

# Cosmological equivalence principle & dark energy without dark energy

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DLW: **New J. Phys.** 9 (2007) 377

**Phys. Rev. Lett.** 99 (2007) 251101

arXiv:0712.3984

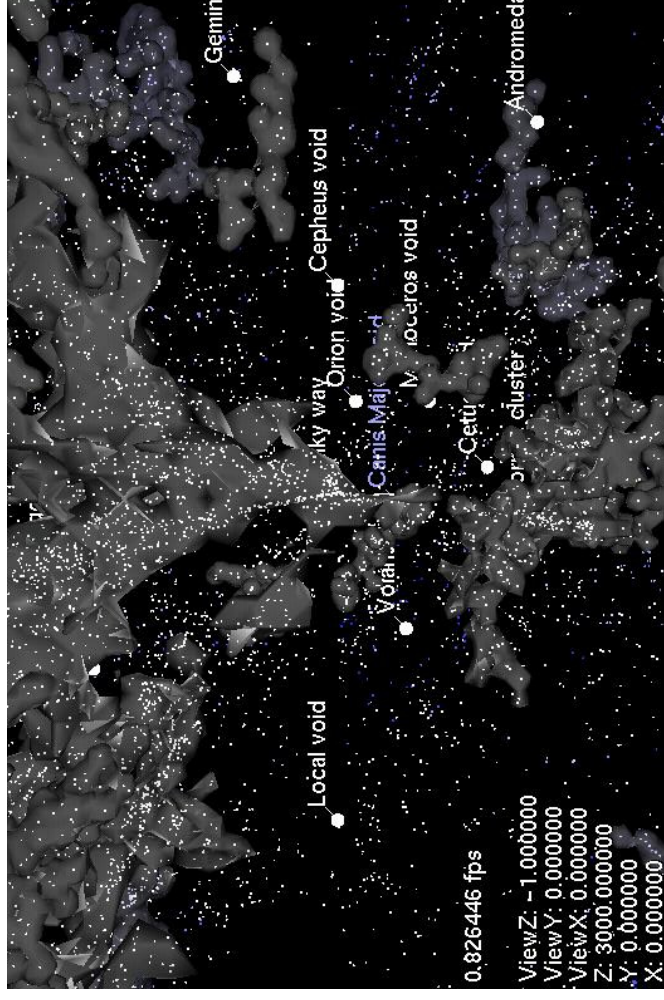
**Phys. Rev. D** 78 (2008) 084032

[arXiv:0809.1183];

**new results, to appear**

B.M. Leith, S.C.C. Ng and DLW:

**ApJ** 672 (2008) L91



# What is “dark energy”?

- Usual explanation: a homogeneous isotropic form of “stuff” which violates the strong energy condition.  
(Locally pressure  $P = w\rho c^2$ ,  $w < -\frac{1}{3}$ ;  
e.g., for cosmological constant,  $\Lambda$ ,  $w = -1$ .)
- My hypothesis: in ordinary general relativity, a manifestation of global *variations* of those aspects of gravitational energy which by virtue of the equivalence principle cannot be localised – the *cosmological quasilocal gravitational energy* associated with *dynamical gradients* in spatial curvature generated by a universe as inhomogeneous as the one we observe.  
[Call this *dark energy* if you like. It involves *energy*; and “nothing” is dark.]

# Overview

I will write down a viable model for the observed universe, with its almost isotropic Hubble flow but inhomogeneous matter distribution, considering

- the definition of gravitational energy;
- the decoupling bound systems from the expansion of space;
- operational issues associated with measurements and averaging in an inhomogeneous universe;
- resolving the Sandage-de Vaucouleurs paradox;
- understanding the problem of obtaining a void-dominated universe;
- how to realistically obtain apparent cosmological acceleration without exotic dark energy

# From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ( $\delta\rho/\rho \sim 10^{-5}$  in photons and baryons;  $\sim 10^{-4}, 10^{-3}$  in non-baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter  $30h^{-1}$  Mpc. [Hubble constant  $H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1}$ ] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void-dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

# The Sandage-de Vaucouleurs paradox...

- Matter homogeneity only observed at  $\gtrsim 200$  Mpc scales
- If “the coins on the balloon” are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than  $\sim 200$  Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically “quiet” .
- Can we explain this as an effect of dark energy? Maybe. Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy).
- Empirical results do not appear to match best-fit  $\Lambda$ CDM parameters (Axenides & Perivolaropoulos, PRD 65 (2002) 127301).

# Inhomogeneous cosmology

- Need an averaging scheme to extract the average homogeneous geometry
- Only exact approaches dealing with *averages* of full non-linear Einstein equations considered here (NOT perturbation theory: Kolb et al...; NOT LTB models etc)
- Still many approaches, with different assumptions
- Do we average tensors on curves of observers (Zalaletdinov 1992, 1993) ... recent work Coley, Pelavas, and Zalaletdinov, PRL 95 (2005) 151102; Coley and Pelavas, PR D75 (2007) 043506
- Can we get away with averaging scalars (density, pressure, shear ...)? (Buchert 2000, 2001) ... recent work Buchert CQG 23 (2006) 817; Astron. Astrophys. 454 (2006) 415; Gen. Rel. Grav. 40 (2008) 467 etc

# Carfora-Piotrkowska-Buchert-Ehlers -Russ-Soffel-Kasai-Börner equations

For irrotational dust cosmologies, with energy density,  $\rho(t, \mathbf{x})$ , expansion,  $\theta(t, \mathbf{x})$ , and shear,  $\sigma(t, \mathbf{x})$ , on a compact domain,  $\mathcal{D}$ , of a suitably defined spatial hypersurface of constant average time,  $t$ , and spatial 3-metric, average cosmic evolution described by exact equations

$$3\frac{\dot{a}^2}{\bar{a}^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle\mathcal{R}\rangle - \frac{1}{2}\mathcal{Q}$$

$$3\frac{\ddot{\bar{a}}}{\bar{a}} = -4\pi G\langle\rho\rangle + \mathcal{Q}$$

$$\partial_t\langle\rho\rangle + 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0$$

$$\partial_t(\bar{a}^6\mathcal{Q}) + \bar{a}^4\partial_t(\bar{a}^2\langle\mathcal{R}\rangle) = 0.$$

$$\mathcal{Q} \equiv \frac{2}{3}(\langle\theta^2\rangle - \langle\theta\rangle^2) - 2\langle\sigma^2\rangle$$

# Back-reaction

Angle brackets denote the spatial volume average, e.g.,

$$\langle \mathcal{R} \rangle \equiv \left( \int_{\mathcal{D}} d^3x \sqrt{\det^3 g} \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

$$\langle \theta \rangle = 3 \frac{\dot{a}}{a}$$

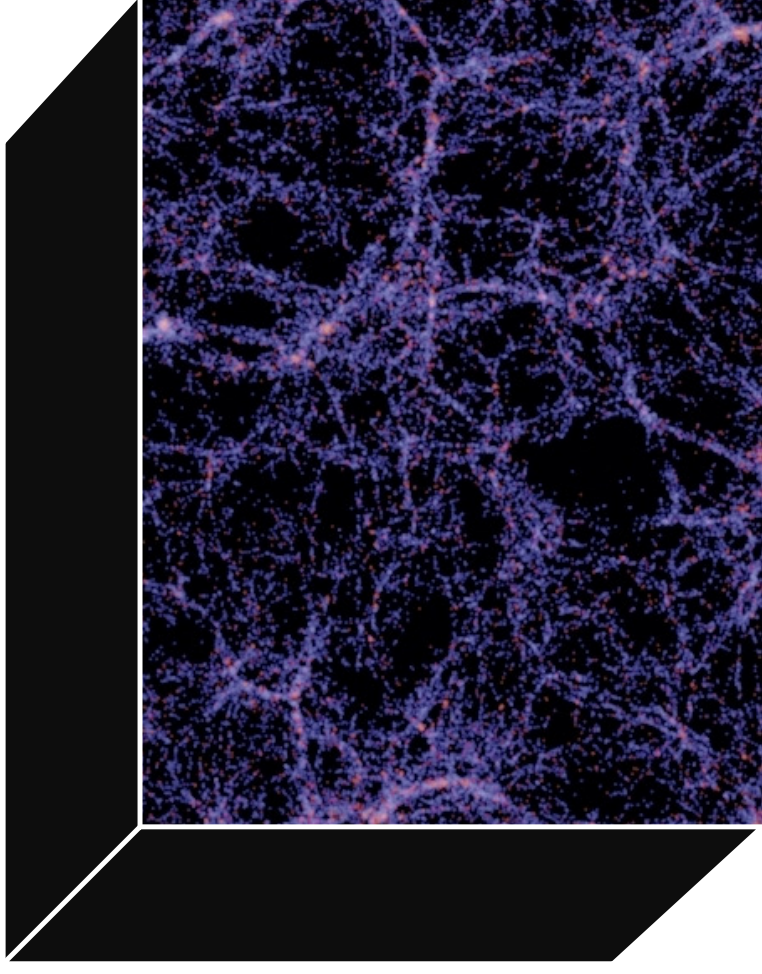
Generally for any scalar  $\Psi$ ,

$$\frac{d}{dt} \langle \Psi \rangle - \left\langle \frac{d\Psi}{dt} \right\rangle = \langle \Psi \theta \rangle - \langle \theta \rangle \langle \Psi \rangle$$

- The extent to which the back-reaction,  $\mathcal{Q}$ , can lead to apparent cosmic acceleration or not has been the subject of much debate.



# Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which  $\delta\rho/\rho \sim -1$ .
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rods and clocks and volume average ones

# The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume-average environment (void)

# Dilemma of gravitational energy...

- In GR spacetime carries energy & angular momentum

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle,  $T_{\mu\nu}$  contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in  $G_{\mu\nu}$ : variations are “quasilocal”!
- Newtonian version,  $T - U = -V$ , of Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where  $T = \frac{1}{2}m\dot{a}^2x^2$ ,  $U = -\frac{1}{2}kmc^2x^2$ ,  $V = -\frac{4}{3}\pi G\rho a^2x^2m$ ;  
 $\mathbf{r} = a(t)\mathbf{x}$ .

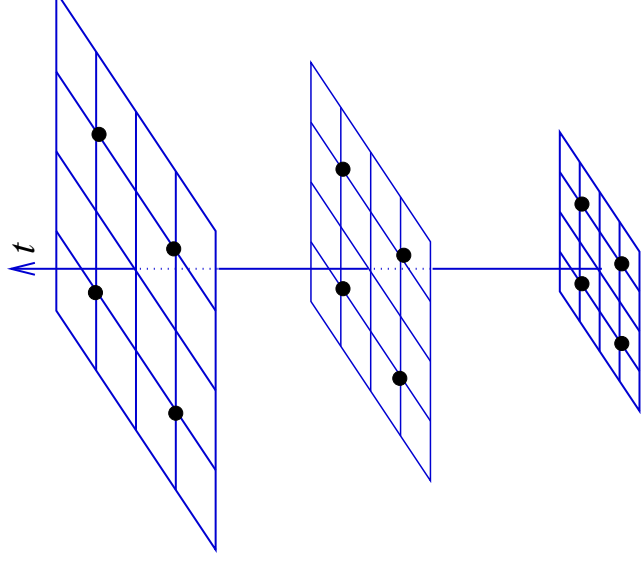
# Ricci curvature and gravitational energy

- For Lemaître–Tolman–Bondi models constant spatial curvature replaced by energy function with  $E(r) > 0$  in regions of negative spatial curvature.
- In quasilocal Hamiltonian approach of Chen, Nester and Liu (MPL A22 (2007) 2039) relative to a fiducial static Cartesian reference frame a comoving observer in  $k = -1$  FLRW universe sees negative quasilocal energy; or relative to the static frame the comoving observer has positive quasilocal energy.
- For perturbation theory I advocate “Machian gauge” of Bičak, Katz and Lynden–Bell (PR D76 (2007) 063501): uniform Hubble flow plus minimal shift distortion condition.

# Back to first principles...

- Standard approach assumes single global FLRW frame plus Newtonian perturbations
- In absence of exact background symmetries, Newtonian approximation requires a weak field approximation about suitable static Minkowski frame
- What is the largest scale on which the Strong Equivalence Principle can be applied?
- Need to address Mach's principle: "*Local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions*"
- How does coarse-graining affect relative calibration of clocks and rods, from local to global, to account for average effects of gravity?

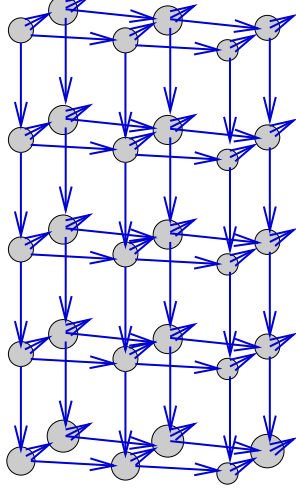
# Back to first principles...



- Homogeneous isotropic volume expansion is locally indistinguishable from equivalent motion in static Minkowski space; on local scales

$$z \simeq \frac{H_0 \ell_r}{c}, \quad H_0 = \left. \frac{\dot{a}}{a} \right|_{t_0}$$

# Semi-tethered lattice



- Extend to decelerating motion over long time intervals by Minkowski space analogue (semi-tethered lattice - indefinitely long tethers with one end fixed, one free end on spool, apply brakes synchronously at each site)
- Brakes convert kinetic energy of expansion to heat and so to other forms
- Brake impulse can be arbitrary pre-determined function of local proper time; but provided it is synchronous deceleration remains homogeneous and isotropic: *no net force on any lattice observer*

# Cosmological Equivalence Principle

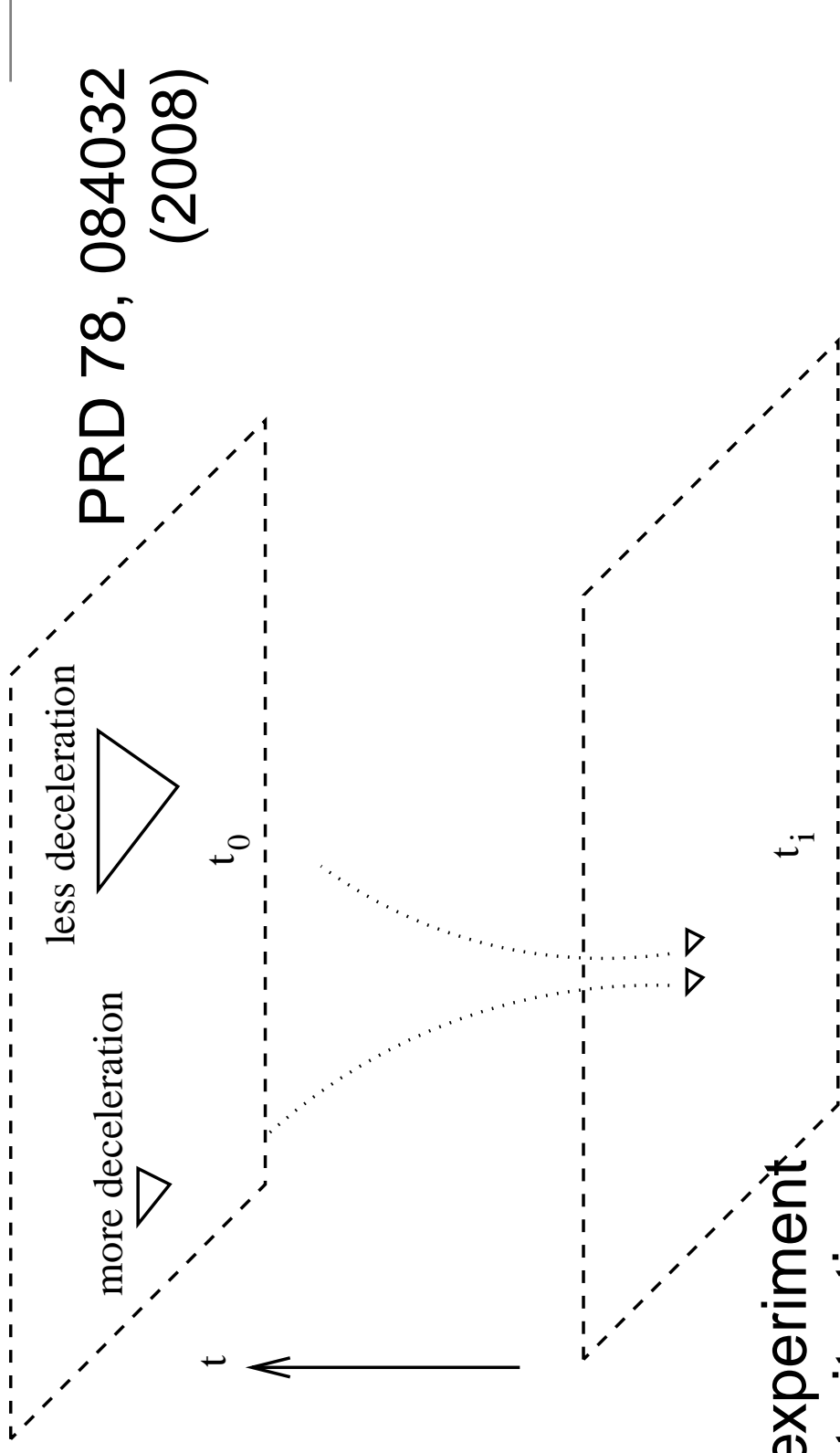
- *At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation,*

$$ds_{\text{CIF}}^2 = a^2(\eta) [-d\eta^2 + dr^2 + r^2 d\Omega^2],$$

- Defines Cosmological Inertial Frame (CIF)
- Accounts for regional average effect of density in terms of frames for which the state of rest in an expanding space is indistinguishable from decelerating expansion of particles moving in a static space



# Cosmological Equivalence Principle

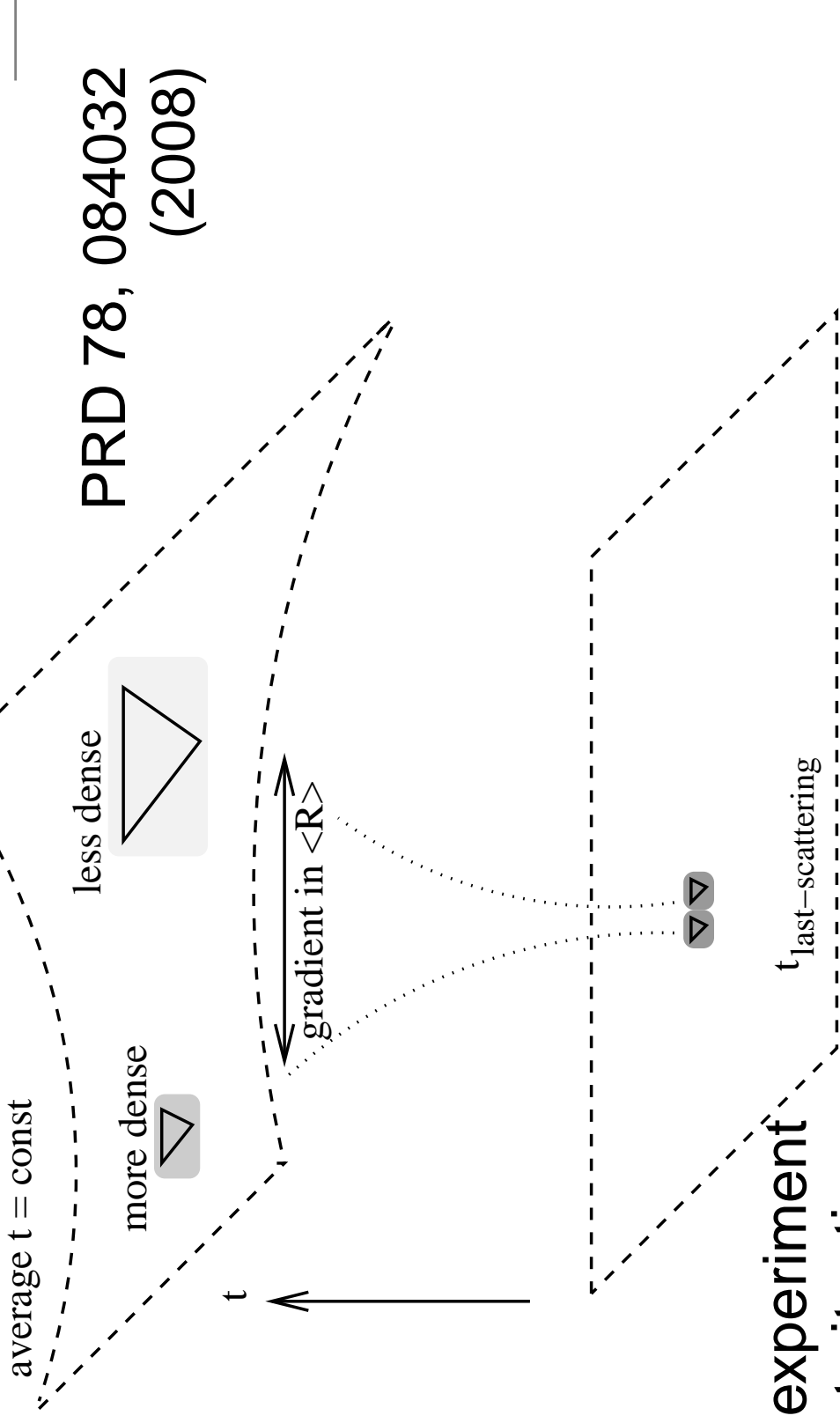


PRD 78, 084032  
(2008)

Thought experiment:  
equivalent situations:

- SR: observers in disjoint regional semi-tethered lattices volume decelerate at different rates
- Those who decelerate more age less

# Cosmological Equivalence Principle



PRD 78, 084032  
(2008)

Thought experiment equivalent situations:

- GR: regions of different density have different volume deceleration (for same initial conditions)
- Those in denser region age less

# Bound and unbound systems...

- Isotropic observers “at rest” within expanding space in voids may have clocks ticking at a rate  $d\tau_v = \gamma(\tau_w, \mathbf{x})d\tau_w$  with respect to static observers in bound systems.  
Volume average:  $dt = \bar{\gamma}_w d\tau_w$ ,  $\bar{\gamma}_w(\tau_w) = \langle -\xi^\mu n_\mu \rangle_{\mathcal{H}}$
- We are not restricted to  $\gamma = 1 + \epsilon$ ,  $\epsilon \ll 1$ , as expected for typical variations of binding energy.
- Observable universe is assumed unbound.
- I find  $\bar{\gamma}_w \simeq 1.38$  at present epoch from relative regional deceleration  $\sim 10^{-10} \text{ms}^{-2}$  integrated over age of universe. (N.B. Absolute upper bound:  $\bar{\gamma}_w < 1.5$ .)
- Where is infinity? In 1984 George Ellis suggested a notion of *finite infinity*: a region within which isolated systems, such as stars or galaxies, or galaxy clusters are approximately independent dynamical systems.

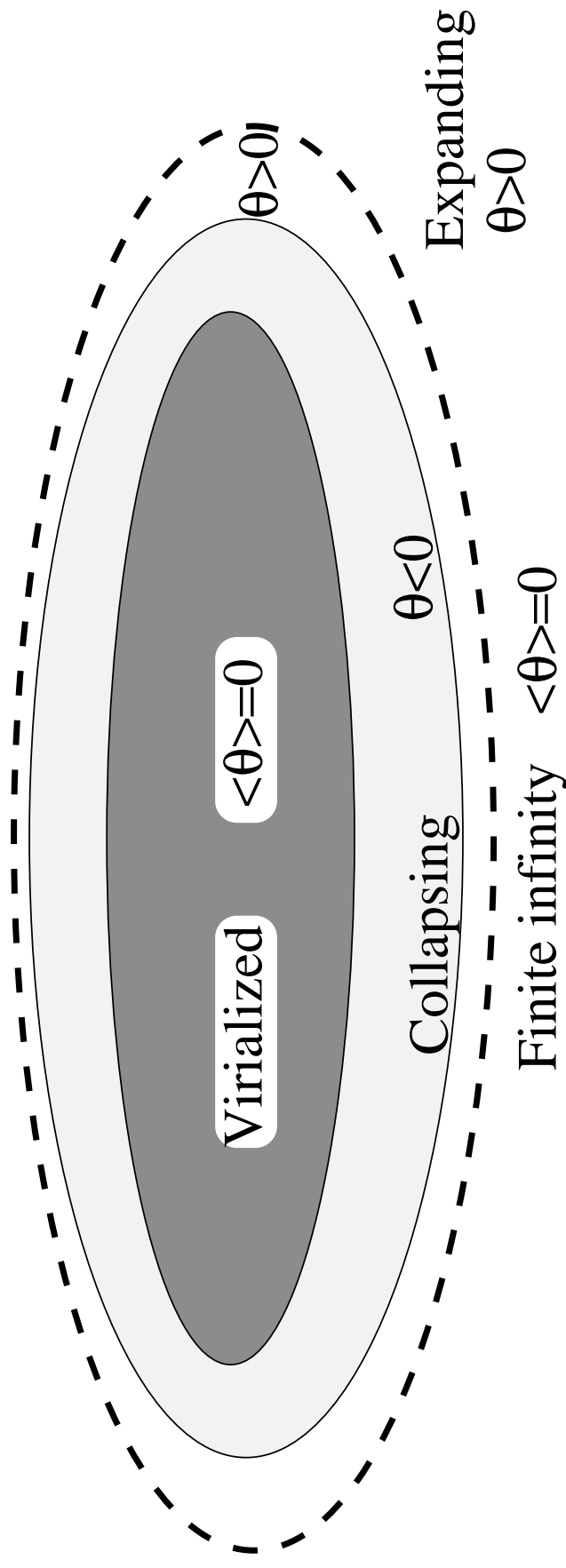
# Where is infinity?

- Inflation provides us with boundary conditions.
- Initial smoothness at last-scattering ensures a uniform initial expansion rate. For gravity to overcome this a universal critical density exists.  
BUT if we assume a smooth average evolution we can overestimate the critical density today.

$$\rho_{\text{cr}} \neq \frac{3H_{\text{av}}^2}{8\pi G}$$

- Identify finite infinity relative to demarcation between bound and unbound systems, depending on the time evolution of the true critical density since last-scattering.
- Normalise *wall time*,  $\tau_w$ , as the time at finite infinity, (close to galaxy clocks) by  $\langle -\xi^\mu n_\mu \rangle_{\mathcal{F}_I} = \langle \gamma(\tau_w, \mathbf{x}) \rangle_{\mathcal{F}_I} = 1$ .

# Finite infinity



- Define *finite infinity*, "*f*" as boundary to minimal connected region within which *average expansion* vanishes  $\langle \theta \rangle = 0$  or average curvature vanishes  $\langle R \rangle = 0$ .
- Shape of *f* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

# Cosmic rest frame

- Patch together CIFs for observers who see an isotropic CMB by taking surfaces of uniform volume expansion

$$\left\langle \frac{1}{\ell_r(\tau)} \frac{d\ell_r(\tau)}{d\tau} \right\rangle = \frac{1}{3} \langle \theta \rangle_1 = \frac{1}{3} \langle \theta \rangle_2 = \dots = \bar{H}(\tau)$$

- Average over regions in which (i) spatial curvature is zero or negative; (ii) space is expanding at the boundaries, at least marginally.
- Solves the Sandage–de Vaucouleurs paradox implicitly.
- Voids appear to expand faster; but their local clocks tick faster, locally measured expansion can still be uniform.
- Global average  $H_{\text{av}}$  on large scales with respect to *any* one set of clocks will differ from  $\bar{H}$

# Two/three scale model

$$\bar{a}^3 = f_{wi} a_w^3 + f_{vi} a_v^3$$

- Splits into void fraction with scale factor  $a_v$  and “wall” fraction with scale factor  $a_w$ . Assume  $\delta^2 H_w = \frac{1}{3} \langle \sigma^2 \rangle_w$ ,  $\delta^2 H_v = \frac{1}{3} \langle \sigma^2 \rangle_v$ .
- Buchert equations for volume averaged observer, with  $f_v(t) = f_{vi} a_v^3 / \bar{a}^3$  (void volume fraction) and  $k_v < 0$

$$\frac{\ddot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_v^2}{9f_v(1-f_v)} - \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3} \bar{\rho}_0 \frac{\bar{a}_0^3}{\bar{a}^3},$$

$$\ddot{f}_v + \frac{\dot{f}_v^2(2f_v - 1)}{2f_v(1-f_v)} + 3\frac{\dot{\bar{a}}}{\bar{a}} \dot{f}_v - \frac{3\alpha^2 f_v^{1/3}(1-f_v)}{2\bar{a}^2} = 0,$$

if  $f_v(t) \neq \text{const}$ ; where  $\alpha^2 = -k_v f_{vi}^{2/3}$ .

# Two/three scale model

- Universe starts as Einstein–de Sitter, from boundary conditions at last scattering consistent with CMB; almost no difference in clock rates initially.
- We must be careful to account for clock rate variations. Buchert's clocks are set at the *volume average* position, with a rate between wall clocks and void clock extreme.

$$\bar{H}(t) = \bar{\gamma}_w H_w = \bar{\gamma}_v H_v; \quad H_w \equiv \frac{1}{a_w} \frac{da_w}{dt}, \quad H_v \equiv \frac{1}{a_v} \frac{da_v}{dt}$$

where  $\bar{\gamma}_v = \frac{dt}{d\tau_v}$ ,  $\bar{\gamma}_w = \frac{dt}{d\tau_w} = 1 + (1 - h_r) f_v / h_r$ ,  
 $h_r = H_w / H_v < 1$ .

- Need to be careful to obtain global  $H_{av}$  in terms of one set of isotropic observer wall clocks,  $\tau_w$ .



# Bare cosmological parameters

- Different sets of cosmological parameters are possible
- Bare cosmological parameters are defined as fractions of the true critical density related to the bare Hubble rate

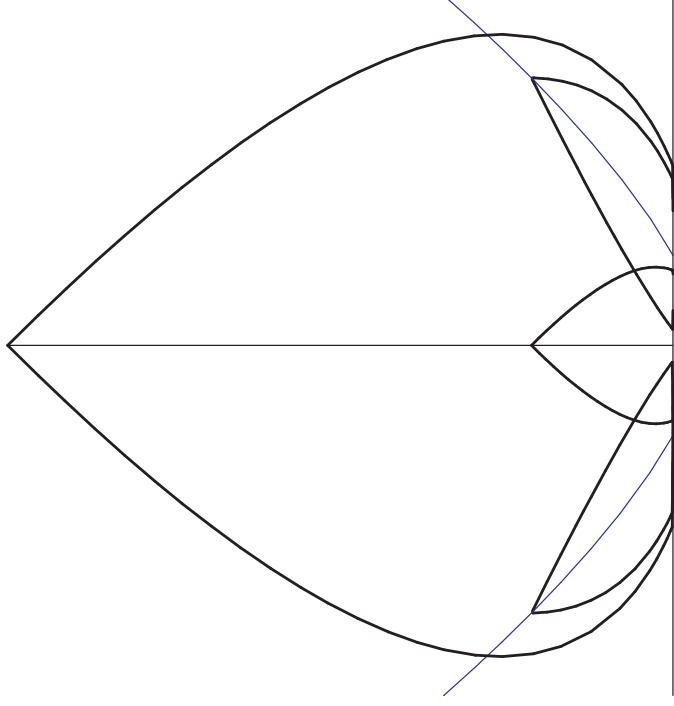
$$\bar{\Omega}_M = \frac{8\pi G\bar{\rho}_{M0}\bar{a}_0^3}{3\bar{H}^2\bar{a}^3},$$

$$\bar{\Omega}_k = \frac{\alpha^2 f_v^{1/3}}{\bar{a}^2 \bar{H}^2},$$

$$\bar{\Omega}_Q = \frac{-\dot{f}_v^2}{9f_v(1-f_v)\bar{H}^2}.$$

- These are the volume-average parameters, with first Buchert equation:  $\bar{\Omega}_M + \bar{\Omega}_k + \bar{\Omega}_Q = 1$ .

# Past light cone average



- Interpret Buchert solution by radial null cone average

$$ds^2 = -dt^2 + \bar{a}^2(t) d\bar{\eta}^2 + A(\bar{\eta}, t) d\Omega^2,$$

- LTB metric but NOT an LTB solution
- Conformally match radial null geodesics to those of finite infinity geometry with uniform local Hubble flow.

# Dressed cosmological parameters

- Conventional parameters for “wall observers” in galaxies: defined by assumption (no longer true) that others in entire observable universe have synchronous clocks and same local spatial curvature

$$\begin{aligned} ds_{\mathcal{F}_I}^2 &= -d\tau_w^2 + a_w^2(\tau_w) [d\eta_w^2 + \eta_w^2 d\Omega^2] \\ \rightarrow ds^2 &= -d\tau_w^2 + \frac{\bar{a}^2}{\bar{\gamma}_w^2} [d\bar{\eta}^2 + r_w^2(\bar{\eta}, \tau_w) d\Omega^2] \end{aligned}$$

where  $r_w \equiv \bar{\gamma}_w (1 - f_v)^{1/3} f_{wi}^{-1/3} \eta_w(\bar{\eta}, \tau_w)$ , and volume-average conformal time  $d\bar{\eta} = dt/\bar{a} = \bar{\gamma}_w d\tau_w/\bar{a}$ .

- This leads to conventional dressed parameters which do not sum to 1, e.g.,

$$\Omega_M = \bar{\gamma}_w^3 \bar{\Omega}_M.$$

# Tracker solution PRL 99, 251101

- General exact solution possesses a “tracker limit”

$$\bar{a} = \frac{\bar{a}_0(3\bar{H}_0 t)^{2/3}}{2 + f_{v0}} \left[ 3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0}) \right]^{1/3}$$

$$f_v = \frac{3f_{v0}\bar{H}_0 t}{3f_{v0}\bar{H}_0 t + (1 - f_{v0})(2 + f_{v0})},$$

- Void fraction  $f_v(t)$  determines many parameters:

$$\bar{\gamma}_w = 1 + \frac{1}{2}f_v = \frac{3}{2}\bar{H}t$$

$$\tau_w = \frac{2}{3}t + \frac{2(1 - f_{v0})(2 + f_{v0})}{27f_{v0}\bar{H}_0} \ln \left( 1 + \frac{9f_{v0}\bar{H}_0 t}{2(1 - f_{v0})(2 + f_{v0})} \right)$$

$$\bar{\Omega}_M = \frac{4(1 - f_v)}{(2 + f_v)^2}$$

# Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2(1 - f_v)^2}{(2 + f_v)^2}.$$

As  $t \rightarrow \infty$ ,  $f_v \rightarrow 1$  and  $\bar{q} \rightarrow 0^+$ .

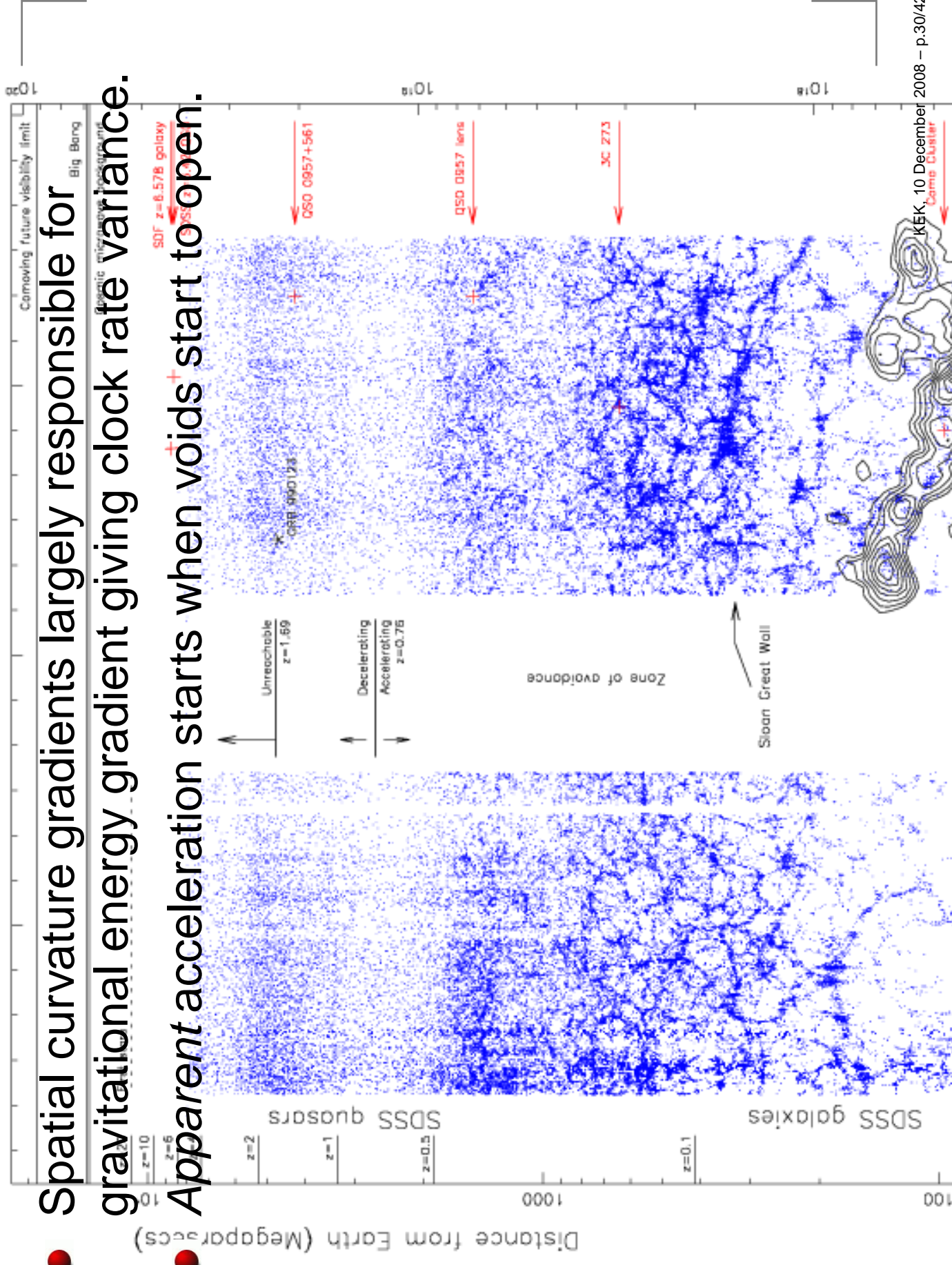
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-(1 - f_v)(8f_v^3 + 39f_v^2 - 12f_v - 8)}{(4 + f_v + 4f_v^2)^2},$$

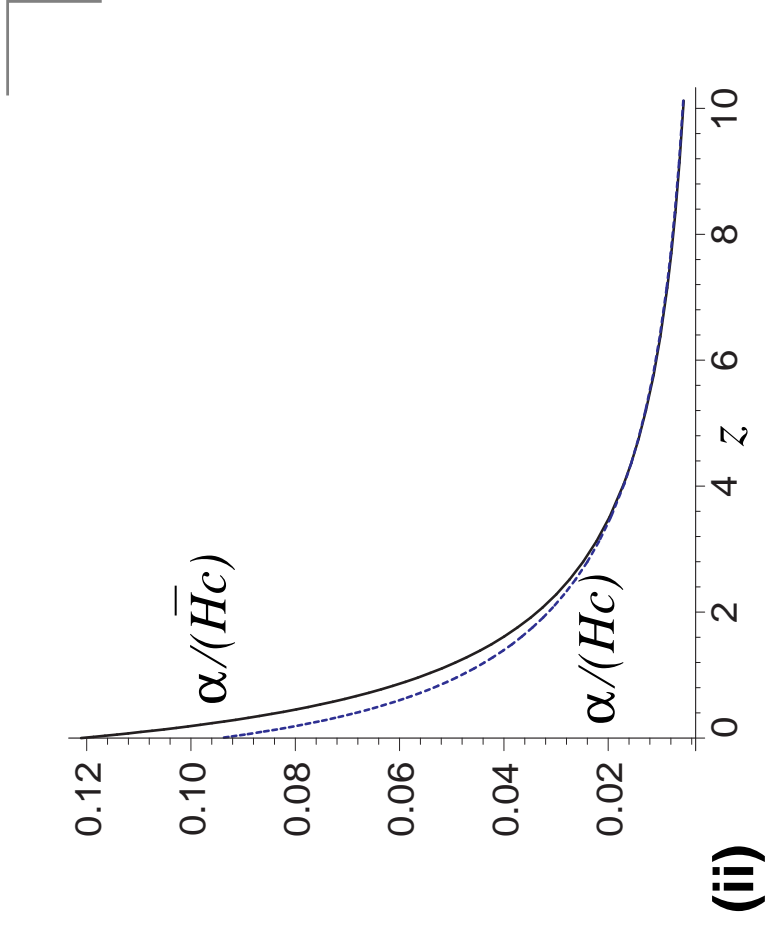
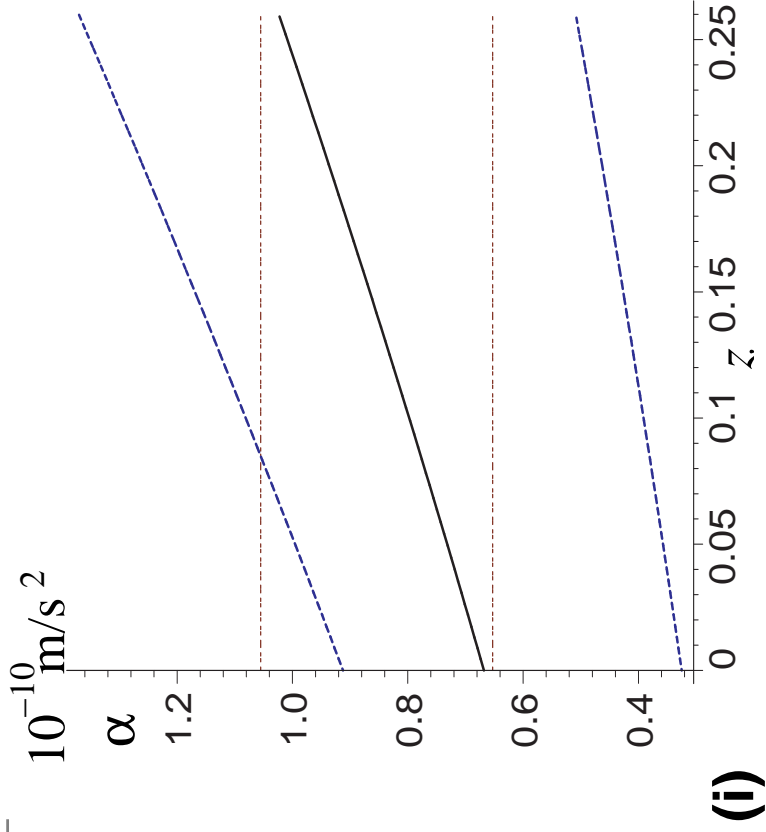
Effective deceleration parameter starts at  $q \sim \frac{1}{2}$ , for small  $f_v$ ; changes sign when  $f_v = 0.58670773\dots$ , and approaches  $q \rightarrow 0^-$  at late times.

# Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- Apparent acceleration starts when voids start to open.



# CEP relative acceleration scale



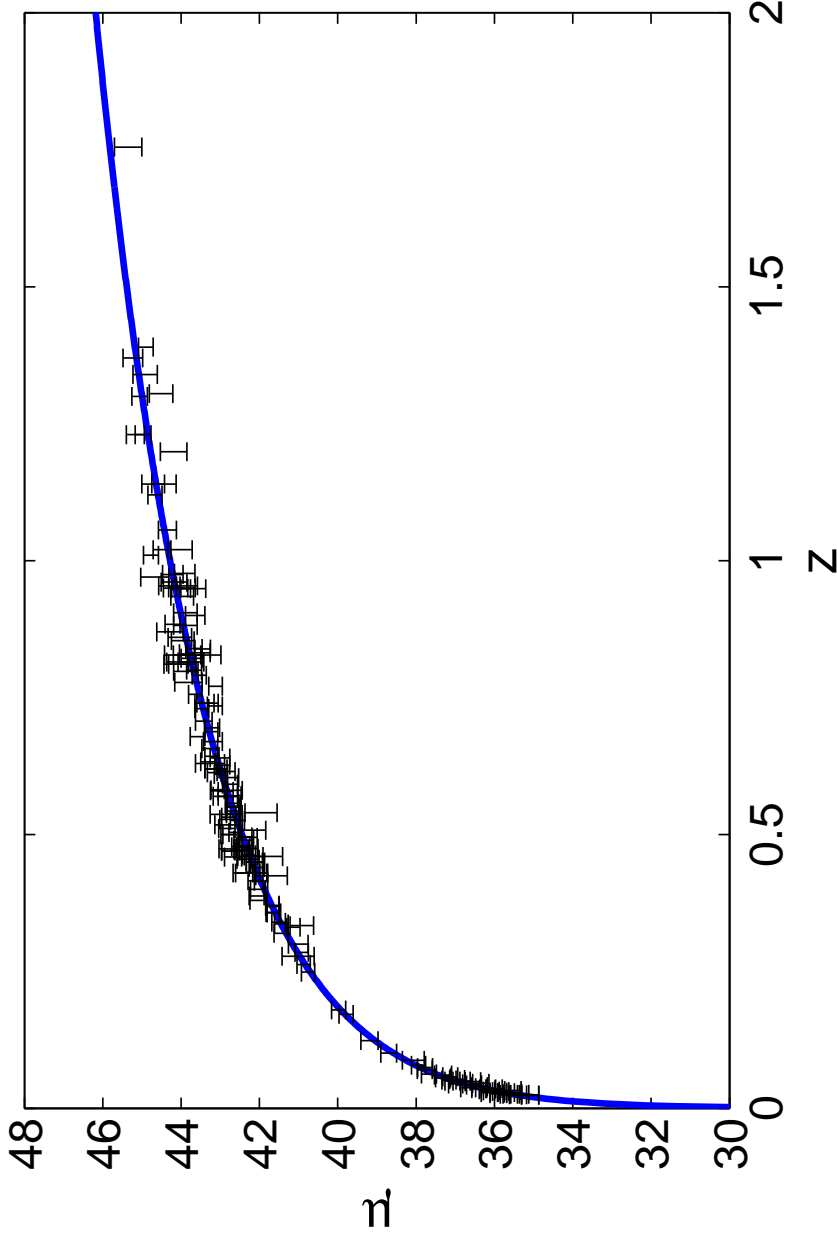
By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude  $\alpha = H_0 c \bar{\gamma}_w \dot{\gamma}_w / (\sqrt{\bar{\gamma}_w^2 - 1})$  beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large  $z$ .

● For  $z \lesssim 0.25$ , coincides with empirical MOND scale

$$\alpha_0 = 1.2^{+0.3}_{-0.2} \times 10^{-10} \text{ ms}^{-2} h_{75}^2 = 8.1^{+2.5}_{-1.6} \times 10^{-11} \text{ ms}^{-2} \text{ for}$$

$$H_0 = 61.7 \text{ km sec}^{-1} \text{ Mpc}^{-1}.$$

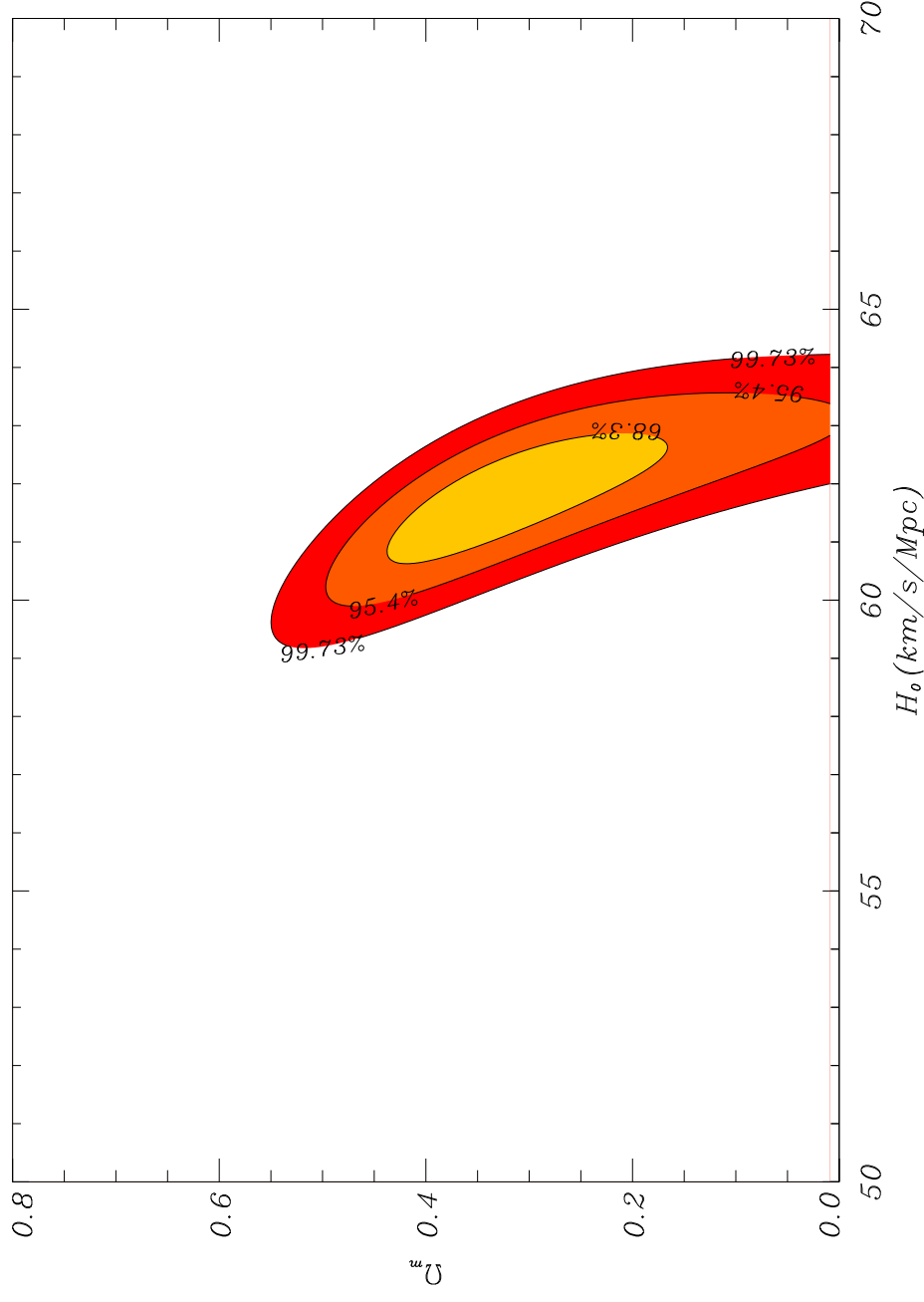
# Test 1: Snela luminosity distances



- Type Ia supernovae of Riess07 Gold data set fit with  $\chi^2$  per degree of freedom = 0.9
- With  $55 \leq H_0 \leq 75 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ ,  $0.01 \leq \Omega_{M0} \leq 0.5$ , find Bayes factor  $\ln B = 0.27$  in favour of FB model (marginally): statistically indistinguishable from  $\Lambda$ CDM.



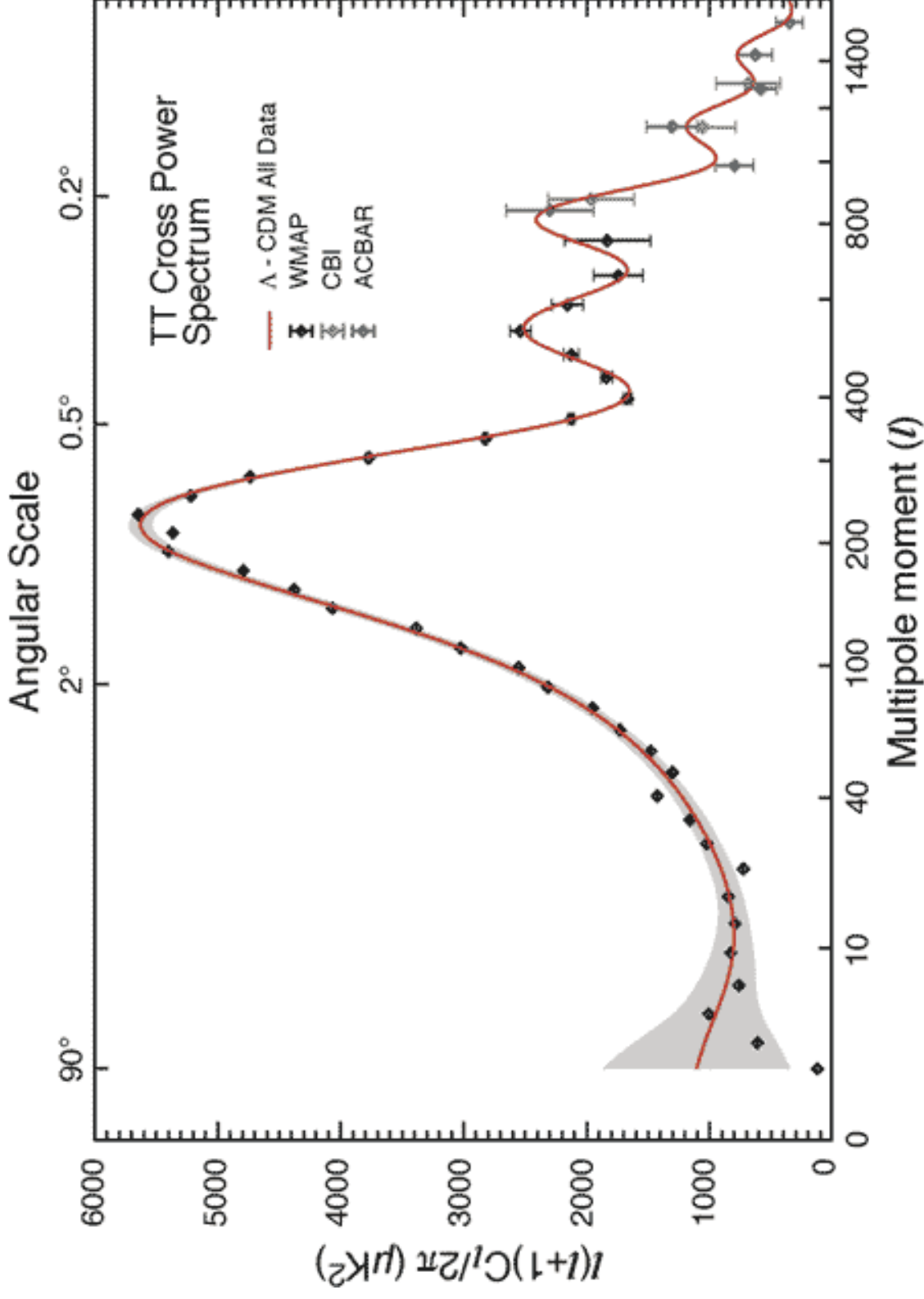
# Test 1: Snela luminosity distances



Best-fit  $H_0$  agrees with HST key team, Sandage et al.,

$H_0 = 62.3 \pm 1.3$  (stat)  $\pm 5.0$  (syst)  $\text{km sec}^{-1} \text{Mpc}^{-1}$  [ApJ 653 (2006) 843].

# Test 2: Angular scale of CMB Doppler peaks

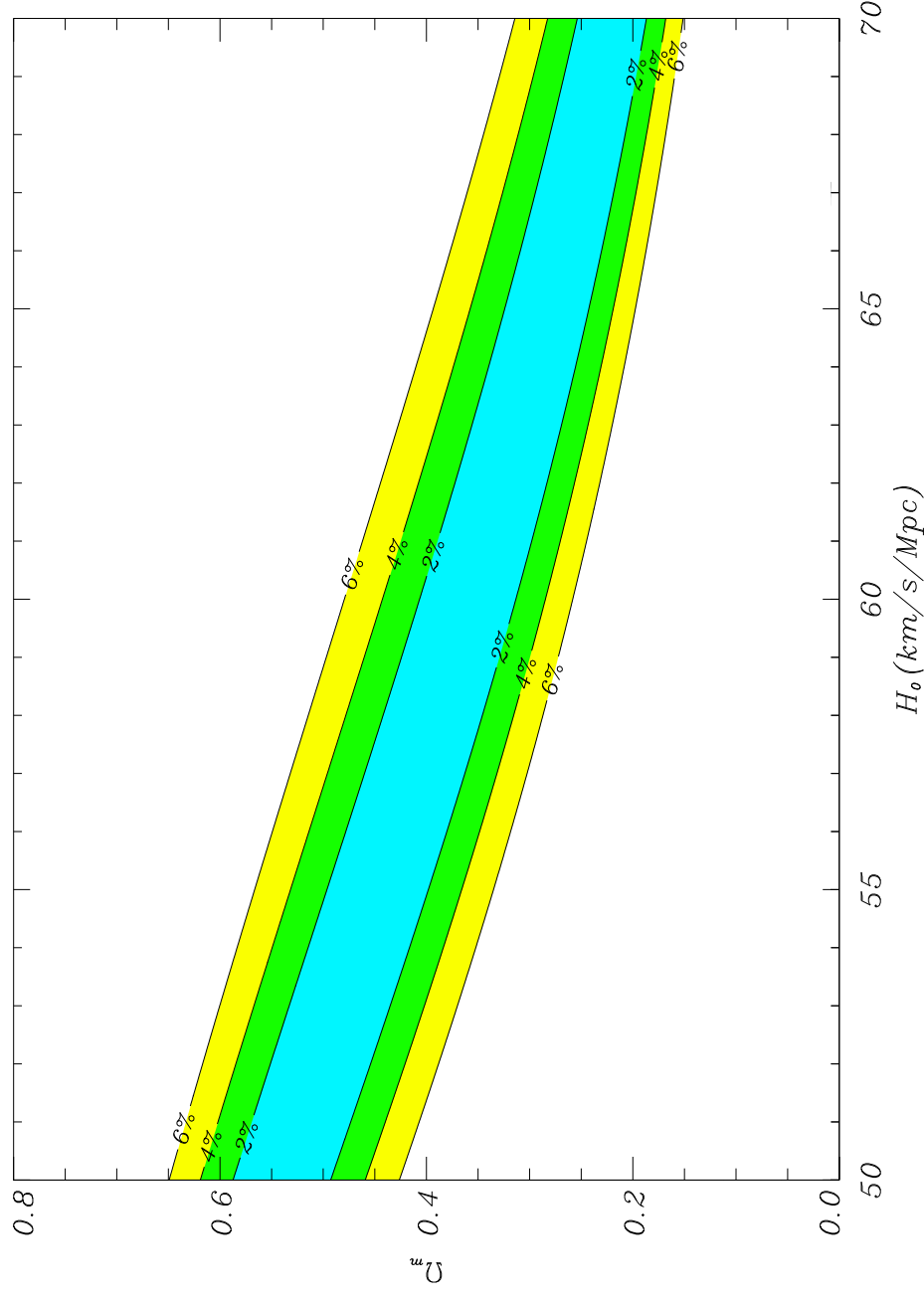


Power in CMB temperature anisotropies versus angular size of fluctuation on sky

# Test 2: Angular scale of CMB Doppler peaks

- Angular scale is related to spatial curvature of FLRW models
- Relies on the simplifying assumption that spatial curvature is same everywhere
- In new approach spatial curvature is not the same everywhere
- Volume-average observer measures lower mean CMB temperature ( $\bar{T}_0 \sim 1.98 \text{ K}$ , c.f.  $T_0 \sim 2.73 \text{ K}$  in walls) and a smaller angular anisotropy scale
- Relative focussing between voids and walls
- Integrated Sachs–Wolfe effect needs recomputation
- Here just calculate angular–diameter distance of sound horizon

# Test 2: Angular scale of CMB Doppler peaks

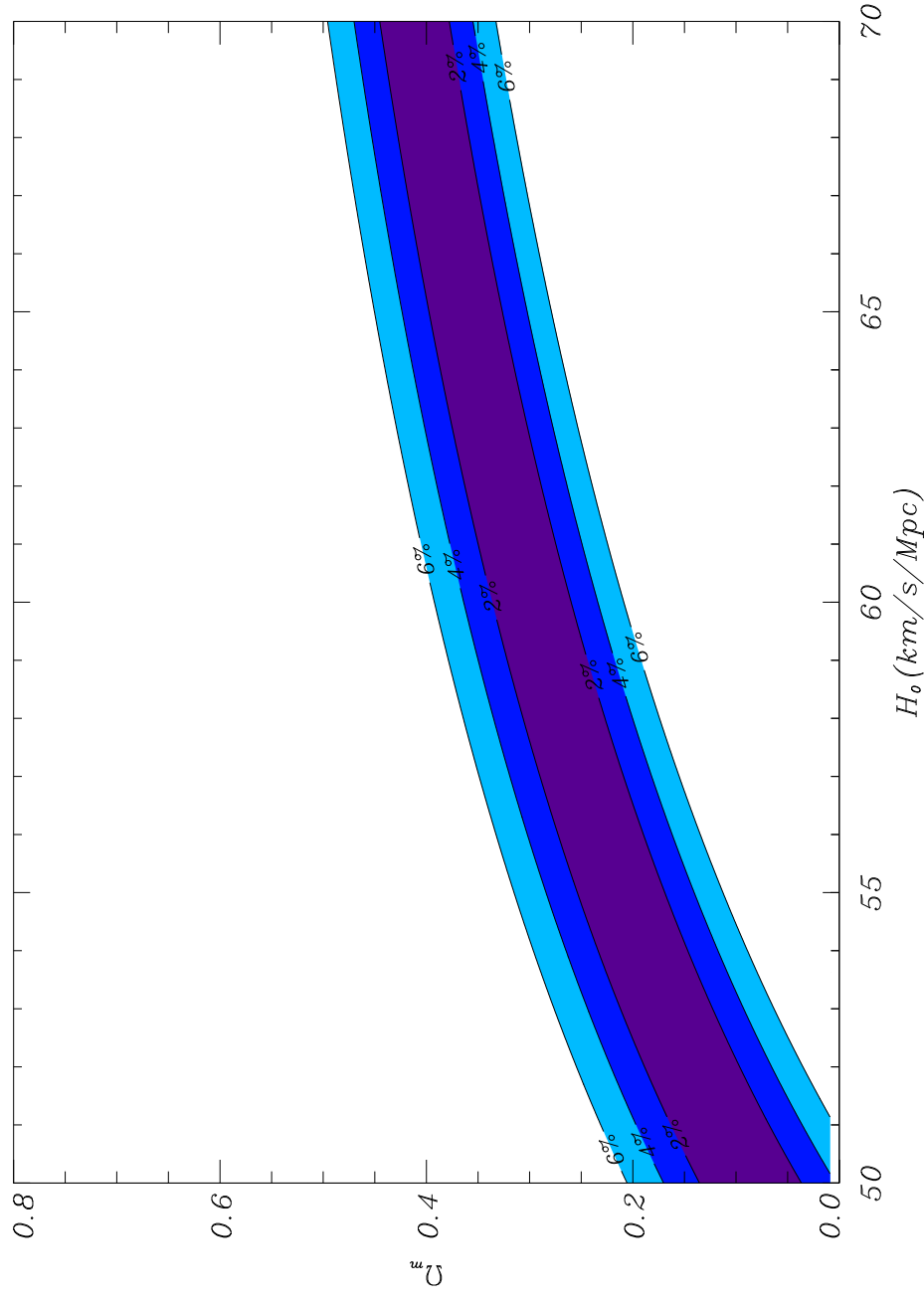


Parameters within the  $(\Omega_m, H_0)$  plane which fit the angular scale of the sound horizon  $\delta = 0.01$  rad deduced for WMAP, to within 2%, 4% and 6%.

# Test 3: Baryon acoustic oscillation scale

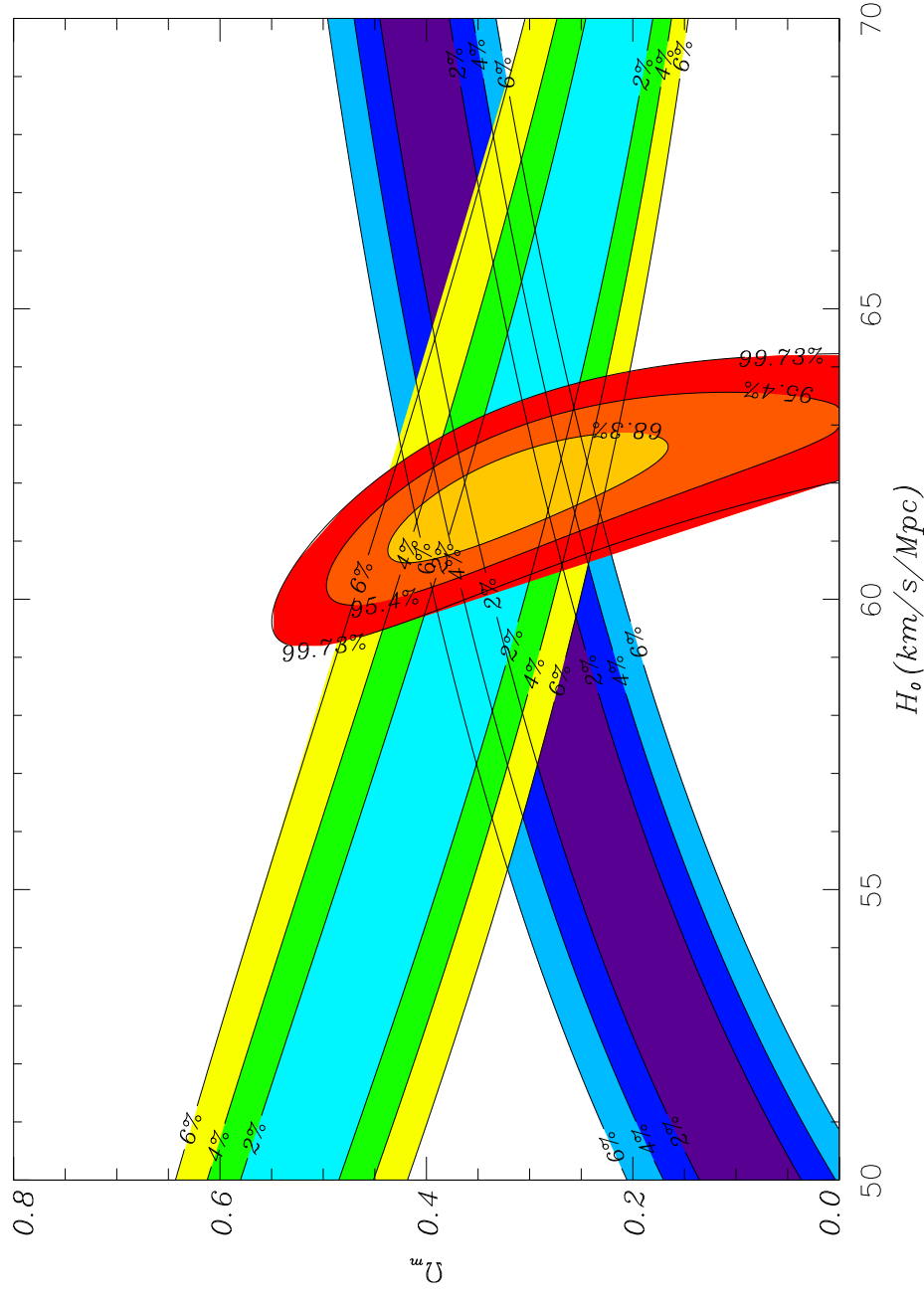
- In 2005 Cole et al. (2dF), and Eisenstein et al. (SDSS) detected the signature of the comoving baryon acoustic oscillation in galaxy clustering statistics
- Powerful independent probe of “dark energy”
- Here the effective dressed geometry should give an equivalent scale

# Test 3: Baryon acoustic oscillation scale



Parameters within the  $(\Omega_m, H_0)$  plane which fit the effective comoving baryon acoustic oscillation scale of  $104h^{-1}$  Mpc, as seen in 2dF and SDSS.

# Agreement of independent tests



Best-fit parameters:  $H_0 = 61.7^{+1.2}_{-1.1} \text{ km sec}^{-1} \text{ Mpc}^{-1}$ ,  
 $\Omega_m = 0.33^{+0.11}_{-0.16}$  ( $1\sigma$  errors for SnIa only) [Leith, Ng &  
Wiltshire, ApJ 672 (2008) L91]

# Best fit parameters

- Hubble constant  $H_0 = 61.7^{+1.2}_{-1.1} \text{ km sec}^{-1} \text{ Mpc}^{-1}$
- present void volume fraction  $f_{v0} = 0.76^{+0.12}_{-0.09}$
- bare density parameter  $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter  $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non-baryonic dark matter / baryonic matter mass ratio  $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0}) / \bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant  $\bar{H}_0 = 48.2^{+2.0}_{-2.4} \text{ km sec}^{-1} \text{ Mpc}^{-1}$
- mean lapse function  $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter  $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe  $\tau_0 = 14.7^{+0.7}_{-0.5} \text{ Gyr}$



# Weyl curvature hypothesis?

- CIFs have negligible Weyl curvature.
- Since Weyl curvature includes any “non-local” curvature, the CEP says that the only “non-local” curvature allowed is that which has accumulated through causal processes – gravitational collapse; gravitational wave production – within the past light cone at any event
- If the CEP can be taken to apply at very early epochs, in the limit of specifying an initial condition it coincides with Penrose’s Weyl curvature hypothesis

# Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi-local gravitational energy*, of *gradients* in spatial curvature etc.
- The “timescape” model passes three major independent tests which support  $\Lambda$ CDM and may resolve significant puzzles and anomalies.
- Every cosmological parameter requires subtle recalibration, but no “new” physics beyond dark matter: no  $\Lambda$ , no exotic scalars, no modifications to gravity.
- Questions raised – otherwise unanswered – should be addressed irrespective of phenomenological success.