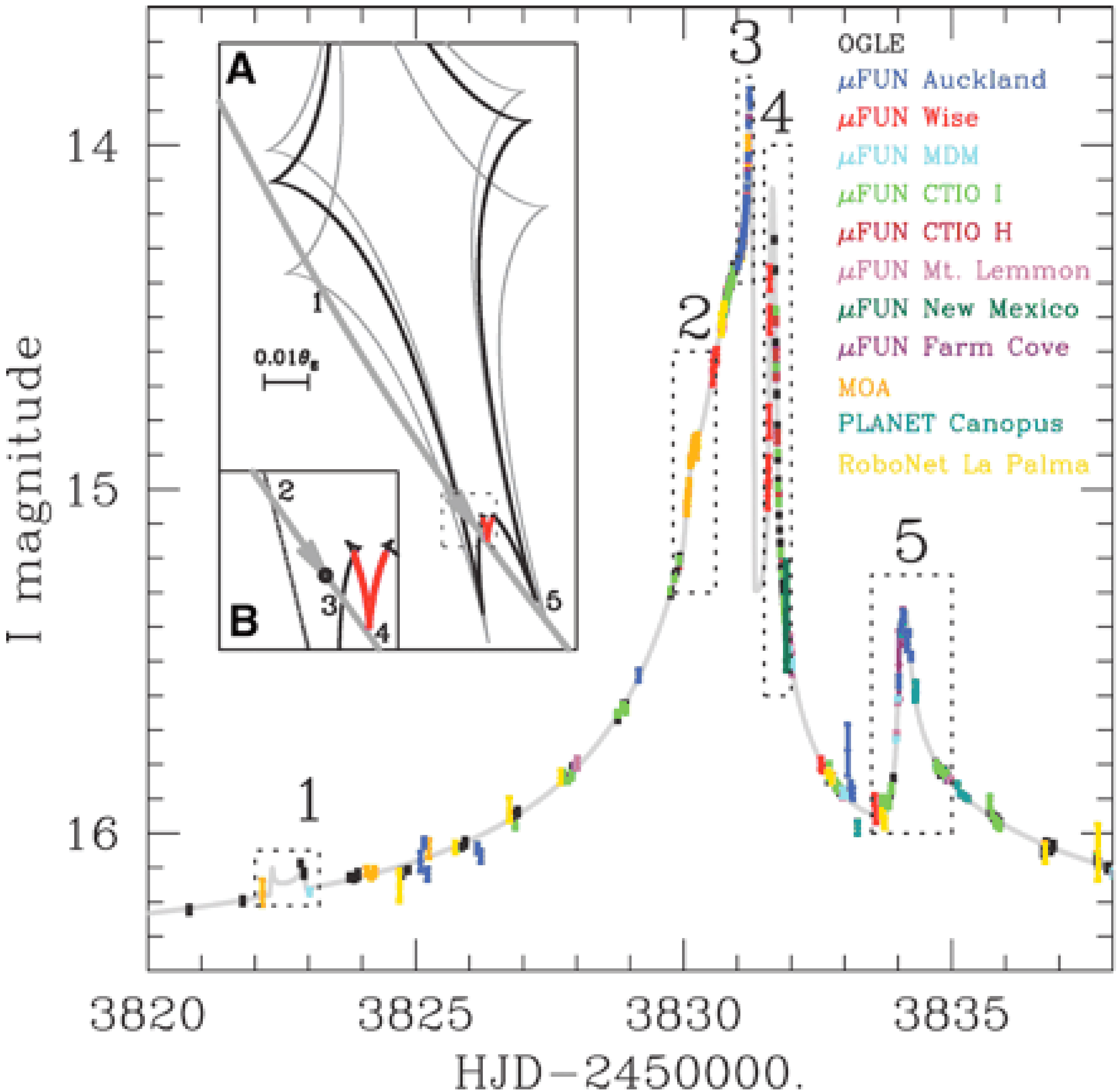
Dark Energy
Dark Matter
Dark Object (Exoplanet etc)

Main Result

HA, arXiv:0809.4122

First systematic attempt to determine lensed image positions for arbitrary N using Perturbation Theory

Analogy of Sun-Jupiter-Saturn



Gaudi et al. Science (08)

Approaches (Modelling)

Fluid Approx.(Continuum)

Cosmological GL

Lens=galaxies, LSS

Particle Approx. (Discrete)

Microlens

Lens=stars, planets, etc

Question (#1)

Lens = N particles

⇓ **N⇒∞**

Lens = Continuum

have to agree (proof?)

N-finite Effects?

N-finite Effects

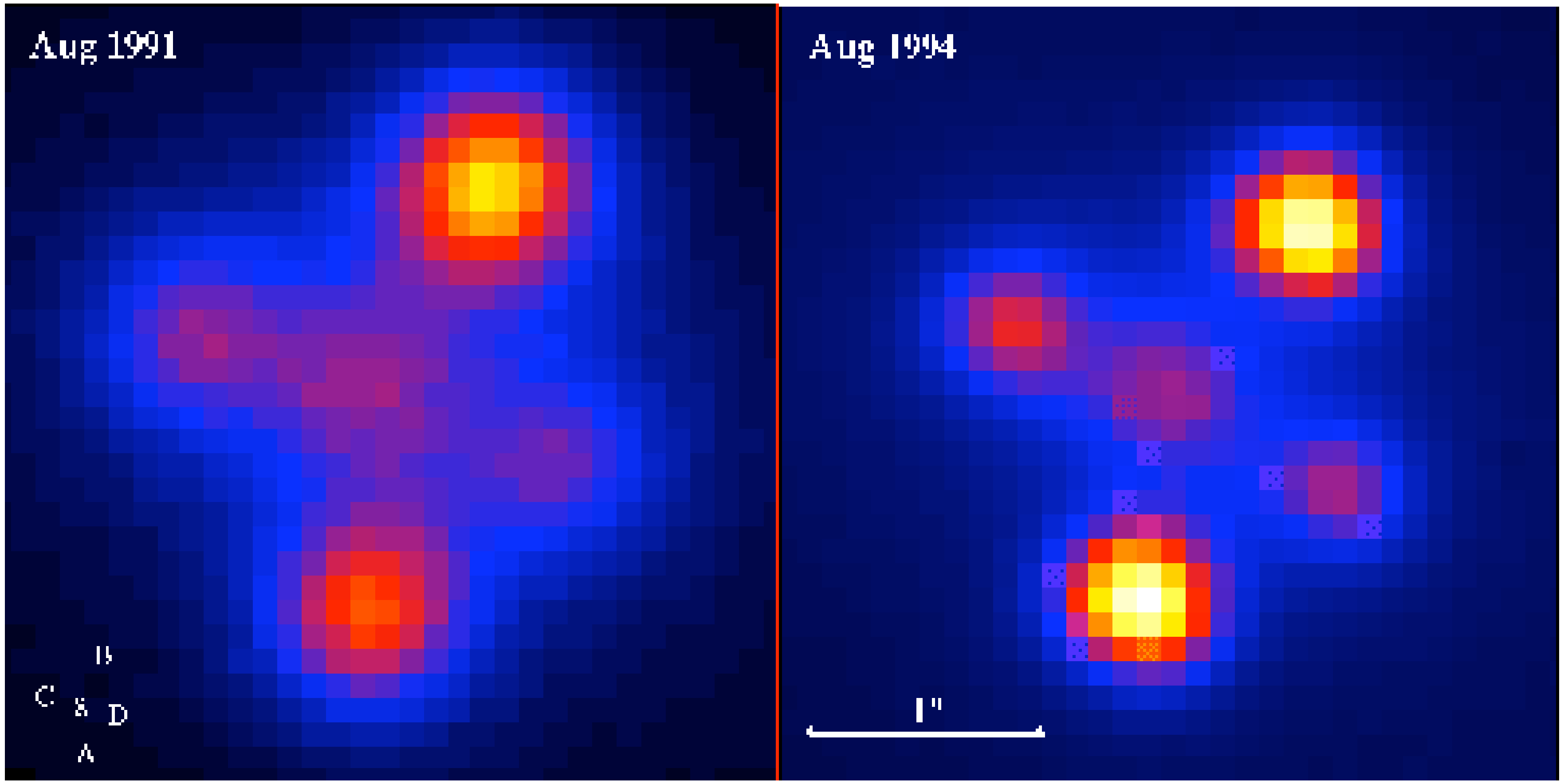
have been observed.

QSO microlens

Lens=galaxy (+star)

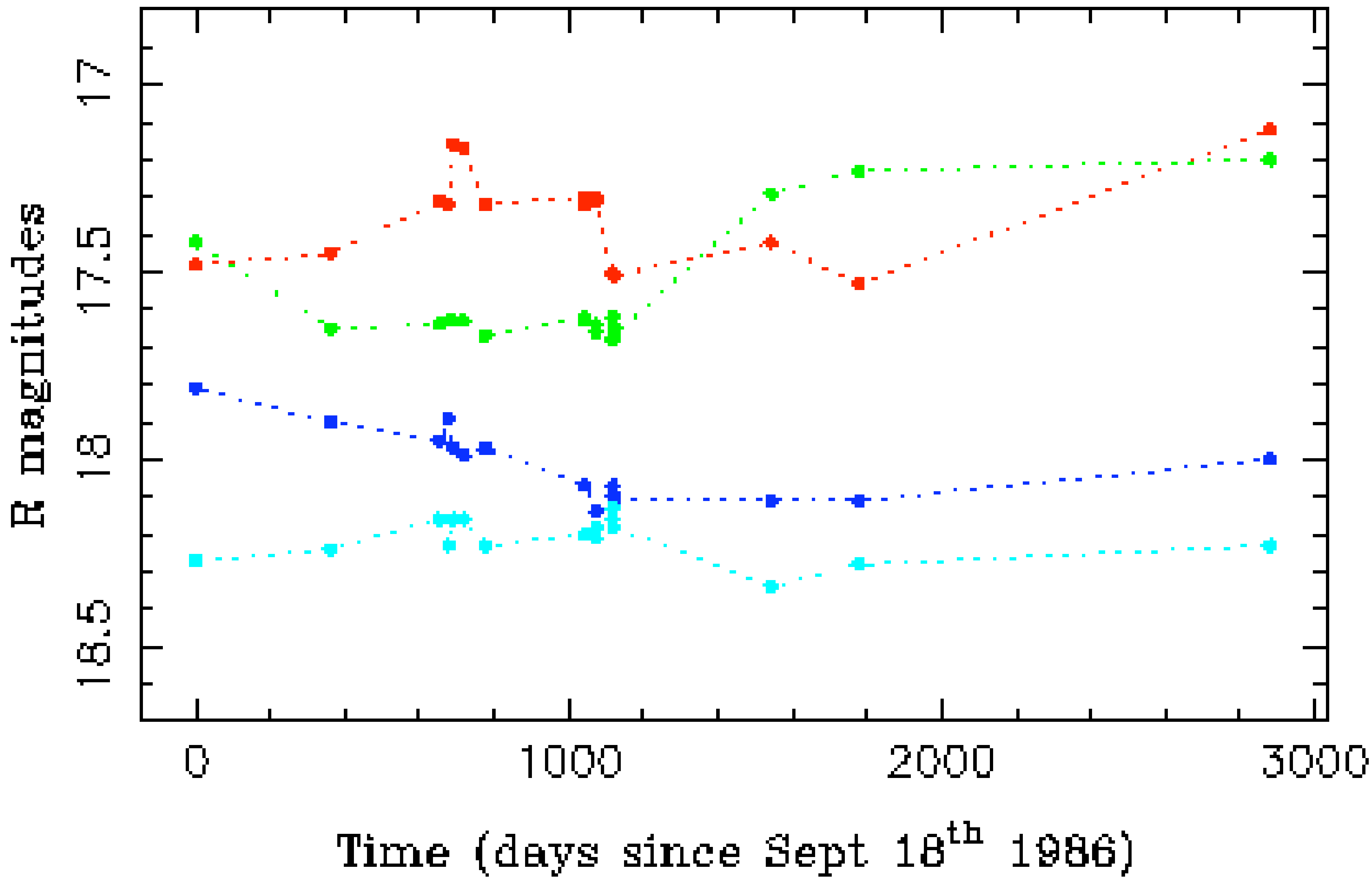
Source=quasar

Q2237+0305
= Einstein Cross



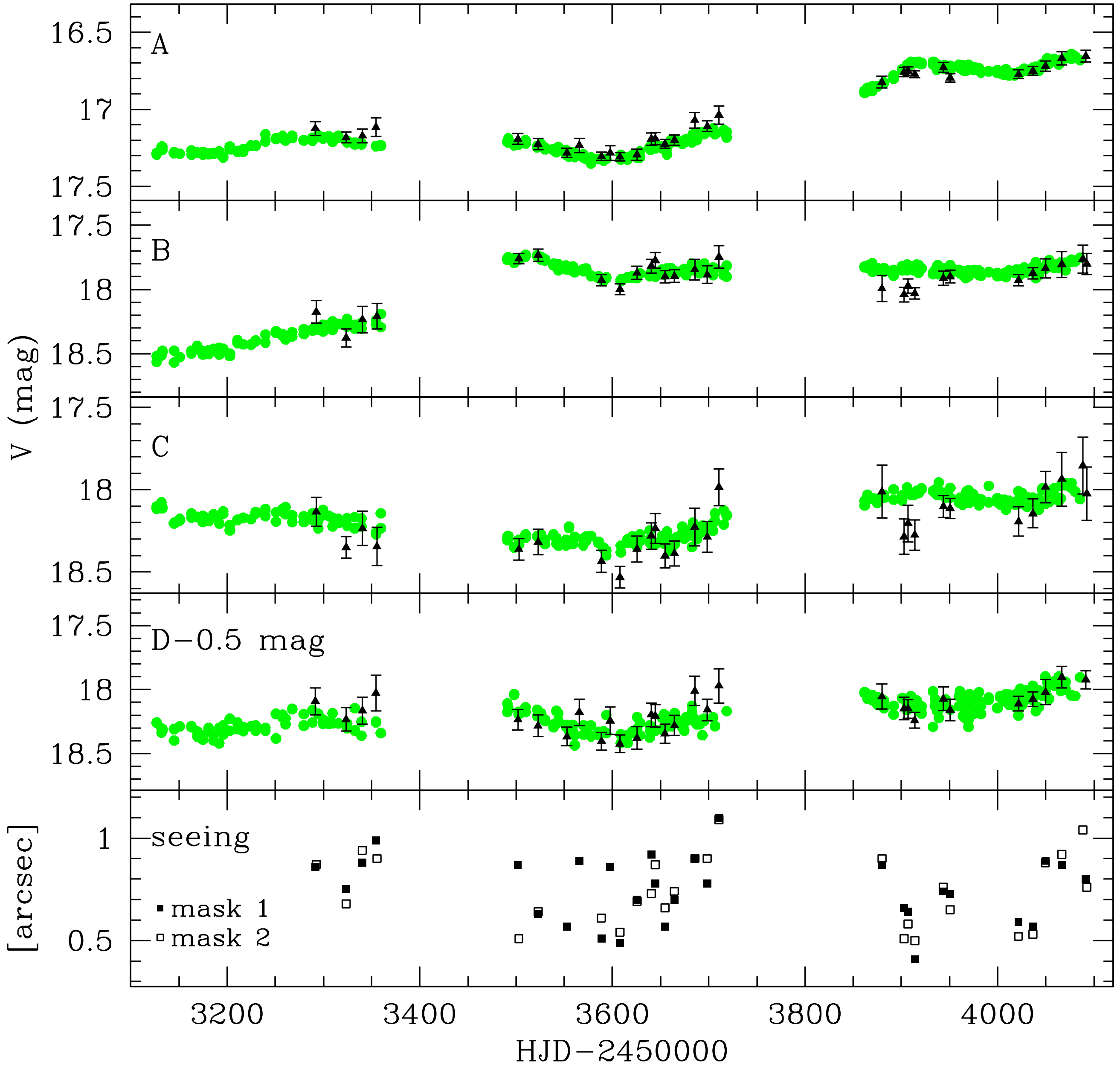
Wambsganss, LRR (91)

Time Variability



Wambsganss, LRR (01)

Q2237+0305
= Einstein Cross
Time Variability



Eigenbrod et al.
ArXiv:0709.2828

Question（#2）

We want to get

Roots for lens eq.

=Positions of images

Analytic expressions

--- unknown

Problem

N=1 ⇒　Quadratic Eq.

N=2 ⇒

Complex Quintic Eq.

(Witt 90)

Real Quintic Eq.

(Asada 02, Asada et al 04)

Theorem (Galois)

Algebraic Eq. cannot be solved algebraically, if 5th or higher order.

Algebraic method

$= +,\ -,\ \times,\ \div,\ \sqrt[n]{\ }$

Thus, Formula is unknown.

Our goal

First attempt to get perturbative roots

Approximate ones can be sufficient for observation

§2-2. Complex Formalism

Bourassa and Kantowski (73,75)

GL

= 2D mapping (thin lens)

Source Pl. ⟷ **Lens Pl.**

$\vec{\beta}$ $\vec{\theta}$

$w = w_x + i w_y$ $z = x + i y$

Assumption

1) thin lens approx.

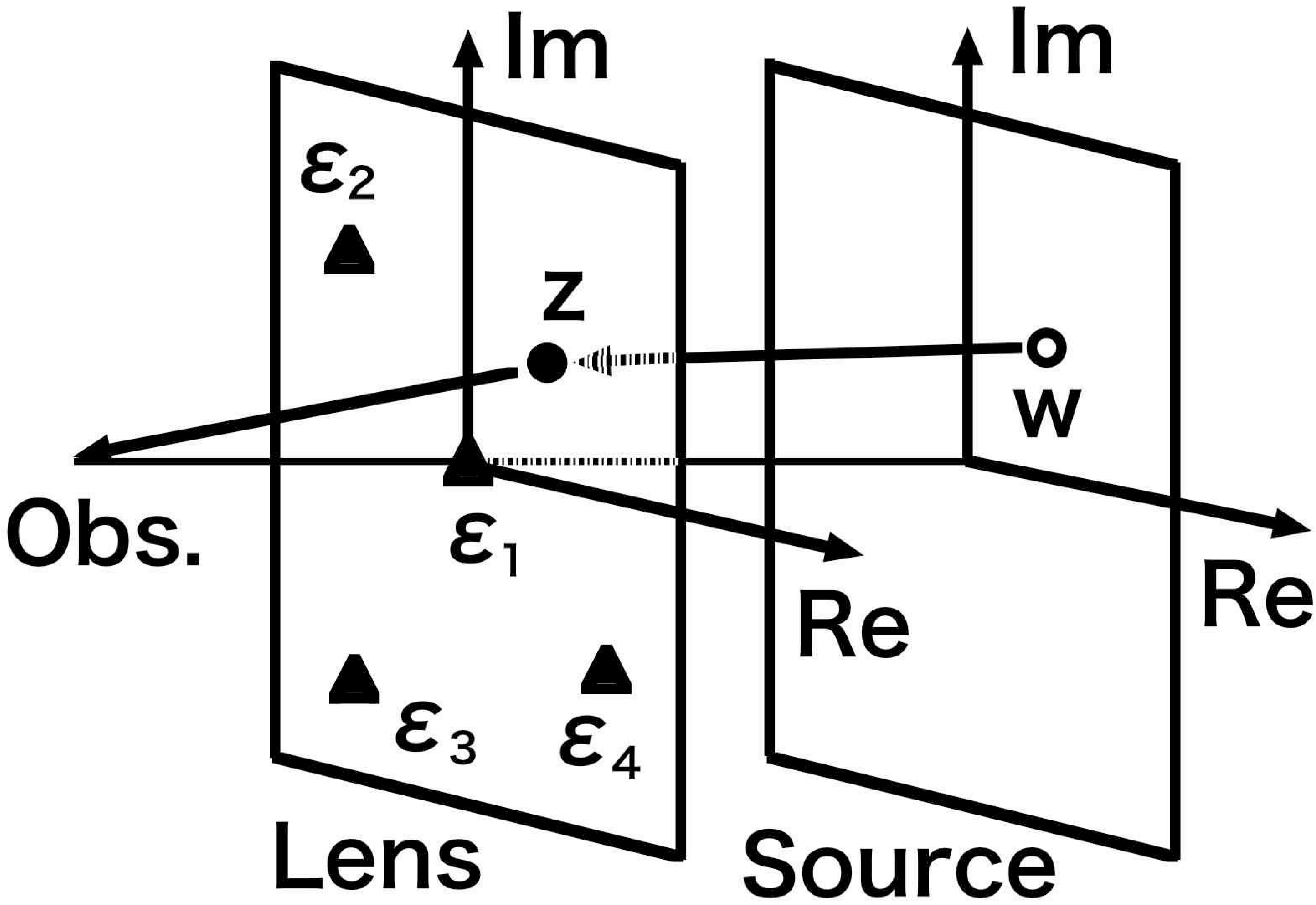
2) arbitrary N
co-planar mass

#) any configuration
without symmetry

Lens Equation (Coupled)

Vector form

$$\boldsymbol{\beta} = \boldsymbol{\theta} - \sum_{i}^{N} \nu_i \frac{\boldsymbol{\theta} - \boldsymbol{e}_i}{|\boldsymbol{\theta} - \boldsymbol{e}_i|^2}$$


Im
Im
ε_2
z
w
Obs.
ε_1
Re
Re
ε_3
ε_4
Lens
Source

Complex Notation

$$w = z - \sum_{i}^{N} \frac{\nu_i}{z^* - \epsilon_i^*}$$

Single-Complex-Variable Polynomial

Witt (90)

z^* **deleted**

Polynomial in Only z

N^2**+1th Order**

$$w = z - \sum_{i}^{N} \frac{\nu_i}{z^* - \epsilon_i^*}$$

c.c.

$$w^* = z^* - \sum_{i}^{N} \frac{\nu_i}{z - \epsilon_i}$$

Only z but no z*

$$(z-w)\prod_{l=1}^{N}\left((w^*-\epsilon_l^*)\prod_{k=1}^{N}(z-\epsilon_k)+\sum_{k=1}^{N}\nu_k\prod_{j\neq k}^{N}(z-\epsilon_j)\right)$$

$$=\sum_{i=1}^{N}\nu_i\prod_{l=1}^{N}(z-\epsilon_l)$$

$$\times\prod_{m\neq i}^{N}\left((w^*-\epsilon_m^*)\prod_{k=1}^{N}(z-\epsilon_k)+\sum_{k=1}^{N}\nu_k\prod_{j\neq k}^{N}(z-\epsilon_j)\right)$$

Perturbation

Mass Ratio

$$\nu_i = M_i / M_{tot} < 1$$

--- expansion parameter

Iterative calculations

$$z = \sum_{p_2=0}^{\infty} \sum_{p_3=0}^{\infty} \cdots \sum_{p_N=0}^{\infty} \nu_2^{p_2} \nu_3^{p_3} \cdots \nu_N^{p_N} z_{(p_2)(p_3)\cdots(p_N)}$$

0th order root

$$\alpha_i \equiv -1/w_i^*$$

$$\alpha_\pm = \frac{w}{2}\left(1 \pm \sqrt{1 + \frac{4}{ww^*}}\right)$$

$$\epsilon_i$$

$$w_i = w - \epsilon_i$$

Problem

$$\alpha_i$$

does not satisfy Lens Eq.

↓

mixed with unphysical roots

Dual-Complex-Variables Formalism

Both Z and Z^*

Merit

Equivalent to Lens Eq.

↓

No unphysical root

$$C(z, z^*) = \sum_{k=2}^{N} \nu_k D_k(z^*)$$

Linear in $\boldsymbol{\nu}$

$$D_k(z^*) = \frac{1}{z^*} - \frac{1}{z^* - \epsilon_k^*}$$

Iteration

$$z = \sum_{p_2=0}^{\infty} \sum_{p_3=0}^{\infty} \cdots \sum_{p_N=0}^{\infty} (\nu_2)^{p_2} (\nu_3)^{p_3} \cdots (\nu_N)^{p_N} z_{(p_2)(p_3)\cdots(p_N)}$$

1st Order

$$z_{(0)\cdots(1_k)\cdots(0)} = \frac{b_{(0)\cdots(1_k)\cdots(0)} - a_{(0)\cdots(1_k)\cdots(0)} b^*_{(0)\cdots(1_k)\cdots(0)}}{1 - a_{(0)\cdots(1_k)\cdots(0)} a^*_{(0)\cdots(1_k)\cdots(0)}}$$

$$a_{(0)\cdots(1_k)\cdots(0)} = \frac{1}{\left(z^*_{(0)\cdots(0)}\right)^2}$$

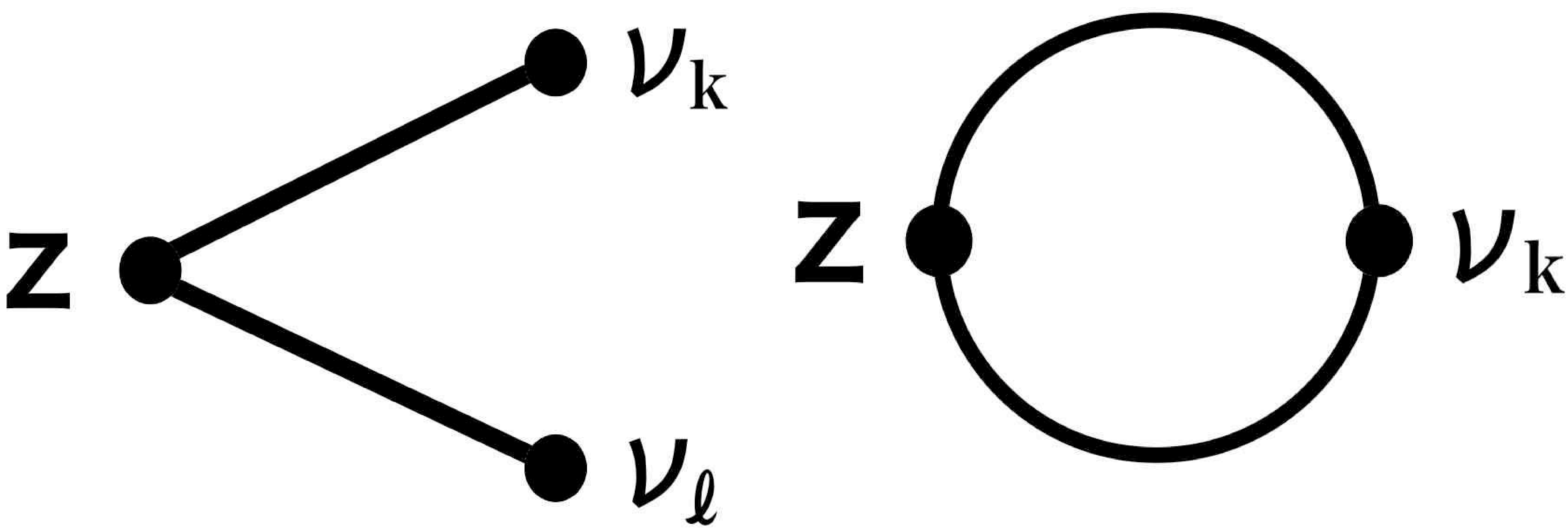
$$b_{(0)\cdots(1_k)\cdots(0)} = \frac{\epsilon^*_k}{z^*_{(0)\cdots(0)}\left(z^*_{(0)\cdots(0)} - \epsilon^*_k\right)}$$

2nd Order

$$z_{(0)\cdots(2_k)\cdots(0)} = \frac{b_{(0)\cdots(2_k)\cdots(0)} - a_{(0)\cdots(2_k)\cdots(0)} b^*_{(0)\cdots(2_k)\cdots(0)}}{1 - a_{(0)\cdots(2_k)\cdots(0)} a^*_{(0)\cdots(2_k)\cdots(0)}}$$

$$z_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)}$$
$$= \frac{b_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)} - a_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)} b^*_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)}}{1 - a_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)} a^*_{(0)\cdots(1_k)\cdots(1_l)\cdots(0)}}$$

Figure 5. Graph representations of interactions among point masses for images at the second order level. The top and bottom graphs represent a mutually-interacting image and a self-interacting one, respectively.



Similarly,

3rd Order, 4th Order・・

--- Systematic !

Convergence: On/Off-axis

Case 1 (On-axis)	$\nu = 0.1$	$e = 1$	$w = 2$
Root	1	2	3
1st.	2.43921	-0.389214	0.95
2nd.	2.43855	-0.388551	0.95
3rd.	2.43858	-0.388519	0.949938
Lens Eq.	2.43858	-0.388517	0.949937

Case 2 (Off-axis)	$\nu = 0.1$	$e = 1$	$w = 1 + i$
Root	1	2	3
1st.	1.33716+1.40546 i	-0.337158-0.355459 i	0.95-0.05 i
2nd.	1.33632+1.40363 i	-0.336316-0.354881 i	0.95-0.05 i
3rd.	1.33634+1.40371 i	-0.336275-0.354839 i	0.95-0.05025 i
Lens Eq.	1.33633+1.40371 i	-0.336272-0.354835 i	0.950015-0.0502659 i

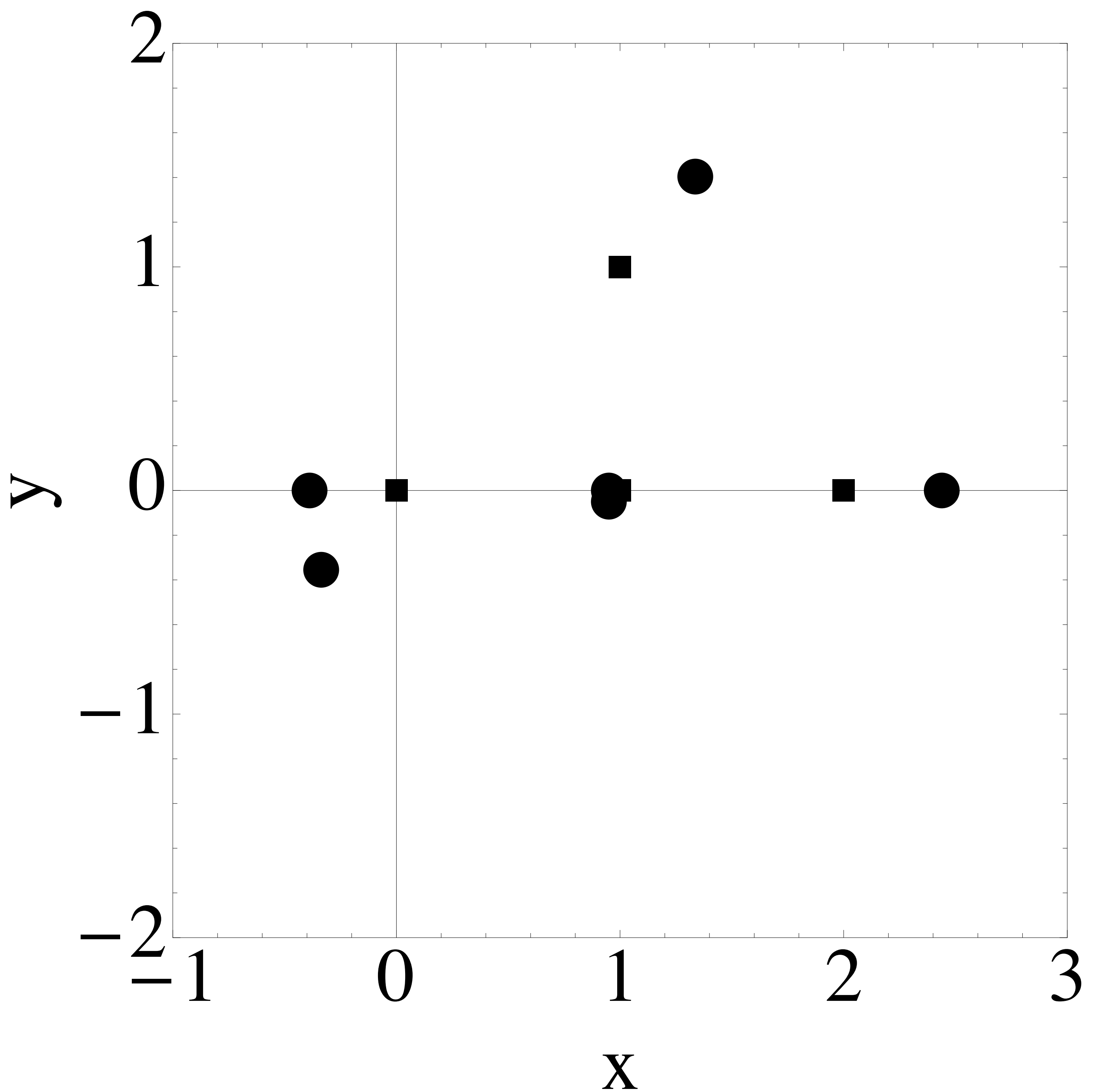
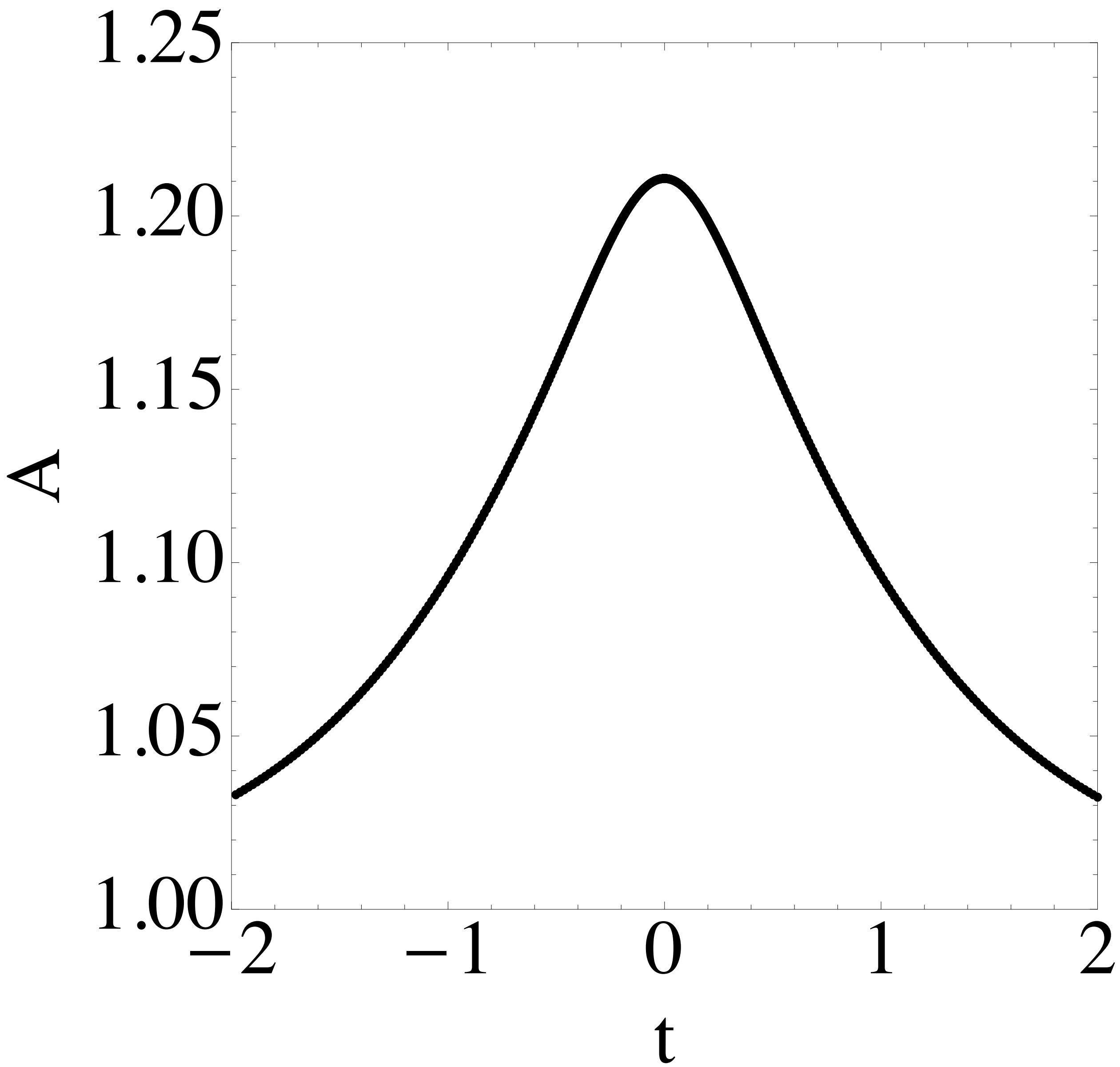
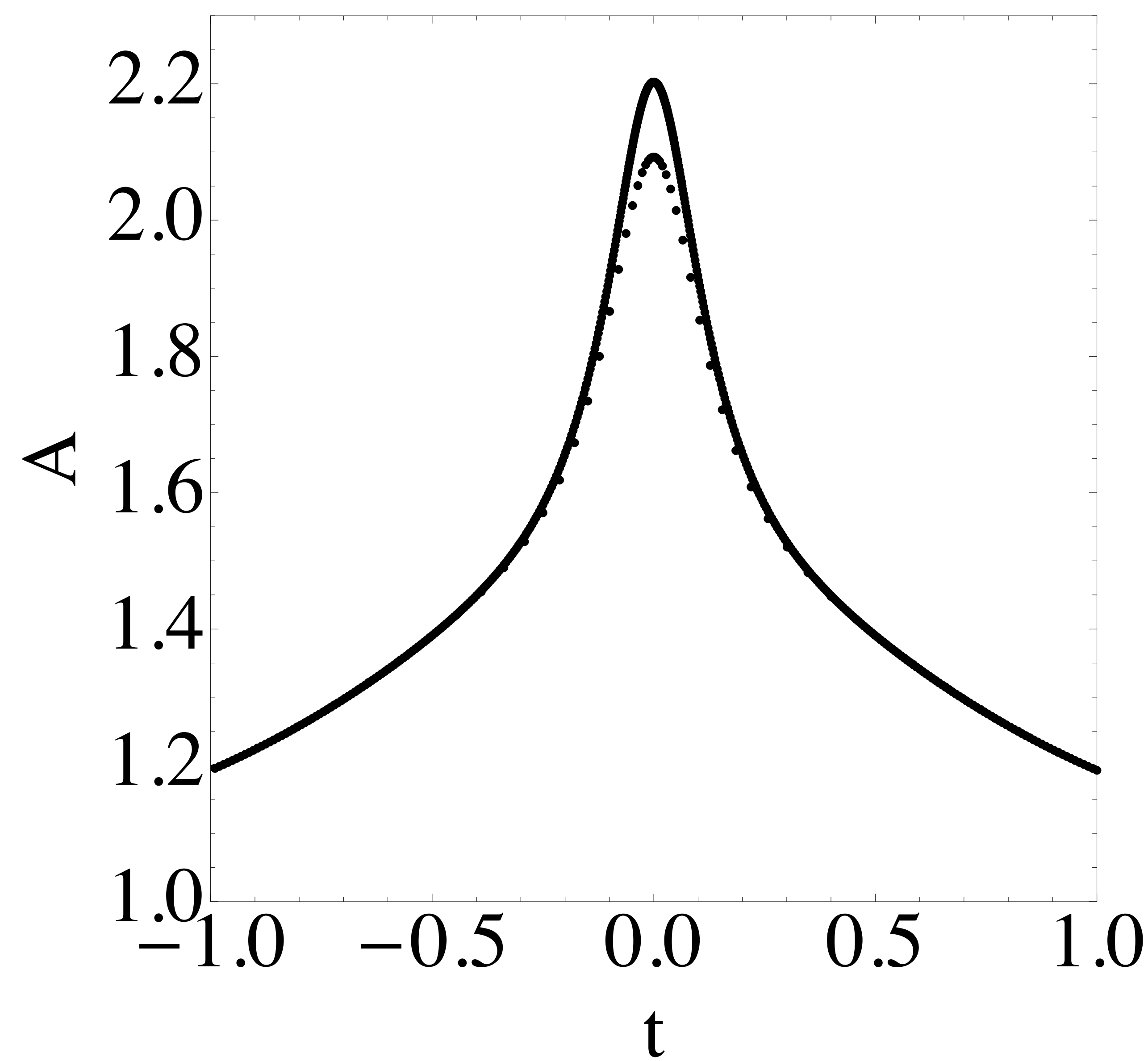


Figure 2. Perturbative image positions for a binary lens case. This plot corresponds to Tables 1 and 2. The lenses ($e_1 = 0, e_2 = 1$) and sources ($w = 2$ and $w = 1+i$) are denoted by filled squares. The image positions are denoted by filled disks. Perturbative images at the 1st, 2nd and 3rd orders are overlapped so that we cannot distinguish them in this figure.

Light Curve -- by 1st

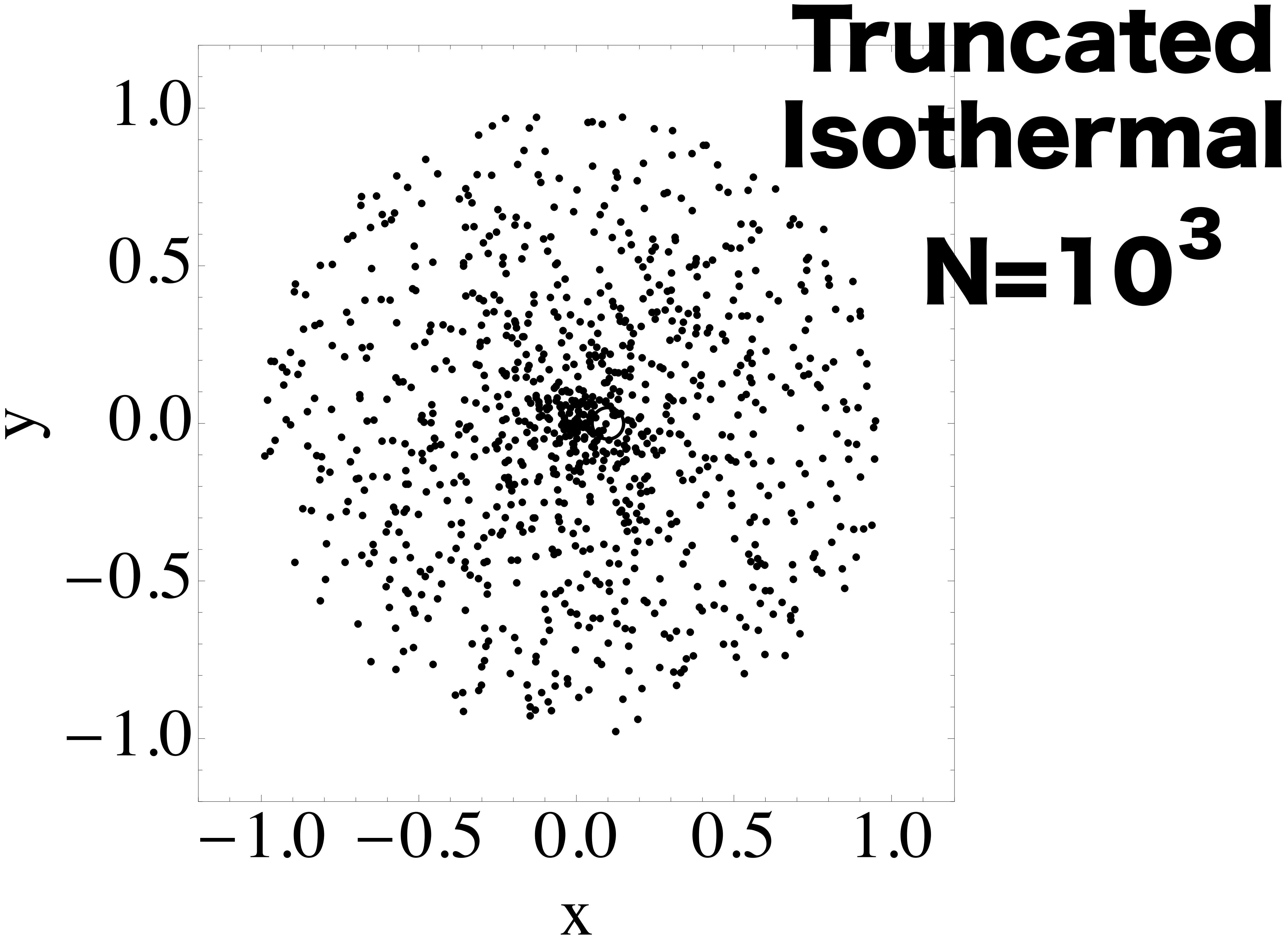




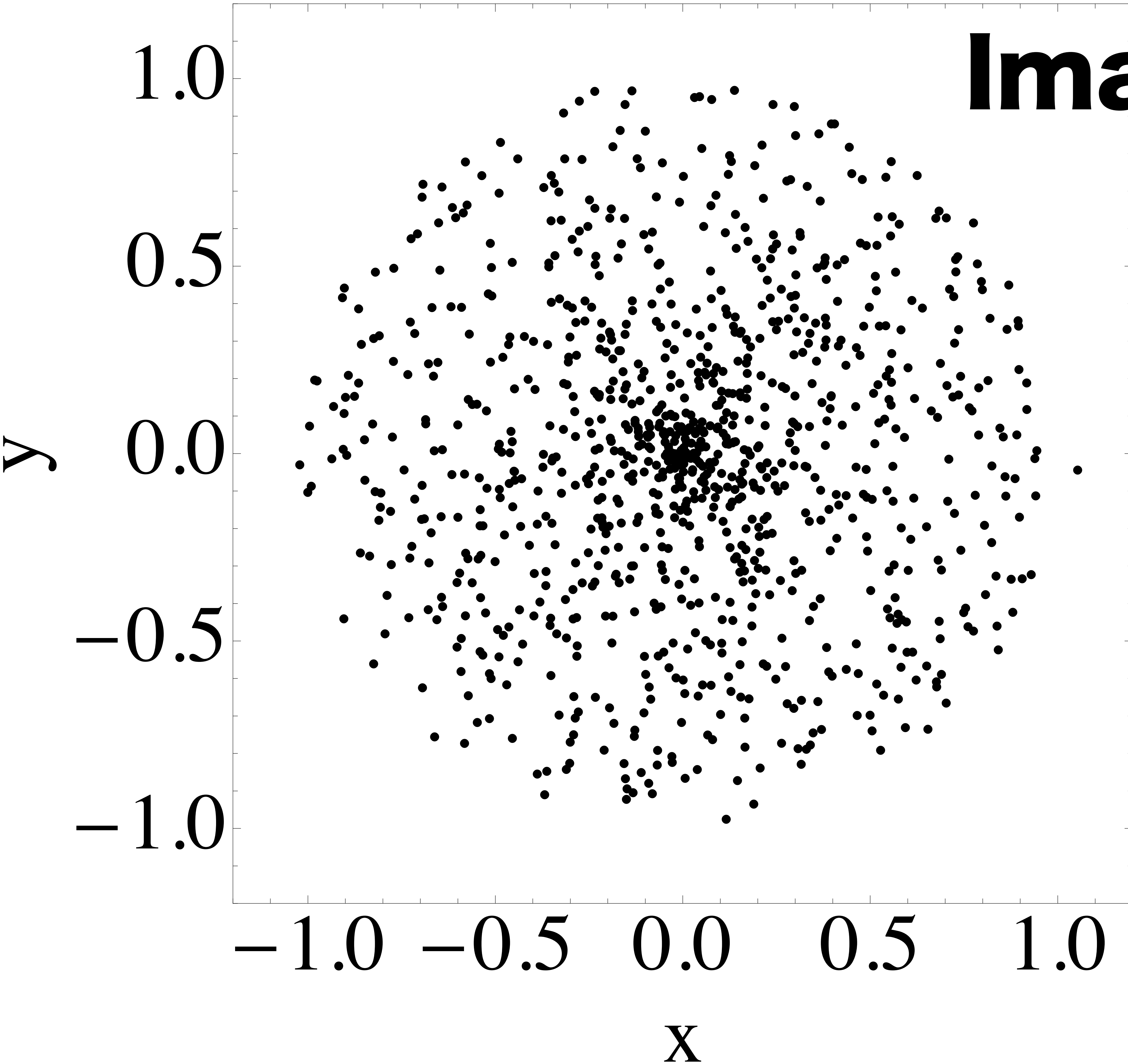
$$w = 1.4 + it$$

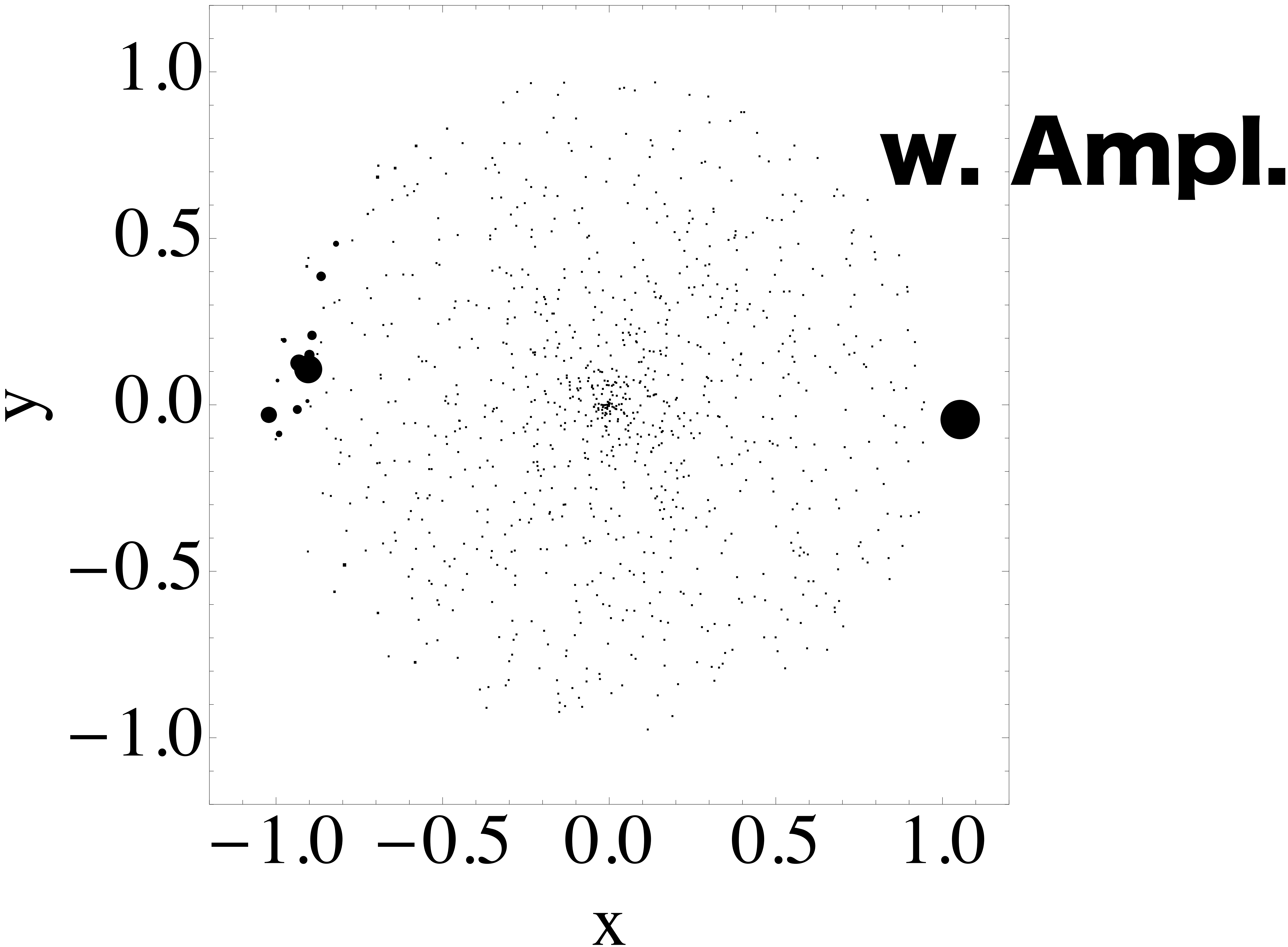
$$w = 0.8 + it$$

$$\nu_2 = 0.1 \qquad e = 1$$

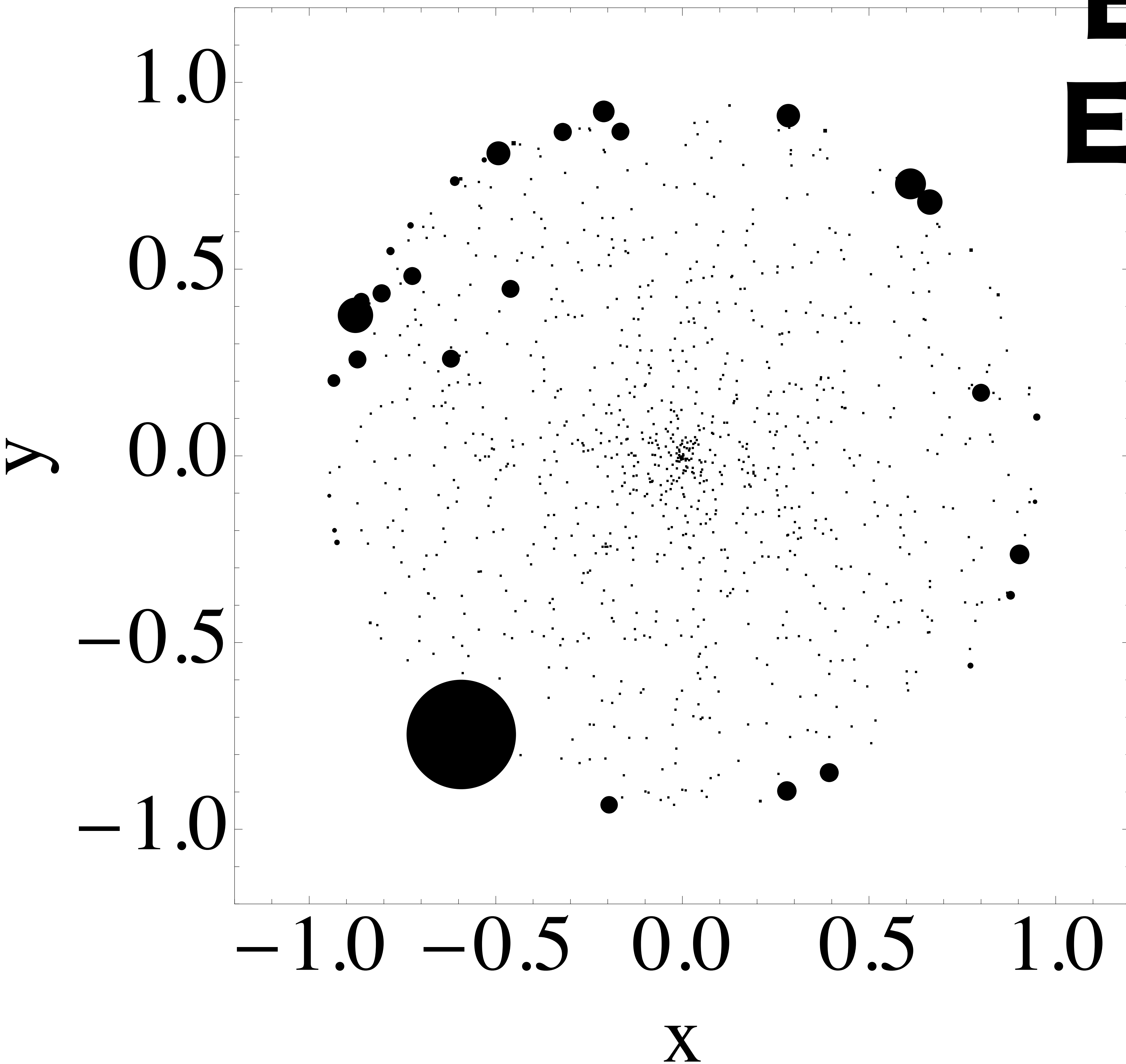

Truncated
Isothermal
N=10^3
y
x
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0

Images




w. Ampl.
1.0
0.5
0.0
−0.5
−1.0
y
−1.0
−0.5
0.0
0.5
1.0
x

Broken Einstein Ring



§3. Summary

HA, arXiv:0809.4122

First attempt to get lensed image positions for arbitrary N

Future Works

1 Applications to N-Finite Effects

Ex) Mean, Variance in Mag.

2 Extension to Multiple Lens Planes

Ex) Cosmological GL

Thank you !

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