SMALL-X RESUMMATION AND FACTORISATION SCHEMES*

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I present a method for computing small-x resummed splitting functions in the $\overline{\mathrm{MS}}$-scheme at subleading $\log x$ level in the context of the renormalisation-group-improved approach.

The relation between the $\overline{\mathrm{MS}}$-scheme (widely used in fixed order perturbation theory) and the Q_0-scheme[1] (which appears the most natural scheme for small-x resummations) is of primary importance if one wants to stabilise at small-x the fixed order partonic anomalous dimensions.

Small-x resummations are obtained by employing the so-called $\boldsymbol{k}$-factorisation formula[2]. For instance, a structure function F_i at small-x (i.e., small moments $\omega \equiv N-1 \simeq 0$), can be factorised in the product of a process-dependent impact factor h_i, and a universal unintegrated gluon density $\mathcal{F}$, both transverse momentum dependent:

$$F_{i,\omega}(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\; h_{i,\omega}(Q^2,\boldsymbol{k})\mathcal{F}_\omega(\boldsymbol{k}) \; . \tag{1}$$

The resummation of the leading logarithms of x is embodied in the unintegrated gluon density which obeys the BFKL equation

$$\mathcal{F}_\omega(\boldsymbol{k}) = \mathcal{F}^{(0)}_\omega(\boldsymbol{k}) + \frac{1}{\omega}\int \mathrm{d}\boldsymbol{k}'^2\; K(\boldsymbol{k},\boldsymbol{k}')\mathcal{F}_\omega(\boldsymbol{k}') \; . \tag{2}$$

The Q_0-scheme is defined by specifying an off-shell initial condition for the gluon density: $\mathcal{F}^{(0)}_\omega(\boldsymbol{k}) = \delta^2(\boldsymbol{k}-Q_0)$; the integrated gluon is then

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defined by integration up to the relevant scale Q^2:

$$g_\omega(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\ \Theta(Q^2 - \boldsymbol{k}^2)\mathcal{F}_\omega(\boldsymbol{k})\ . \tag{3}$$

The relation between the Q_0-scheme and the $\overline{\mathrm{MS}}$ one is obtained by solving the BFKL equation in dimensional regularisation $D = 4 + 2\varepsilon$. I use a method[3] based on an integral representation for the solution which is suitable also for running coupling and NL corrections. In the limit of vanishing ε, the integral is dominated by a saddle-point $\bar\gamma(\alpha_{\mathrm{s}}/\omega)$ given by the familiar relation $1 = \frac{\alpha_{\mathrm{s}}}{\omega}\left(\frac{Q^2}{\mu^2}\right)^\varepsilon \chi_0(\bar\gamma)$. The result

$$g_\omega^{(Q_0)}(Q^2) = \frac{\exp\{\int_0^{\bar\gamma}\chi_1/\chi_0\}}{\bar\gamma\sqrt{-\chi_0'(\bar\gamma)}}\exp\left\{\frac{1}{\varepsilon}\int_0^{\frac{\alpha_{\mathrm{s}}}{\omega}\left(\frac{Q^2}{\mu^2}\right)^\varepsilon}\frac{\mathrm{d}a}{a}\ \bar\gamma(a)\right\} \equiv R\cdot g_\omega^{(\overline{\mathrm{MS}})}$$

is an explicit factorisation of the collinear singularities in a minimal subtraction form, which has to be identified with the $\overline{\mathrm{MS}}$ gluon. The ε-finite prefactor, usually denoted $R(\bar\gamma(\alpha_{\mathrm{s}}/\omega))$, is just the coefficient needed to change the scheme, and depends in particular on the first two terms in the ε-expansion of the BFKL kernel "eigenvalue" in $4 + 2\varepsilon$ dimensions $\chi(\gamma,\varepsilon) = \chi_0(\gamma) + \varepsilon\chi_1(\gamma) + \cdots$. The problem of this relation is that the coefficient R, depending on the LL anomalous dimension $\bar\gamma$, has leading Pomeron singularities of increasing weight with the perturbative order, indicating that a small-x resummation is in principle required for the scheme change too.

This can be achieved[4] by implementing the scheme change not just as a product with the coefficient R, but in $\boldsymbol{k}$-factorised form, namely as a transverse momentum convolution of $\mathcal{F}$ with some function ρ:

$$g_\omega^{(\overline{\mathrm{MS}})}(Q^2) = \int \mathrm{d}^2\boldsymbol{k}\ \rho_\omega(Q^2/\boldsymbol{k}^2)\mathcal{F}_\omega(\boldsymbol{k}) \tag{4}$$

By properly choosing the function ρ we can obtain the scheme change to any degree of subleading accuracy. In the simplest approximation, the requirement of consistency at LLx level constrains the function ρ to be just the inverse Mellin transform of the coefficient function $1/\gamma R(\gamma)$.

For values of the transverse momentum $\boldsymbol{k}^2$ smaller than the external scale Q^2, ρ is almost constant (see Fig. 1) and close to the unity; in the opposite range, namely at large transverse momenta, it shows wide oscillations and negative values. The function ρ should be compared with the Θ function which, in place of ρ, provides by definition the integrated gluon in the Q_0-scheme in Eq. (3).

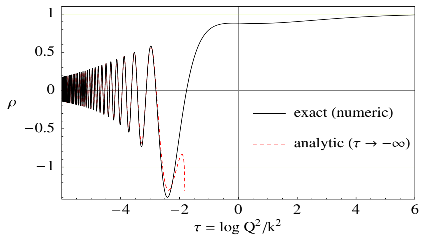

Figure 1.

The big advantage of using $\boldsymbol{k}$-factorisation for the scheme change is that, by computing the unintegrated gluon $\mathcal{F}$ in the renormalisation group improved (RGI) approach[5], the effective anomalous dimension which dominates the integral (4) is much smoother than the LLx one and is expected to provide a much more stable result.

In Fig. 2 I compare the RGI NL gluon in the Q_0-scheme in green with the gluon in the $\overline{\text{MS}}$-scheme in red. The fitted gluons of the CTEQ and MRST collaborations are also shown. Our gluons have been obtained by evolving the RGI NLx BFKL equation starting from a valence-like initial condition which has not been fine-tuned so as to get good agreement at large x. In particular, the small-x growth is just a consequence of the small-x evolution. We observe that the difference between $\overline{\text{MS}}$ and Q_0 is modest compared to that between CTEQ and MRST. This could justify somehow a phenomenological use of the Q_0 gluon, for instance, in saturation studies. Furthermore, there is no tendency for the MSbar gluon to go negative, as one could have suspected from the oscillations of the ρ function shown in the previous slide.

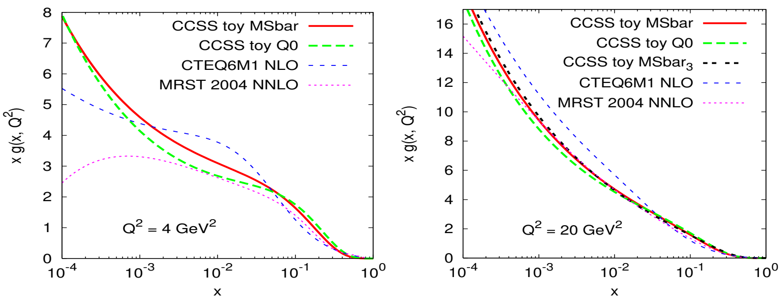

Figure 2.

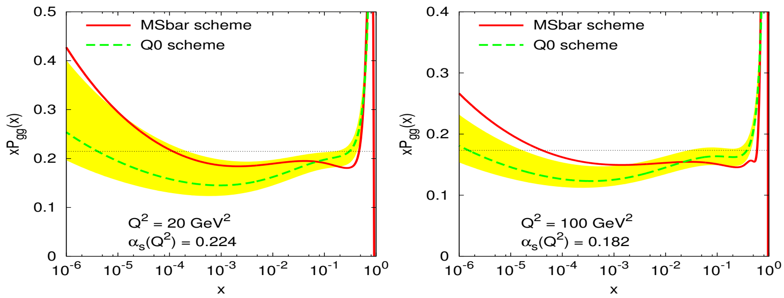

Figure 3.

We can also extract the splitting functions in both schemes. As before, these results can be taken seriously only at small-$x \lesssim 10^{-1}$. It appears (see Fig. 3) that the splitting function is more sensitive to the scheme change than the density itself. At low Q^2 the scheme difference is nearly the same as the renormalisation scale uncertainty, and so might not be considered a major effect. However, since the renormalisation scale uncertainty is NNLx, it decreases more rapidly than the scheme difference. Therefore at large Q^2 the effect of the factorisation scheme change is not negligible.

Quarks can also be included in the resummed flavour singlet evolution with $\boldsymbol{k}$-factorisation. We have performed in this case a preliminary study based on an approximation which includes only the rational coefficients of the γ_{qg} anomalous dimension. We find that resummation effects are in this case sizeable even around x of order 10^{-3} and are somewhat larger than the gluonic ones. However they are much smaller than pure NLx ones.

In conclusion, we have proposed a $\boldsymbol{k}$-factorised form of the $Q_0 \to \overline{\text{MS}}$ scheme-change which is stable and does not suffer of the leading Pomeron singularity of the usual leading $\log(x)$ BFKL hierarchy. Its implementation in a full quark-gluon evolution will provide $\overline{\text{MS}}$ small-x resummed splitting functions which matches with the fixed order perturbative calculations.

References

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