HOW RESUMMATION DEPRESSES THE GLUON AT SMALL X

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We summarize recent progress in the resummation of perturbative evolution at small x. We show that the problem of incorporating BFKL small x logs in GLAP evolution is now completely solved, and that the main effect of small x resummation is to reduce the growth of the gluon at small x in the HERA and LHC regions.

1. BFKL logs in GLAP evolution

It has been thought for a long time that the resummation of small x contributions to perturbative evolution might require techniques that go beyond standard perturbative factorization. However, dramatic theoretical progress,[1,2] largely prompted by the unexpected success of NLO GLAP evolution in describing the growth of structure functions observed at HERA,[3,4] has led to a complete understanding of this issue within a perturbative framework. Namely, it is now clear that small x logs, as described by the BFKL equation, can be fully incorporated in the standard GLAP framework, that their inclusion stabilizes the behaviour of perturbation theory at small x, and that, somewhat surprizingly, they suppress the growth of parton distributions at small x down to the smallest values of x and the largest values of Q^2 accessible at HERA and the LHC.

2. Theoretical progress: the three ingredients

The resummation of logarithmically enhanced small x contributions to perturbative evolution can be performed within two different approaches, which share several basic physical assumptions but differ in the implementation, most notably because one (CCSS[5,6]) is rooted in the BFKL equation and extracts the anomalous dimension numerically from the gluon green function, while the other (ABF,[7,8,9] on which we will concentrate) improves the standard GLAP anomalous dimension by including in it infinite series of logarithmically enhanced terms. A detailed comparison is in Ref.[4].

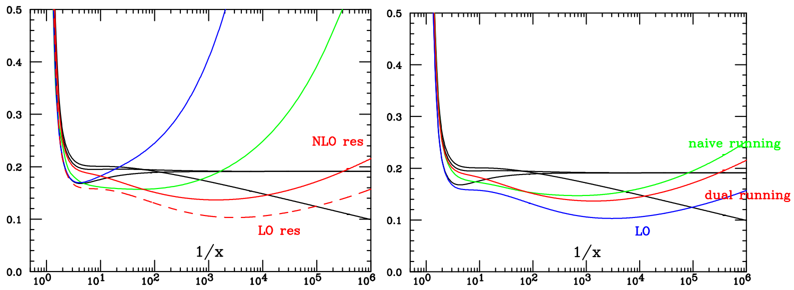

Figure 1. (Left) Resummed and unresummed splitting functions with $\alpha_s = 0.2$ and $n_f = 0$. The curves shown are (from top to bottom on the right): double–leading resummation, running coupling resummation thereof, full NLO resummation, fixed LO and NLO, full LO resummation, fixed NNLO. (Right) Effect of the choice of argument of the running coupling.

The ABF resummation is based on three basic ingredients. The first ingredient is duality (first introduced in Ref.[11,7] and recently[12] proven to all orders at the running–coupling level), which states that at leading twist the BFKL and GLAP equations admit the same solutions if their respective kernels are suitably matched. Using duality, the information on leading, subleading,... x logs from BFKL and that on leading, subleading,... Q^2 logs from GLAP can be combined in a double–leading expansion of either (or both) the GLAP and BFKL kernels. The double–leading splitting function thus reabsorbs the fixed–order small–x instability, but it leads to a splitting function (see Fig. 1) which grows too rapidly at small x in comparison to the data,[13] thereby signalling the need for further resummation.

The second ingredient is the all–order resummation of small x running coupling effects: these are formally subleading, but their contribution to the splitting function diverges as $x \to 0$. Their resummation[8] qualitatively changes the small N behaviour of the anomalous dimension: the double–leading square–root cut at relatively large $N \sim 0.5$ (at the HERA scale) is replaced by a simple pole with very small residue at small $N \sim 0.2$. This considerably softens the small x behaviour of the splitting function, as shown in Fig. 1.

Finally, the third ingredient is the symmetry of the BFKL kernel,[14] which relates the collinear GLAP region to an anti-collinear region, where incoming and outgoing gluon virtualities are exchanged. If combined with the running coupling resummation,[9] this symmetry further softens the small x behaviour and, more importantly, it leads to a stable perturbative expansion at the resummed level. The interplay of this symmetry with running coupling effects is quite subtle, because the running of the coupling breaks the collinear–anticollinear symmetry. In Fig. 1 the NLO resummed splitting function is compared with that where the argument of the strong coupling is chosen "naively" (i.e. just Q^2) instead of that obtained from running coupling duality: the effect is almost as large as the whole NLO correction.

3. Resummed evolution: stability and softening

The resummation has the effect of completely stabilizing the splitting function at small x"[9] the NLO and LO resummed results are quite close, they all depend weakly on the value of α_s and on the choice of factorization scale, and resummed evolution is not very different from unresummed one, thereby explaining the success of standard GLAP evolution at HERA.

In fact, the qualitative behaviour of fully resummed results completely differs from standard BFKL folklore, according to which leading small x logs should lead to a strong growth of the structure function at small x, while subleading logs change this behaviour completely, by replacing it with a softer growth (or possibly no growth at all). Quite on the contrary, it is clear from Figure 1 that the resummed splitting function lies below the unresummed down to very small $x \lesssim 10^{-5}$, and that the NLO resummed result is similar to the LO one, but in fact somewhat above it.

These conclusions are particularly evident if one considers the impact of resummation (in the $n_f = 0$ case) on the evolution of the gluon distribution (Figure 2). The resummed result for the convolution of the splitting function with a gluon distribution of the form $xg(x, 4\mathrm{GeV}^2) = x^{-0.18}(1-x)^5$ is below the unresummed LO, NLO, and NNLO results down to very small x. The effect of this suppression of the scale dependence is moderate but visible in the evolution of the gluon up to rather large values of Q^2 and

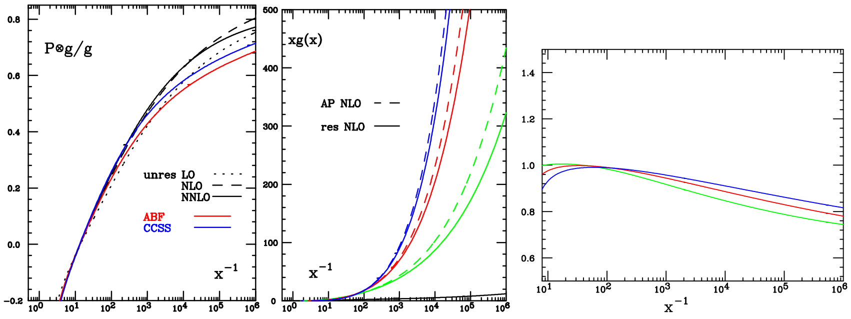

Figure 2. (left) Log derivative of the gluon with respect to scale at $Q^2 = 4$ GeV^2; (center) gluon evolution (from bottom to top: $Q^2 = 4$, 10^2, 10^4, 10^6 GeV^2); (right) resummed/fixed NLO K–factor (from bottom to top: $Q^2 = 10^2$, 10^4, 10^6 GeV^2) .

$1/x$, also shown. The corresponding K–factor (defined as the ratio of the NLO resummed to NLO fixed order evolved gluon) depends rather weakly on scale, as the figure shows.

4. Outlook

Resummed results at small x show remarkable stability, and suppress the growth of parton distributions in a moderate but visible way. It will be interesting to work out their phenomenological implications, both at the LHC and in other relevant context such as high–energy cosmic rays.

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