

COLOR DIPOLES FROM BREMSSTRAHLUNG IN HIGH ENERGY EVOLUTION

Y. HATTA

*RIKEN BNL Research Center,
Brookhaven National Laboratory,
Upton, NY 11973, USA,
E-mail: hatta@quark.phy.bnl.gov*

We show that the recently developed Hamiltonian theory for high energy QCD evolution in the presence of gluon number fluctuation is consistent with the color dipole picture. We construct the color glass weight function and the dipole densities of an onium, and derive evolution equations for these quantities by acting with the Bremsstrahlung Hamiltonian.

1. Introduction

Recently there has been renewed interest in the small- x QCD evolution equation in the *dilute*, non-saturated regime. The main reason of this is the recognition that the gluon number fluctuations¹ developed in this regime significantly affect the asymptotic behavior of scattering amplitudes. As is well established, the recombination of gluon cascades (ladders), to leading log approximation, is included in the BK-JIMWLK equation.³ [Figure 1(a)] However, it does not describe how a hadron develops many cascades and eventually comes to saturation because it misses the corresponding *Bremsstrahlung* diagrams Fig. 1(b). These diagrams are important only in the dilute regime, and are responsible for the event-by-event fluctuation of the gluon number.² In this talk I will explain how to include the diagrams Fig. 1(b) in the Hamiltonian approach to high energy evolution.

2. Bremsstrahlung Hamiltonian

In Ref. ⁴, an effective action summing all order Bremsstrahlung diagrams has been derived in the Color Glass Condensate (CGC) formalism. [See

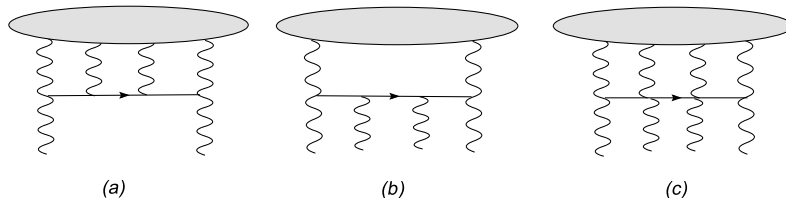


Figure 1. (a) Gluon recombination in a high energy hadron (upper blob). (b) Gluon splitting.

Ref. ⁷ for a very different approach.] It reads

$$\Delta H_{\text{BREM}} = \frac{1}{(2\pi)^3} \int_{\mathbf{x}\mathbf{y}\mathbf{z}} \mathcal{K}_{\mathbf{x}\mathbf{y}\mathbf{z}} [\rho_{\infty}(\mathbf{x})\rho_{\infty}(\mathbf{y}) + \rho_{-\infty}(\mathbf{x})\rho_{-\infty}(\mathbf{y}) - \rho_{\infty}(\mathbf{x})W(\mathbf{z})\rho_{-\infty}(\mathbf{y}) - \rho_{-\infty}(\mathbf{x})W^{\dagger}(\mathbf{z})\rho_{\infty}(\mathbf{y})], \quad (1)$$

with the two dimensional kernel

$$\mathcal{K}(\mathbf{x}, \mathbf{y}, \mathbf{z}) \equiv \frac{(\mathbf{x} - \mathbf{z}) \cdot (\mathbf{y} - \mathbf{z})}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2}, \quad (2)$$

and the Wilson line

$$W(\mathbf{x}) = P \exp \left(-g \int_{-\infty}^{\infty} dx^+ \frac{\delta}{\delta \rho^a(x^+, \mathbf{x})} T^a \right). \quad (3)$$

The action is quadratic in the charge ρ of the right-moving hadron (represented as a blob in Fig. 1(b)), and is all order in A^- corresponding to the gluon legs. The subscript $\pm\infty$ refers to the x^+ coordinate. In the previous formulation of the CGC, the charges ρ were effectively x^+ -independent. But in the presence of Bremsstrahlung, one has to explicitly keep track of the x^+ -coordinate. This is tantamount to treat the color charges as non-commutative matrix.^{7,8}

3. Dipole model limit

In general, evolution equations derived from H_{BREM} are very complicated due to the non-commutativity problem. However, as shown in Ref. ⁵, in the dipole model in the large N_c limit, the non-commutativity of charges becomes irrelevant and one can derive tractable evolution equations. In fact, the diagram Fig. 1(b) is naturally included in the dipole model as the splitting of a dipole and subsequent gluon emission from the child dipoles, see Fig. 2. Because of this reason, most of the recent developments in gluon fluctuations have been made in the dipole model.

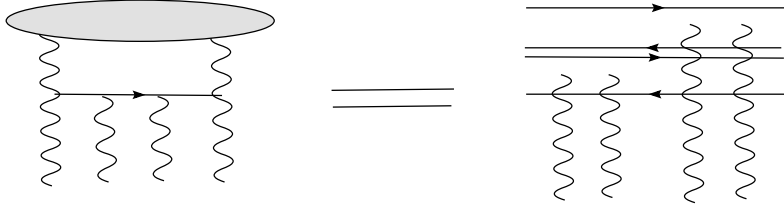


Figure 2. Gluon Bremsstrahlung corresponds to the dipole splitting.

The point is that one can explicitly construct the color glass weight function for an onium (=collection of dipoles):

$$Z_\tau[\rho] = \sum_{N=1}^{\infty} \int d\Gamma_N P_N(\{z_i\}; \tau) \prod_{i=1}^N D^\dagger(z_{i-1}, z_i) \delta[\rho], \quad (4)$$

where τ is the rapidity and P_N is the N -dipole probability distribution which can be computed numerically.¹ D^\dagger is the dipole creation operator

$$D^\dagger(\mathbf{x}, \mathbf{y}) = \frac{1}{N_c} \text{tr}(W(\mathbf{x})W^\dagger(\mathbf{y})), \quad (5)$$

acting on the dipole ‘vacuum’ state $\delta[\rho]$.

The evolution equation for an arbitrary operator $X[\rho]$ is given by

$$\frac{\partial}{\partial \tau} \langle X[\rho] \rangle = \int [D\rho] Z_\tau[\rho] H_{BREM} X[\rho]. \quad (6)$$

It is straightforward to work out the action of H_{BREM} on Z_τ keeping only large- N_c surviving terms^{6,5}. The result is consistent with the known evolution equation for P_N . Then one can evaluate the remaining integral over ρ . Here we give two examples. The evolution equation for the dipole density operator

$$D(\mathbf{x}, \mathbf{y}) \equiv -\frac{1}{g^2 N_c} \rho_\infty^a(\mathbf{x}) \rho_\infty^a(\mathbf{y}), \quad (7)$$

is the BFKL equation. The evolution equation for the dipole pair density

$$\langle D(\mathbf{x}_1, \mathbf{y}_1) D(\mathbf{x}_2, \mathbf{y}_2) \rangle_\tau = \frac{1}{g^4 N_c^2} \langle \rho^a(\mathbf{x}_1) \rho^a(\mathbf{y}_1) \rho^b(\mathbf{x}_2) \rho^b(\mathbf{y}_2) \rangle_\tau, \quad (8)$$

is given by

$$\begin{aligned} \frac{\partial}{\partial \tau} \langle D_{\mathbf{x}_1, \mathbf{y}_1} D_{\mathbf{x}_2, \mathbf{y}_2} \rangle_{\tau} &= [H_{\text{BFKL}}^{(1)} + H_{\text{BFKL}}^{(2)}] \langle D_{\mathbf{x}_1, \mathbf{y}_1} D_{\mathbf{x}_2, \mathbf{y}_2} \rangle_{\tau} \\ &+ \frac{\bar{\alpha}_s}{4\pi} \left\{ \mathcal{M}_{\mathbf{x}_1 \mathbf{y}_2 \mathbf{x}_2} \delta_{\mathbf{x}_2 \mathbf{y}_1} \langle D_{\mathbf{x}_1 \mathbf{y}_2} \rangle_{\tau} + \mathcal{M}_{\mathbf{x}_1 \mathbf{x}_2 \mathbf{y}_1} \delta_{\mathbf{y}_1 \mathbf{y}_2} \langle D_{\mathbf{x}_1 \mathbf{x}_2} \rangle_{\tau} \right. \\ &\left. + \mathcal{M}_{\mathbf{y}_1 \mathbf{y}_2 \mathbf{x}_1} \delta_{\mathbf{x}_1 \mathbf{x}_2} \langle D_{\mathbf{y}_1 \mathbf{y}_2} \rangle_{\tau} + \mathcal{M}_{\mathbf{y}_1 \mathbf{x}_2 \mathbf{x}_1} \delta_{\mathbf{x}_1 \mathbf{y}_2} \langle D_{\mathbf{y}_1 \mathbf{x}_2} \rangle_{\tau} \right\}. \quad (9) \end{aligned}$$

In addition to the BFKL part, the rhs. contains terms linear in $\langle D \rangle$. These terms exactly correspond to the $2 \rightarrow 4$ process, Fig. 2. Likewise, the n -ple dipole density couples to the n' -ple ($n' < n$) dipole densities. Combined with the terms describing gluon saturation, they constitute the Pomeron loop equation. The solution to this equation and its phenomenological consequences are discussed elsewhere.²

Acknowledgments

This work has been done in collaboration with E. Iancu, L. McLerran and A. Stasto.

References

1. G. P. Salam, *Nucl. Phys.* **B449**, 589 (1995).
2. E. Iancu, in these proceedings; G. Soyez, in these proceedings.
3. I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996); Y. V. Kovchegov, *Phys. Rev.* **D60**, 034008 (1999); J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, *Phys. Rev.* **D59**, 014014 (1999); E. Iancu, A. Leonidov and L. D. McLerran, *Nucl. Phys.* **A692**, 583 (2001).
4. Y. Hatta, E. Iancu, L. McLerran, A. Stasto and D. N. Triantafyllopoulos, *Nucl. Phys.* **A764**, 423 (2006).
5. Y. Hatta, E. Iancu, L. McLerran and A. Stasto, *Nucl. Phys.* **A762**, 272 (2005).
6. A. Kovner and M. Lublinsky, *Phys. Rev.* **D72**, 074023 (2005).
7. A. Kovner and M. Lublinsky, *Phys. Rev.* **D71**, 085004 (2005).
8. Y. Hatta, *Nucl. Phys.* **A768**, 222 (2006).