

Solving the inverse problem with inhomogeneous universes

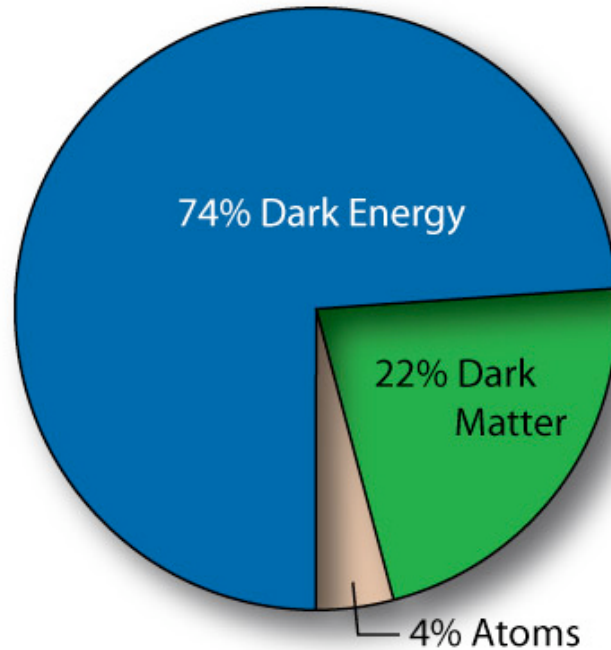
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Tomohiro Kai(Osaka City University)

and

[Ken-ichi Nakao](#)(Osaka City University)

Cosmic Pie



Recent observations imply that there is a mysterious matter component called dark energy.

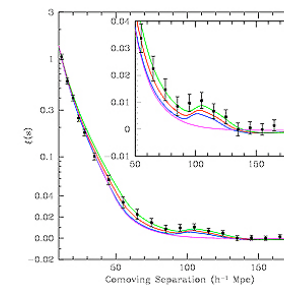
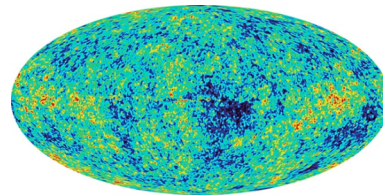
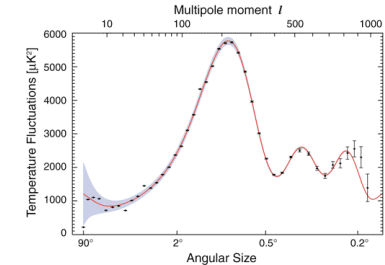
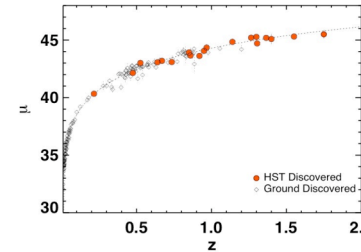
$$\rho + 3P < 0$$

The reason why the dark energy is necessary.

Observational facts

- Hubble parameter : $H_0 \approx 71 \text{ km/s/Mpc}$
- Distance-redshift relation given by the observations of type Ia super novae
- Baryon acoustic oscillations in cosmic microwave background
- Baryon acoustic oscillations in large scale structure of galaxies
- Integrated Sachs-Wolfe effects

and further?



&

the assumption of homogeneity and isotropy of our universe

Base of our conviction (confidence?) in homogeneity and isotropy of our universe

Cosmic microwave background
highly isotropic except for dipole component

Copernican principle

“We stay a typical place in the universe”



Everywhere isotropic = homogeneous and isotropic

- We stay at very special place of our universe

Observations + GR + Homogeneity and isotropy = Dark Energy

- Inhomogeneities may cause acceleration of cosmic volume expansion

Observations + GR + Homogeneity and isotropy = Dark Energy

- Large scale gravitational interactions cannot be described by General Relativity (GR)

Observations + GR + Homogeneity and isotropy = Dark Energy

Copernican principle is a stain on the precision cosmology.

We should confirm by observations
whether our position in the universe
is not really special or not.

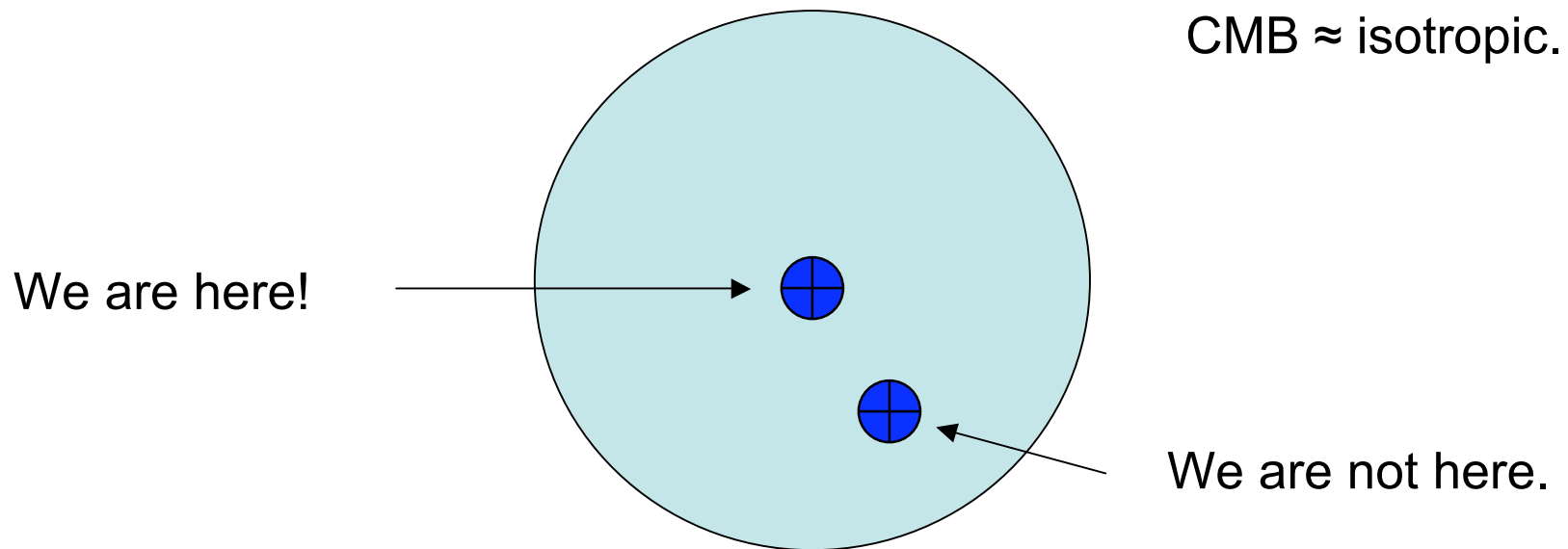
Now we might get a chance to accomplish it
by virtue of the dark energy fever.

For this purpose, we need to understand the inhomogeneous universe.

How is it observationally different from
homogeneous and isotropic universe?

We assume that our position in the universe is very special.

We are in the vicinity of the center of an almost isotropic but inhomogeneous universe.



Can we explain all of the observational data by this model?

Can we exclude such a model by the observational data?

We try to construct an
exactly spherically symmetric dust filled model
with a distance-redshift relation of the concordance Λ CDM model.

Lemaitre-Tolman-Bondi(LTB) solution
Exact solution for the Einstein equations,
which describes a spherically symmetric dust universe.

$$ds^2 = -dt^2 + \frac{(\partial_r R)^2}{1 - k(r)r^2} dr^2 + R^2 d\Omega^2$$

$$(\partial_t R)^2 = \frac{2M(r)}{R} - k(r)r^2$$

Curvature function

$$M(r) = 4\pi \int_0^r \rho(t, x) R^2(t, x) \partial_x R(t, x) dx \quad : \text{Mass function}$$

Solution

Three arbitrary functions

$$R(t,r) = (6M(r))^{1/3} (t - t_B(r))^{2/3} S(x)$$

$$x \equiv k(r)r^2 \left(\frac{t - t_B(r)}{6M(r)} \right)^{2/3}$$

where

$$S(x) = \frac{\operatorname{ch}y - 1}{6^{1/3}(\operatorname{sh}y - y)^{2/3}} \quad \text{with } x = -\frac{1}{6^{2/3}}(\operatorname{sh}y - y)^{2/3} \quad \text{for } x < 0$$

$$S(x) = \left(\frac{3}{4} \right)^{1/3} \quad \text{for } x = 0$$

$$S(x) = \frac{1 - \operatorname{cos}y}{6^{1/3}(y - \operatorname{siny})^{2/3}} \quad \text{with } x = \frac{1}{6^{2/3}}(y - \operatorname{siny})^{2/3} \quad \text{for } x > 0$$

At $t = t_B(r)$, $R=0$ for all r \longrightarrow $t = t_B(r)$ is the Big Bang time

Two of three functional degrees $k(r)$, $M(r)$ and $t_B(r)$ are the physical degrees of freedom.

One of three functional degrees corresponds to the degree of rescaling the radial coordinate r

Inverse problem

Iguchi et. al.(2002), Chung and Romano(2006),
Vanderveld, Flanagan & Wasserman(2006), Tanimoto and Nambu(2008)

Impose two theoretical or observational conditions.



Two physical functional freedoms are fixed.

We obtain an inhomogeneous universe model
which is possible to explain two current observations

Two of previous works

Iguchi, Nakamura & Nakao(2002)

- Distance -redshift relation of the concordance Λ CDM model
- $t_B(r) = 0$: simultaneous Big Bang
for the consistency with Inflationary scenario.

They could not solve the inverse problem beyond $z \approx 1.7$.

The reason is not physical but purely technical; there is a regular-singular point of the differential equation at $z \approx 1.7$.

Vanderveld, Flanagan & Wassermann(2006)

Tanimoto & Nambu(2007)

- Distance -redshift relation of the concordance Λ CDM model
- $t_B(r)$ is inhomogeneous : delayed Big Bang

They could solve the inverse problem for whole domain of redshift by virtue of the appropriate prescription at the regular-singular point.

Present Case

We impose two conditions identical to those of Iguchi-Nakamura-Nakao

- Distance-redshift relation of concordance Λ CDM model
- Simultaneous Big Bang $t_B(r) = 0$

If we can solve the inverse problem, we have the LTB universe model which is possible to explain the SNe data in all domain of redshift and further is consistent with inflationary scenario.

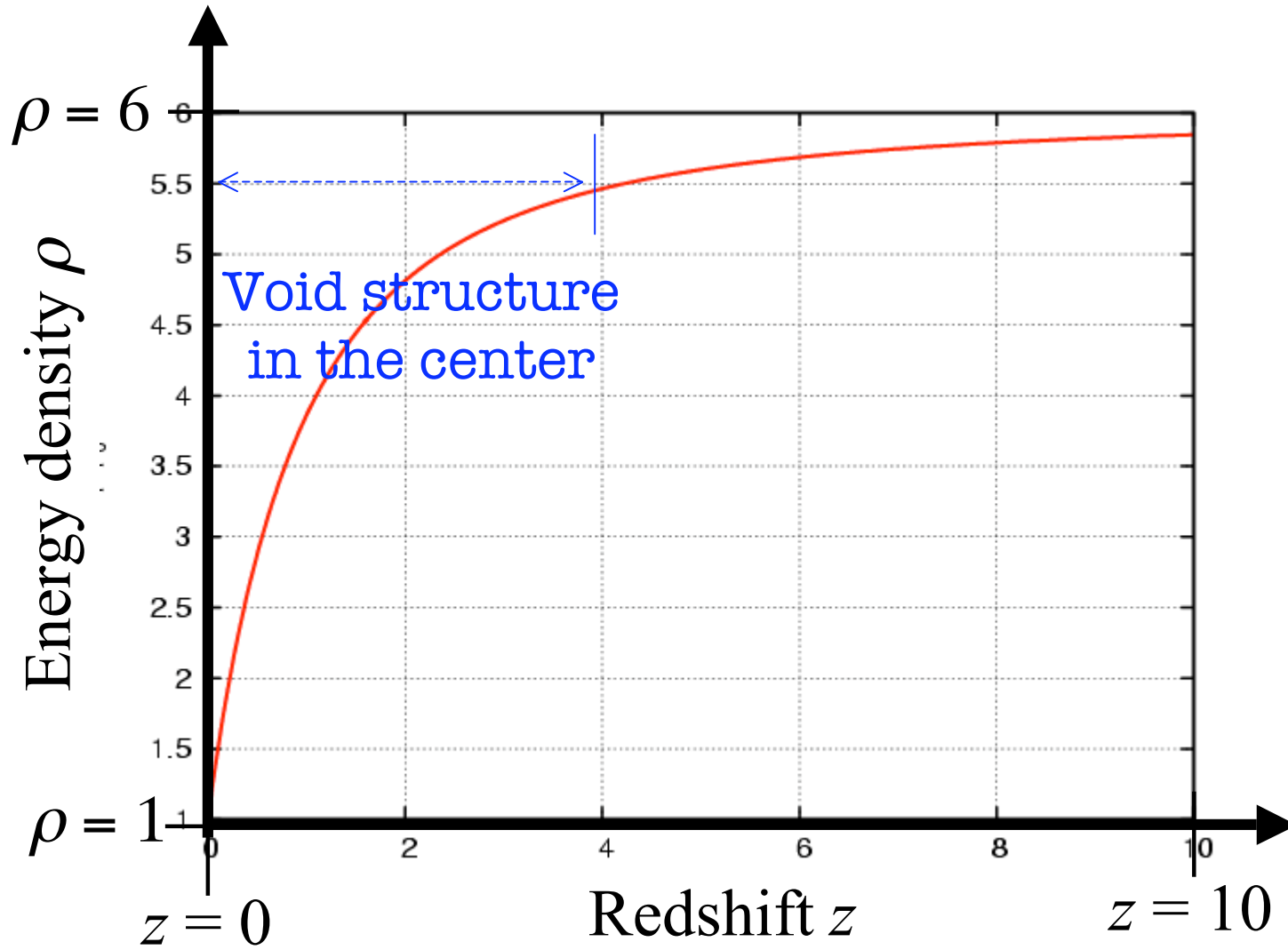
Solving the Inverse Problem

The problem reduces to numerical integration of five coupled ordinary differential equations with regular-singular points

skip details...

We succeeded in getting the LTB universe model which satisfies previous conditions for all redshift.

Void structure

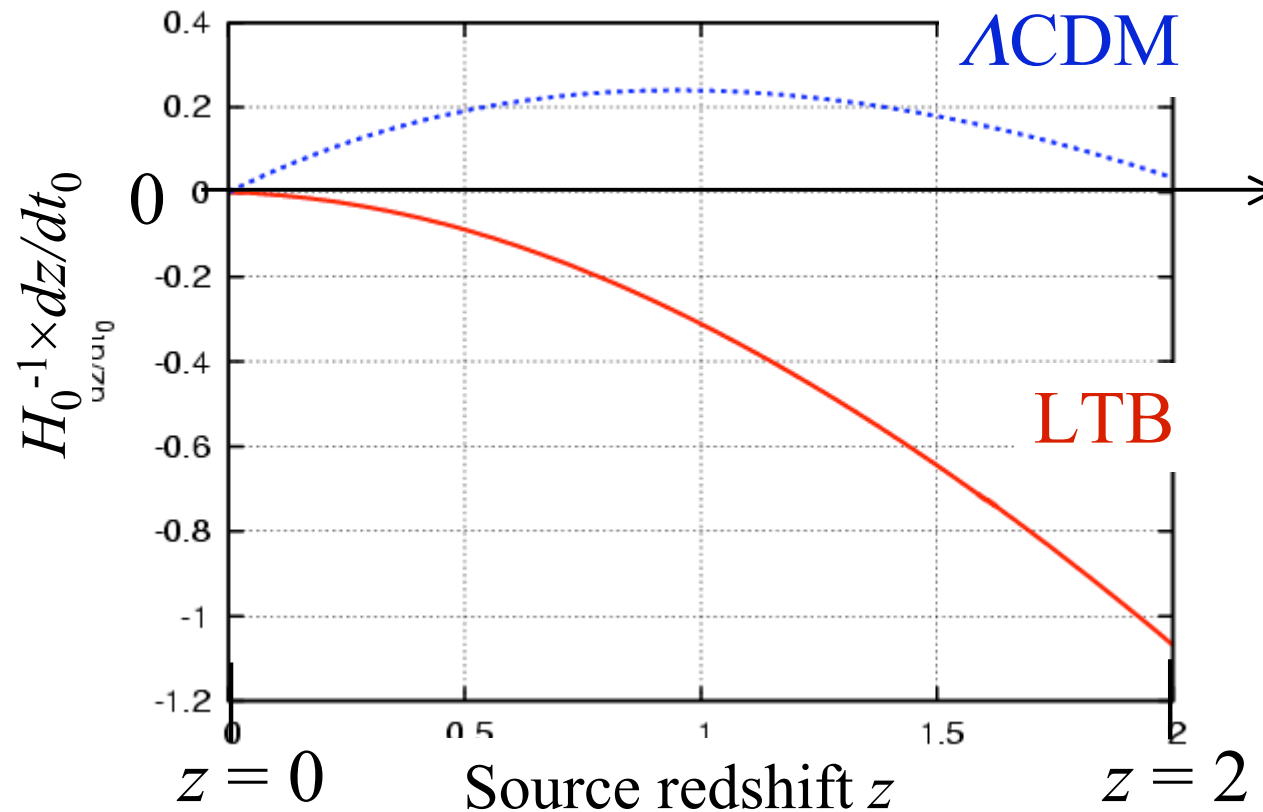


The most important observed quantity

Time derivative of the cosmological redshift

Uzan, Clarkson & Ellis(2007)

We correctly calculated it of our present model.



$dz/dt > 0$ for Λ CDM, whereas $dz/dt < 0$ for the LTB in $0 < z < 2$

Is it possible to observe?

$$\Delta v \approx c\Delta z \approx 1\text{cm/s} \quad \text{for 1 year}$$

$$\Delta v \approx c\Delta z \approx 10\text{cm/s} \quad \text{for 10 years}$$

Nature **452**, 610-612 (3 April 2008)

A laser frequency comb that enables radial velocity measurements with a precision of 1 cm s^{-1}

Chih-Hao Li^{1,2}, Andrew J. Benedick³, Peter Fendel^{3,4}, Alexander G. Glenday^{1,2}, Franz X. Kärtner³, David F. Phillips¹, Dimitar Sasselov¹, Andrew Szentgyorgyi¹ & Ronald L. Walsworth^{1,2}

According to the authors,

“The astro-comb could also be used to directly measure the expansion of the early universe”

Conclusion I

If we observe whether dz/dt is negative or not
for the domain of $0 < z < 2$,
we can distinguish the LTB model from Λ CDM model.

We can test the Copernican principle.

Recent related works

J. Garcia-Berillid and T. Haugbolle [JCAP09(2008)016]

<Looking the void in the eyes-the kinematic Sunyaev-Zeldovich effect in LTB models>

“current observations of only nine clusters with large error bars already rule out LTB models with void size greater than ~ 1.5 Gpc”

J. P. Zibin, A. Moss and D. Scott [arXiv:0809.3761]

<Can we avoid dark energy>

“An appropriate void profile can fit both the latest CMB, supernova data and radial growth rate of BAO scale inhomogeneities. However, this requires either a fine-tuned primordial spectrum or a Hubble rate so low as to rule these model out.”

J. Garcia-Berillid and T. Haugbolle [arXiv:0810.4939]

<The radial BAO scale and Cosmic Shear, a new observable for Inhomogeneous Cosmologies>

“Even though the new data sets are significantly more constraining, LTB models – albeit with slightly larger voids – are still in excellent agreement with observations”

In these analysis, the initial data for perturbations in LTB universes are statistically homogeneous.

We need more study about inhomogeneous universes.

In these analysis, homogeneity of CMB is assumed.

How do we check the homogeneity of CMB?

If we wish to check the Copernican principle, we must not use the Copernican principle.

The time derivative of the cosmological redshift
 dz/dt seems to be the most useful information

4-dimensional information.

Other observational data need some assumptions
(e.g. homogeneity of CMB)

Effects of smaller scale clumpiness

If there are objects in foreground...

Contaminations such as
absorption, scattering, gravitational lensing...

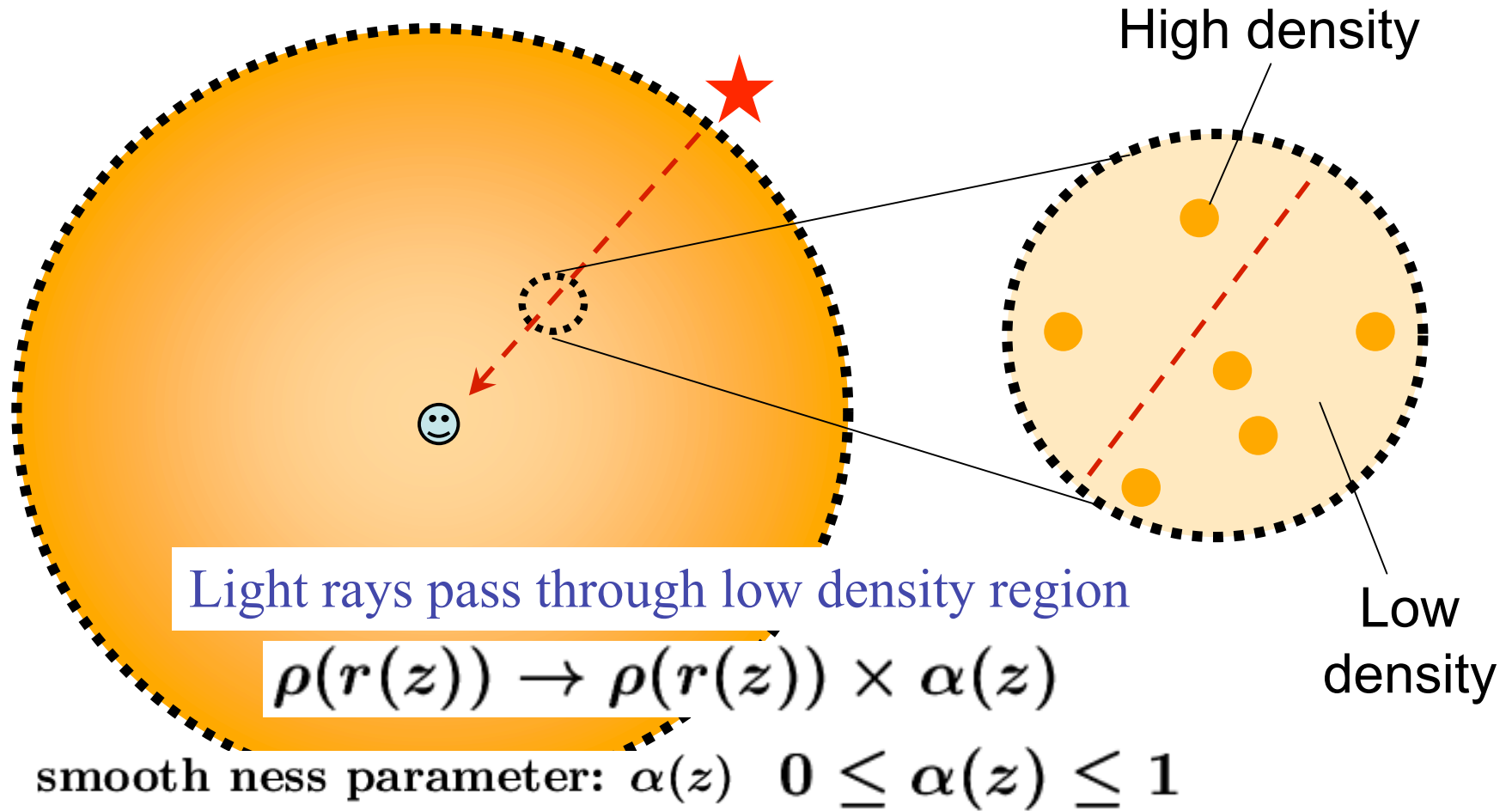
SNe data to determine the distance-redshift relation
includes super novae with few foreground objects.



How dose this selection affect on the inverse problem?

Hybrid Inhomogeneous Model

LTB universe + local clumpiness



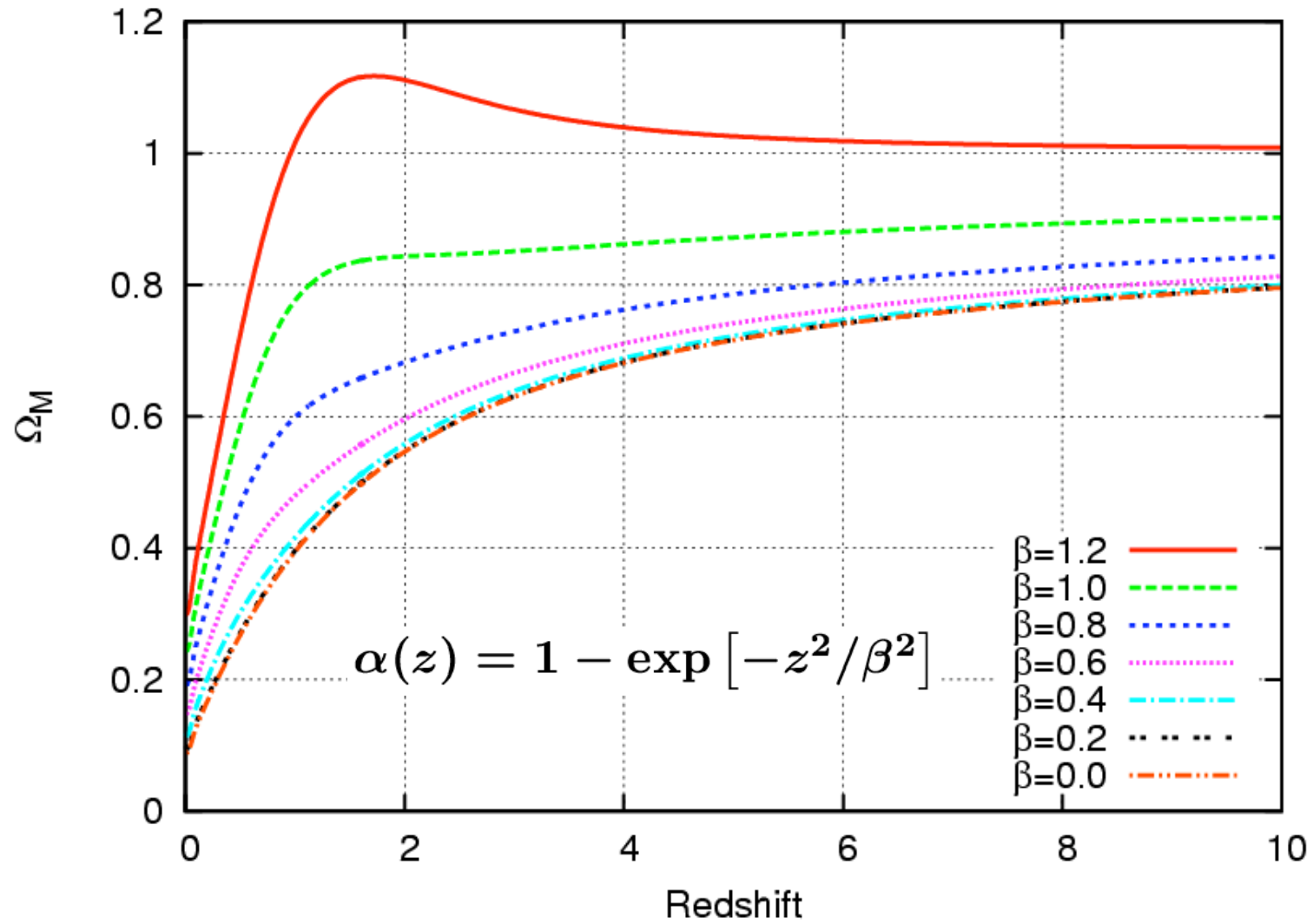
We calculate the following quantities

$$H(t,r) = \frac{\partial_t R}{R} \quad : \text{local Hubble function}$$

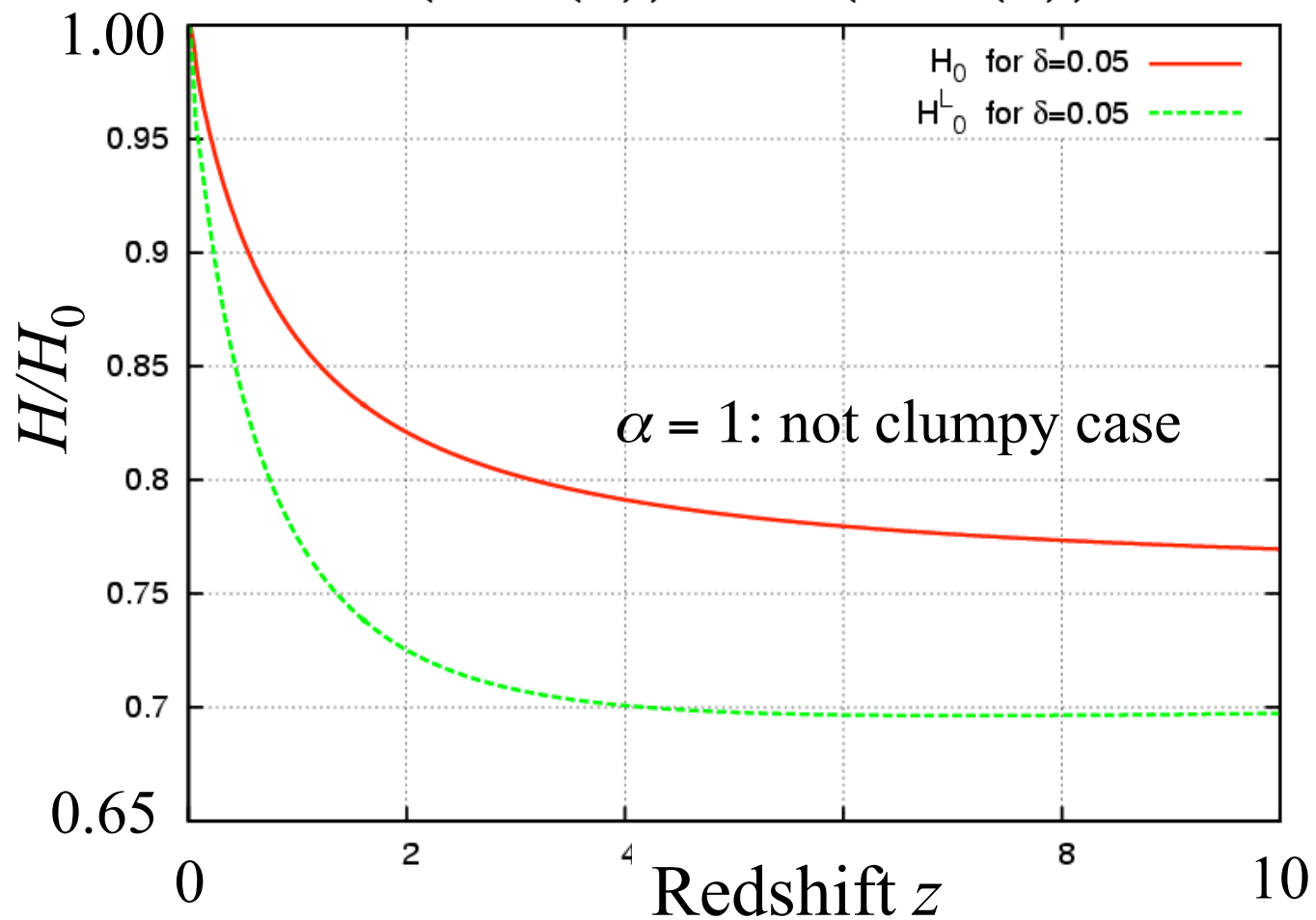
$$H^L(t,r) = \frac{\partial_t \partial_r R}{\partial_r R} \quad : \text{longitudinal expansion rate}$$

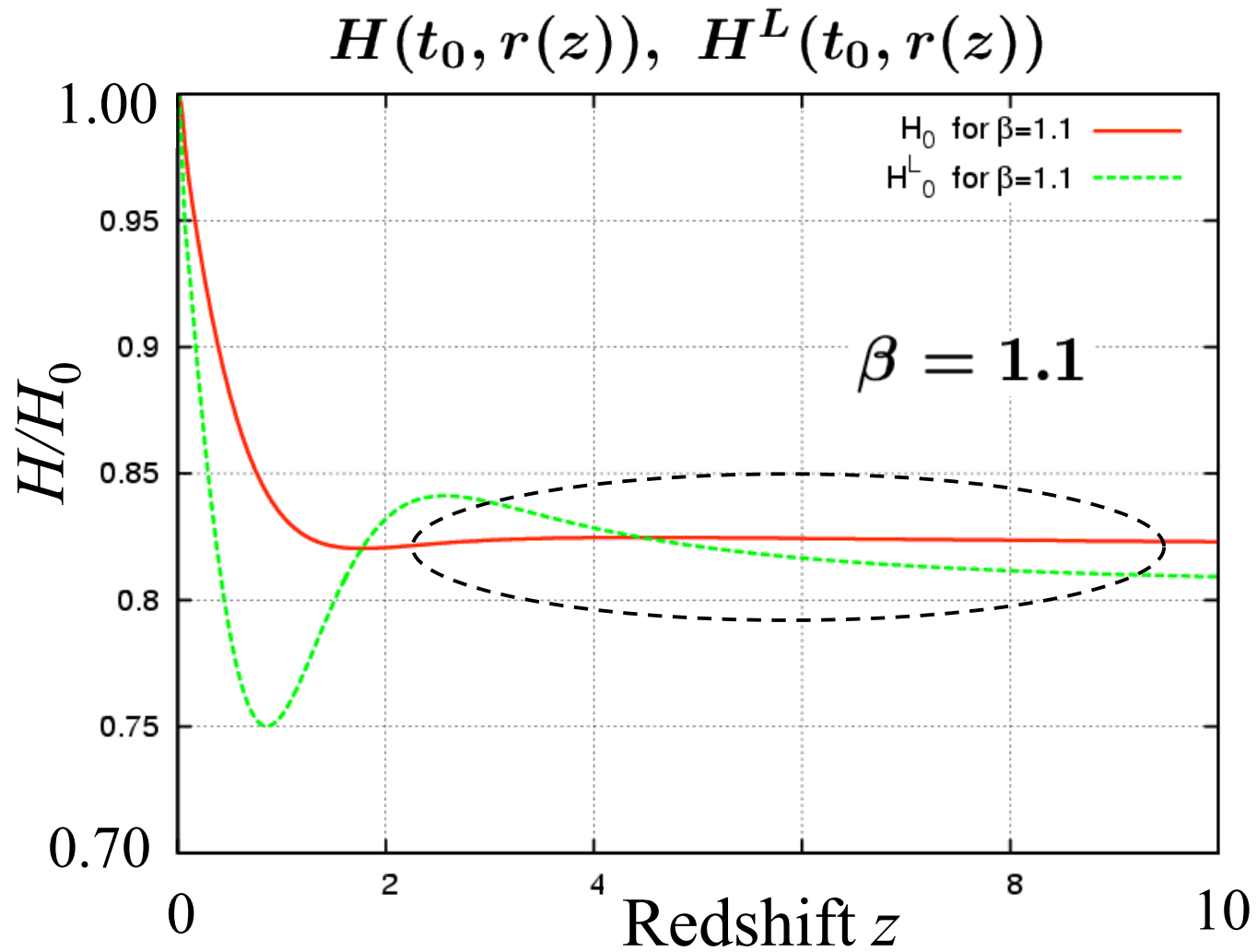
$$\Omega_M(t,r) = \frac{2M(r)}{H^2 R^3(t_0,r)} \quad : \text{density parameter function}$$

$$\Omega_M(t_0, r(z))$$



$$H(t_0, r(z)), H^L(t_0, r(z))$$





Appropriate β reduces the strength of inhomogeneities

We have reached the stage at which “the Copernican principle” will be confirmed observationally?

Thank you for your attention.



▪ The effects of inhomogeneities on cosmic volume expansion

- Post Newtonian; Futamase(1988)
- Relativistic; Buchert(1999,...) ,
Abramo, Brandenberger & Mukanov(1997..) ,
Nambu(2005),
Notari(2005), Kolb, Matarrese, Notari & Riotto(2005),...
Flanagan(2005),
Hirata & Seljak(2005),
Ishibashi & Wald(2005),
Kasai, Asada & Futamase(2006),
Kai, Kozaki, Nakao, Nambu & Yoo(2007), ,
Paranjape & Singh(2008)