

## IMPACT PARAMETER DEPENDENT GLUON DENSITY FROM THE BK EQUATION. \*

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In this contribution we analyze an impact parameter dependent gluon density that follows from the Balitsky-Kovchegov equation. Nonmonotonical behavior of the impact parameter gluon density leads to a natural definition of the saturation line. Implications for  $F_2$  are also presented.

The simplest theoretical tool within perturbative QCD which takes into account recombination of gluons when density of partonic systems is high, is the Balitsky-Kovchegov equation<sup>1,2</sup>. This nonlinear evolution equation is a natural generalization of the BFKL evolution equation and due to presence of the Triple Pomeron Vertex (TPV)<sup>3</sup>, it sums up pomeron fan diagrams. That equation can be applied to determine unintegrated gluon density and then observables via integration over the impact parameter (distance from the center of the target) and application of the  $k_T$  factorization theorem. In the present contribution we also consider the structure function  $F_2$ , which can be expressed as:  $F_2 = F_2^{\gamma g} \otimes f$  where  $\otimes$  stands for a convolution in longitudinal and transverse momentum.  $F_2^{\gamma g}$  is given by the quark box and crossed-box contributions  $\gamma g \rightarrow \bar{q}q$ .

The BK equation with subleading N $L_x$  corrections within the KMS<sup>4</sup> framework for the impact parameter dependent unintegrated gluon density assumes following form:

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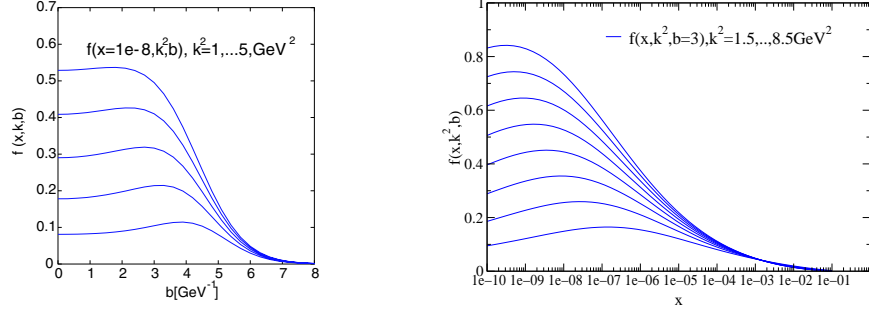


Figure 1. Unintegrated gluon density as a function of  $b$  for fixed  $k^2$  and  $x = 10^{-8}$  (left). Unintegrated gluon density as a function of  $x$  for  $b=3$  and fixed  $k^2$  (right)

$$\begin{aligned}
f(x, k^2, b) &= \tilde{f}^{(0)}(x, k^2, b) \\
&+ \frac{\alpha_s(k^2) N_c}{\pi} k^2 \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk'^2}{k'^2} \left\{ \frac{f(\frac{x}{z}, k'^2, b) \Theta(\frac{k^2}{z} - k'^2) - f(\frac{x}{z}, k^2, b)}{|k'^2 - k^2|} + \frac{f(\frac{x}{z}, k^2, b)}{|4k'^4 + k^4|^{\frac{1}{2}}} \right\} \\
&+ \frac{\alpha_s(k^2) N_c}{\pi} \int_x^1 dz \left[ \bar{P}_{gg}(z) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f(\frac{x}{z}, k'^2, b) + P_{gq} \Sigma\left(\frac{x}{z}, k'^2, b\right) \right] \\
&- 2\pi\alpha_s^2(k^2) \left[ k^2 \left( \int_{k^2}^{\infty} \frac{dk'^2}{k'^4} f(x, k'^2) \right)^2 + f(x, k^2) \int_{k^2}^{\infty} \frac{dk'^2}{k'^4} \ln\left(\frac{k'^2}{k^2}\right) f(x, k'^2) \right]
\end{aligned} \tag{1}$$

the inhomogeneous term stands for the input gluon distribution and is given by:  $\tilde{f}^{(0)}(x, k^2, b) = \frac{\alpha_s(k^2)}{2\pi} S(b) \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, k_0^2\right)$ . Two first lines of (1) correspond to the BFKL evolution. The theta function,  $\Theta(\frac{k^2}{z} - k'^2)$ , reflects the consistency constraint. The third line corresponds to the DGLAP effects generated by the part of the gluon splitting function,  $P_{gg}(z)$ , that is not singular in the limit  $z \rightarrow 0$ . The  $\Sigma(x, k^2, b^2)$  term is the contribution of the singlet quark distribution to the gluon. The fourth line represents action of the TPV on the gluon density. The usual unintegrated gluon density is obtained via integration over distance from the center of the target:  $f(x, k^2) = \int d^2b f(x, k^2, b)$  the input profile is assumed to be Gaussian,  $S(b) = \exp(-b^2/R^2)/\pi R^2$  where  $R$  is radius of area in which gluons are concentrated. We take  $R = 2.8 \text{GeV}^{-1}$  which follows from the measurement of diffractive  $J/\psi$  photo-production off proton. The equation (1) is local in  $b$  but due to presence of nonlinearity the  $b$  dependence cannot be factorised. The method of solving it was developed in <sup>6</sup> and details of the solution method can be found there. In fig (1a) we plot unintegrated gluon

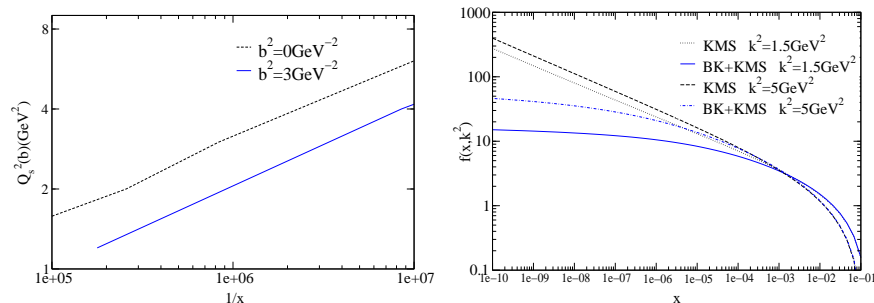


Figure 2. The impact parameter dependent saturation line (left). Integrated over the impact parameter gluon density obtained from (1) compared to the linear BFKL/DGLAP evolution in the framework of KMS (right).

density as a function of the impact parameter for fixed values of  $k^2$  and for  $x = 10^{-8}$ . We observe that at small values of  $x$  and central values of  $b$  the saturation effects are strongest and lead to depletion of gluon density. On the other hand due to large distance phenomena as for example confinement which we model via Gaussian input there are less gluons for peripheral  $b$ . The net result is clear. Impact parameter dependent gluon density has a maximum as a function of distance from the target.

Fig (1b) visualizes unintegrated gluon density as a function of  $x$  fig(2). At large  $x$  for fixed gluon momentum BFKL/DGLAP effects lead to a strong growth of the gluon density. At certain value of  $x$  the nonlinear term becomes equal to linear and cancels out with it. That effect leads to occurrence of a maximum which leads to a natural definition of a saturation scale i.e. we define the saturation scale (2a) as  $Q_s^2$  for which:  $\frac{\partial f(x, Q_s^2, b)}{\partial \ln 1/x} = 0$ . A similar maximum is not seen for the gluon density integrated over the impact parameter (2b) it flattens but does not fall as very small  $x$  is approached. This is due to the large contribution to the integral from the peripheral region where density of gluons has not saturated yet. In fig.(1b) we observe well known fact that the lower  $k$  is the earlier saturation effects manifest themselves. This can be understood in our approach from the structure of the integral in the nonlinear part. The lower limit of integration is given by value of  $k^2$  at which we probe the gluon and extends to the infinity. The lower the momentum is the longer is the path of integration. Finally we present an implication of recombination effects on the proton structure function  $F_2$  fig. (3). Those effects are almost negligible at HERA kinematical region. From our analysis we conclude that saturation effects may be more visible for quantities which are sensitive to the momentum

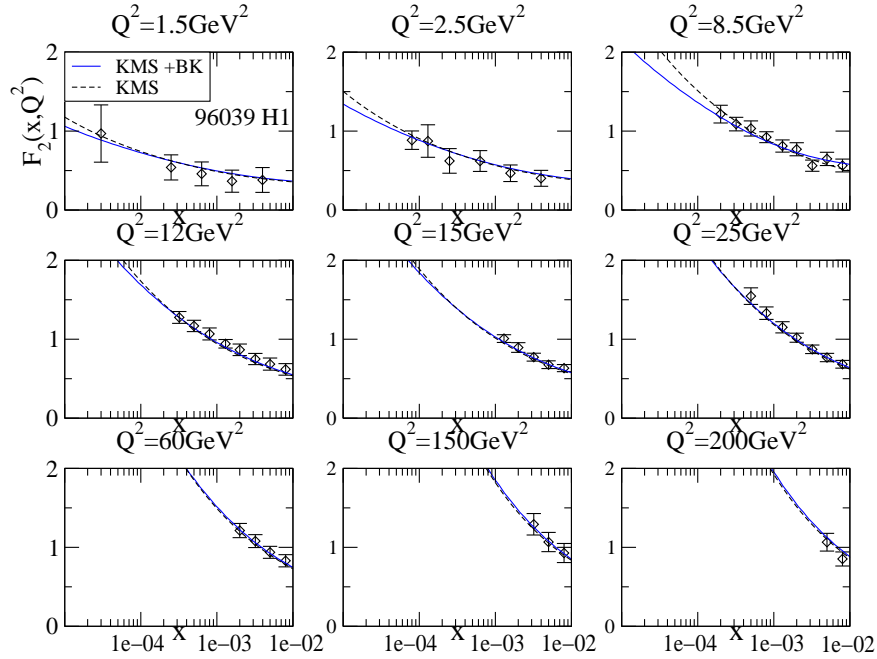


Figure 3. The  $F_2$  structure function obtained from (1) compared to prediction based on the KMS evolution

transfer during the interaction, which is conjugate via Fourier transform to the impact parameter  $b$ .

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